Embracing Low Inertia for Power System Frequency Control
A Dynamic Droop Approach

Enrique Mallada

4th Grid Science Winter School and Conference
January 14, 2021
Acknowledgements
Dynamic Degradation

In the United States:

“While the three [contiguous] U.S. interconnections currently exhibit adequate frequency response performance above their interconnection frequency response obligations, there has been a significant decline in the frequency response performance of the Western and Eastern Interconnections,” FERC said.

[FERC, Nov. 16]
Dynamic Degradation

In the United States:

FERC, Nov. 16

“While the frequency response of renewable generators is critical for power systems to remain stable and reliable, there has been a significant decline in the frequency response performance of the Western and Eastern Interconnections,” FERC said.

[FERC, Nov. 16]
Inverter-based Control

Challenges

• Measurements with noise and delays
• Stability + robustness (plug & play)
• Lack of incentives

Current approach: Use inverter-based control to mimic generators response
Inverter-based Control

Challenges
• Measurements with noise and delays
• Stability + robustness (plug & play)
• Lack of incentives

Current approach: Use inverter-based control to mimic generators response
Inverter-based Control

Challenges
• Measurements with noise and delays
• Stability + robustness (plug & play)
• Lack of incentives

Our approach: Design and tune of controllers rooted on sound control principles

Dynamic Droop Control (iDroop)

Design Objectives:
• Exploit power electronics capabilities
• Improve Dynamic Performance
• Minimize control effort
• Stability and Robustness
Merits and Trade-offs of Inertia

\[ m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f \]
Merits and Trade-offs of Inertia

\[ \ddot{\theta} = -\frac{d}{m} \dot{\theta} - g \sin \theta + \frac{f}{m} \]
Merits and Trade-offs of Inertia

\[ \ddot{\theta} = -\frac{d}{m} \dot{\theta} - g \sin \theta + \frac{f}{m} \]

**Pros:** Provides natural disturbance rejection

**Cons:** Hard to regain steady-state
Merits and Trade-offs of Low Inertia

\[ \ddot{\theta} = -\frac{d}{m} \dot{\theta} - g \sin \theta + \frac{f}{m} \]

**Cons:** Susceptible to disturbances

**Pros:** Regains steady-state faster
Roadmap to Low Inertia Frequency Control

• Performance Specification

• Limits of Virtual Inertia and Droop Control

• Dynamic Droop Control: iDroop
Global analysis of synchronization performance for power systems: bridging the theory-practice gap
Fernando Paganini, Fellow, IEEE, and Enrique Mallada, Senior Member, IEEE
IEEE Transactions on Automatic Control, July 2020

Dynamic Droop Control in Low-inertia Power Systems
Yan Jiang, Richard Pates, and Enrique Mallada
IEEE Transactions on Automatic Control, October 2020

Dynamic Droop Approach for Storage-based Frequency Control
Yan Jiang*, Eliza Cohn*, Petr Vorobei†, and Enrique Mallada*
Roadmap to Low Inertia Frequency Control

• Performance Specification

• Limits of Virtual Inertia and Droop Control

• Dynamic Droop Control: iDroop
Power System Performance

 Depends on several factors: generators, network, disturbance

good performance metric must identify the source of the degradation!
<table>
<thead>
<tr>
<th>Performance Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Response</td>
</tr>
</tbody>
</table>
Decomposition of Step Response

System Frequency

\[ \bar{w}(t) = \frac{\sum_{i=1}^{n} M_i \omega_i(t)}{\sum_{i=1}^{n} M_i} \]

[Synchronization Error]

\[ \tilde{\omega}_i(t) = \omega_i(t) - \bar{w}(t) \]

[Center of Inertia: Kundur '94]
**Step Disturbance Performance**

**System Frequency**

\[
\dot{\bar{w}}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}
\]

- Nadir
- RoCoF
- Steady-state

---

**Deviation from Mean**

\[
\bar{w}_i(t) = w_i(t) - \bar{w}(t)
\]

---

**Nadir**

\[
||\bar{w}||_{\infty} := \sup_{t \geq 0} |\bar{w}(t)|
\]

**RoCoF**

\[
||\dot{\bar{w}}||_{\infty} := \sup_{t \geq 0} |\dot{\bar{w}}(t)|
\]

**Steady-state**

\[
||\dot{w}||_{\infty} := \sup_{t \geq 0} |\dot{w}(t)|
\]

**Synchronization Cost**

\[
\|\bar{w}\|_2 = \left( \int_{0}^{+\infty} \sum_{i=1}^{n} \bar{w}_i^2(t) dt \right)^{\frac{1}{2}}
\]
Performance Specification

Frequency Response

Control Effort

System Freq. : \( \bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i} \)

Sync. Error : \( \tilde{w}_i(t) = w_i(t) - \bar{w}(t) \)
Control Effort

Inverter Power

\[ \bar{x}(t) = \sum_i x_i(t) \]

- Power Rating
- Steady-State Effort
  \[ |\bar{x}(\infty)| = \sum_i R_{r,i}^{-1} |\bar{w}(\infty)| \]
- Injected Energy
  \[ \dot{E}(t) = \bar{x}(t) \]

Power Rating

\[ \|\bar{x}\|_\infty := \sup_{t \geq 0} |\bar{x}(t)| \]

Max Energy

\[ \|E\|_\infty := \sup_{t \geq 0} |E(t)| \]

Steady-State Effort

\[ |\bar{x}(\infty)| = \sum_i R_{r,i}^{-1} |\bar{w}(\infty)| \]
Performance Specification

Frequency Response

\[ \bar{\omega}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i} \]

Sync. Error: \[ \tilde{w}_i(t) = w_i(t) - \bar{\omega}(t) \]

Benchmark: Quantify control ability to eliminate overshoot in the Nadir

Control Effort

Injected Power: \[ \bar{x}(t) = \sum_i x_i(t) \]

Injected Energy: \[ \dot{E}(t) = \bar{x}(t) \]
Roadmap to Low Inertia Frequency Control

• Performance Specification

• Limits of Virtual Inertia and Droop Control

• Controller Design: iDroop
Power Network Model

Bus Dynamics
\[ P = \text{diag}(p_i) \]

Step: \[ u = \frac{1}{s} u_0 \]

Step disturbance

Network Dynamics
\[ \frac{1}{s} L \]

Laplacian Matrix
\[ L_{ij} = \begin{cases} -B_{ij} & \text{if } ij \in E \\ \sum_k B_{ik} & \text{if } i = j \\ 0 & \text{o.w.} \end{cases} \]

[Bergen Hill ‘81]
Bus Dynamics

**Model:** Swing Equations + Turbine

\[
\begin{align*}
\dot{\theta}_i &= w_i \\
M_i \dot{w}_i &= -D_i w_i + q_i + (u_i - p_{e,i} + x_i) \\
\tau_i \dot{q}_i &= -R_{g,i}^{-1} w_i - q_i 
\end{align*}
\]

**Generator:** \( g_i : (u_i - p_{e,i} + x_i) \mapsto w_i \)
**Bus Dynamics**

**Generator:** \( g_i : (u_i - p_{e,i} + x_i) \rightarrow w_i \)

**Model: Swing Equations + Turbine**

\[
\begin{align*}
\dot{\theta}_i &= \omega_i \\
M_i \dot{\omega}_i &= -D_i \omega_i + q_i + (u_i - p_{e,i} + x_i) \\
\tau_i \dot{q}_i &= -R_{g,i}^{-1} \omega_i - q_i \\
g_i(s) &= \frac{\tau_i s + 1}{M_i \tau_i s^2 + (M_i + D_i \tau_i) s + D_i + r_{g,i}^{-1}}
\end{align*}
\]
Bus Dynamics

\[ \dot{\theta}_i = w_i \]

\[ p_i : \begin{cases} 
(M_i + \nu_i) \dot{w}_i = -(D_i + R_{r,i}^{-1}) w_i + q_i + (u_i - p_{e,i}) \\
\tau_{r,i} \dot{q}_i = -q_i - R_{g,i}^{-1} w_i
\end{cases} \]

**Inverter:** \( C_i : w_i \mapsto x_i \)

**Droop Control and Virtual Inertia:**

\[ c_i : \begin{cases} 
\dot{x}_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \\
c_i(s) = -(\nu_i s + R_{r,i}^{-1})
\end{cases} \]
Control of Low Inertia Pendulum

\[ m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + u \]

**Cons:** Susceptible to disturbances

**Pros:** Regains steady-state faster
Control of Low Inertia Pendulum

Virtual Mass Control: \[ m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - \nu\dot{\theta} \]

**Pros:**
Provides disturbance rejection

**Cons:**
Hard to regain steady-state + excessive control effort
Control of Low Inertia Pendulum

Virtual Friction Control: \[ m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - r^{-1}\dot{\theta} \]

**Pros:** Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

**Cons?** None, at least for pendulum
Performance Specification

**Frequency Response**

- SystemFreq.: \( \bar{\omega}(t) = \sum_{i=1}^{n} \frac{M_i w_i(t)}{\sum_{i=1}^{n} M_i} \)
- Sync. Error: \( \tilde{\omega}_i(t) = w_i(t) - \bar{\omega}(t) \)

**Control Effort**

- Injected Power: \( \bar{x}(t) = \sum_i x_i(t) \)
- Injected Energy: \( \dot{E}(t) = \bar{x}(t) \)

**Benchmark**: Quantify control ability to eliminate overshoot in Nadir
Modal Decomposition for Multi-Rated Machines

**Assumption:** Let \( f_i \) be the machine relative inertia (\( f_i = \frac{M_i}{\max_j M_j} \)), and assume

\[
\begin{align*}
g_i(s) &= \frac{1}{f_i} g_0(s) \\
c_i(s) &= f_i c_0(s)
\end{align*}
\]

**Swing Equations + Turbine**

\[
g_0(s) = \frac{\tau s + 1}{m \tau s^2 + (m + d \tau) s + d + r^{-1}}
\]

**Virtual Inertia**

\[
c_0(s) = -(\nu s + r^{-1})
\]

\[
\begin{align*}
M_i &= f_i m, & D_i &= f_i d, & R_{g,i} &= \frac{1}{f_i} r_g, & \tau_i &= \tau \\
\nu_i &= f_i \nu, & R_{r,i} &= \frac{1}{f_i} r_r
\end{align*}
\]
Modal Decomposition for Multi-Rated Machines

Assumption: Let $f_i$ be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and assume

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

Swing Equations + Turbine

$$g_0(s) = \frac{\frac{\tau s + 1}{m \tau s^2 + (M + d \tau) s + d + r^{-1}}}{s}$$

Virtual Inertia

$$c_0(s) = -(\nu s + r_{\tau}^{-1})$$

$$M_i = f_i m, \quad D_i = f_i d, \quad R_{g,i} = \frac{1}{f_i} r_g, \quad \tau_i = \tau$$

$$\nu_i = f_i \nu \quad R_\tau, i = \frac{1}{f_i} r_{\tau}$$
Modal Decomposition for Multi-Rated Machines

**Assumption:** Let $f_i$ be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

\[
\begin{align*}
g_i(s) &= \frac{1}{f_i} g_0(s) \\
c_i(s) &= f_i c_0(s)
\end{align*}
\]

\[
F = \text{diag}(f_i)
\]
Modal Decomposition for Multi-Rated Machines

**Assumption:** Let $f_i$ be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

\begin{align*}
g_i(s) &= \frac{1}{f_i} g_0(s) \\
c_i(s) &= f_i c_0(s)
\end{align*}

![Diagram](image)

$F = \text{diag}(f_i)$

Change of Vars.

System Frequency

\[
\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}
\]

Change of Vars.

\[
\bar{w}_i(t) = w_i(t) - \bar{w}(t)
\]

Sync Error

**Eigenvalues of:**

\[L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}\]

\[0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}\]

[Paganini M ‘17, Guo Low 18’]
**Performance Specification**

**Frequency Response**

System Freq. : $\bar{\omega}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{\omega}(t)$

**Control Effort**

Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

Injected Energy: $\dot{E}(t) = \bar{x}(t)$

**Benchmark:** Quantify control ability to eliminate overshoot in Nadir
System Frequency w/ Virtual Inertia

\[ c_i : x_i = -f_i (\nu \dot{w}_i + r_r^{-1} w_i) \]

Nadir Overshoot Elimination:

\[ \nu \geq \nu_{\text{min}} := r_g \left( \sqrt{r_g^{-1}} + \sqrt{d + r_g^{-1} + r_r^{-1}} \right)^2 - m \]

No Control

\[ r_r^{-1} = 0, \ \nu = 0 \]

Virtual Inertia

\[ r_r^{-1} = 0, \ \nu = \nu_{\text{min}} \]

Droop Control

\[ r_r^{-1} \neq 0, \ \nu = 0 \]

requires \( \nu > 0 \) in low inertia systems (low \( m \))
Performance Specification

Frequency Response

System Freq. : \( \bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i} \)

Sync. Error : \( \tilde{w}_i(t) = w_i(t) - \bar{w}(t) \)

Control Effort

Injected Power: \( \bar{x}(t) = \sum_i x_i(t) \)

Injected Energy: \( \dot{E}(t) = \bar{x}(t) \)

Benchmark: Quantify control ability to eliminate overshoot in Nadir
Control Effort

\[ \overline{x}(t) = \sum_i x_i(t) \]

\[ c_i : x_i = -f_i (\nu \dot{w}_i + r_r^{-1} \dot{w}_i) \]

No Control \hspace{1cm} Virtual Inertia \hspace{1cm} Droop Control

\[ r_r^{-1} = 0, \; \nu = 0 \]

\[ r_r^{-1} = 0, \; \nu = \nu_{\text{min}} \]

\[ r_r^{-1} \neq 0, \; \nu = 0 \]
Roadmap to Low Inertia Frequency Control

• Performance Specification

• Limits of Virtual Inertia and Droop Control

• Dynamic Droop Control: iDroop
Control of Low Inertia Pendulum

Virtual Friction Control: \[ m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f - r^{-1}\dot{\theta} \]

**Pros:** Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

**Cons?** Large steady-state effort in power systems.
iDroop: Dynamic Droop Control

Instead of Virtual Inertia:

\[ c_i : \left\{ x_i = -\left( \nu_i \dot{w}_i + R_{r,i}^{-1} w_i \right), \quad c_i(s) = -\left( \nu_i s + R_{r,i}^{-1} \right) \right\} \]
iDroop: Dynamic Droop Control

Instead of Virtual Inertia: \[ c_i \colon \begin{cases} \dot{x}_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), & c_i(s) = -(\nu_i s + R_{r,i}^{-1}) \end{cases} \]

We use

\[ c_i \colon \begin{cases} \dot{x}_i = -\delta_i (R_{r,i}^{-1} w_i + x_i) - \nu_i \dot{w}_i \\
\end{cases} \]

\[ c_i(s) = -\nu_i \frac{s + \delta_i (R_{r,i}^{-1} / \nu_i)}{s + \delta_i} \]

\[ |c_i(i\omega)|_{dB} \]

\[ iDroop – Lead \quad \nu_i > R_{r,i}^{-1} \]

\[ \delta_i \left( R_{r,i}^{-1} / \nu_i \right) \]

\[ \log \omega \]

\[ iDroop – Lag \quad \nu_i < R_{r,i}^{-1} \]

\[ \delta_i \left( R_{r,i}^{-1} / \nu_i \right) \]

\[ \log \omega \]
**Bus Dynamics /w iDroop**

**Inverter:**  
\[ C_i : w_i \rightarrow x_i \]

**iDroop Control:**  
\[ c_i(t) = \begin{cases} x_i & = -\delta_i \left( R_{r,i}^{-1} w_i + x_i \right) - \nu_i \dot{w_i} \\ c_i(s) & = -\nu_i \frac{s + \delta_i \left( R_{r,i}^{-1} / \nu_i \right)}{s + \delta_i} \end{cases} \]

**Closed-loop Bus Dynamics w/ iDroop:**  
\[ \begin{cases} \dot{\theta}_i = w_i \\ M_i \dot{w}_i = -D_i w_i + p_i + (u_i - p_{e,i} + x_i) \\ \tau_i \dot{p}_i = -p_i + R_{r,i}^{-1} w_i \\ \dot{x}_i = -\delta_i \left( R_{r,i}^{-1} w_i + x_i \right) - \nu_i \dot{w_i} \end{cases} \]
Control of Low Inertia Pendulum

**Dynamic Friction Control:**

\[ m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f + x \]

\[ \dot{x} = -\delta \left( r^{-1} \dot{w} + x \right) - \nu \dot{w} \]
Control of Low Inertia Pendulum

No Control

\[ m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f \]

Dynamic Friction Control

\[ m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f + x \]
\[ \dot{x} = -\delta(r^{-1}_T w + x) - \nu \dot{w} \]
**Performance Specification**

**Frequency Response**

System Freq.: \( \bar{\omega}(t) = \sum_{i=1}^{n} M_i \omega_i(t) / \sum_{i=1}^{n} M_i \)

Sync. Error: \( \tilde{\omega}_i(t) = \omega_i(t) - \bar{\omega}(t) \)

**Control Effort**

Injected Power: \( \bar{x}(t) = \sum_i x_i(t) \)

Injected Energy: \( \dot{E}(t) = \bar{x}(t) \)

**Benchmark:** Quantify control ability to eliminate overshoot in Nadir
Overshoot Elimination w/ iDroop

\[ c_i : \dot{x}_i = -f_i(\delta(r_r^{-1}w_i + x_i) + \nu \dot{w}_i) \]

Whenever \( \delta = \tau \) and \( \nu = r_r^{-1} + r_g^{-1} \)

**Virtual Inertia**

\( r_r^{-1} = 0, \nu = \nu_{min} \)

**Droop Control**

\( r_r^{-1} \neq 0, \nu = 0 \)

**iDroop**

\( r_r^{-1} = 0, \nu = r_g^{-1}, \delta = \tau^{-1} \)
Overshoot Elimination w/ iDroop

Whenever

\[ \delta = T \]

and

\[ \nu = r_r^{-1} + r_g^{-1} \]

\[ \bar{w}(t) = \frac{\sum_i u_{0,i}}{\sum_i f_i d + r_g^{-1} + r_r^{-1}} \left(1 - e^{\frac{d + r_g^{-1} + r_r^{-1}}{m} t}\right) \]

first order step response
System Frequency w/ iDroop

Nadir Cancellation

\[ \delta = \tau^{-1} \]
\[ \nu = r_r^{-1} + r_g^{-1} \]

\[ \bar{w}(t) = \frac{\sum_i u_{0,i}}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}} \left(1 - e^{-\frac{d + r_g^{-1} + r_r^{-1}}{m} t}\right) \]

first order step response

iDroop can reduce system frequency step-response to a first order system - no Nadir!
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>generator inertia coefficient</td>
<td>$m$</td>
<td>$0.0192 \text{s}^2 \text{rad}^{-1}$</td>
</tr>
<tr>
<td>generator damping coefficient</td>
<td>$d$</td>
<td>$0.01 \text{s} \text{rad}^{-1}$</td>
</tr>
<tr>
<td>turbine time constant</td>
<td>$\tau$</td>
<td>$3.02 \text{s}$</td>
</tr>
<tr>
<td>turbine droop coefficient</td>
<td>$r_g$</td>
<td>$184.65 \text{s} \text{rad}^{-1}$ for SW, $369.3 \text{s} \text{rad}^{-1}$ for VI, DC, and iDroop</td>
</tr>
<tr>
<td>inverter droop coefficient</td>
<td>$r_r$</td>
<td>$369.3 \text{s} \text{rad}^{-1}$</td>
</tr>
</tbody>
</table>

Base Case: No Inverter Control

Icelandic Grid
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)
- Droop equally set for inverters in all cases
- Virtual inertia tuned for critically damped response $\nu = \nu_{\text{min}}$
- iDroop tuned for Nadir elimination
Example: Icelandic Power Grid

• Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

iDroop Benefit Summary

• Overshoot Elimination in Nadir
• Noise Attenuation
• Disturbance Rejection
• Reduce Inter-area Oscillations
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)
Step Disturbance - Frequency

- Droop equally shared between gens. and inverters.
- Virtual inertia tuned for critically damped response $\nu = \nu_{\text{min}}$
- iDroop tuned for Nadir elimination

| $\delta^*$ | 0.331 |
| $\nu^*$  | 0.0054 |
| $\nu_{VI}$ | 0.035 |

iDroop Tuning

$\delta = \tau^{-1}$

$\nu = r_r^{-1} + r_g^{-1}$

\[ r_r^{-1} = \frac{1}{2} r_{g,0} \]

\[ r_r^{-1} = 0, \nu = \nu_{\text{min}} \]

\[ r_r^{-1} = 0, \nu = r_t^{-1}, \delta = \tau^{-1} \]
Step Disturbance – Control Effort

Inertia parameters of iDroop and VI are set to achieve zero overshoot.

| $\delta^*$ | 0.331 |
| $\nu^*$   | 0.0054 |
| $\nu_{VI}$ | 0.035 |

Zero Nadir Tuning

$\delta = \tau^{-1}$
$\nu = r_r^{-1} + r_g^{-1}$
Parameter Uncertainty

![Graph showing the relationship between maximum frequency deviation and the ratio of real to nominal turbine time constants.](image-url)
Other Benefits of iDroop

\[ c_i : \begin{cases} \dot{x}_i = -\delta_i \left( R_{r,i}^{-1} w_i + x_i \right) - \nu_i \dot{w}_i \end{cases} \]

Benefits Summary

- Overshoot Elimination in Nadir
- Noise Attenuation
- Disturbance Rejection
- Reduce Inter-area Oscillations

\[ c_i(s) = -\nu_i \frac{s + \delta_i \left( R_{r,i}^{-1} / \nu_i \right)}{s + \delta_i} \]

---

Jan 14 2021

Enrique Mallada (JHU)
Thanks!

Related Publications:
• Jiang, Cohn, Vorobev, M “Dynamic Droop Approach for Storage-based Frequency Control,” IEEE TPS, conditionally accepted (arXiv)
• Min, Paganini, M, “Accurate Reduced Order Models for Coherent Synchronous Generators,” L-CSS 2020
• Y. Jiang, E. Cohn, P. Vorobev, and E. M, ”Storage-Based Frequency Shaping Control,” L-CSS 2020

Enrique Mallada
mallada@jhu.edu
http://mallada.ece.jhu.edu
Backup Slides
Power Engineering Metrics

Step Response
Nadir, RoCoF, Steady-State

Eigenvalue Analysis [Vergheese ‘89]
Dom. Eig, Damping Ratios, Part. Fact.

+ Domain specific, capture system degradation
- Relation with cause? Eigenvalue sensitivity? More than one ”dominant” eig.?
System Theoretic Metrics

Deltas: \( \delta(t)u_0 \)

Noise: \( \eta(t) \)

\( G \)

\( L \)

Frequency: \( w(t) \)

Coherence: \( \theta(t) - \bar{\theta}(t)1 \)

Losses: \( L_g^{\frac{1}{2}} \theta(t) \)

\[ \mathcal{L}_\infty \text{-norm:} \quad \| y \|_\infty := \sup_{t \geq 0} \max_i |y_i(t)| \]

\[ \mathcal{L}_2 \text{-norm:} \quad \| y \|_2 := \left( \int_0^\infty y(t)^T y(t) dt \right)^{\frac{1}{2}} \]

+ Close form solutions, qualitative analysis, computational methods

- Restrictive assumptions, not direct connection with RoCoF, Nadir, step disturbances

[Tegling... '15, Poolla... '15, Grunberg... '16, Simpson-Porco... ’16, Wu et al ’16, Adreasson ’17, Coletta ‘17... ]
Modal Decomposition for Multi-Rated Machines

Assumption: Let \( f_i \) the machine relative inertia (\( f_i = \frac{M_i}{\max_j M_j} \)), and assume

\[
g_i(s) = \frac{1}{f_i} g_0(s) \quad \quad c_i(s) = f_i c_0(s)
\]

Swing Equations + Turbine

\[ g_0(s) = \frac{\tau s + 1}{m \tau s^2 + (m + d \tau) s + d + r^{-1}} \]

Virtual Inertia

\[ c_0(s) = -(\nu s + r_r^{-1}) \]

\[
M_i = f_i m, \quad D_i = f_i d, \quad R_{g,i} = \frac{1}{f_i} r_g, \quad \tau_i = \tau
\]

\[
\nu_i = f_i \nu, \quad R_{r,i} = \frac{1}{f_i} r_r
\]
Modal Decomposition for Multi-Rated Machines

Assumption: Let $f_i$ the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

$$g_i(s) = \frac{1}{f_i} g_0(s)$$
$$c_i(s) = f_i c_0(s)$$

Change of Vars.

![Diagram of modal decomposition for multi-rated machines](image)

[Paganini M '17, Guo Low 18']
Modal Decomposition for Multi-Rated Machines

**Assumption:** Let \( f_i \) the machine relative inertia (\( f_i = \frac{M_i}{\max_j M_j} \)), and

\[
g_i(s) = \frac{1}{f_i} g_0(s) \\
c_i(s) = f_i c_0(s)
\]

System Frequency:

\[
\bar{\omega}(t) = \frac{\sum_{i=1}^{n} M_i \bar{w}_i(t)}{\sum_{i=1}^{n} M_i}
\]

Change of Vars.

\[
\hat{w} = V F^{-\frac{1}{2}} w
\]

**Sync Error**

\[
\bar{w}_i(t) = w_i(t) - \bar{\omega}(t)
\]

**Eigenvalues of:**

\[
\lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}
\]

[Paganini M ‘17, Guo Low 18’]
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>generator inertia coefficient</td>
<td>$m$</td>
<td>0.0192 s$^2$ rad$^{-1}$</td>
</tr>
<tr>
<td>generator damping coefficient</td>
<td>$d$</td>
<td>0.01 s rad$^{-1}$</td>
</tr>
<tr>
<td>turbine time constant</td>
<td>$\tau$</td>
<td>3.02 s</td>
</tr>
<tr>
<td>turbine droop coefficient</td>
<td>$r_\gamma$</td>
<td>184.65 s rad$^{-1}$ for SW, 369.3 s rad$^{-1}$ for VI, DC, and iDroop</td>
</tr>
<tr>
<td>inverter droop coefficient</td>
<td>$r_r$</td>
<td>369.3 s rad$^{-1}$</td>
</tr>
</tbody>
</table>
Step Disturbance - Frequency

- Droop equally shared between gens. and inverters.
- Virtual inertia tuned for critically damped response $\nu = \nu_{min}$
- iDroop tuned for Nadir elimination

<table>
<thead>
<tr>
<th>$\delta^*$</th>
<th>0.331</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu^*$</td>
<td>0.0054</td>
</tr>
<tr>
<td>$\nu_{VI}$</td>
<td>0.035</td>
</tr>
</tbody>
</table>

iDroop Tuning

$\delta = r^{-1}$

$\nu = r^{-1} + r_g^{-1}$

\[ r_g^{-1} = \frac{1}{2} r_g,0 \]

\[ r_r^{-1} = 0, \ \nu = \nu_{min} \]

\[ r_r^{-1} = 0, \ \nu = r_t^{-1}, \ \delta = r^{-1} \]
Step Disturbance – Control Effort

Inertia parameters of iDroop and VI are set to achieve zero overshoot.

\[ \delta^* = 0.331 \]
\[ \nu^* = 0.0054 \]
\[ \nu_{VI} = 0.035 \]

Zero Nadir Tuning
\[ \delta = \tau^{-1} \]
\[ \nu = r_r^{-1} + r_g^{-1} \]
Parameter Uncertainty

![Graph showing the relationship between Maximum Frequency Deviation \( \Delta \omega \) (mHz) and the Ratio of Real to Nominal Turbine Time Constants \( \frac{\tau_r}{\tau_i} \).]
Step Disturbance

Inertia parameters of iDroop and VI are set to achieve zero overshoot.

| $\delta^*$ | 0.331 |
| $\nu^*$   | 0.0054 |
| $\nu_{VI}$ | 0.035 |

**Zero Nadir Tuning**

$$\delta = \tau^{-1}$$

$$\nu = \frac{1}{\tau} + \frac{1}{r_g}$$

![Graph showing system frequency and synchronization cost over time](image_url)

![Graph comparing SW, DC, VI, and iDroop](image_url)
**Step Disturbance**

Inertia parameters of iDroop and VI are set to achieve zero overshoot.

<table>
<thead>
<tr>
<th></th>
<th>$\delta^*$</th>
<th>$\nu^*$</th>
<th>$\nu_{VI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>iDroop</td>
<td>0.331</td>
<td>0.0054</td>
<td>0.035</td>
</tr>
<tr>
<td>VI</td>
<td>$\delta^*$</td>
<td>0.331</td>
<td>$\nu^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0108</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\nu_{VI}$</td>
</tr>
</tbody>
</table>

**System Frequency**

**Synchronization Cost**