Wave-Particle Interactions with Highly Oblique Whistler Waves

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Ref: Albert (2017), J. Geophys. Res. Space Physics, 122, 5339–5354.
See also: Artemyev et al. (2016), Space Sci. Rev., 200, 261–355.

Radiation belt electrons are well-described by multidimensional quasi-linear diffusion:

$$
\begin{aligned}
\frac{\partial f}{\partial t} &= \frac{1}{G}\frac{\partial}{\partial \alpha_0} G\left(\frac{D_{\alpha_0\alpha_0}}{p^2}\frac{\partial f}{\partial \alpha_0} + \frac{D_{\alpha_0 p}}{p}\frac{\partial f}{\partial p}\right) \\
&+ \frac{1}{G}\frac{\partial}{\partial p} G\left(\frac{D_{\alpha_0 p}}{p}\frac{\partial f}{\partial \alpha_0} + D_{pp}\frac{\partial f}{\partial p}\right) + L^2\frac{\partial}{\partial L}\frac{D_{LL}}{L^2}\frac{\partial f}{\partial L}
\end{aligned}
$$

The D's are largely driven by cyclotron-resonant whistler-mode waves:

plasmaspheric hiss

chorus waves

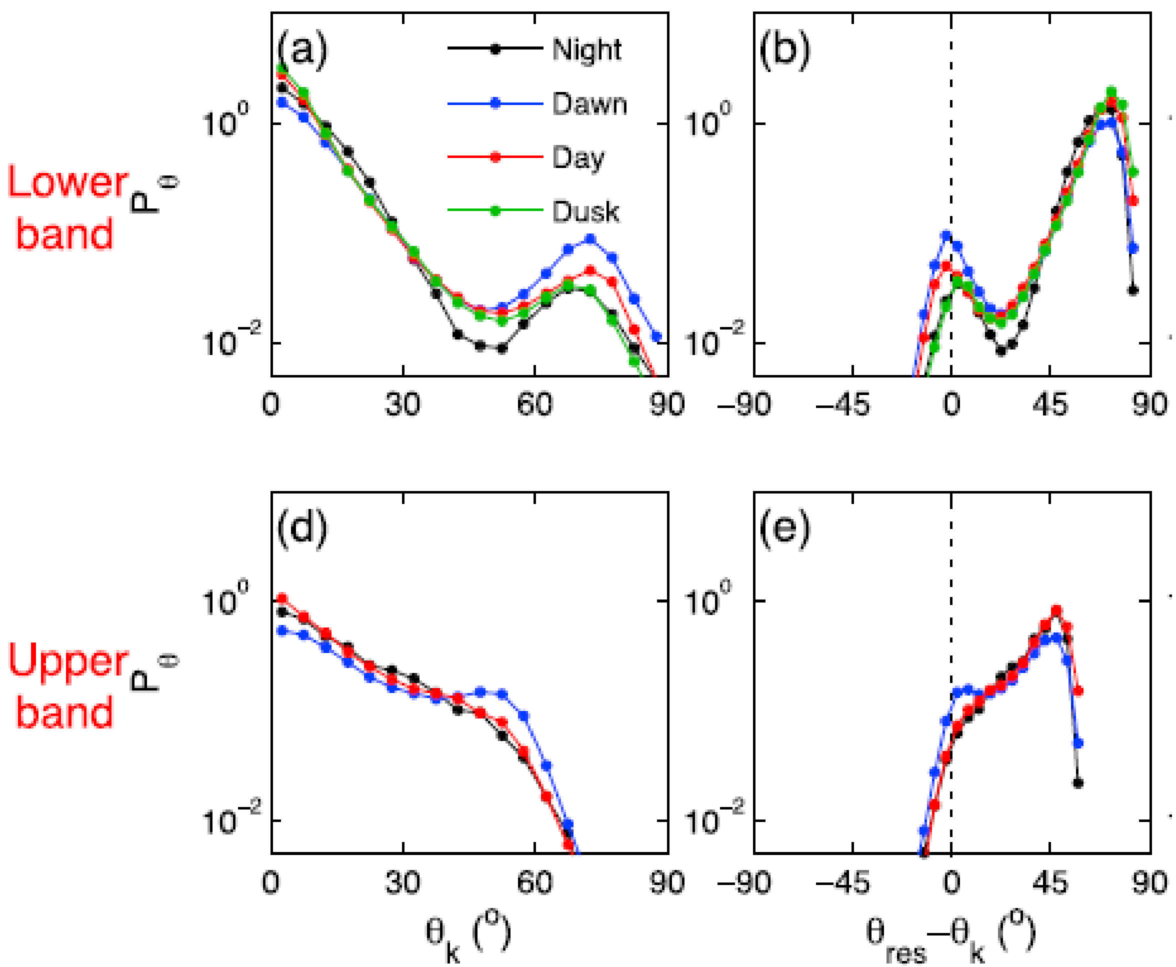
magnetosonic waves

lightning-generated whistlers

Navy VLF transmitters

These waves $\sim B_W \sin(\mathbf{k}\cdot\mathbf{x} - \omega t)$ are characterized by amplitude, frequency ω, and wave vector $\mathbf{k}$, with wave normal angle θ (between $\mathbf{k}$ and $\mathbf{B_0}$).

Typically, θ is small. But some chorus and hiss have $\theta \sim \theta_{RC}$, where $\mu = kc/\omega \to \infty$.



chorus statistics from THEMIS

[Li et al., JGR 2013]

Dipole antennas in space (e.g., DSX) are also predicted to radiate whistlers very near θ_{RC}.

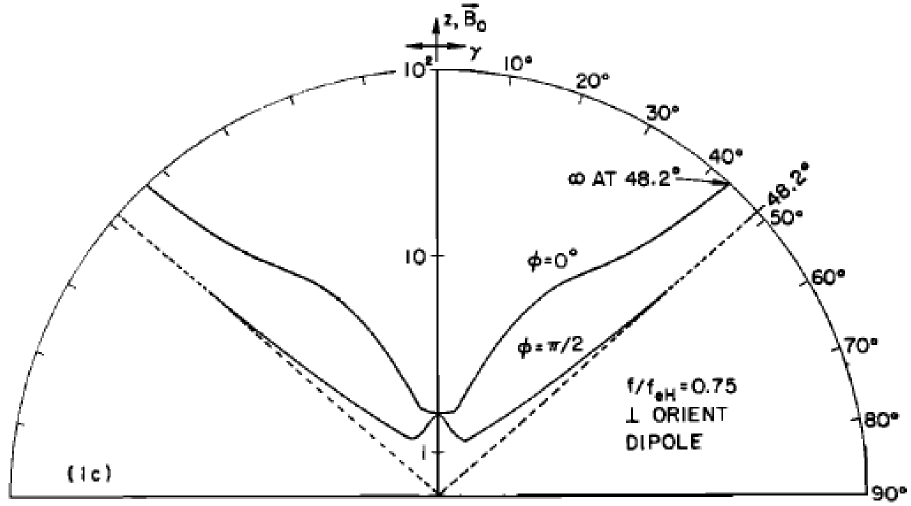


Fig. 1. Normalized radiation patterns for magnetic (L) and electric (D) dipoles at the frequency $f = 0.75\ f_{eH}$. The patterns shown are typical of those that occur in the frequency range $f_{eH} > f > \frac{1}{2}\ f_{eH}$, assuming a moderate to high density plasma.

[Wang and Bell, JGR 1972]

It has recently been suggested that D is "enhanced," though earlier analysis suggests $D \to 0$.

The [Lyons 1974] QLT diffusion coefficient formulas are:

$$D_{\alpha\alpha} = \sum_{n=-\infty}^{\infty} \frac{\Omega_e}{\gamma^2}\frac{B_w^2}{B^2}\int_{\theta_{\min}}^{\theta_{\max}} d\theta \sin\theta\, \Delta_n G_1 G_2 \quad \text{integral over resonances}$$

$$\Delta_n = \frac{\pi}{2}\frac{\sec\theta}{|v_{\|}/c|^3}\Phi_n^2\frac{(-\sin^2\alpha+\Omega_n/\omega)^2}{|1-(\partial\omega/\partial k_{\|})_\theta/v_{\|}|} \quad \text{individual W} - \text{P interaction} \quad \Phi_n \sim \{J_n, J_{n\pm1}\}(k_\perp\rho)$$

$$G_1 = \frac{\Omega_e B^2(\omega)}{\int_{\omega_{LC}}^{\omega_{UC}} B^2(\omega')\,d\omega'}, \quad G_2 = \frac{g(\omega,\theta)}{N(\omega)} \quad \text{wave power distribution in } (\omega,\theta)$$

$$N(\omega) = \int_{\theta_{\min}}^{\theta_{\max}} d\theta' \sin\theta' g(\omega,\theta')\Gamma(\omega,\theta') \quad \text{normalization integral}$$

$$\Gamma = \mu^2\left|\mu+\omega\frac{\partial\mu}{\partial\omega}\right| \quad \mu(\omega,\theta) = \text{ refractive index}$$

where $\Omega_n = -n\dfrac{\Omega_e}{\gamma}$, and (ω,θ) are linked by the resonance condition.

As $\theta_{\max} \to \theta_{RC}$, $\mu \to \infty$, $D \to$?

Not obvious.

Theory and numerics show that

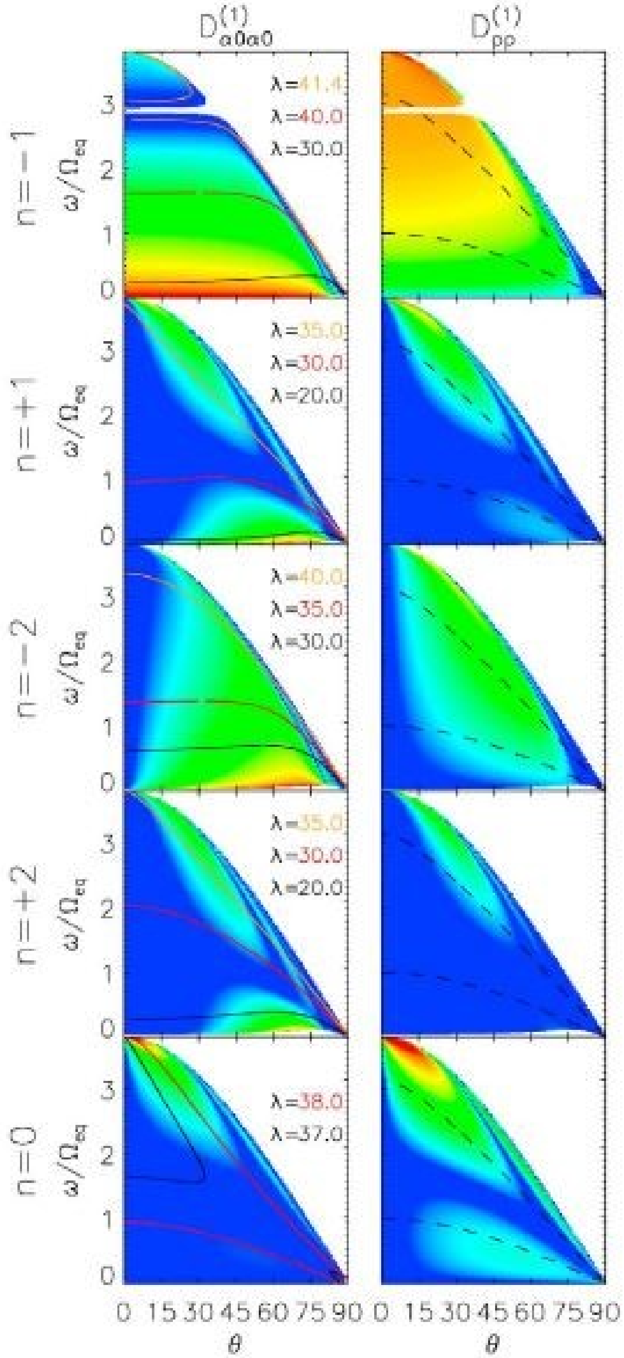
for each n, $D^n_{\alpha\alpha}$ and $D^n_{pp} \to 0$

as $\theta \to \theta_{RC}$ [Albert, JGR, 2012].

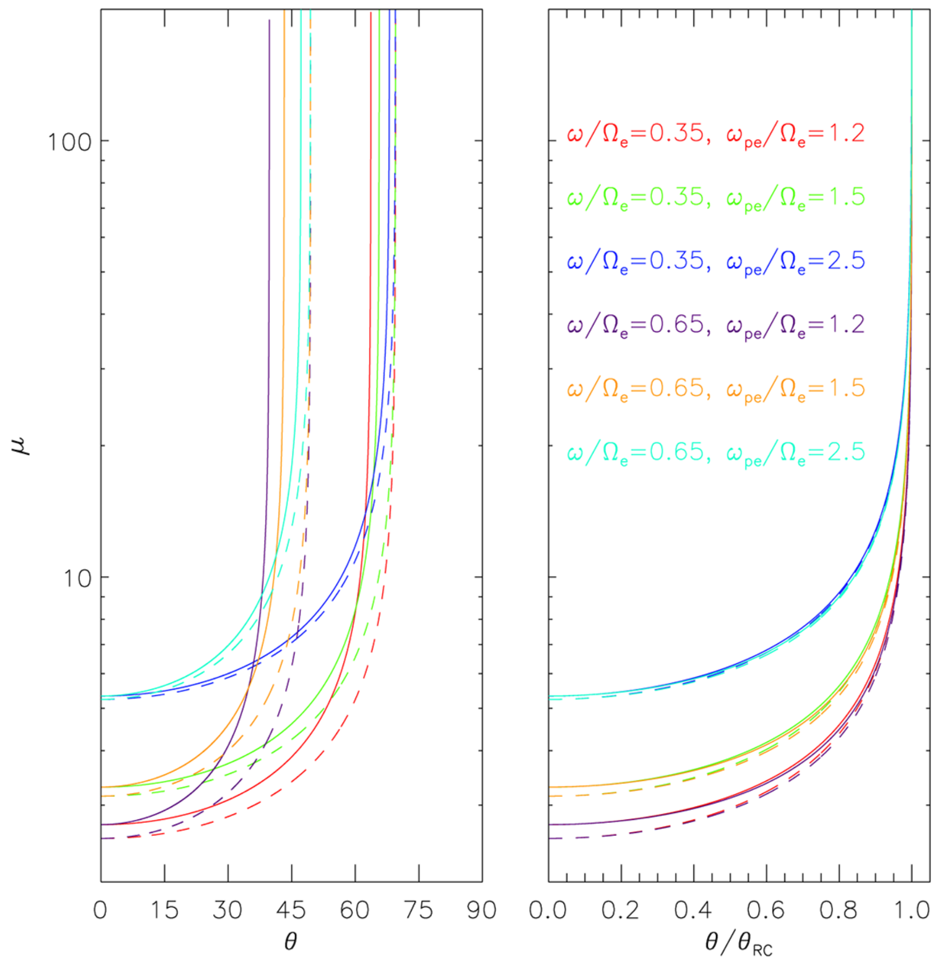
But this isn't the whole story.

Need to worry about $\sum_{n=-\infty}^{\infty} D^n$

(as $\theta \to \theta_{RC}$, the range of contributing n grows)



To study this, μ^2 can be approximated by $\mu^2 = \dfrac{\omega_{pe}^2}{\omega(\Omega_e \cos\theta - \omega)}$
which is qualitatively similar near the RC.



Then $\Gamma = \dfrac{\omega\Omega_e}{\omega_{pe}^2}\mu^5$. With $g(\omega,\theta) \approx \begin{cases} g_0, & \theta_{\min} \le \theta \le \theta_{\max} \\ 0, & \text{otherwise} \end{cases}$

$$N(\omega) = \left(\frac{\omega_{pe}^2}{\omega\Omega_e}\right)^{3/2} g_0 \left[\frac{\cos\theta_{\max} - \frac{2}{3}\cos\theta_{RC}}{(\cos\theta_{\max} - \cos\theta_{RC})^{3/2}} - \frac{\cos\theta_{\min} - \frac{2}{3}\cos\theta_{RC}}{(\cos\theta_{\min} - \cos\theta_{RC})^{3/2}}\right].$$

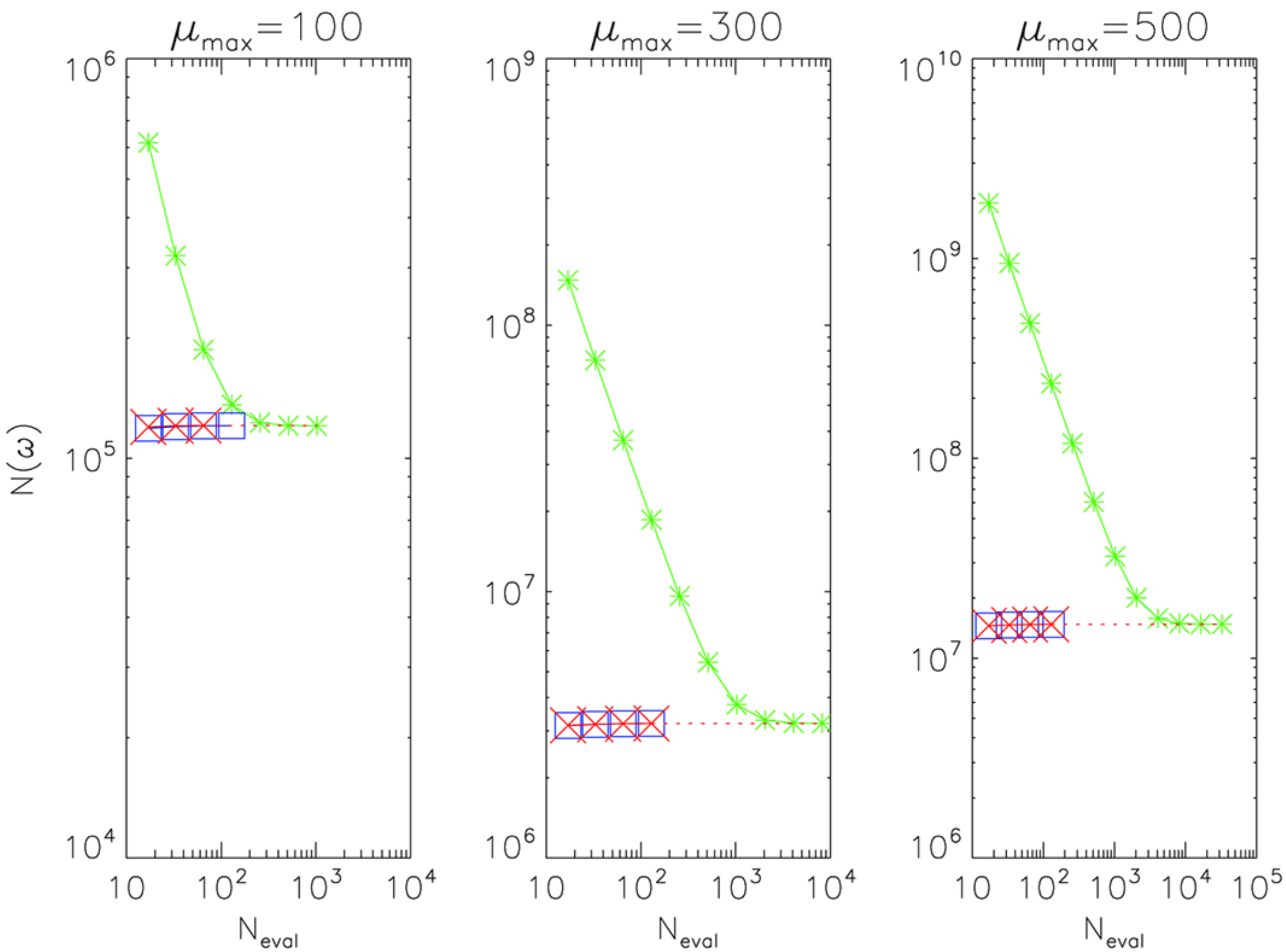
Compare to full-blown $N(\omega) = \int_{\theta_{\min}}^{\theta_{\max}} d\theta' \sin\theta' g(\omega,\theta')\Gamma(\omega,\theta')$,

or Lyons [1974]: $N(\omega) = \omega^2 \left(\dfrac{\omega_{pe}^2}{c^2\Omega_e^2}\dfrac{1+M}{M}\right)^{3/2} \dfrac{2}{(2\pi)^2} I(\omega)$,

$$\begin{aligned} I(\omega) &= \int_0^\infty g(\tan^{-1}x)x\{(1+x^2)\Psi\}^{-3/2} \\ &\times \left\{1 + \frac{1}{\Psi}\left[\frac{\omega^2}{\Omega_p\Omega_e} - \left\{\frac{1}{2}\frac{\omega^2}{\Omega_p^2}(1-M)^2\right\}\left\{(1+x^2)\left(\Psi - 1 + \frac{\omega^2}{\Omega_p\Omega_e}\right) + \frac{1}{2}x^2\right\}^{-1}\right]\right\}dx, \end{aligned}$$

with $M = \dfrac{m_e}{m_p}$, $\Psi = \dfrac{\omega_{pe}^2}{\Omega_e^2}\dfrac{1+M}{M}\mu^{-2}$.

The change of vars leading to this also helps with the full $N(\omega)$, giving large giving large speedup near θ_{RC}.



This approximate $N(\omega)$ can also be written as

$$N(\omega) = \frac{g_0}{3}\frac{\omega}{\Omega_e}\left[\mu_{\max}^3\left(1+3\frac{\mu_G^2}{\mu_{\max}^2}\right) - \mu_{\min}^3\left(1+3\frac{\mu_G^2}{\mu_{\min}^2}\right)\right]$$

where $\mu_G = \mu(\omega, \theta_G)$.

Idea: use (ω, μ) instead of (ω, θ) as the main variables.

This replaces $\theta_{\min} < \theta < \theta_{\max}$ with $\mu_{\min} < \mu < \mu_{\max}$.

For large μ, resonant $\cos\theta \approx \dfrac{\omega}{\Omega_e} \approx \sqrt{\dfrac{n}{\gamma(v_\parallel/c)\mu}}$.

Then all the terms in D can be approximated, giving

$$D_{\alpha\alpha} \approx A_0 \sum_n \int z^4 \frac{J_n^2(z)}{|n|} dz$$

where $z = k_\perp \rho \sim \mu^{1/2}$ and

$$A_0 = \frac{3\pi}{4} \frac{\Omega_e^6/\omega_{pe}^4}{\omega_{UC} - \omega_{LC}} \frac{c^7/v_\perp^7 \gamma^6}{\mu_{\max}^3 - \mu_{\min}^3} \frac{B_w^2}{B^2}.$$

The range of n is restricted by ω_{LC}, ω_{UC}, $\mu_{\min}$, $\mu_{\max}$.

This is much simpler than the original expressions.

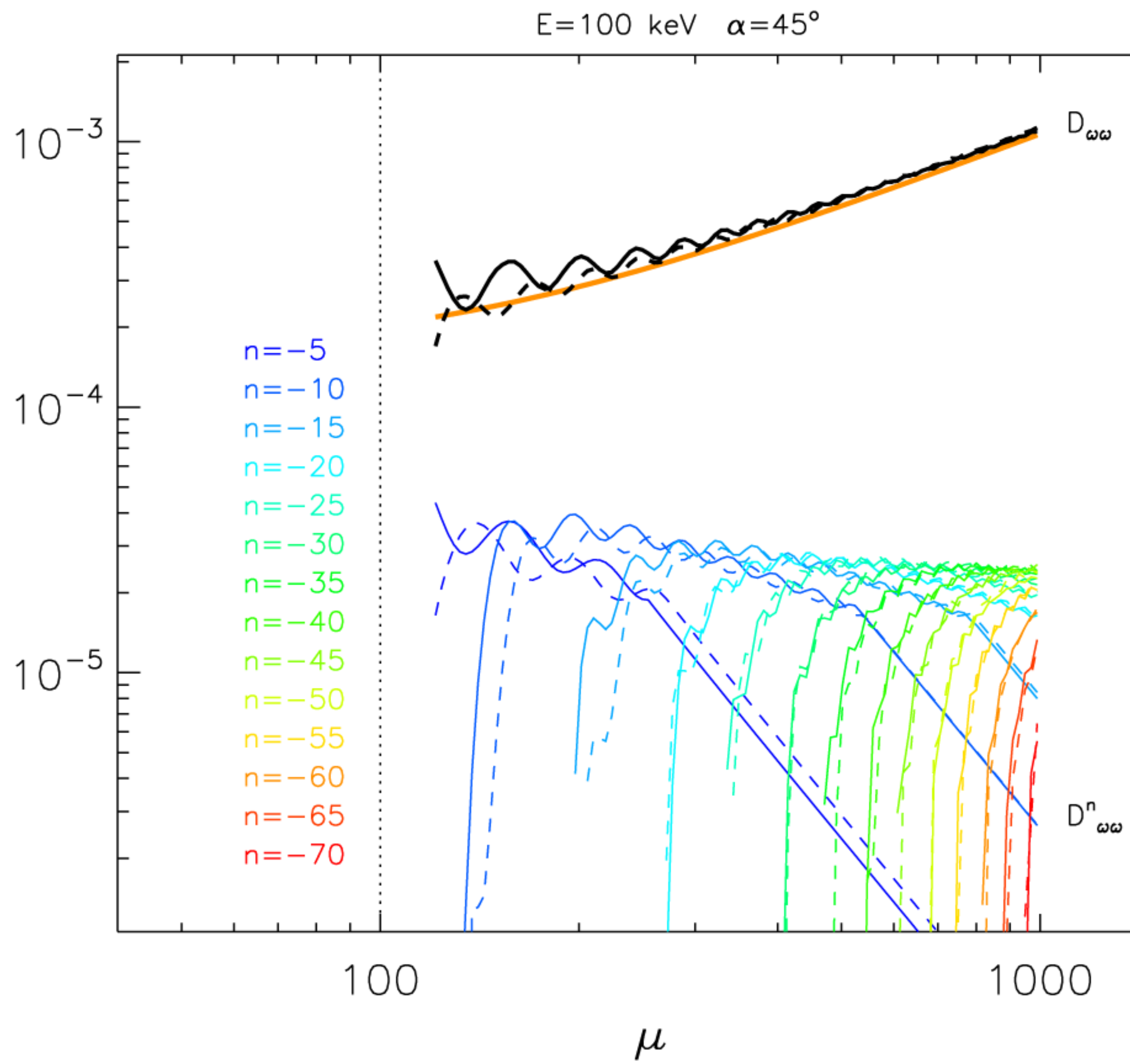
More: the large-arg. approx. for J_n applies if $\tan\alpha > \omega_{UC}/\Omega_e$.

Then $\sum_n$ and $\int dz$ can be done analytically, giving

$$
\begin{aligned}
D_{\alpha\alpha} &\approx \frac{3}{64}\frac{c^3}{v_\perp^3}\frac{\omega_{UC}^3}{\Omega_e^3} D_0 \mu_{\max}, \\
D_{\alpha p} &\approx -\frac{1}{12}\frac{c^4}{v_\perp^2 v^2}\frac{\omega_{UC}^2}{\Omega_e^2} D_0, \\
D_{pp} &\approx \frac{3}{16}\frac{c^5}{v_\perp v^4}\frac{\omega_{UC}}{\Omega_e}\frac{D_0}{\mu_{\max}}
\end{aligned}
\qquad D_0 = \frac{\Omega_e^4}{\omega_{pe}^4}\frac{\Omega_e}{\gamma^2}\frac{B_w^2}{B^2}
$$

As $\mu_{\max} \to \infty$, $D_{\alpha\alpha} \to \infty$ and $D_{pp} \to 0$.

These results have been checked numerically:



Note: $\mu_{\max} \to \infty$ was taken with B_w fixed, so $E_w \to \infty$.

Holding E_w fixed instead gives $B_w \to 0$ and

$$D_{\alpha\alpha} \sim \frac{E_w^2}{\mu_{\max}}, \quad D_{\alpha p} \sim \frac{E_w^2}{\mu_{\max}^2}, \quad D_{pp} \sim \frac{E_w^2}{\mu_{\max}^3}$$

Then $D_{\alpha\alpha} \to 0$, not ∞.

This may be a more realistic model as $\theta \to \theta_{RC}$.

If $\tan\alpha \ll \omega_{LC}/\Omega_e$, the small-arg. approx. for J_n leads to

$$D_{\alpha\alpha} \sim \frac{B_w^2}{\mu_{\max}^3}, \qquad D_{pp} \sim \frac{B_w^2}{\mu_{\max}^3}$$

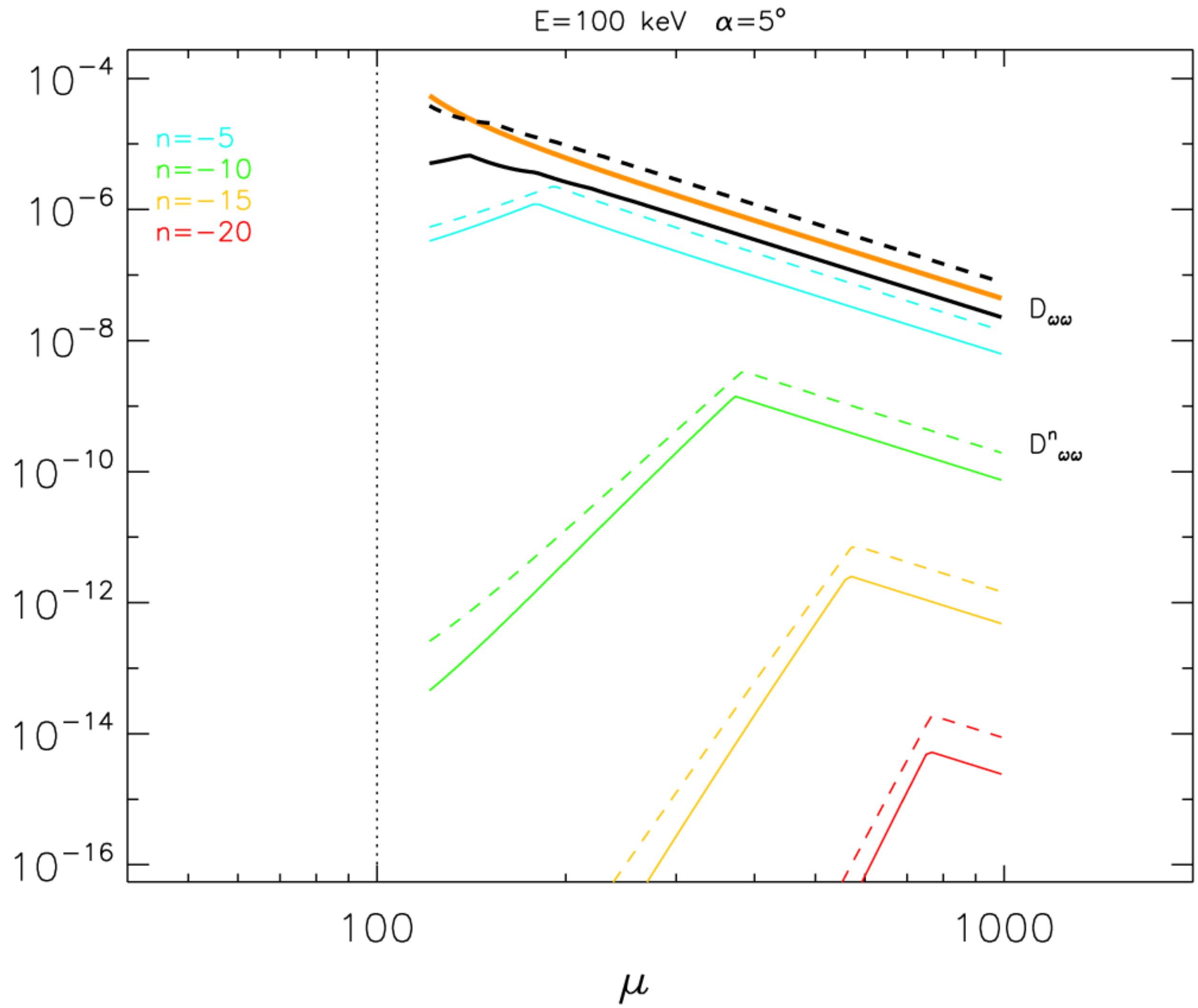
or

$$D_{\alpha\alpha} \sim \frac{E_w^2}{\mu_{\max}^3}, \qquad D_{pp} \sim \frac{E_w^2}{\mu_{\max}^3}$$

which both give $D_{\alpha\alpha} \rightarrow 0$ for highly oblique waves.

(Can redo all of the above using $\mu_{||}$ instead of μ.)

This result has also been checked numerically.



Summary

D for highly oblique whistlers is different from moderately oblique.

Analytical estimates show what to expect, and help with the full calcs.

$D \rightarrow \infty$ vs. $D \rightarrow 0$? It depends on the model:

Parameterize by B_w^2 or E_w^2 or S_w or ...?

(Scale B_w by E_{em}/E_w? Horne et al., JGR 2013)

Limited by $\mu_{\max}$ or $\mu_{\|\max}$? Set by $T_{e\|}$ or something else?

We're reaching the validity limits of cold plasma theory.

Other possible applications:

EMIC: $\mu \rightarrow \infty$ as $\omega \rightarrow \Omega_i$. MS: no RC, but θ and μ are large.