Embracing Low Inertia for Power System Frequency Control

A Dynamic Droop Approach

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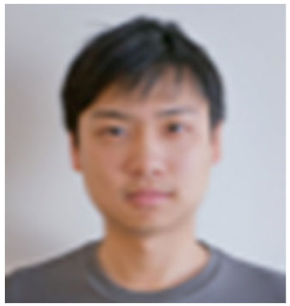
Acknowledgements

Yan Jiang



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Richard Pates

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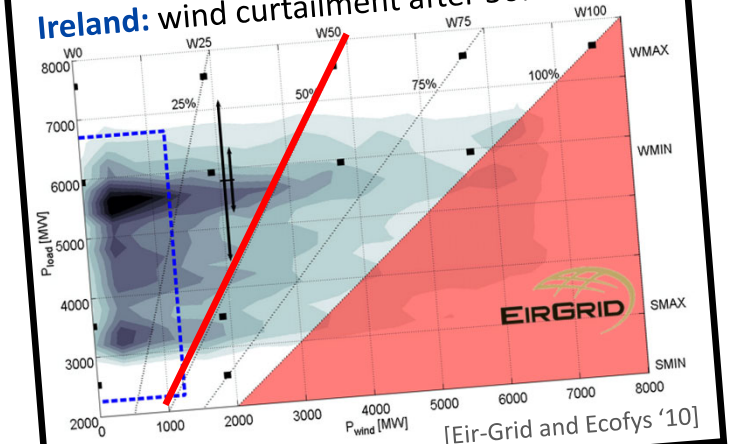
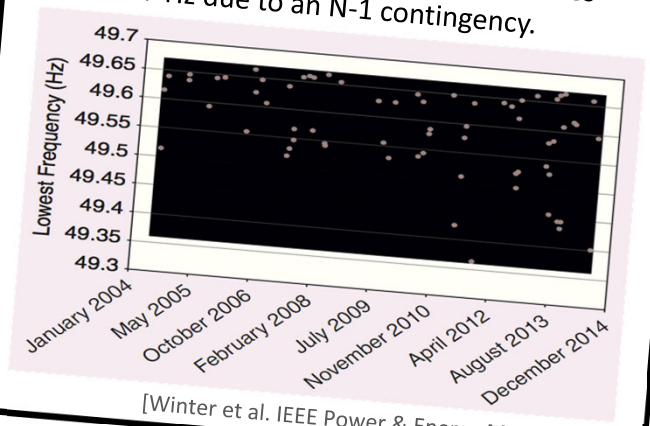
Dynamic Degradation

Ireland: wind curtailment after 50% of load



[Eir-Grid and Ecofys '10]

Nordic system: disturbances with frequencies under 49.7 Hz due to an N-1 contingency.



[Winter et al. IEEE Power & Energy Magazine '15]

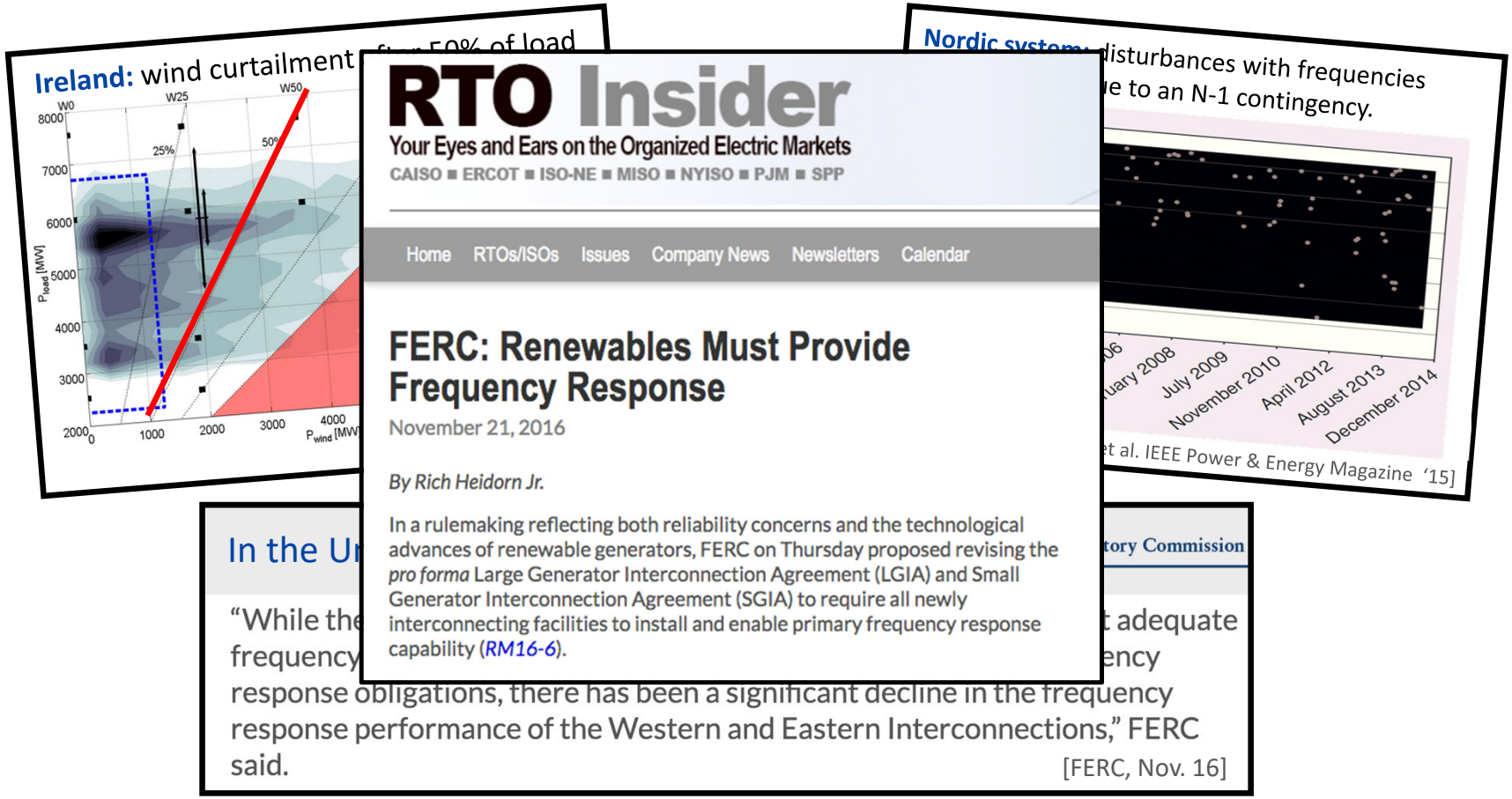
In the United States:

FERC Federal Energy Regulatory Commission

"While the three [contiguous] U.S. interconnections currently exhibit adequate frequency response performance above their interconnection frequency response obligations, there has been a significant decline in the frequency response performance of the Western and Eastern Interconnections," FERC said.

[FERC, Nov. 16]

Dynamic Degradation



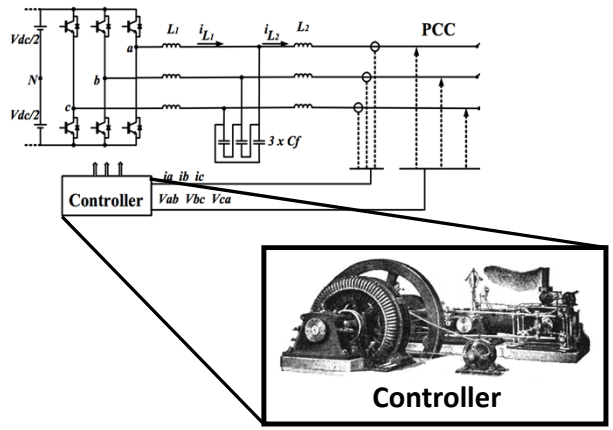
Inverter-based Control

Challenges

- Measurements with noise and delays
- Stability + robustness (plug & play)
- Lack of incentives

Current approach: Use inverter-based control to **mimic generators response**

Virtual Synchronous Generator



Telecom Analogy

Inverter-based Control

Challenges

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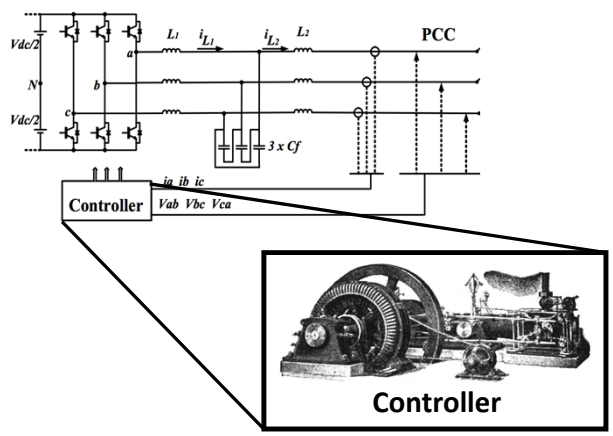
Virtual Synchronous Generator



Telecom Analogy

It works, but perhaps there is something better…

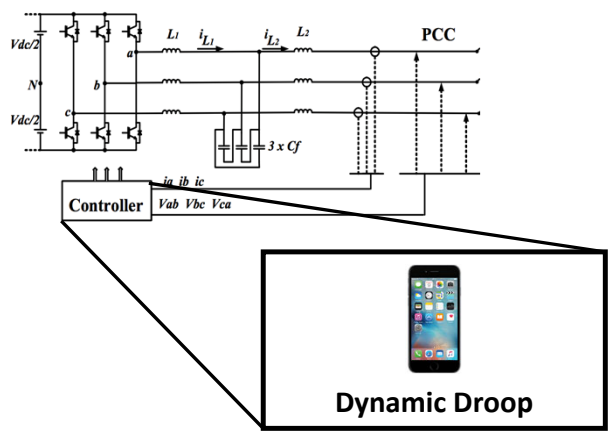
Inverter-based Control

Challenges

- Measurements with noise and delays
- Stability + robustness (plug & play)
- Lack of incentives

Our approach: Design and tune of controllers rooted on **sound control principles**

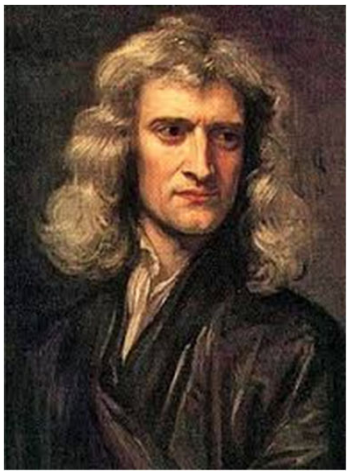
Dynamic Droop Control (iDroop)

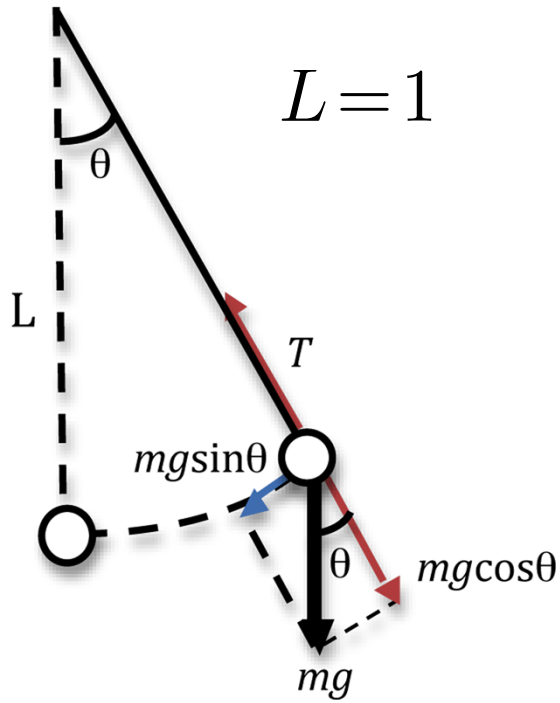


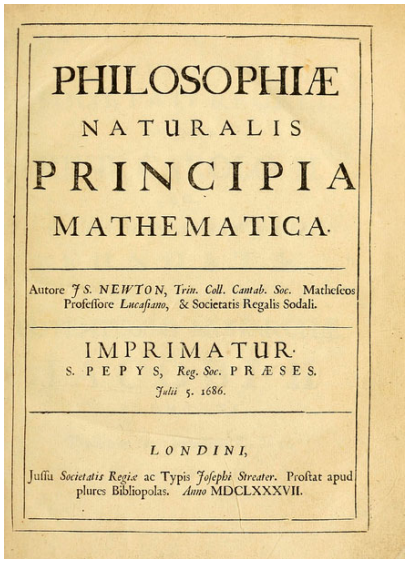
Design Objectives:

- Exploit power electronics capabilities
- Improve Dynamic Performance
- Minimize control effort
- Stability and Robustness

Merits and Trade-offs of Inertia



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S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

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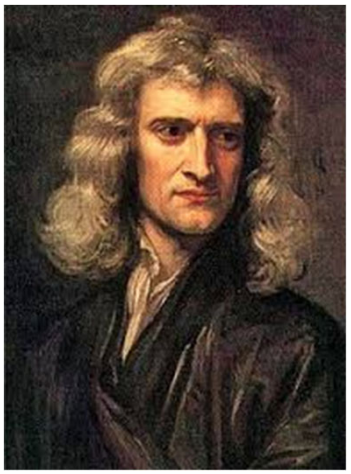
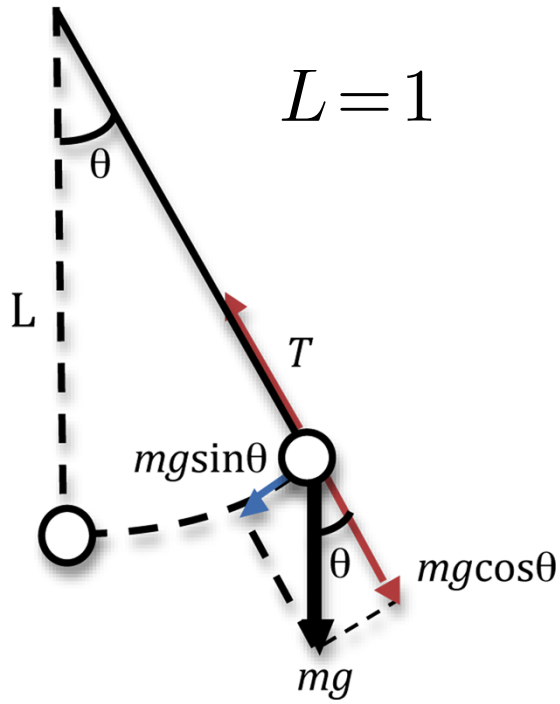
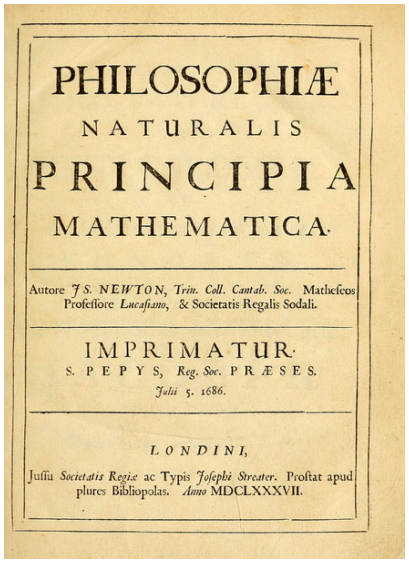
$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f$$

Merits and Trade-offs of Inertia



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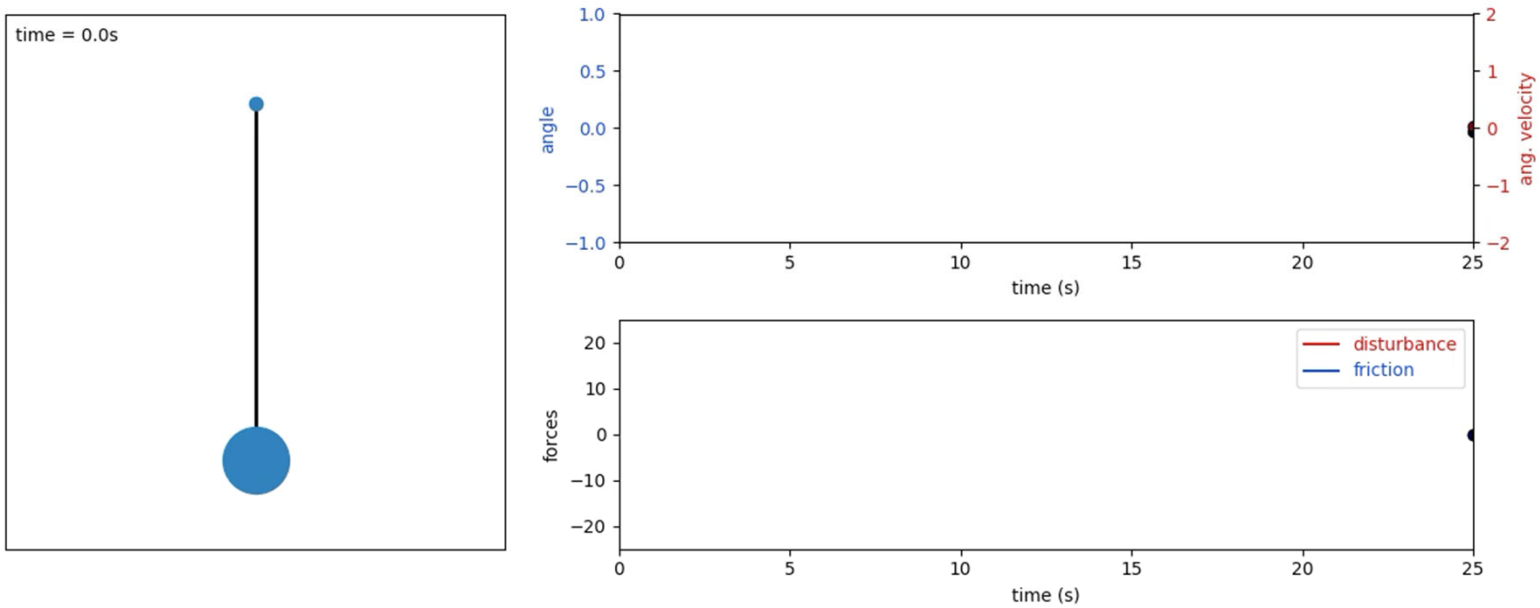
$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$

Merits and Trade-offs of Inertia

$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$

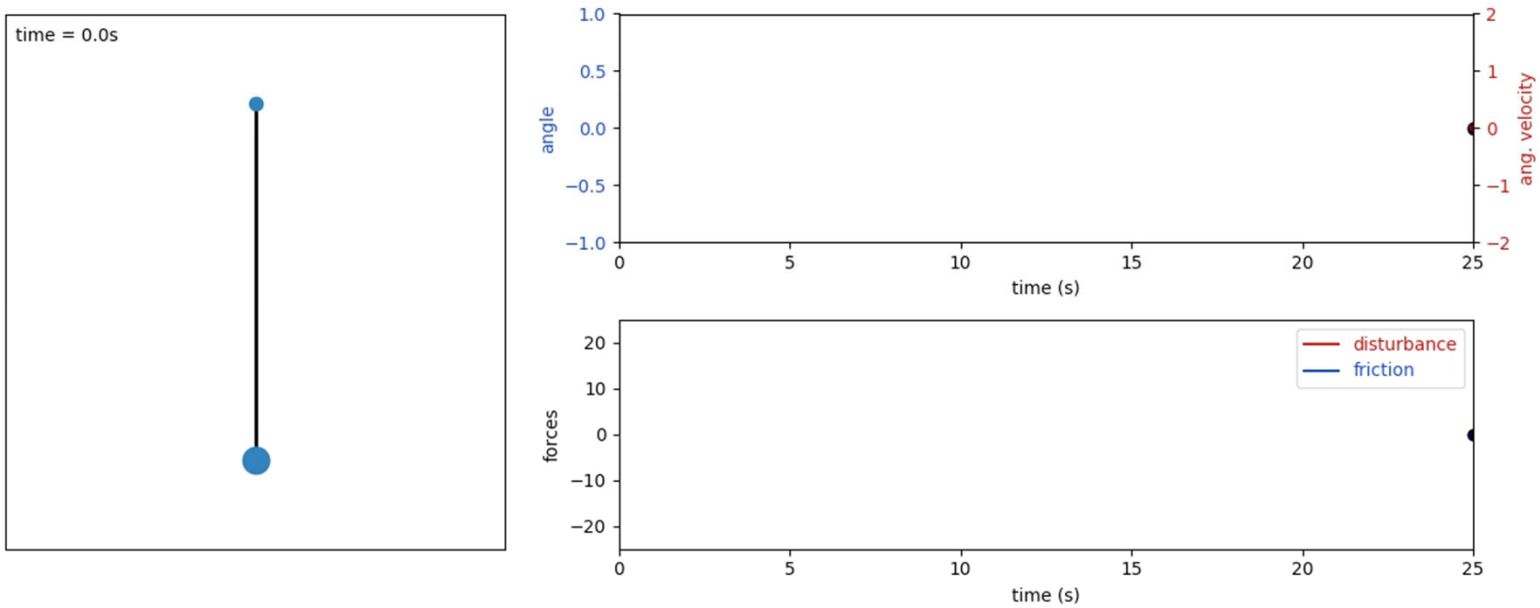


Pros: Provides natural disturbance rejection

Cons: Hard to regain steady-state

Merits and Trade-offs of Low Inertia

$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$



Cons: Susceptible to disturbances

Pros: Regains steady-sate faster

Roadmap to Low Inertia Frequency Control

- Performance Specification
- Limits of Virtual Inertia and Droop Control
- Dynamic Droop Control: iDroop

Global analysis of synchronization performance for power systems: bridging the theory-practice gap

Fernando Paganini, *Fellow, IEEE*, and Enrique Mallada, *Senior Member, IEEE*

IEEE Transactions on Automatic Control, July 2020

Dynamic Droop Control in Low-inertia Power Systems

Yan Jiang, Richard Pates, and Enrique Mallada

IEEE Transactions on Automatic Control, October 2020

Dynamic Droop Approach for Storage-based Frequency Control

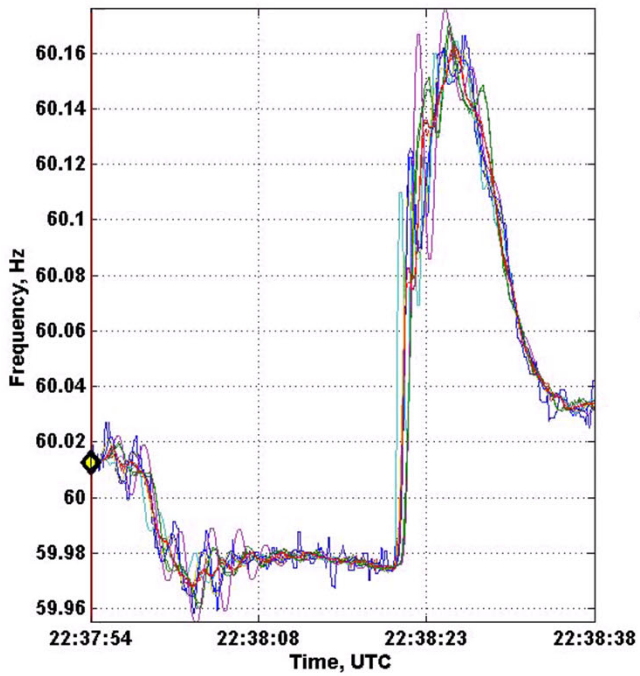
Yan Jiang*, Eliza Cohn*, Petr Vorobev†, and Enrique Mallada*

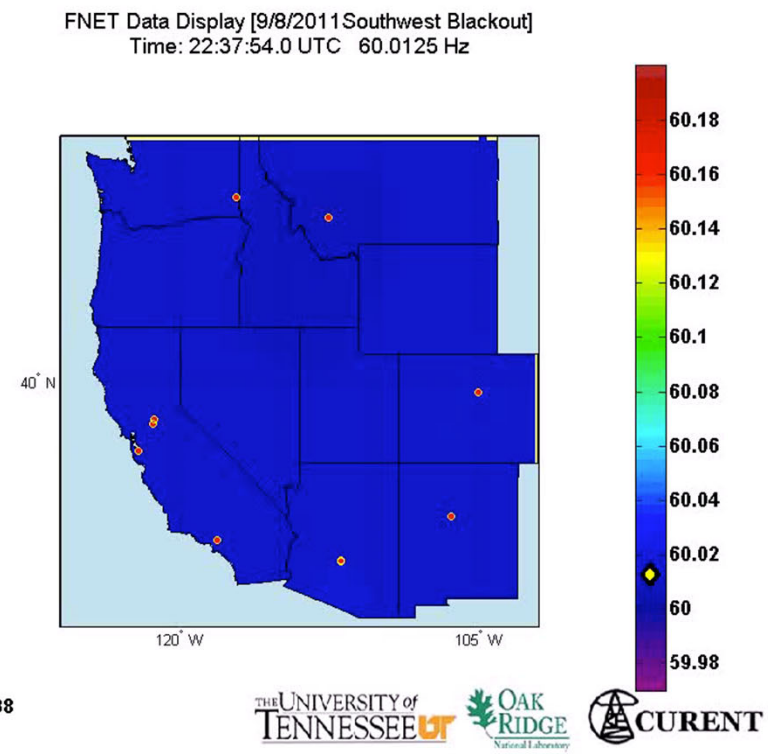
arXiv preprint arXiv:1910.04954 (2019)

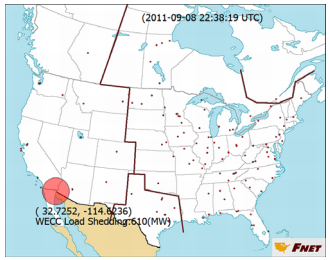
Roadmap to Low Inertia Frequency Control

- Performance Specification
- Limits of Virtual Inertia and Droop Control
- Dynamic Droop Control: iDroop

Power System Performance







Depends on several factors: generators, network, disturbance

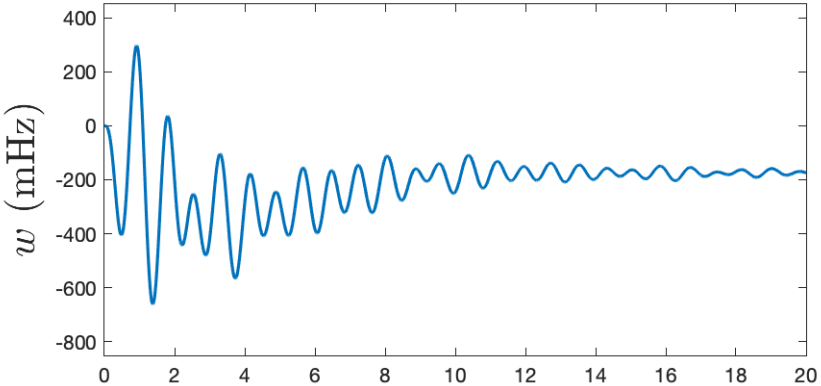
good performance metric must identify the source of the degradation!

Performance Specification

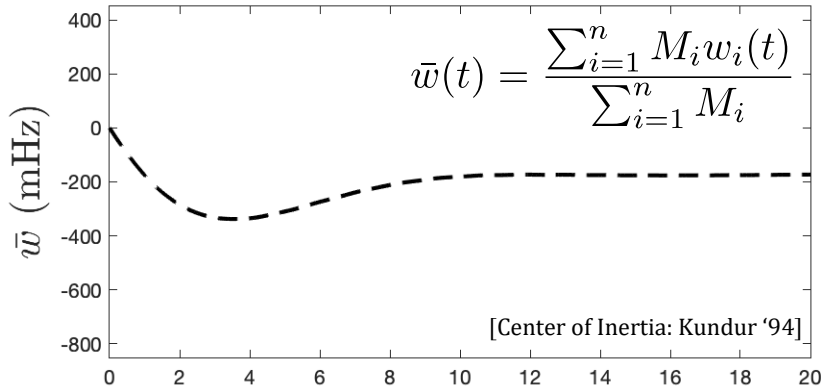
Frequency Response

Control Effort

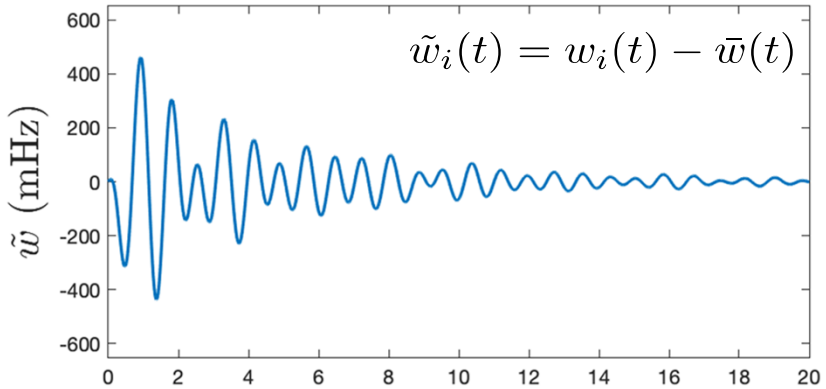
Decomposition of Step Response



System Frequency

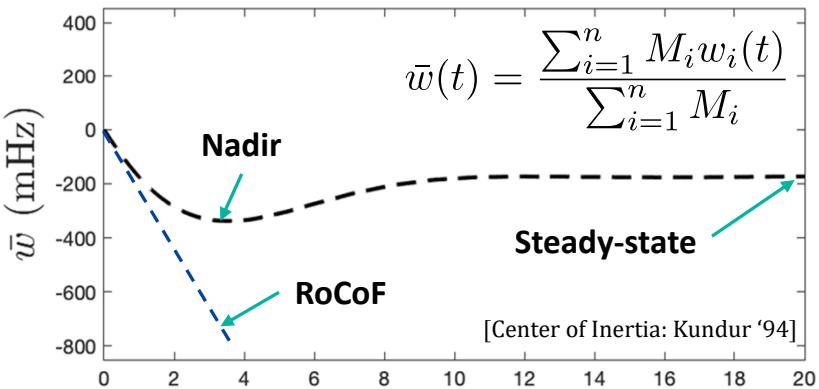


Synchronization Error

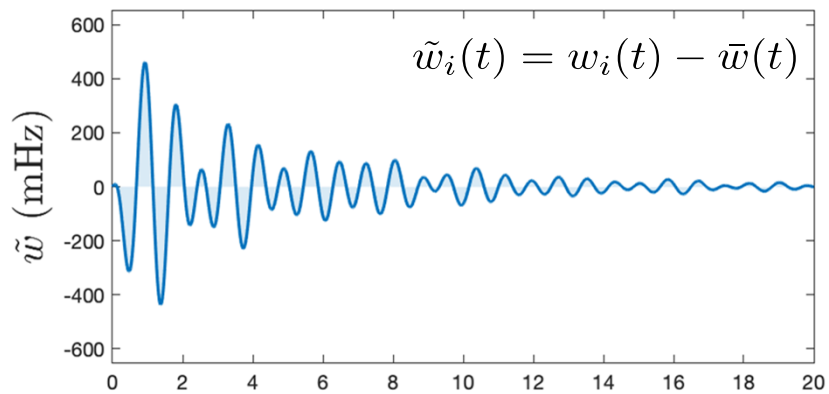


Step Disturbance Performance

System Frequency



Deviation from Mean



Nadir

$$||\bar{w}||_\infty := \sup_{t \geq 0} |\bar{w}(t)|$$

RoCoF

$$||\dot{\bar{w}}||_\infty := \sup_{t \geq 0} |\dot{\bar{w}}(t)|$$

Steady-state

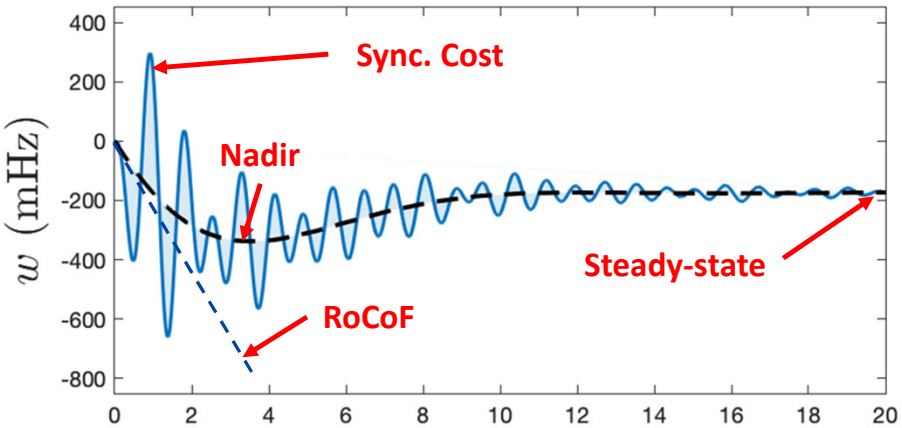
$$||\dot{\bar{w}}||_\infty := \sup_{t \geq 0} |\dot{\bar{w}}(t)|$$

Synchronization Cost

$$||\tilde{w}||_2 = \left(\int_0^{+\infty} \sum_{i=1}^{n} \tilde{w}_i^2(t) dt \right)^{\frac{1}{2}}$$

Performance Specification

Frequency Response



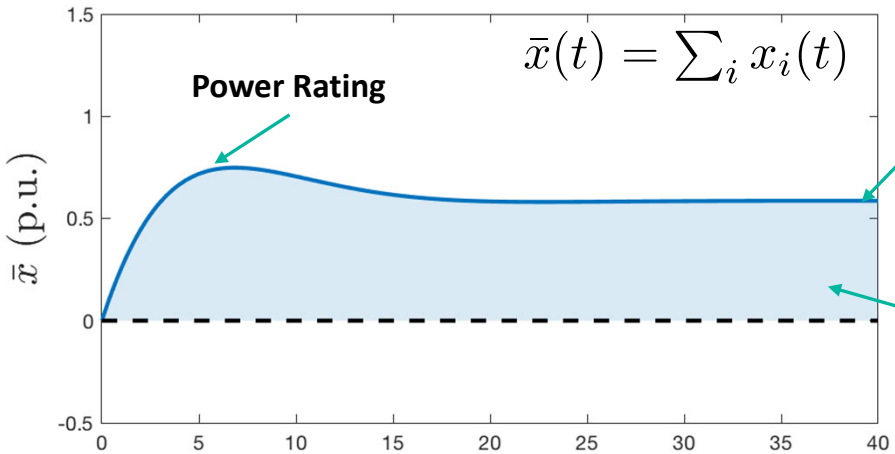
System Freq. : $\bar{w}(t) = \dfrac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort

Control Effort

Inverter Power



Steady-State Effort

$$|\bar{x}(\infty)| = \sum_i R_{r,i}^{-1} |\bar{w}(\infty)|$$

Injected Energy

$$\dot{E}(t) = \bar{x}(t)$$

Power Rating

$$\|\bar{x}\|_\infty := \sup_{t \geq 0} |\bar{x}(t)|$$

Max Energy

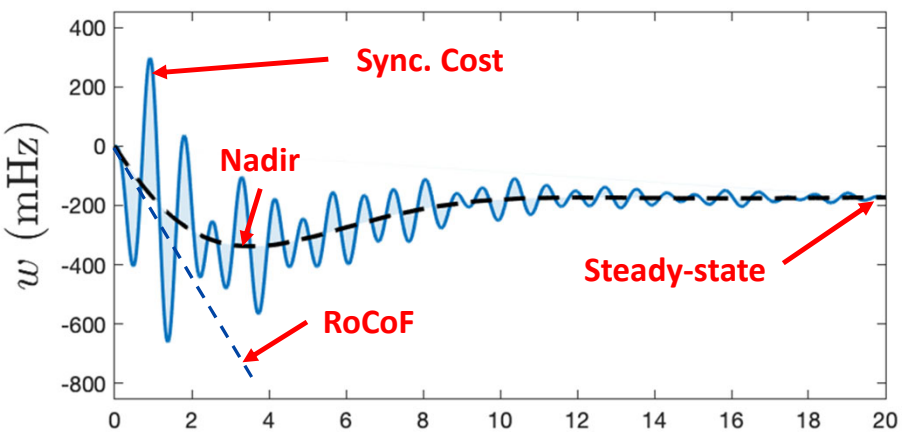
$$\|E\|_\infty := \sup_{t \geq 0} |E(t)|$$

Steady-State Effort

$$|\bar{x}(\infty)| = \sum_i R_{r,i}^{-1} |\bar{w}(\infty)|$$

Performance Specification

Frequency Response



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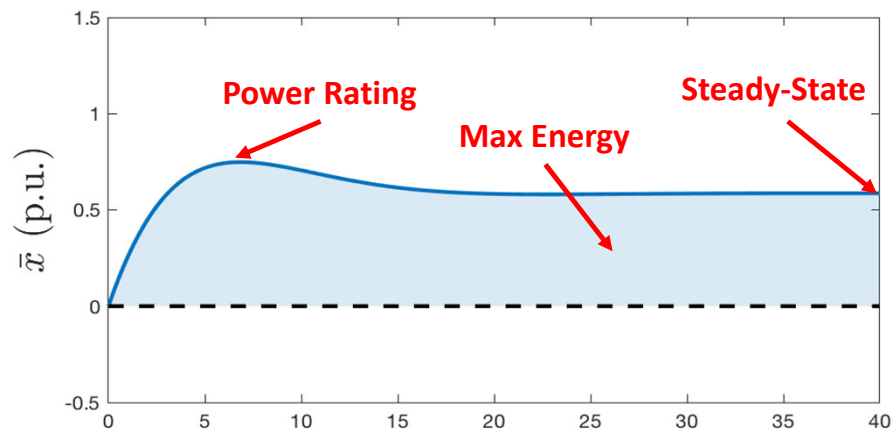
Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

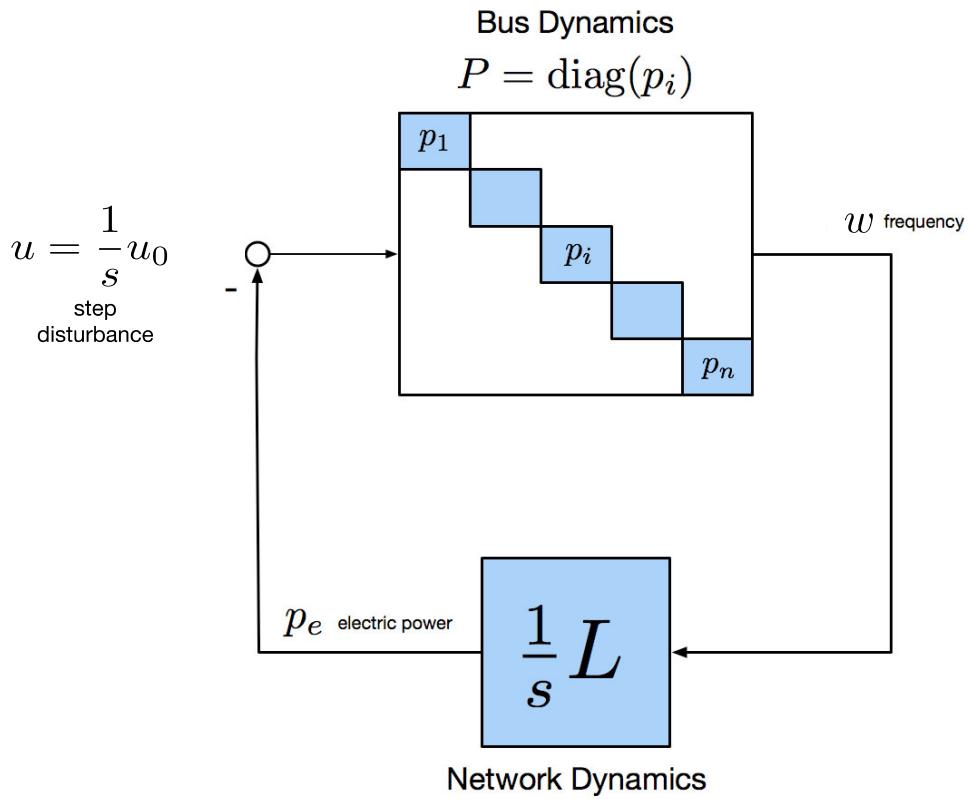
Injected Energy: $\dot{E}(t) = \bar{x}(t)$

Benchmark: Quantify **control ability** to eliminate overshoot in the **Nadir**

Roadmap to Low Inertia Frequency Control

- Performance Specification
- Limits of Virtual Inertia and Droop Control
- Controller Design: iDroop

Power Network Model



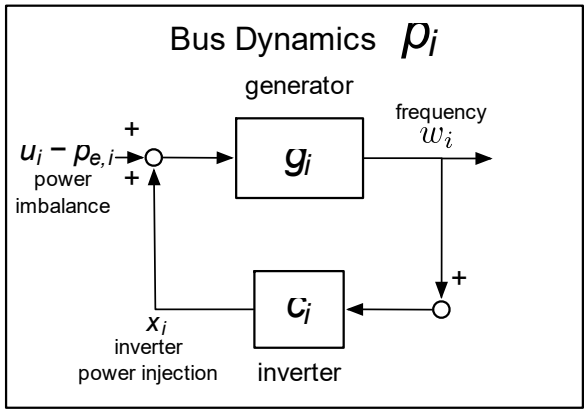
Laplacian Matrix

$$L_{ij} = \begin{cases} -B_{ij} & \text{if } ij \in E \\ \sum_k B_{ik} & \text{if } i = j \\ 0 & \text{o.w.} \end{cases}$$

Step: $u = \frac{1}{s} u_0$

[Bergen Hill '81]

Bus Dynamics

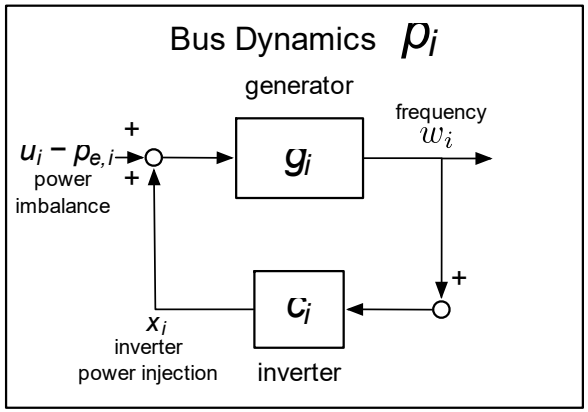


Generator: $g_i : (u_i - p_{e,i} + x_i) \mapsto w_i$

Model: Swing Equations + Turbine

$$g_i : \begin{cases} \dot{\theta}_i &= w_i \\ M_i \dot{w}_i &= -D_i w_i + q_i + (u_i - p_{e,i} + x_i) \\ \tau_i \dot{q}_i &= -R_{g,i}^{-1} w_i - q_i \end{cases}$$

Bus Dynamics



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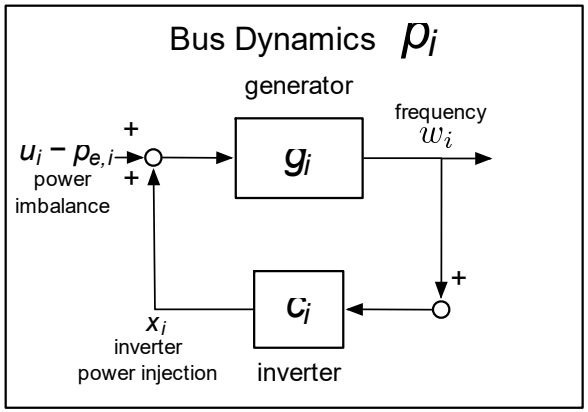
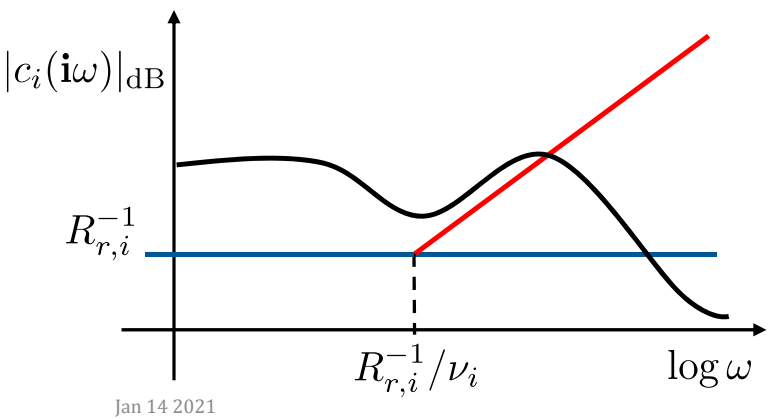
$$g_i(s) = \frac{\tau_i s + 1}{M_i \tau_i s^2 + (M_i + D_i \tau_i) s + D_i + r_{g,i}^{-1}}$$

Bus Dynamics





Inverter: $c_i : w_i \mapsto x_i$

Droop Control and Virtual Inertia:

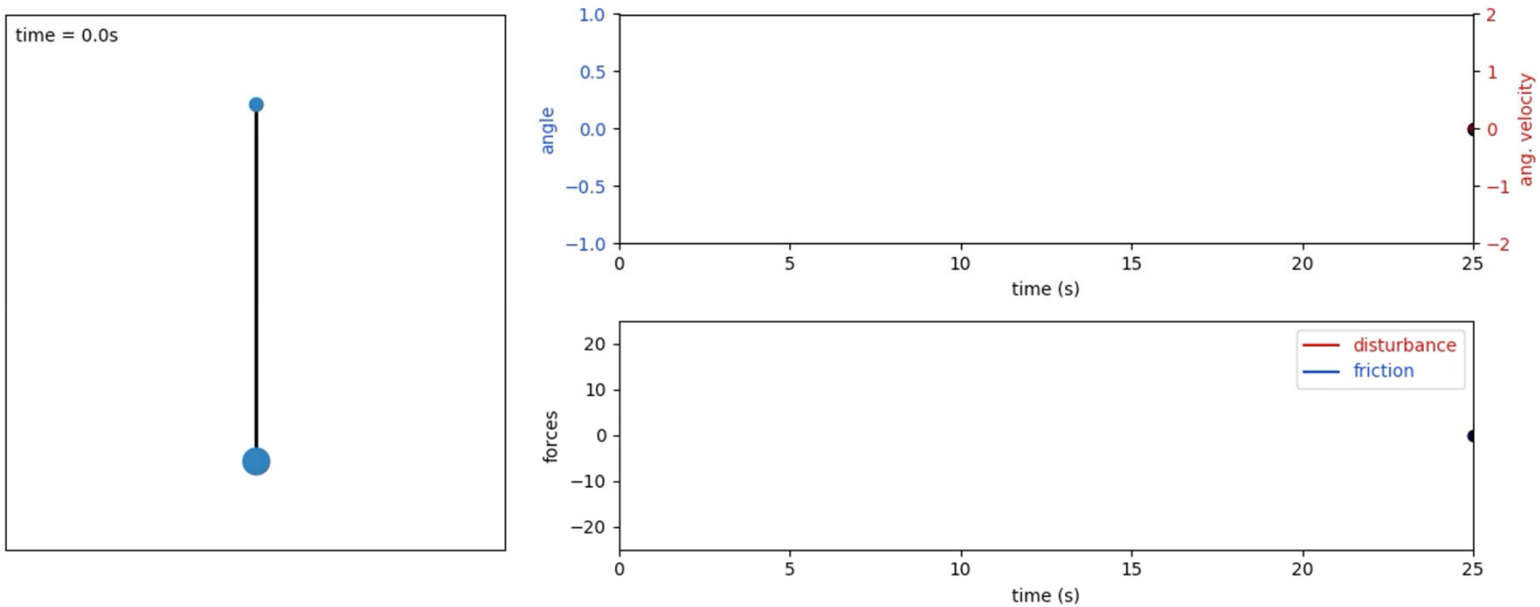
$$c_i : \left\{ x_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \qquad c_i(s) = -(\nu_i s + R_{r,i}^{-1}) \right.$$

Closed-loop Bus Dynamics:

$$p_i : \begin{cases} \dot{\theta}_i = w_i \\ (M_i + \nu_i)\dot{w}_i = -(D_i + R_{r,i}^{-1})w_i + q_i + (u_i - p_{e,i}) \\ \tau_i \dot{q}_i = -q_i - R_{g,i}^{-1} w_i \end{cases}$$

Control of Low Inertia Pendulum

$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + u$$

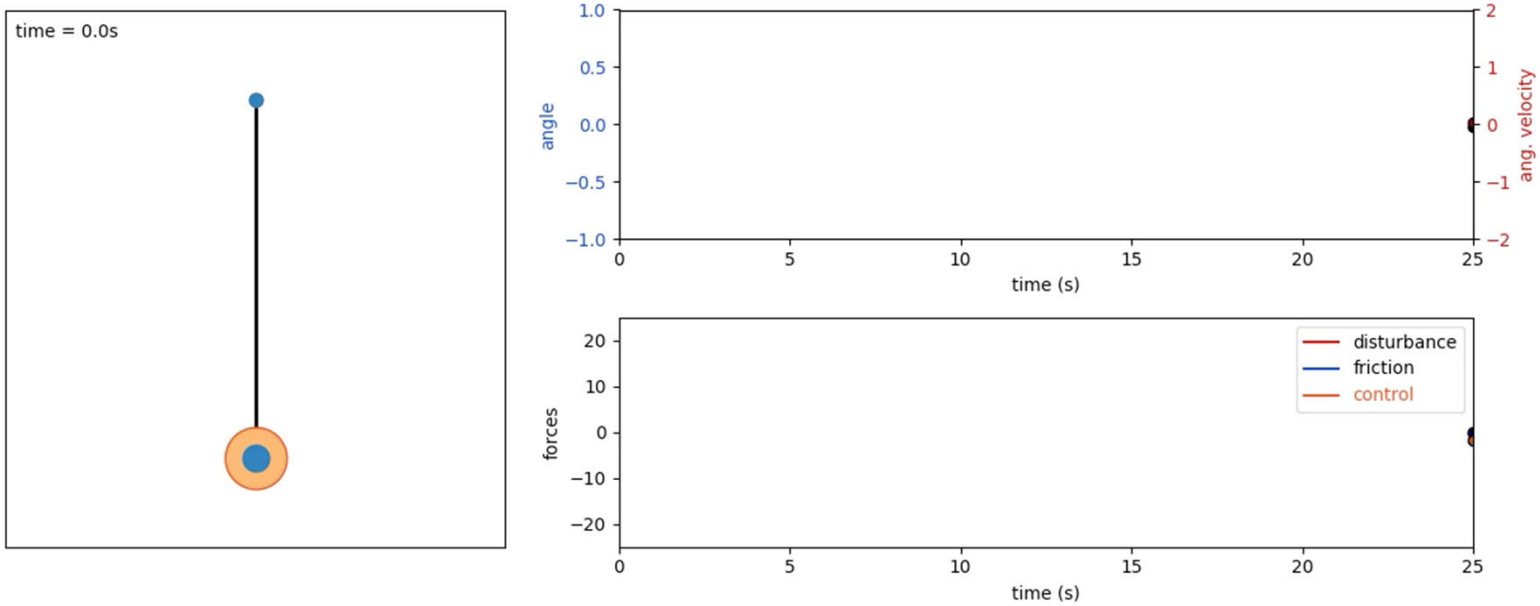


Cons: Susceptible to disturbances

Pros: Regains steady-sate faster

Control of Low Inertia Pendulum

Virtual Mass Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - \nu\ddot{\theta}$

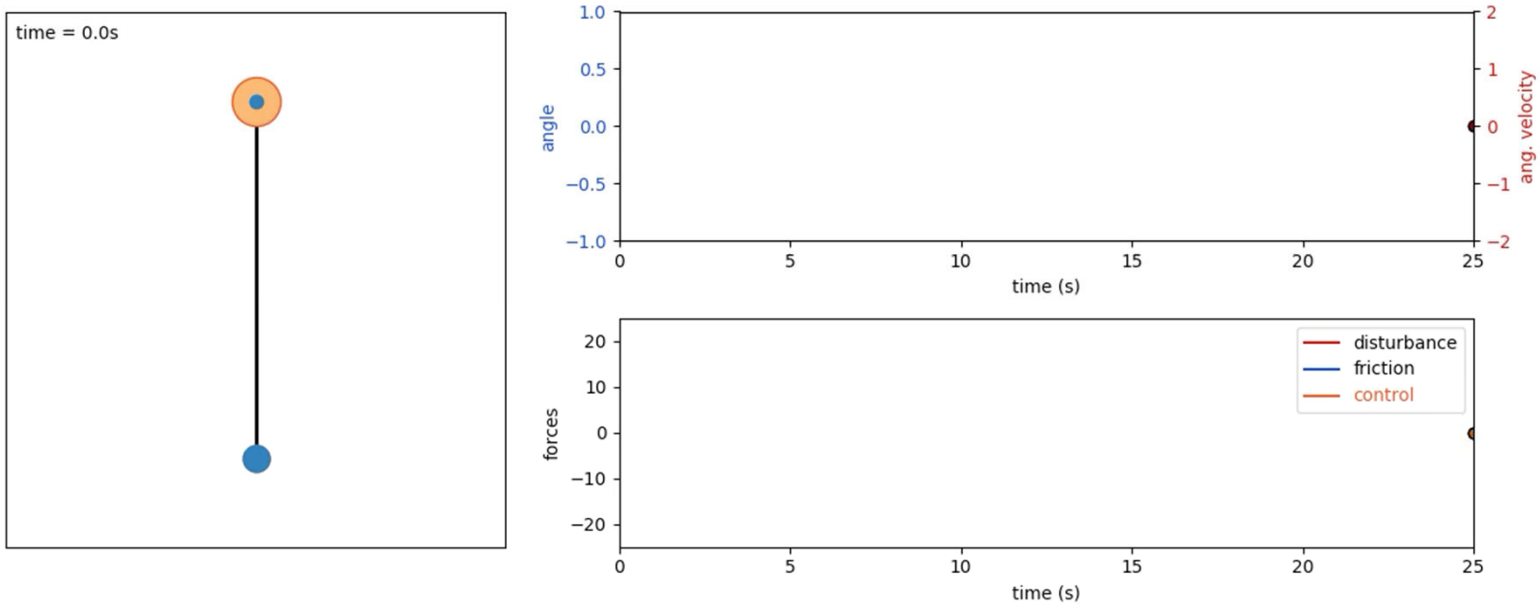


Pros:
Provides disturbance rejection

Cons:
Hard to regain steady-state + **excessive control effort**

Control of Low Inertia Pendulum

Virtual Friction Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - r^{-1}\dot{\theta}$



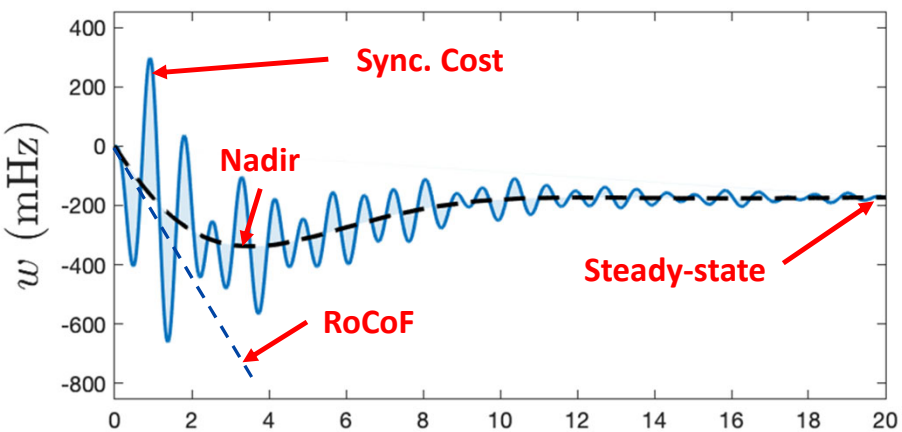
Pros: Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

Cons?

None, at least for pendulum

Performance Specification

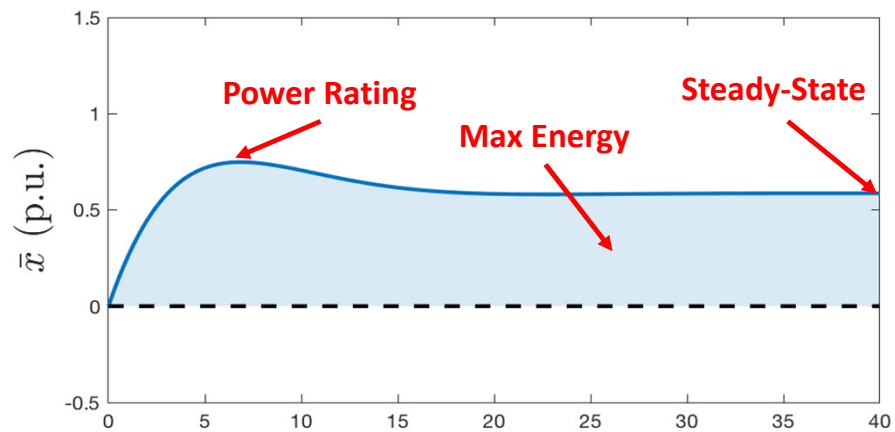
Frequency Response



System Freq. : $\bar{w}(t) = \dfrac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

Injected Energy: $\dot{E}(t) = \bar{x}(t)$

Benchmark: Quantify **control ability** to eliminate overshoot in **Nadir**

Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and assume

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

Swing Equations + Turbine

$$g_0(s) = \frac{\tau s + 1}{m\tau s^2 + (m + d\tau)s + d + r^{-1}}$$

Virtual Inertia

$$c_0(s) = -(\nu s + r_r^{-1})$$

$$M_i = f_i m, \quad D_i = f_i d, \quad R_{g,i} = \frac{1}{f_i} r_g, \quad \tau_i = \tau$$

$$\nu_i = f_i \nu \quad R_{r,i} = \frac{1}{f_i} r_r$$

Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and assume

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

Swing Equations + Turbine

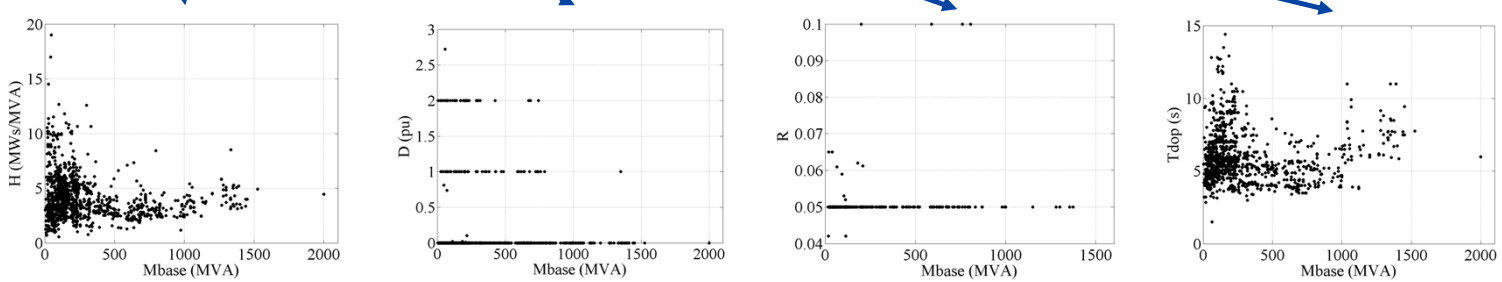
$$g_0(s) = \frac{\tau s + 1}{m\tau s^2 + (m + d\tau)s + d + r^{-1}}$$

Virtual Inertia

$$c_0(s) = -(\nu s + r_r^{-1})$$

$$M_i = f_i m, \quad D_i = f_i d, \quad R_{g,i} = \frac{1}{f_i} r_g, \quad \tau_i = \tau$$

$$\nu_i = f_i \nu \quad R_{r,i} = \frac{1}{f_i} r_r$$



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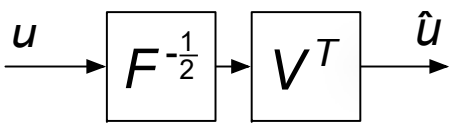
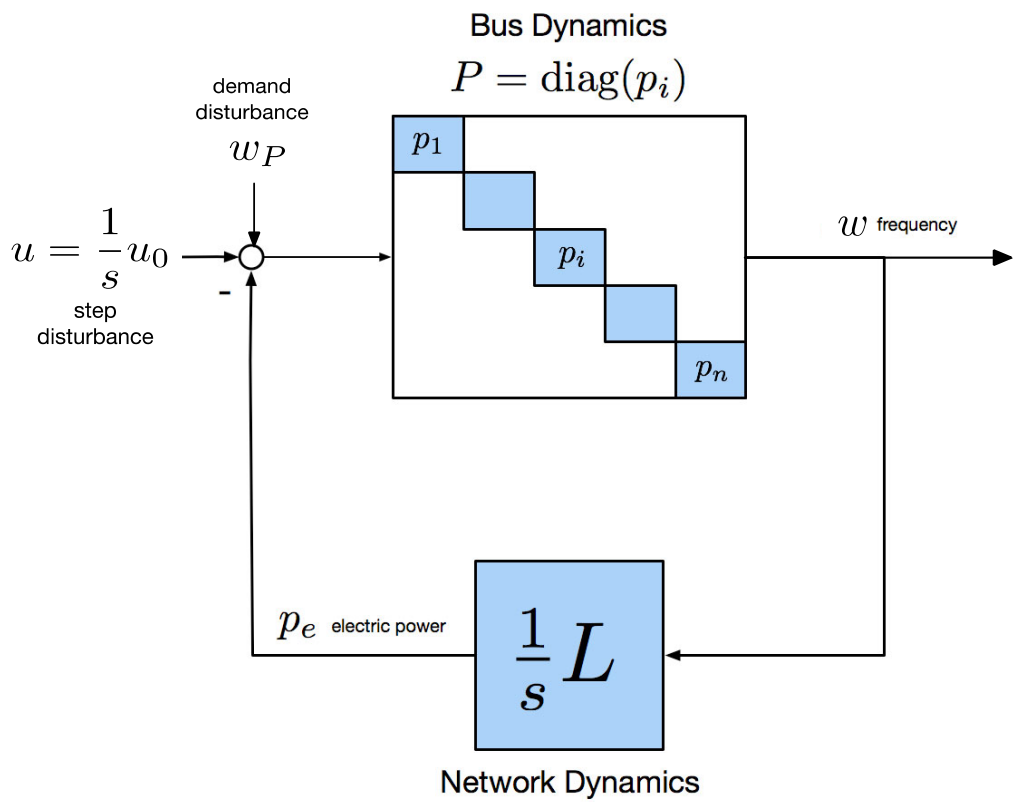
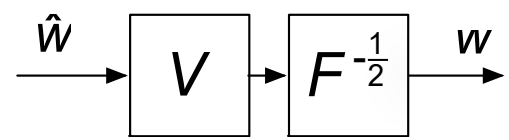
Change of Vars.





Change of Vars.



$$F = \mathrm{diag}(f_i)$$

[Paganini M '17 , Guo Low 18']

Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

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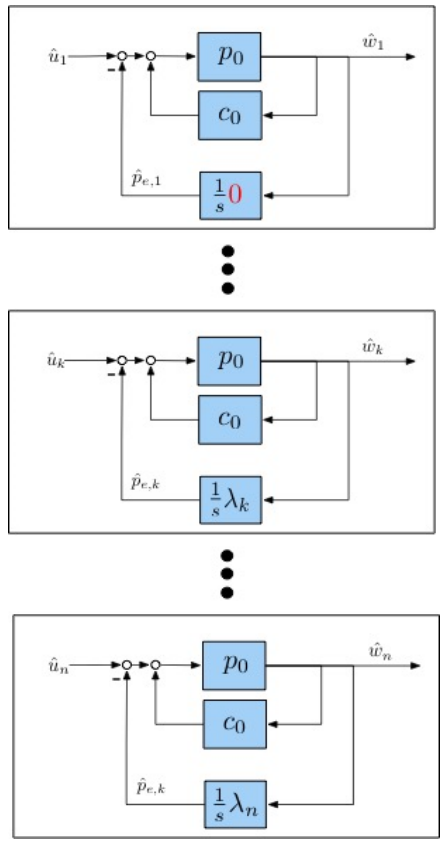


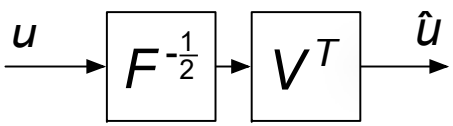
System Frequency

$$\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$$

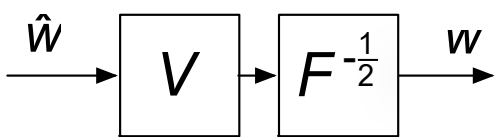
Sync Error

$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Change of Vars.



Change of Vars.



$$F = \mathrm{diag}(f_i)$$

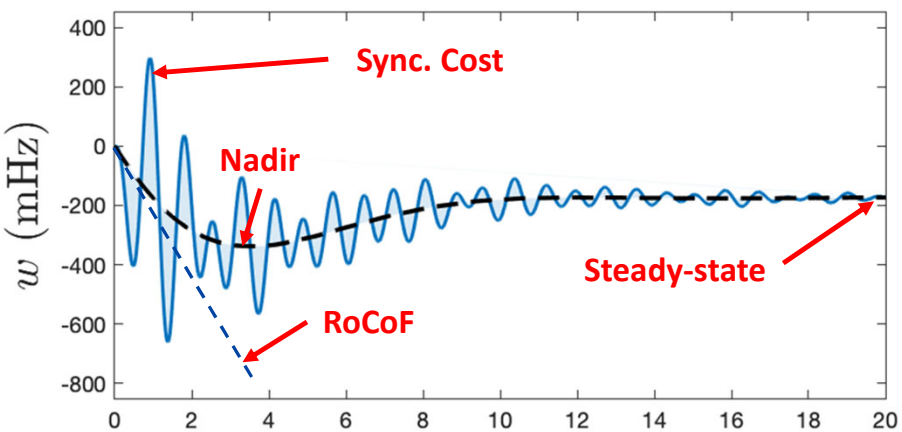
Eigenvalues of: $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

$$0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$$

[Paganini M '17 , Guo Low 18']

Performance Specification

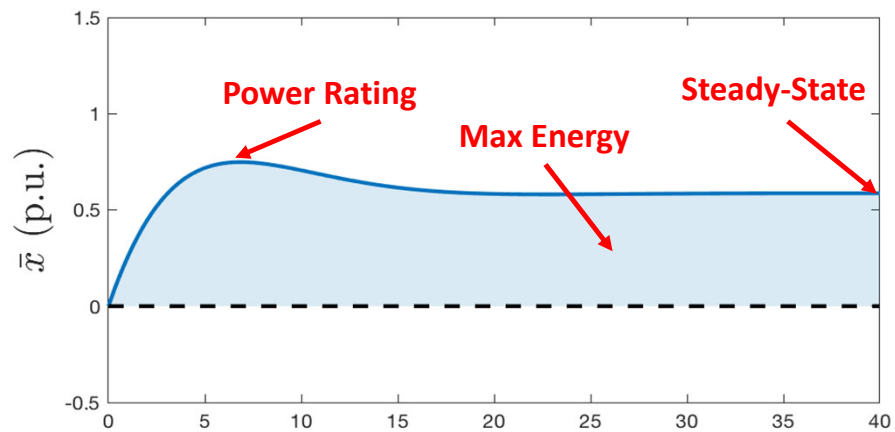
Frequency Response



System Freq. : $\bar{w}(t) = \dfrac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

Injected Energy: $\dot{E}(t) = \bar{x}(t)$

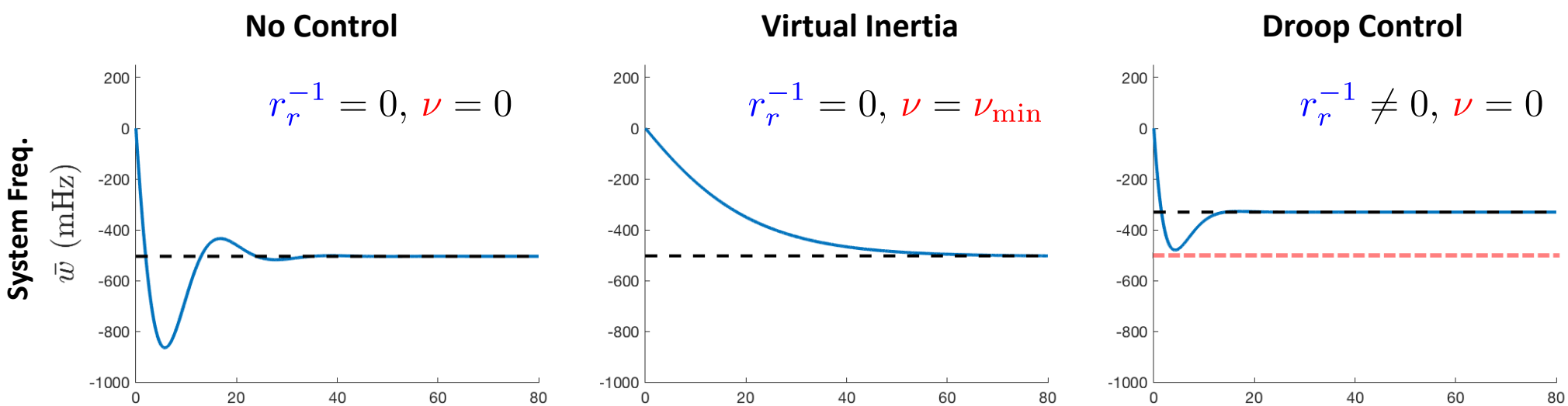
Benchmark: Quantify **control ability** to eliminate overshoot in **Nadir**

System Frequency w/ Virtual Inertia

$$c_i: \; x_i = -f_i(\nu \dot{w}_i + r_r^{-1} w_i)$$

Nadir Overshoot Elimination:

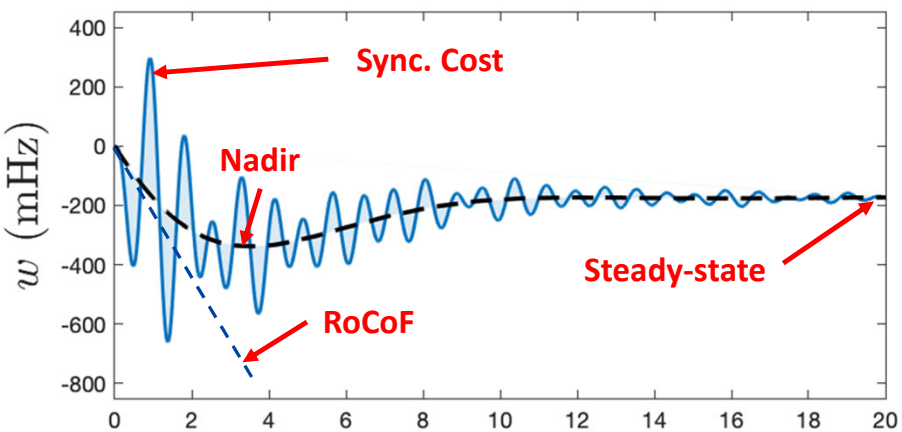
$$\nu \geq \nu_{\min} := \tau_g \left(\sqrt{r_g^{-1}} + \sqrt{d + r_g^{-1} + r_r^{-1}} \right)^2 - m$$



requires $\nu > 0$ in low inertia systems (low m)

Performance Specification

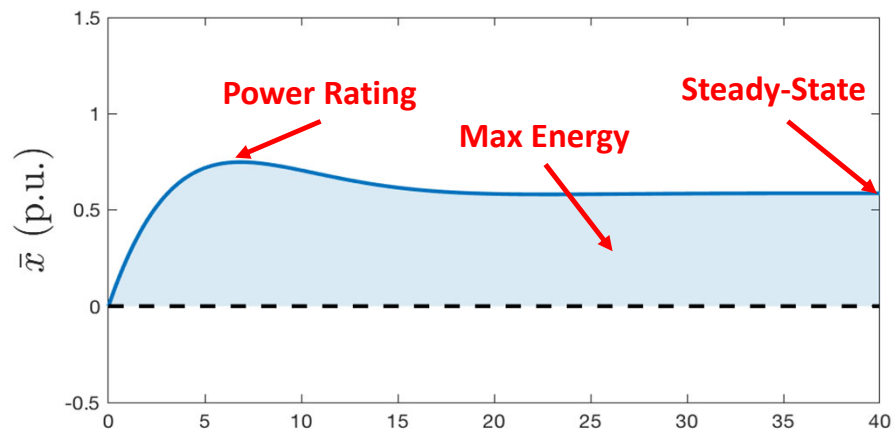
Frequency Response



System Freq. : $\bar{w}(t) = \dfrac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

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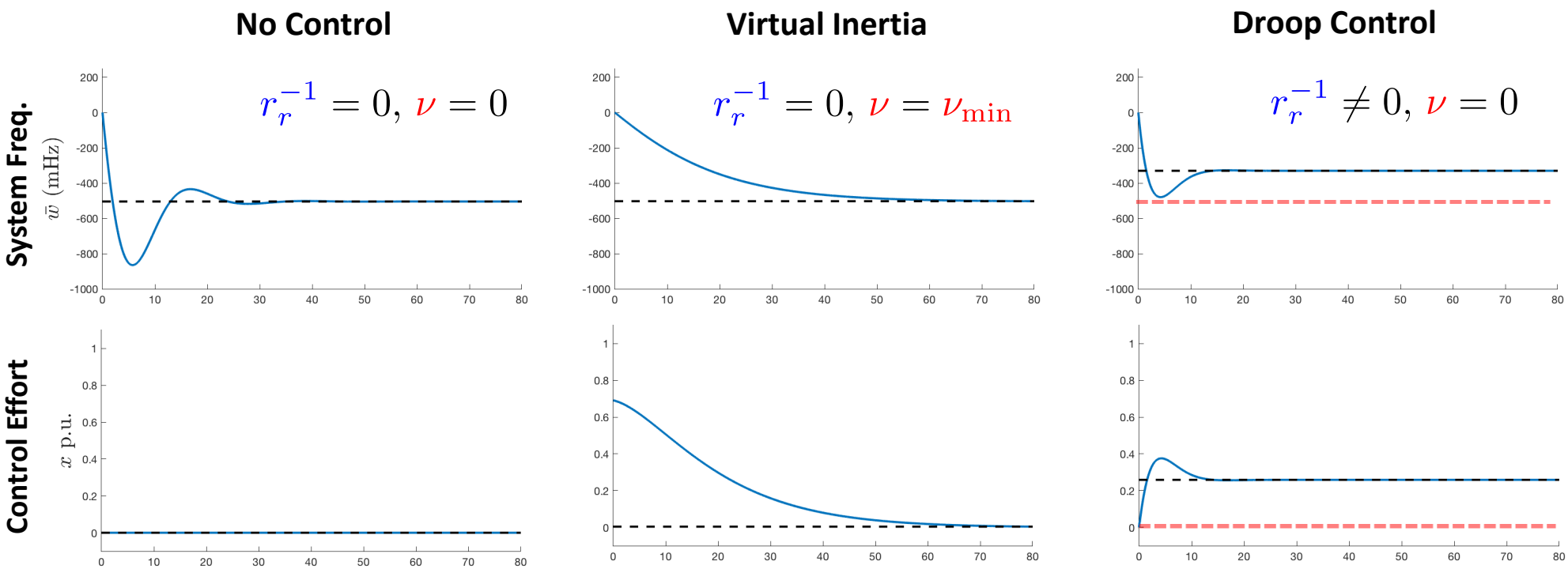
Benchmark: Quantify **control ability** to eliminate overshoot in **Nadir**

Control Effort

$$c_i: \; x_i = -f_i(\nu \dot{w}_i + r_r^{-1} w_i)$$

$$\bar{x}(t) = \sum_i x_i(t)$$

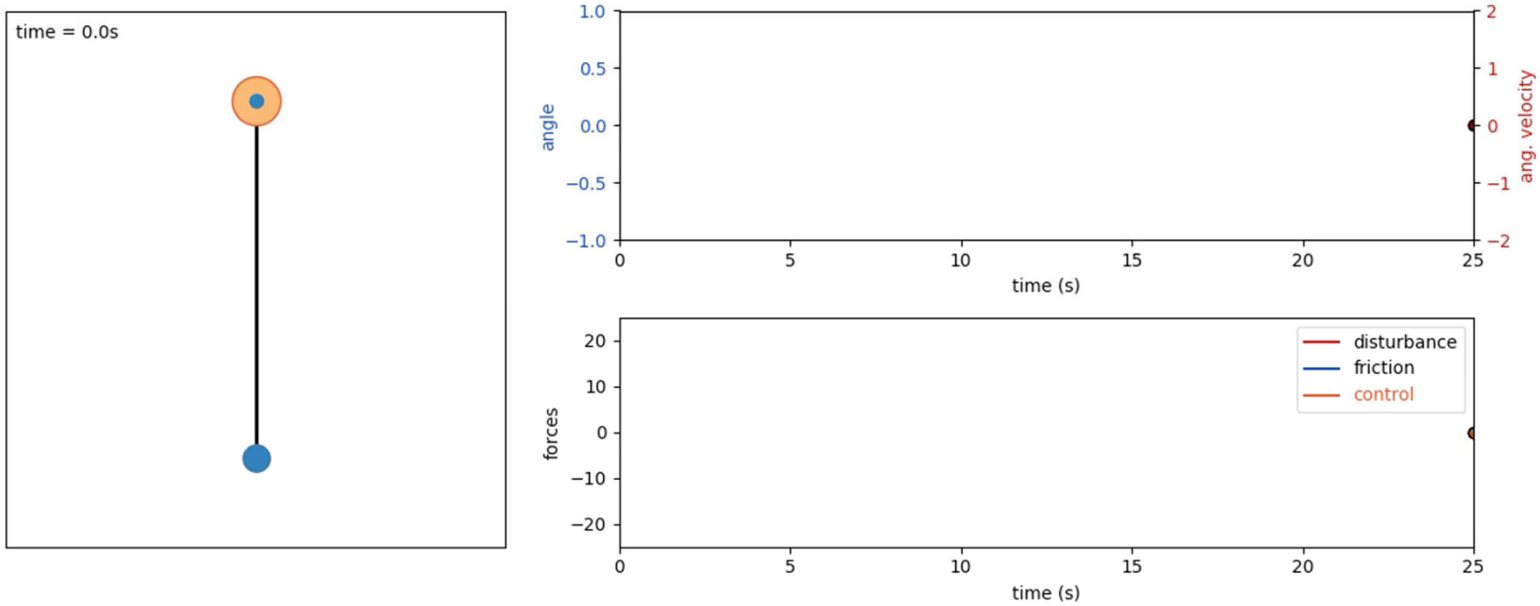


Roadmap to Low Inertia Frequency Control

- Performance Specification
- Limits of Virtual Inertia and Droop Control
- Dynamic Droop Control: iDroop

Control of Low Inertia Pendulum

Virtual Friction Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - r^{-1}\dot{\theta}$

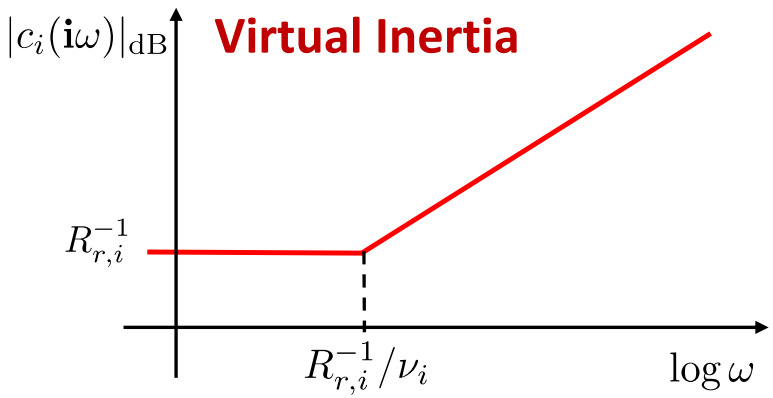


Pros: Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

Cons? Large steady-state effort in power systems

iDroop: Dynamic Droop Control

Instead of Virtual Inertia: $c_i : \left\{ x_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \right.$ $\quad c_i(s) = -(\nu_i s + R_{r,i}^{-1})$



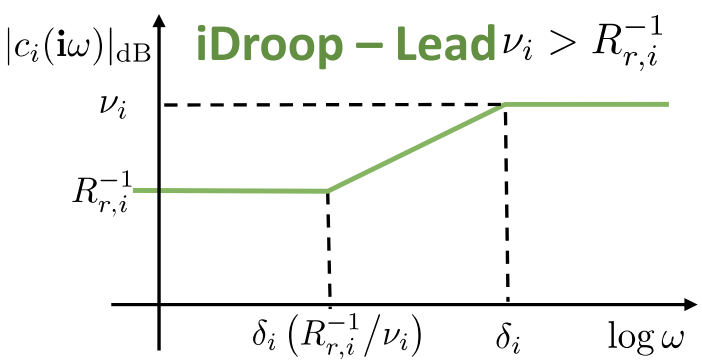
iDroop: Dynamic Droop Control

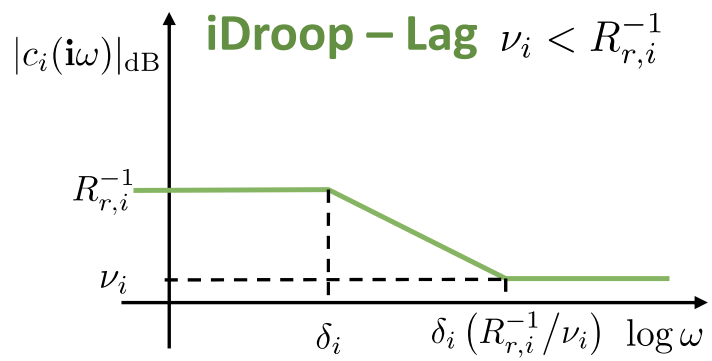
Instead of Virtual Inertia: $c_i : \left\{ x_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \right.$ $\quad c_i(s) = -(\nu_i s + R_{r,i}^{-1})$

We use

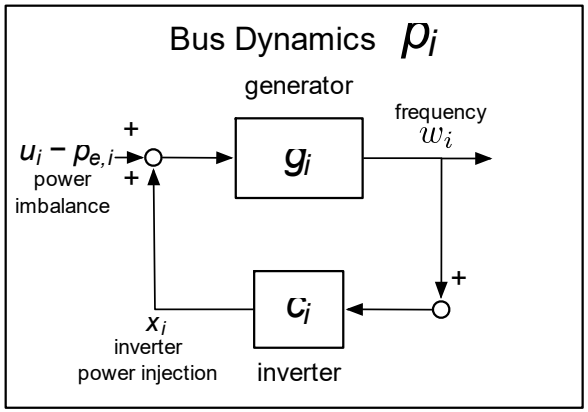
$$c_i : \left\{ \dot{x}_i \quad = -\delta_i (R_{r,i}^{-1} w_i + x_i) - \nu_i \dot{w}_i \right.$$

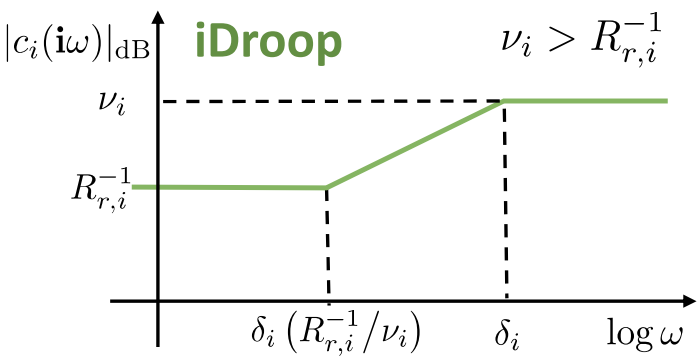
$$\boxed{c_i(s) = -\nu_i \frac{s + \delta_i (R_{r,i}^{-1}/\nu_i)}{s + \delta_i}}$$





Bus Dynamics /w iDroop





Inverter: $c_i : \; w_i \mapsto x_i$

iDroop Control:

$$c_i : \begin{cases} \dot{x}_i &= -\delta_i(R_{r,i}^{-1} w_i + x_i) - \nu_i \dot{w}_i \end{cases}$$

$$c_i(s) = -\nu_i \frac{s + \delta_i(R_{r,i}^{-1}/\nu_i)}{s + \delta_i}$$

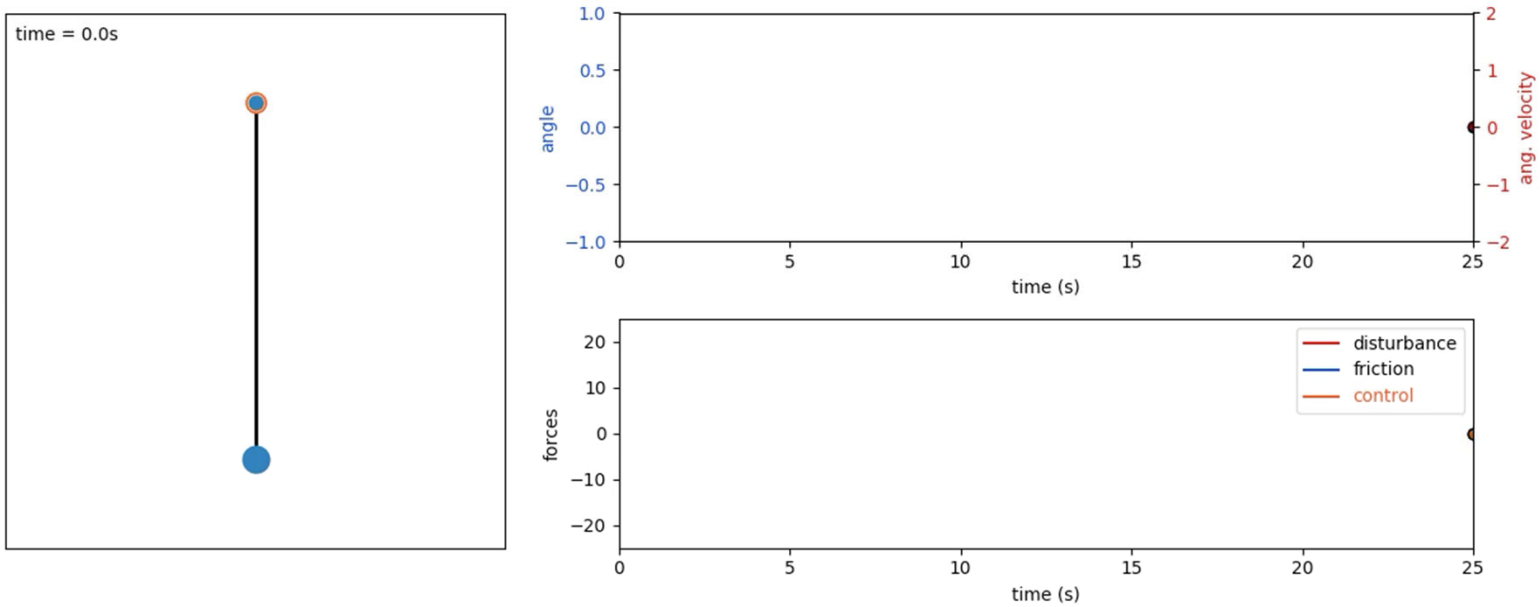
Closed-loop Bus Dynamics w/ iDroop:

$$p_i : \begin{cases} \dot{\theta}_i &= w_i \\ M_i \dot{w}_i &= -D_i w_i + p_i + (u_i - p_{e,i} + x_i) \\ \tau_i \dot{p}_i &= -p_i + R_{g,i}^{-1} w_i \\ \dot{x}_i &= -\delta_i(R_{r,i}^{-1} w_i + x_i) - \nu_i \dot{w}_i \end{cases}$$

Control of Low Inertia Pendulum

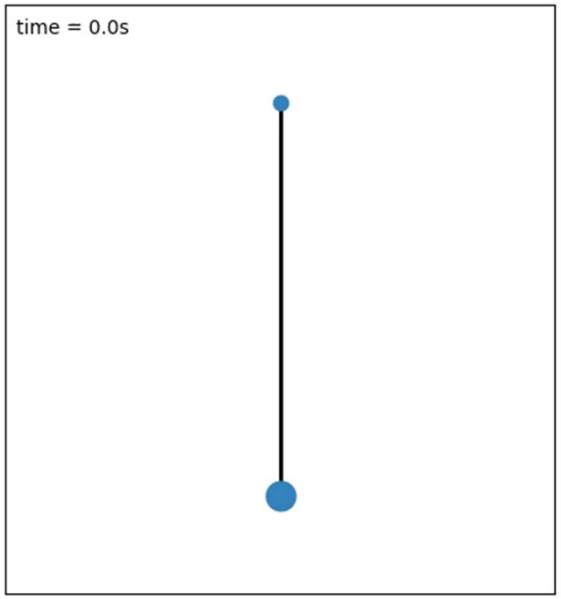
Dynamic Friction Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + x$

$$\dot{x} = -\delta(r_r^{-1}w + x) - \nu\dot{w}$$



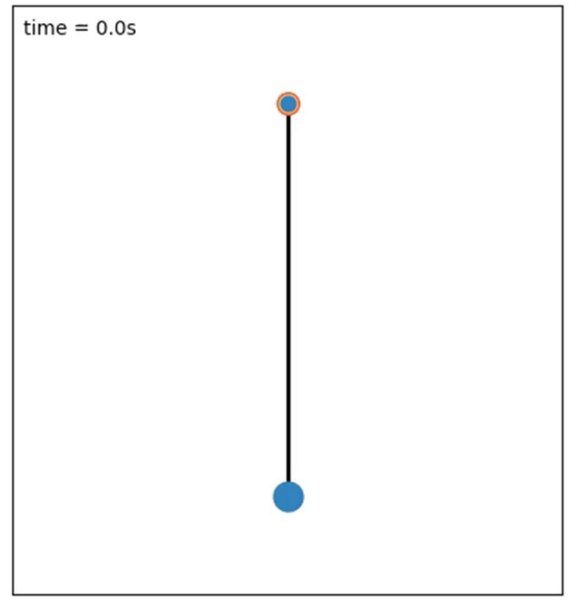
Control of Low Inertia Pendulum

No Control



$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f$$

Dynamic Friction Control

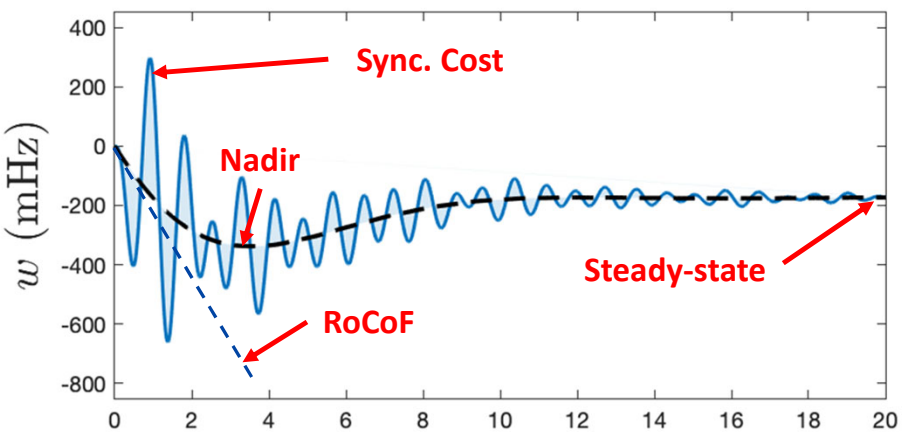


$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + x$$

$$\dot{x} = -\delta(r_r^{-1}w + x) - \nu\dot{w}$$

Performance Specification

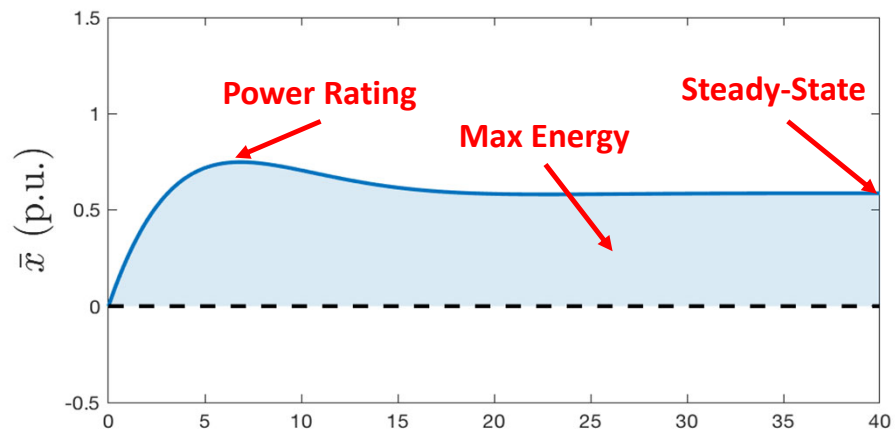
Frequency Response



System Freq. : $\bar{w}(t) = \dfrac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

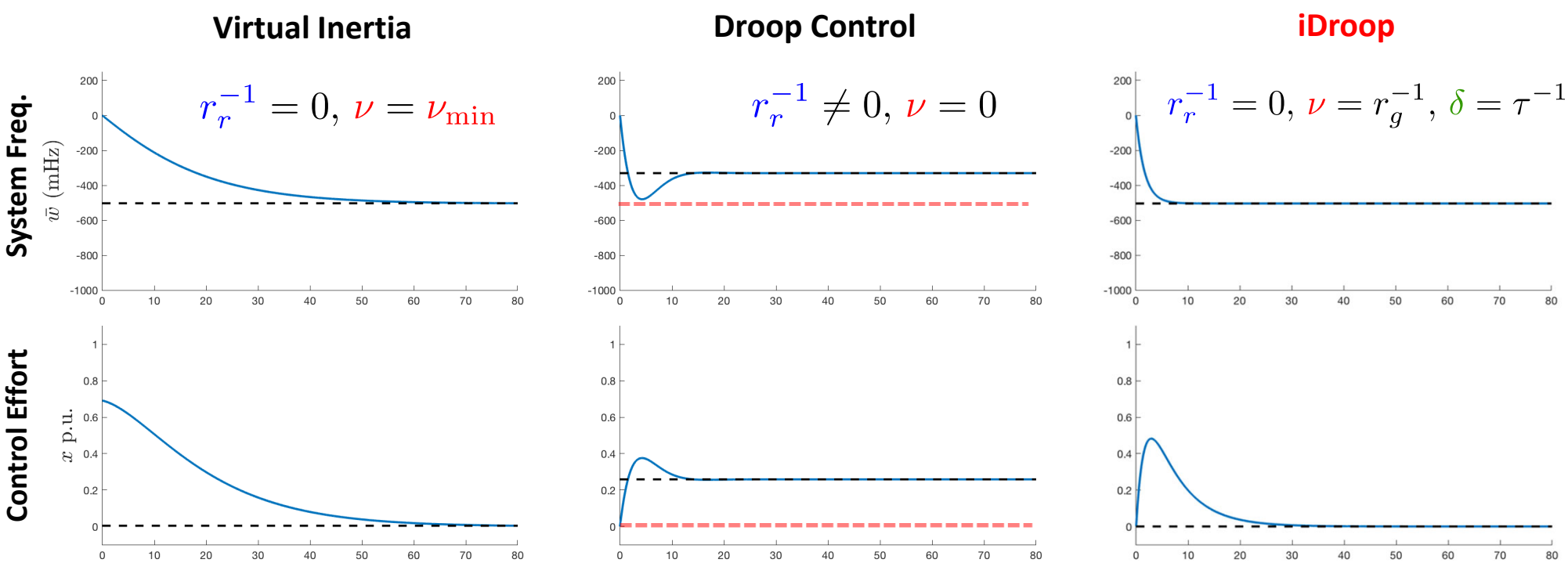
Injected Energy: $\dot{E}(t) = \bar{x}(t)$

Benchmark: Quantify **control ability** to eliminate overshoot in **Nadir**

Overshoot Elimination w/ iDroop

$$c_i : \dot{x}_i = -f_i(\delta(r_r^{-1} w_i + x_i) + \nu \dot{w}_i)$$

Whenever $\delta = \tau$ **and** $\nu = r_r^{-1} + r_g^{-1}$



Overshoot Elimination w/ iDroop

$$c_i : \ \dot{x}_i = -f_i(\delta(r_r^{-1} w_i + x_i) + \nu \dot{w}_i)$$

Whenever

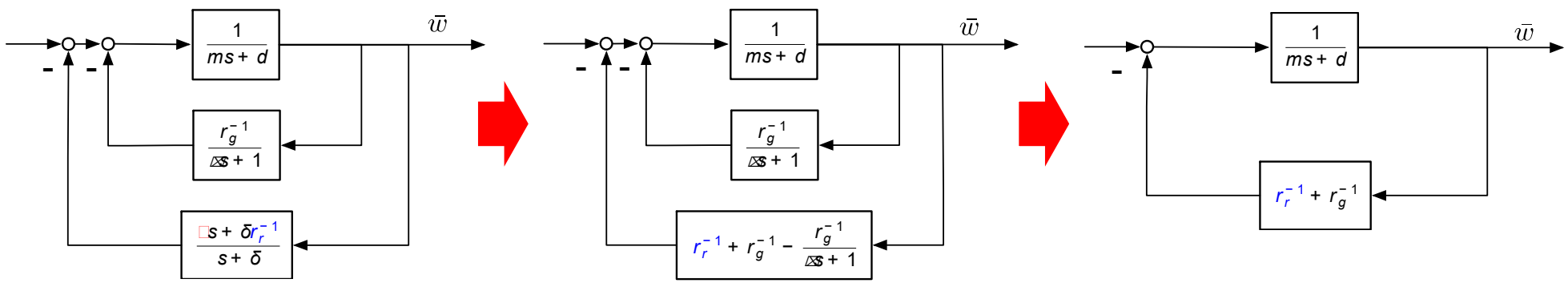
$$\delta = \tau$$

and

$$\nu = r_r^{-1} + r_g^{-1}$$

$$\bar{w}(t) = \frac{\sum_i u_{0,i}}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}} \left(1 - e^{-\frac{d + r_g^{-1} + r_r^{-1}}{m} t}\right)$$

first order step response

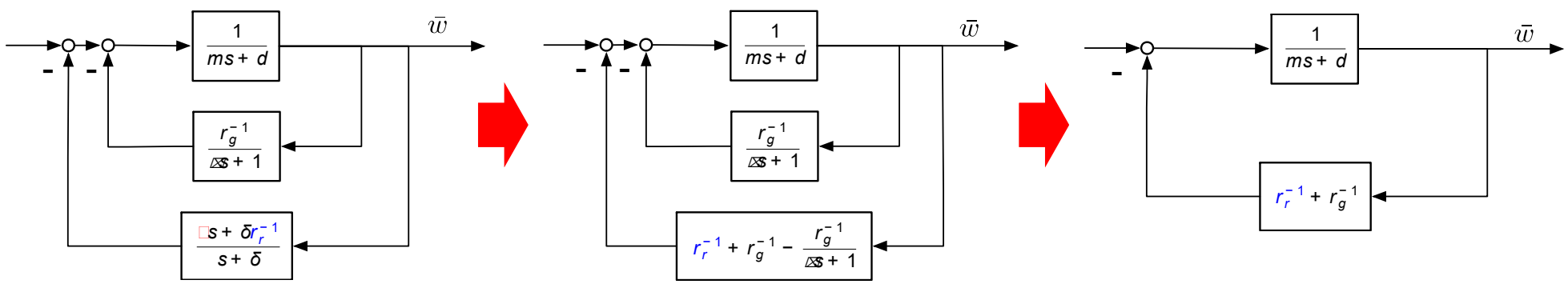


System Frequency w/ iDroop

Nadir Cancellation

$$\left.\begin{aligned}\delta &= \tau^{-1}\\ \nu &= r_r^{-1} + r_g^{-1}\end{aligned}\right\} \Rightarrow \bar{w}(t) = \frac{\sum_i u_{0,i}}{\sum_i f_i}\frac{1}{d + r_g^{-1} + r_r^{-1}}\left(1 - e^{-\frac{d + r_g^{-1} + r_r^{-1}}{m}t}\right)$$

first order step response



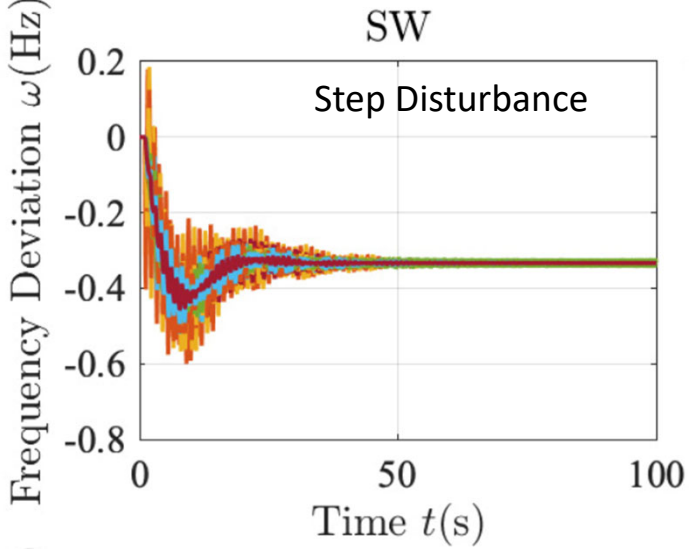
iDroop can reduce system frequency step-response to a first order system - no Nadir!

Example: Icelandic Power Grid

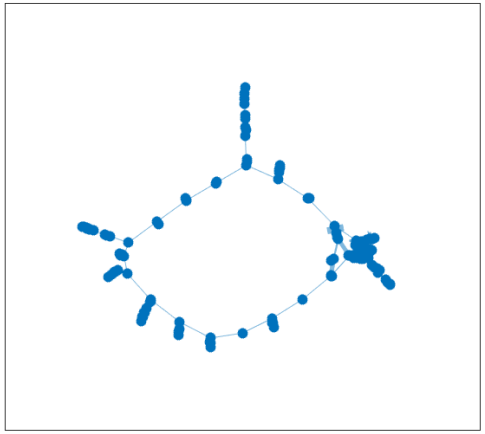
- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

Parameters	Symbol	Value
generator inertia coefficient	m	$0.0192\,\mathrm{s}^2\,\mathrm{rad}^{-1}$
generator damping coefficient	d	$0.01\,\mathrm{s}\,\mathrm{rad}^{-1}$
turbine time constant	τ	$3.02\,\mathrm{s}$
turbine droop coefficient	r_g	$184.65\,\mathrm{s}\,\mathrm{rad}^{-1}$ for SW, $369.3\,\mathrm{s}\,\mathrm{rad}^{-1}$ for VI, DC, and iDroop
inverter droop coefficient	r_r	$369.3\,\mathrm{s}\,\mathrm{rad}^{-1}$

Base Case: No Inverter Control



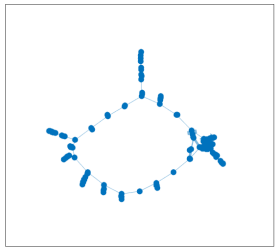
Icelandic Grid

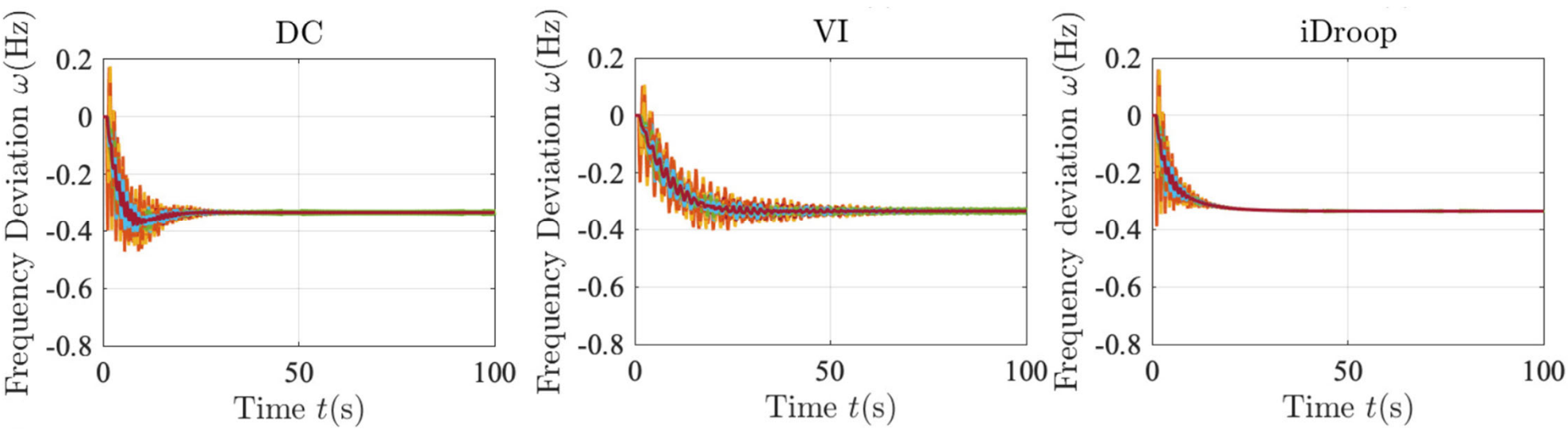


Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)
- Droop equally set for inverters in all cases
- Virtual inertia tuned for **critically damped response** $\boldsymbol{\nu = \nu_{min}}$
- **iDroop** tuned for Nadir elimination

Icelandic Grid





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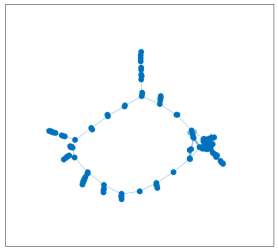
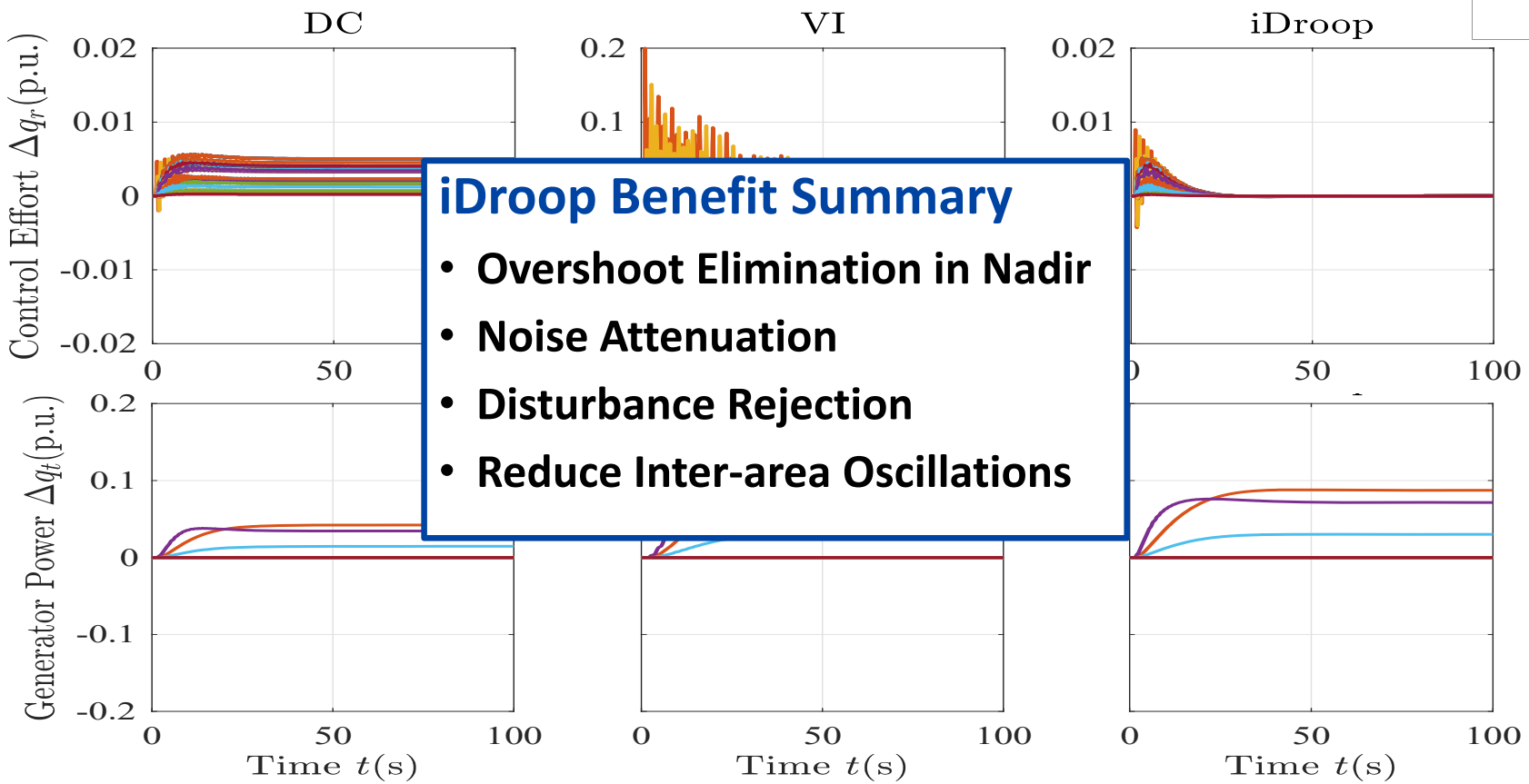
- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

Icelandic Grid





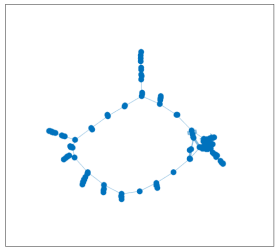
iDroop Benefit Summary

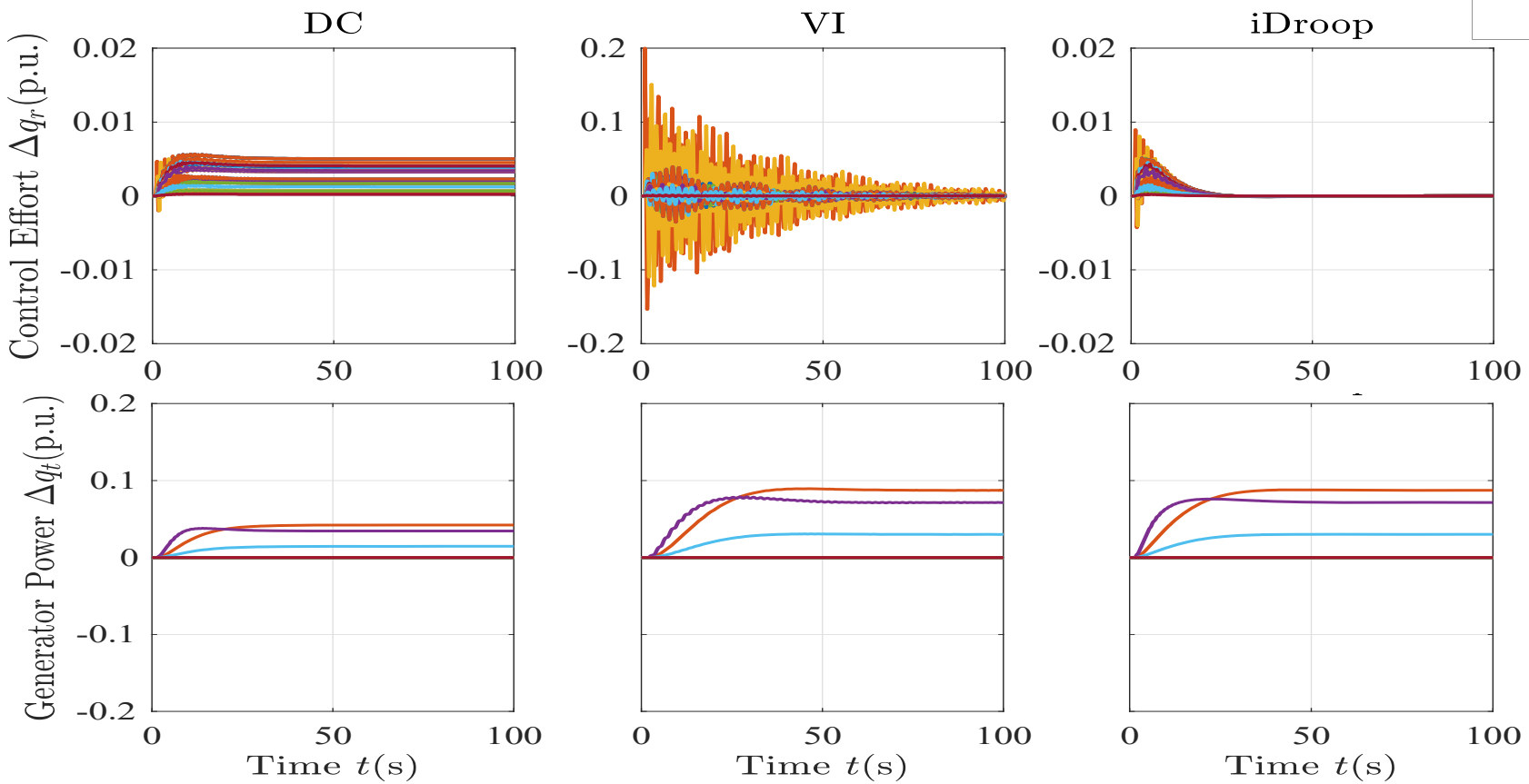
- **Overshoot Elimination in Nadir**
- **Noise Attenuation**
- **Disturbance Rejection**
- **Reduce Inter-area Oscillations**

Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

Icelandic Grid





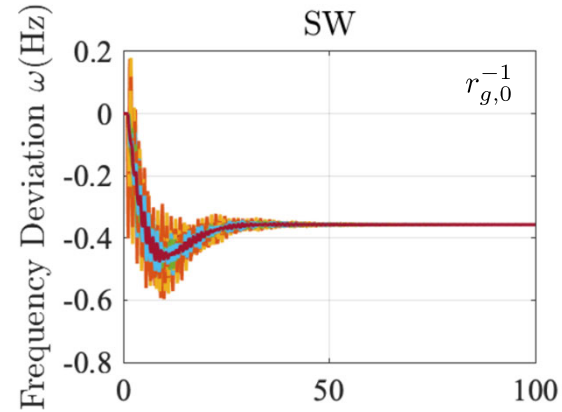
Step Disturbance - Frequency

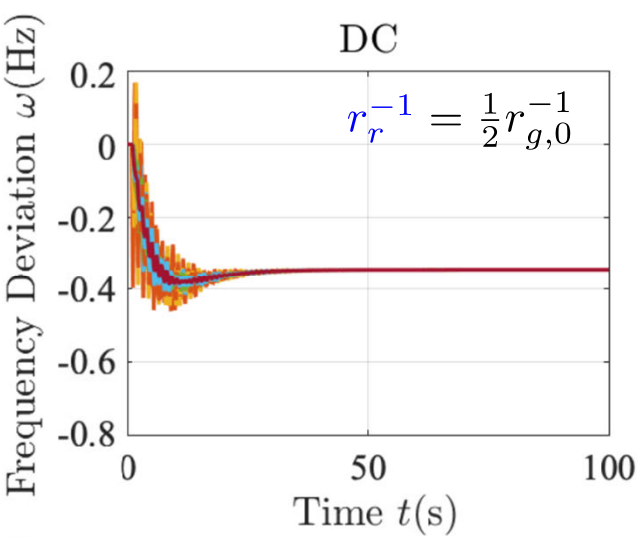
- Droop equally shared between gens. and inverters.
- Virtual inertia tuned for **critically damped response** $\boldsymbol{\nu = \nu_{min}}$
- **iDroop** tuned for Nadir elimination

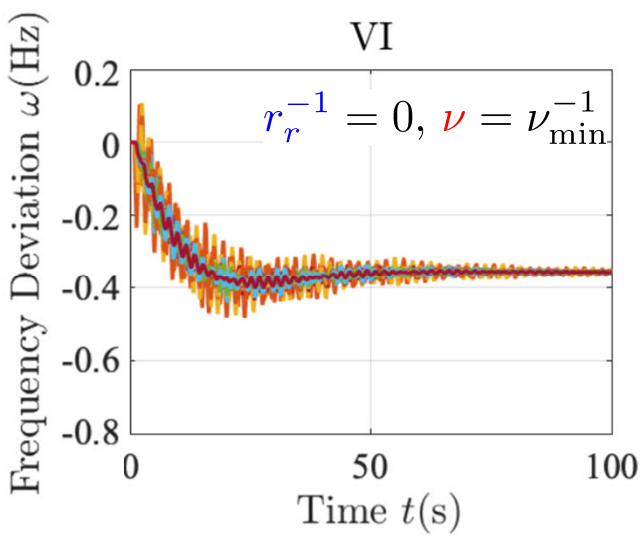
δ^*	0.331
ν^*	0.0054
ν_{VI}	0.035

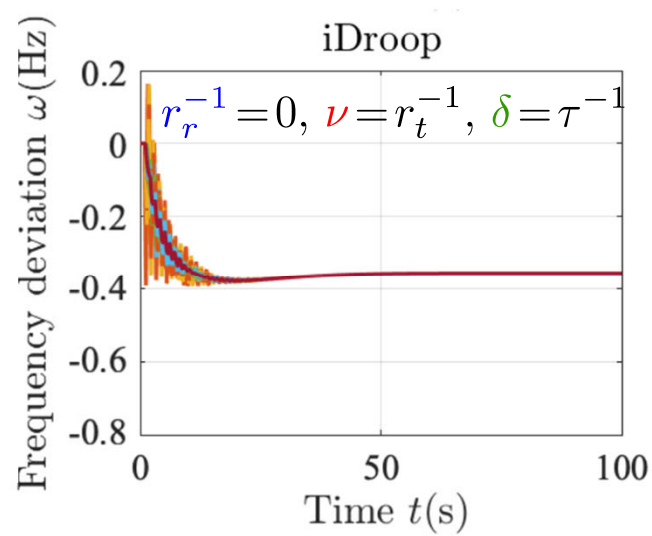
iDroop Tuning

$$\delta = \tau^{-1}$$
$$\nu = r_r^{-1} + r_g^{-1}$$









Step Disturbance – Control Effort

Inertia parameters of iDroop and VI are set to achieve zero overshoot.

δ^*	0.331
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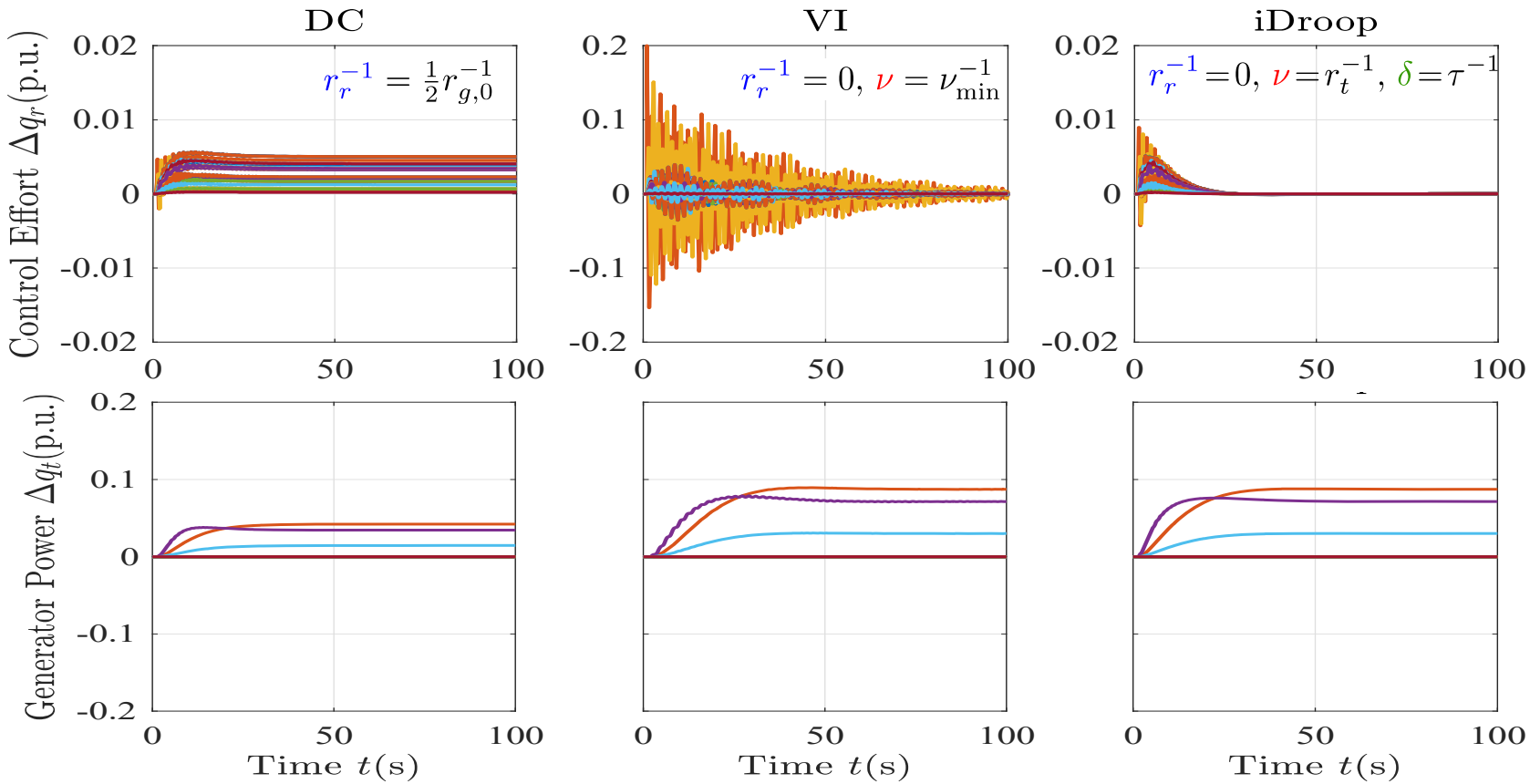
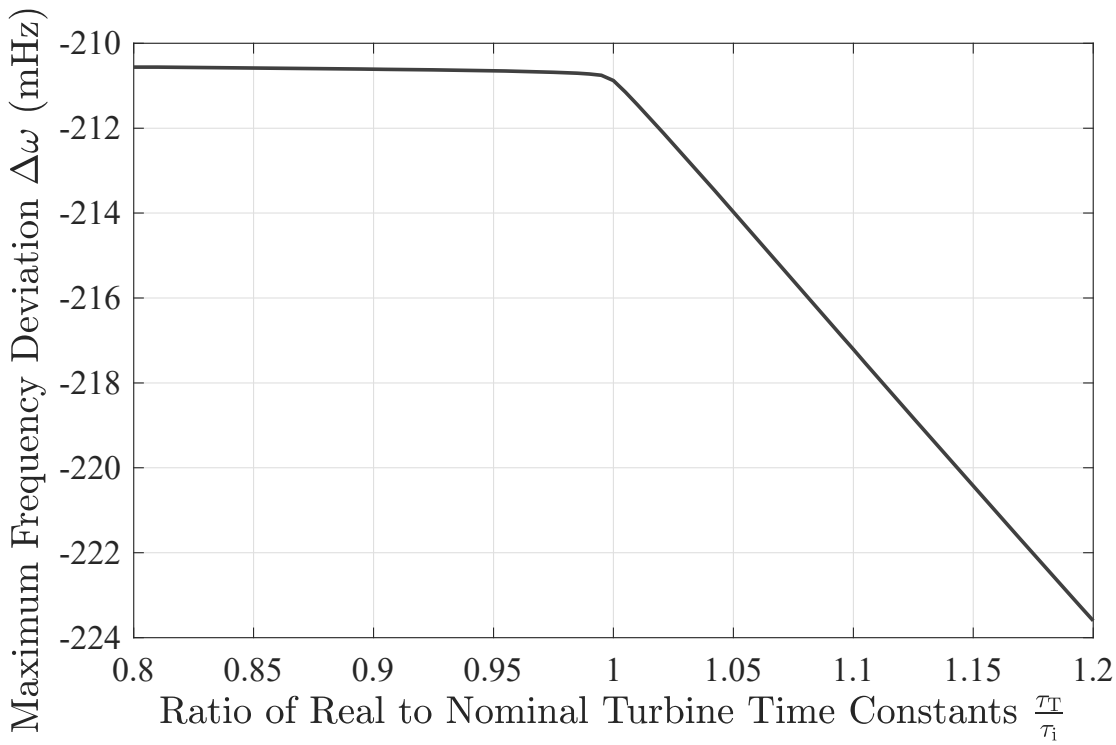
Zero Nadir Tuning

$\delta = \tau^{-1}$

$\nu = r_r^{-1} + r_g^{-1}$



Parameter Uncertainty



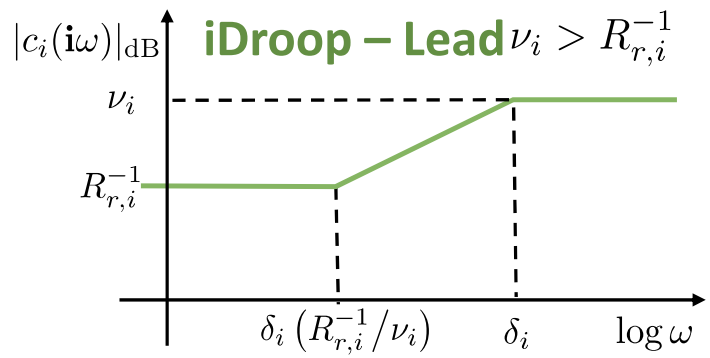
Other Benefits of iDroop

$$c_i : \left\{ \dot{x}_i \quad = -\delta_i (R_{r,i}^{-1} w_i + x_i) - \nu_i \dot{w}_i \right.$$

$$\boxed{c_i(s) = -\nu_i \frac{s + \delta_i (R_{r,i}^{-1}/\nu_i)}{s + \delta_i}}$$

Benefits Summary

- **Overshoot Elimination in Nadir**
- **Noise Attenuation**
- **Disturbance Rejection**
- **Reduce Inter-area Oscillations**



Thanks!

Related Publications:

- Paganini and M, “Global analysis of synchronization performance for power systems: bridging the theory-practice gap,” **IEEE TAC 2020**
- Jiang, Pates, M, “Dynamic Droop Control for Low Inertia Power Systems,” **IEEE TAC 2021**
- Jiang, Cohn, Vorobev, M “Dynamic Droop Approach for Storage-based Frequency Control,” **IEEE TPS, conditionally accepted (arXiv)**
- Min, Paganini, M, “Accurate Reduced Order Models for Coherent Synchronous Generators,” **L-CSS 2020**
- Y. Jiang, E. Cohn, P. Vorobev, and E. M, ”Storage-Based Frequency Shaping Control,” **L-CSS 2020**

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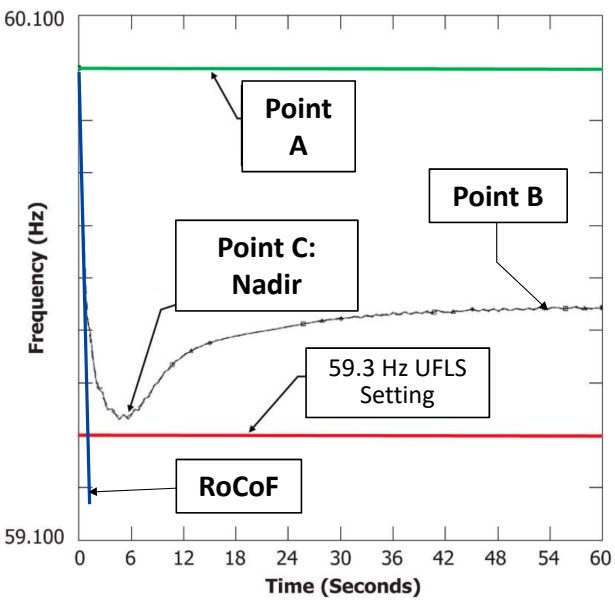
Backup Slides

Power Engineering Metrics

based on classical control theory…

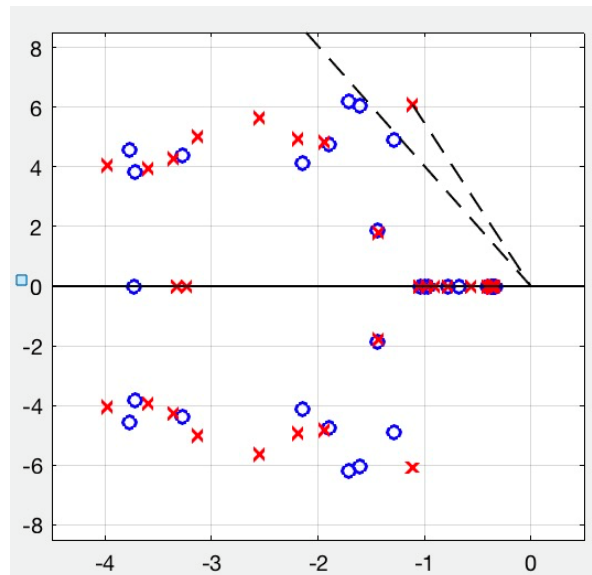
Step Response

Nadir, RoCoF, Steady-State



Eigenvalue Analysis [Verghese '89]

Dom. Eig, Damping Ratios, Part. Fact.

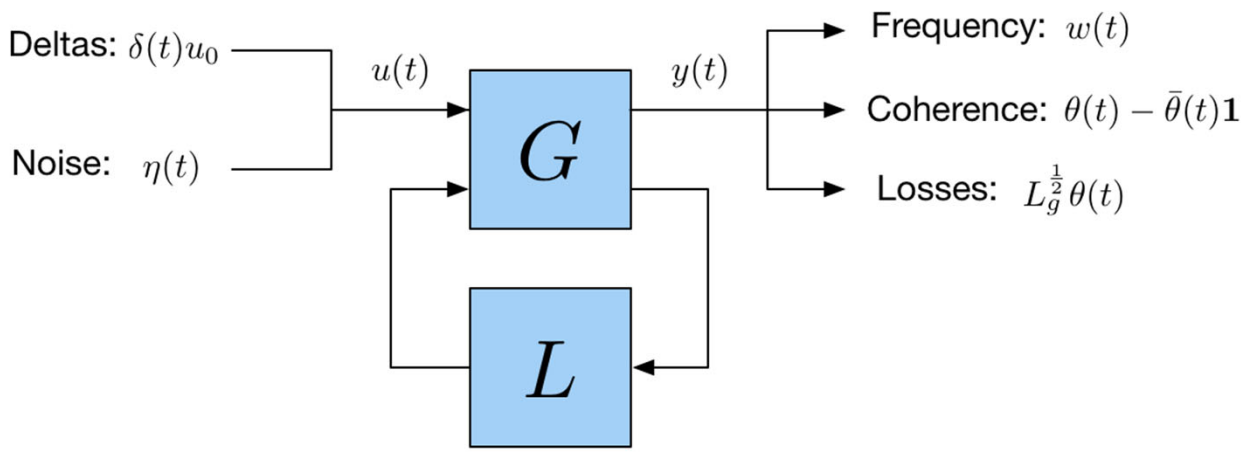


+ Domain specific, capture system degradation

- Relation with cause? Eigenvalue sensitivity? More than one "dominant" eig.?

System Theoretic Metrics

[Tegling… '15, Poolla… '15, Grunberg… '16, Simpson-Porco…'16, Wu et al '16, Adreasson '17, Coletta '17…]



$\mathcal{L}_\infty$-norm:

$$||y||_\infty := \sup_{t \geq 0} \max_i |y_i(t)|$$

$\mathcal{L}_2$-norm:

$$||y||_2 := \left(\int_0^\infty y(t)^T y(t) dt \right)^{\frac{1}{2}}$$

+ Close form solutions, qualitative analysis, computational methods
- Restrictive assumptions, not direct connection with RoCoF, Nadir, step disturbances

Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and assume

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

Swing Equations + Turbine

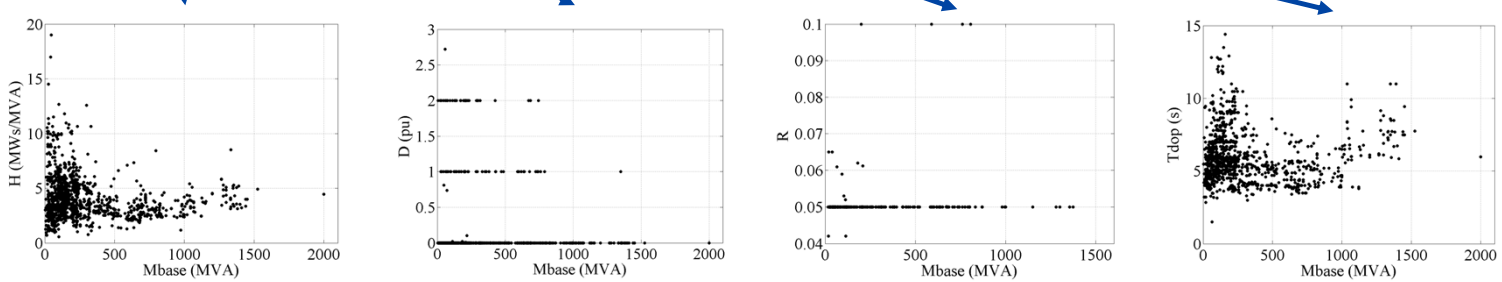
$$g_0(s) = \frac{\tau s + 1}{m\tau s^2 + (m + d\tau)s + d + r^{-1}}$$

Virtual Inertia

$$c_0(s) = -(\nu s + r_r^{-1})$$

$$M_i = f_i m, \quad D_i = f_i d, \quad R_{g,i} = \frac{1}{f_i} r_g, \quad \tau_i = \tau$$

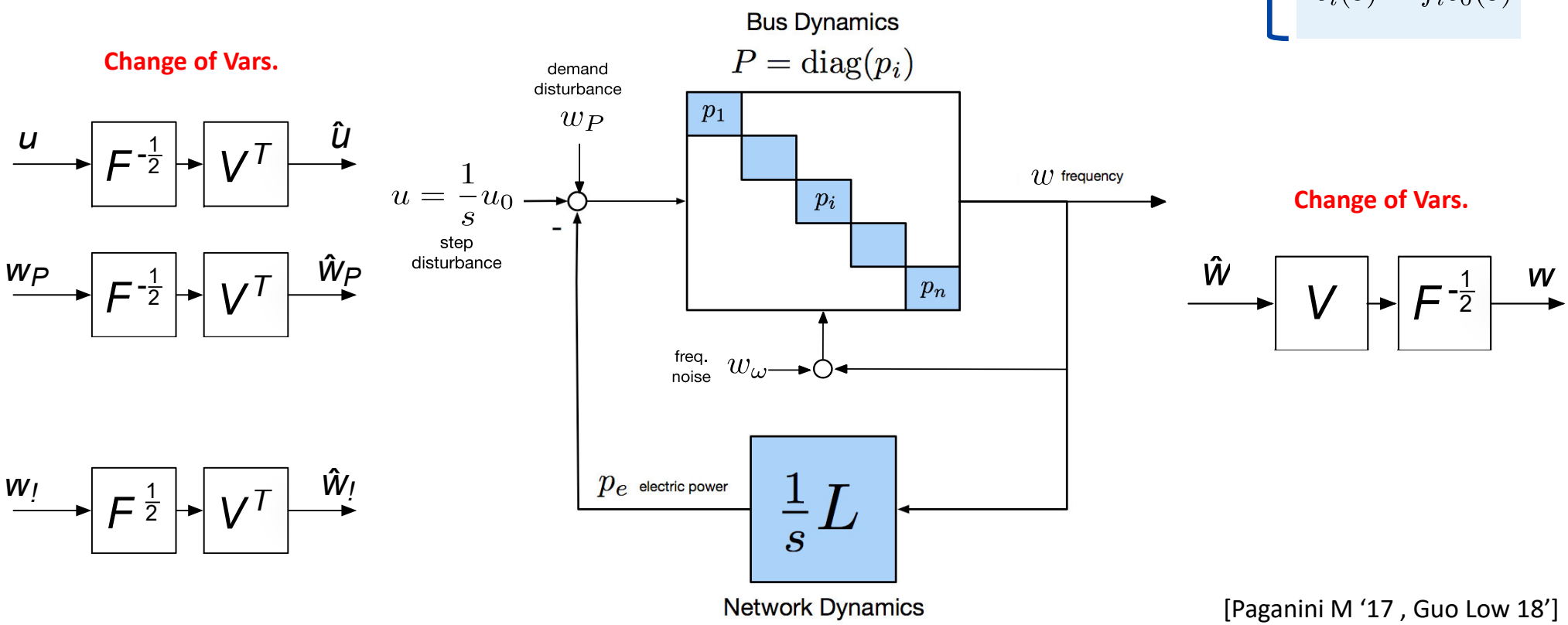
$$\nu_i = f_i \nu \quad R_{r,i} = \frac{1}{f_i} r_r$$



Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

$$\begin{cases} g_i(s) = \frac{1}{f_i} g_0(s) \\ c_i(s) = f_i c_0(s) \end{cases}$$



[Paganini M '17 , Guo Low 18']

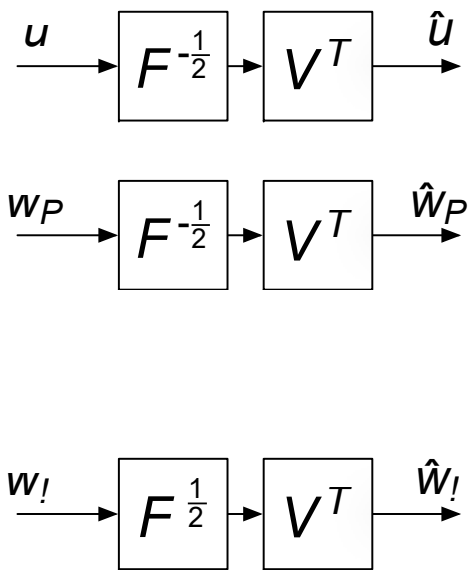
Modal Decomposition for Multi-Rated Machines

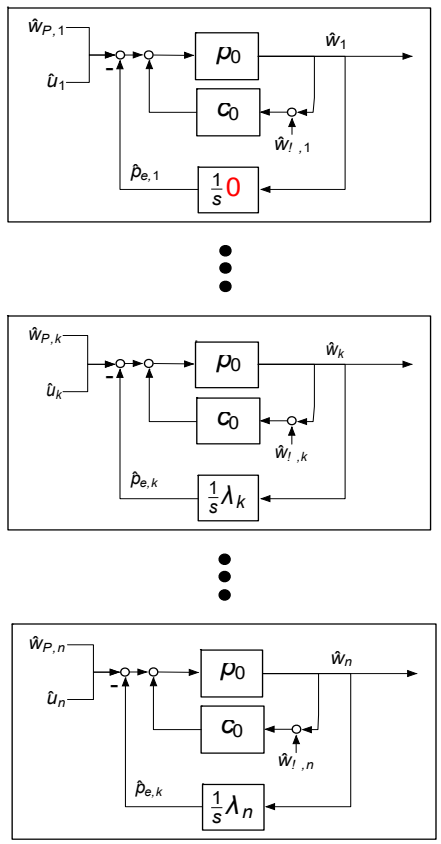
Assumption: Let f_i the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

Change of Vars.

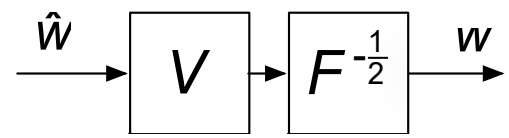




System Frequency

$$\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$$

Change of Vars.



Sync Error

$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Eigenvalues of: $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

$$0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$$

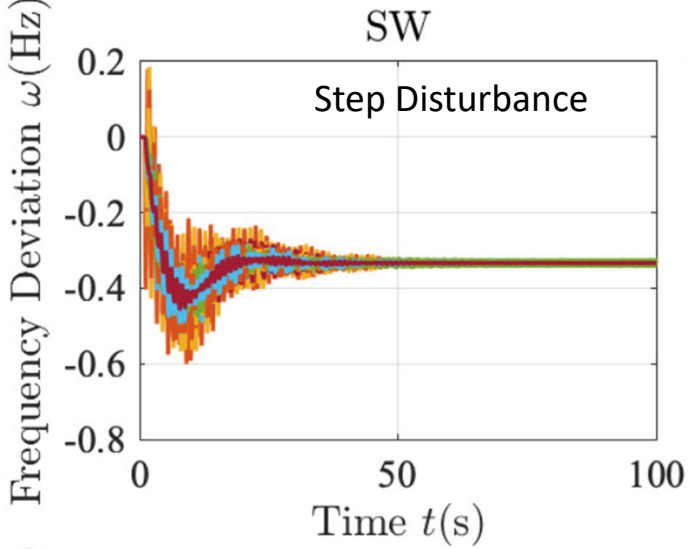
[Paganini M '17 , Guo Low 18']

Example: Icelandic Power Grid

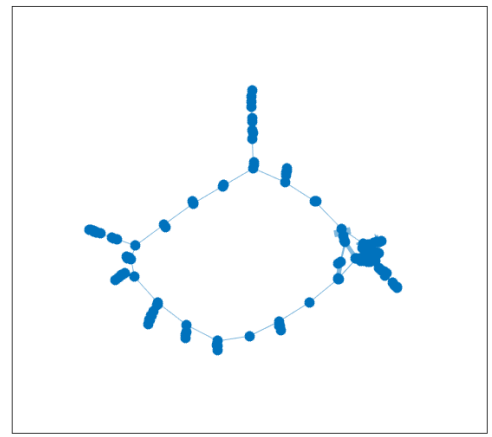
- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

Parameters	Symbol	Value
generator inertia coefficient	m	$0.0192\,\mathrm{s}^2\,\mathrm{rad}^{-1}$
generator damping coefficient	d	$0.01\,\mathrm{s}\,\mathrm{rad}^{-1}$
turbine time constant	τ	$3.02\,\mathrm{s}$
turbine droop coefficient	r_g	$184.65\,\mathrm{s}\,\mathrm{rad}^{-1}$ for SW, $369.3\,\mathrm{s}\,\mathrm{rad}^{-1}$ for VI, DC, and iDroop
inverter droop coefficient	r_r	$369.3\,\mathrm{s}\,\mathrm{rad}^{-1}$

Base Case: No Inverter Control



Icelandic Grid



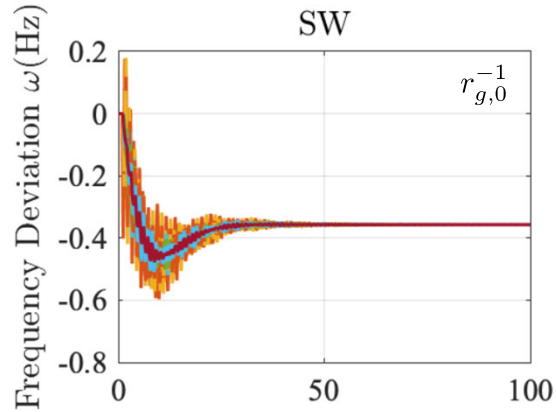
Step Disturbance - Frequency

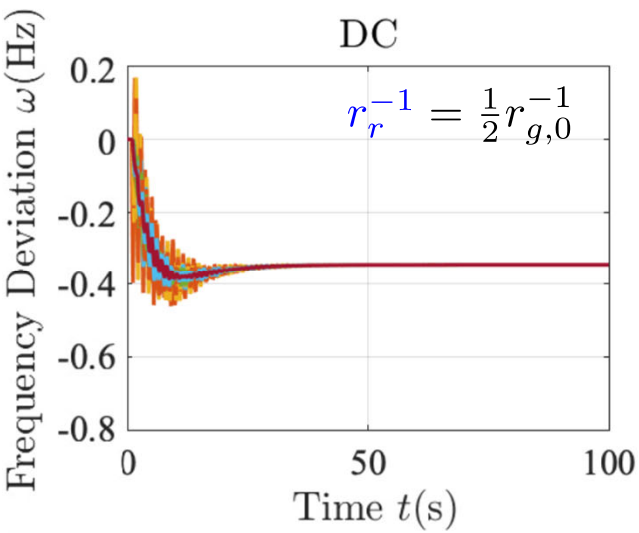
- Droop equally shared between gens. and inverters.
- Virtual inertia tuned for **critically damped response** $\boldsymbol{\nu = \nu_{min}}$
- **iDroop** tuned for Nadir elimination

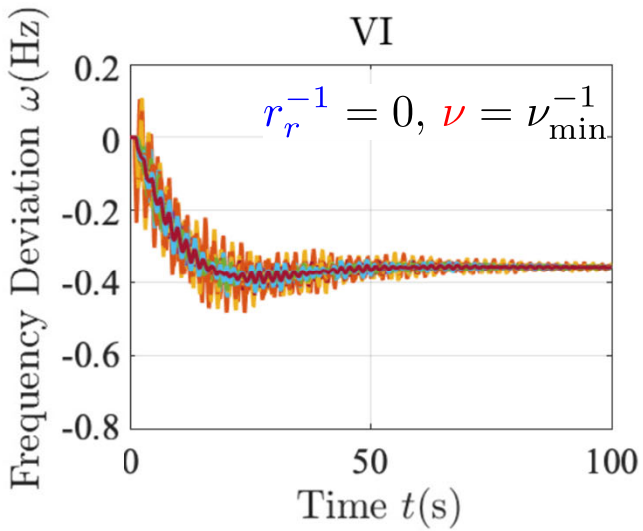
δ^*	0.331
ν^*	0.0054
ν_{VI}	0.035

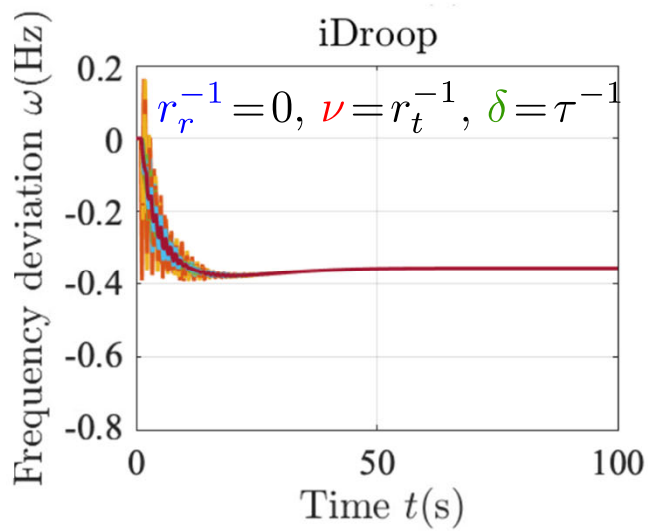
iDroop Tuning

$$\delta = \tau^{-1}$$
$$\nu = r_r^{-1} + r_g^{-1}$$









Step Disturbance – Control Effort

Inertia parameters of iDroop and VI are set to achieve zero overshoot.

δ^*	0.331
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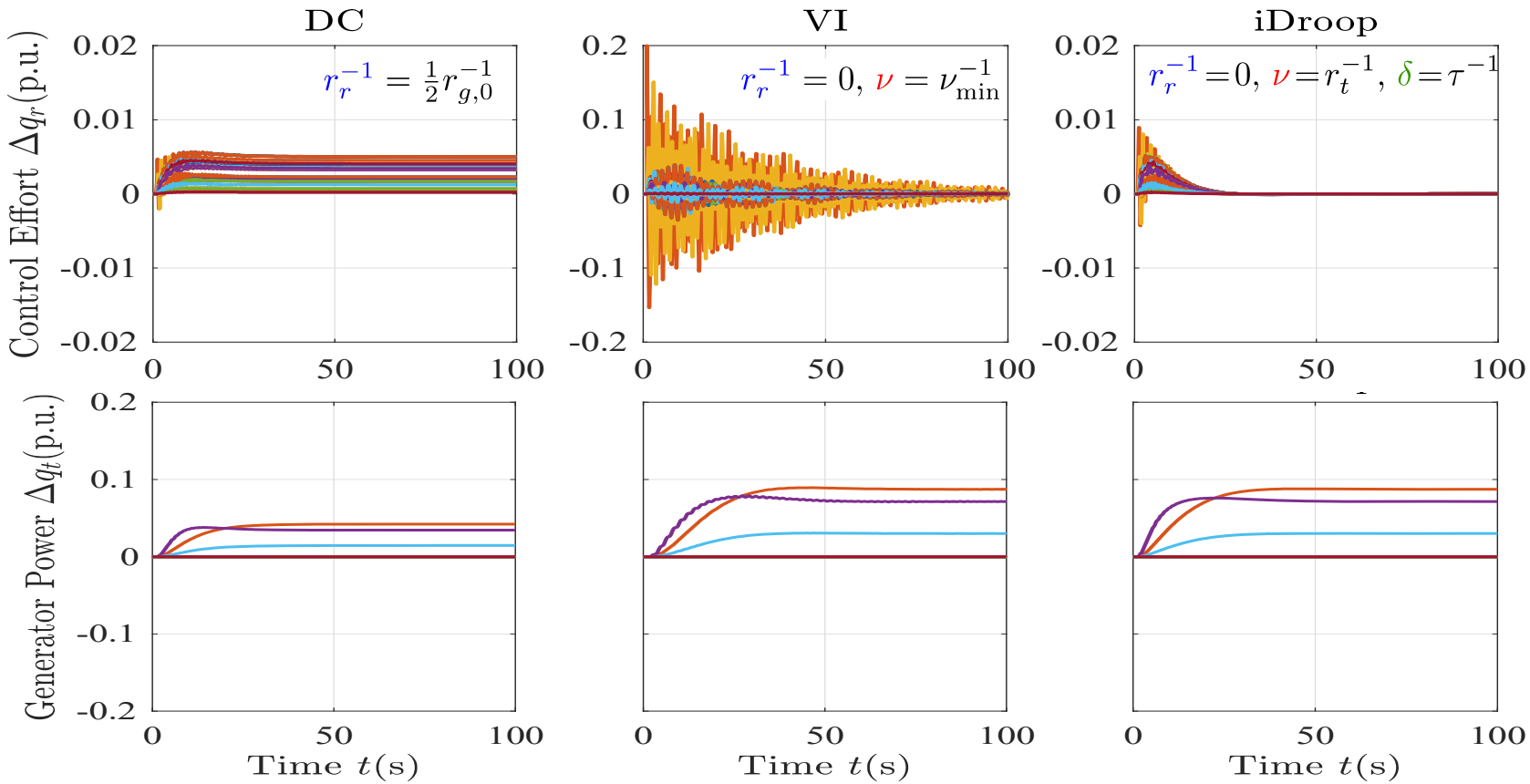
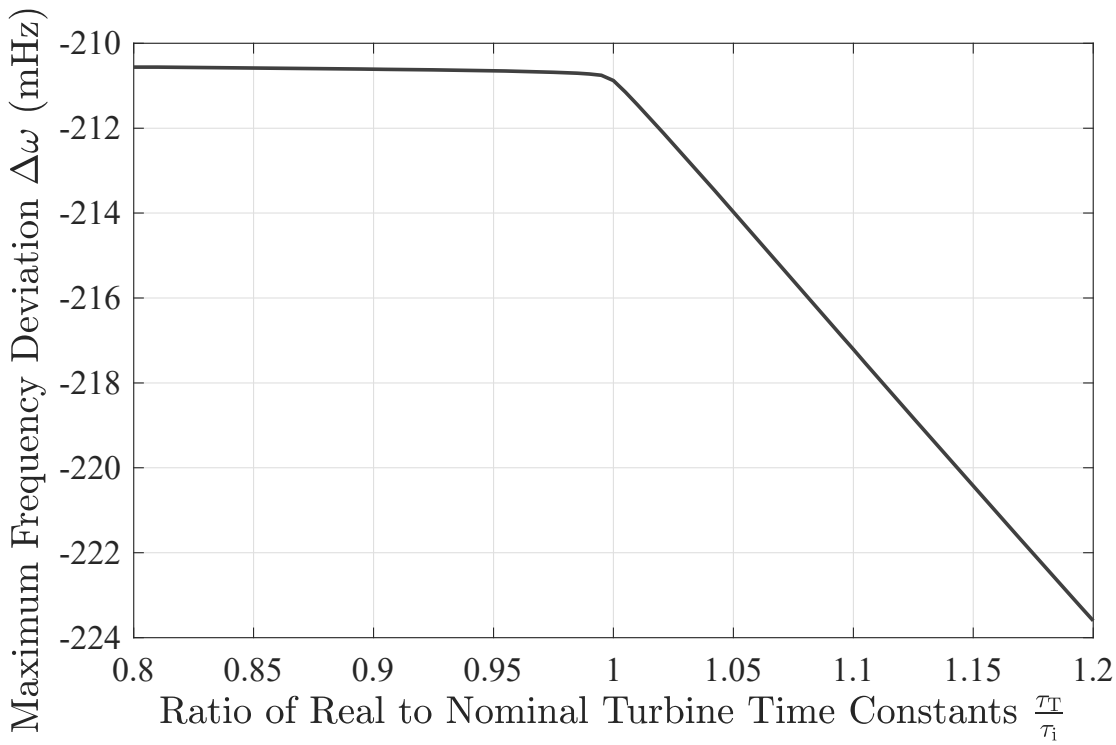
ν_{VI}	0.035

Zero Nadir Tuning

$$\delta = \tau^{-1}$$
$$\nu = r_r^{-1} + r_g^{-1}$$



Parameter Uncertainty



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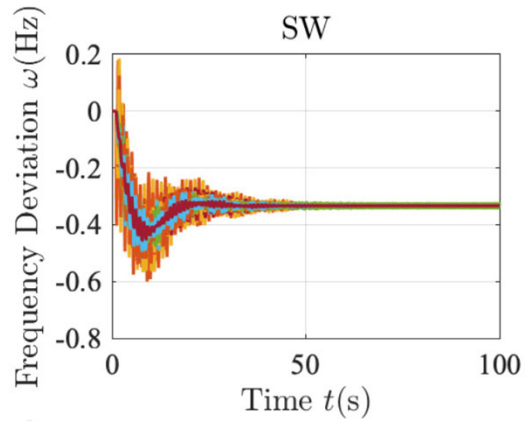

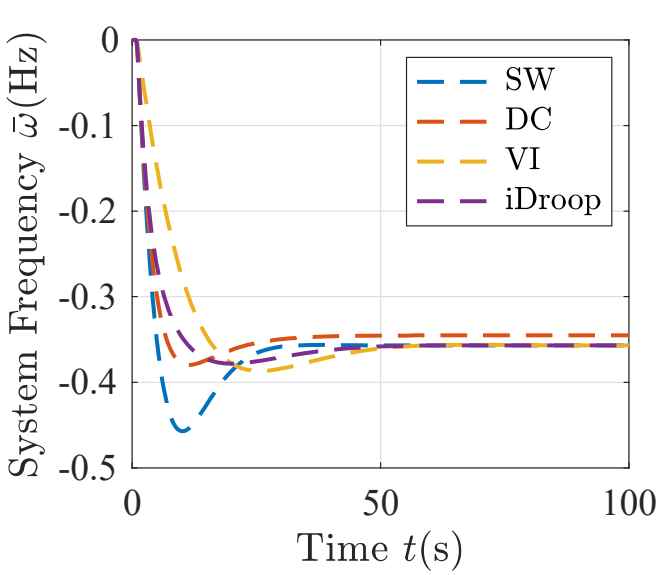

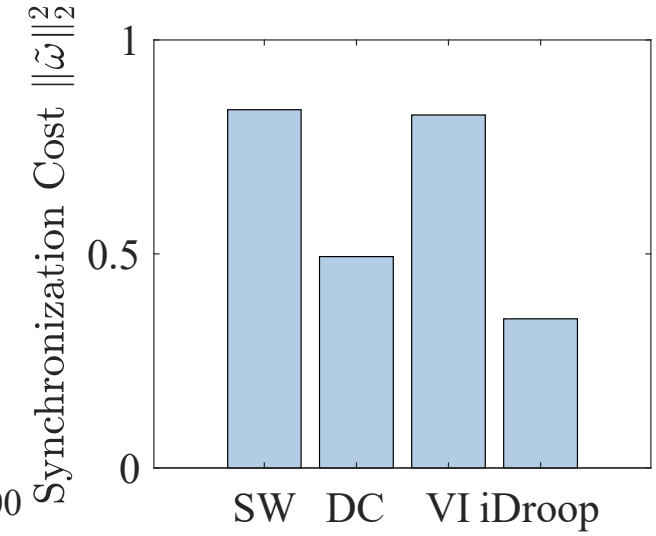

Step Disturbance

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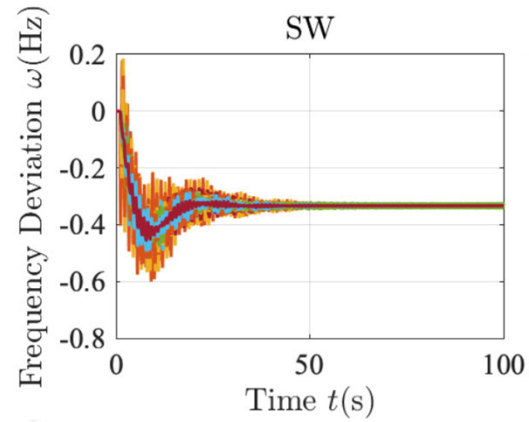
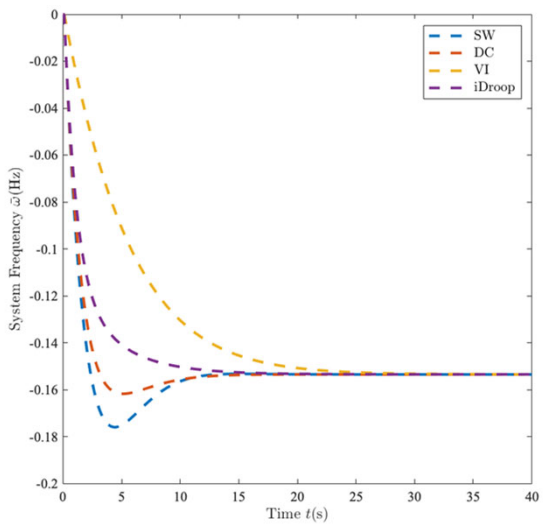
ν_{VI}	0.035

δ^*	0.331
ν^*	0.0108
ν_{VI}	0.070



System Frequency



Synchronization Cost