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Balancibility: Existence and Uniqueness of Power Flow Solutions under Voltage Balance Requirements

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Phase Unbalance

- Increasing power injection variability
  - DERs
  - Dispatchable loads
- Great impact and risks (DOE 2012)
  - Device-level impact
  - System-level impact
- Solution: secure criteria
  - Power system operation under uncertainty
  - Voltage balance guarantee

Quantify Unbalance

- Phase voltage unbalance rate (PVUR) (IEEE standard 141, 1993)

$$PVUR = \Delta_V^{\max} / V_{\text{avg}} \qquad V_{\text{avg}} = \frac{|\mathbf{V}_a| + |\mathbf{V}_b| + |\mathbf{V}_c|}{3}$$

$$\Delta_V^{\max} = \max\{||\mathbf{V}_a| - V_{\text{avg}}|, ||\mathbf{V}_b| - V_{\text{avg}}|, ||\mathbf{V}_c| - V_{\text{avg}}|\}$$

- Line voltage unbalance rate (LVUR) (NEMA standard MG1, 1993)
  - Similar to PVUR with line-to-line voltage magnitudes

- Voltage Unbalance Factor (VUF) (IEEE standard 1159, 2009)
  - Defines on negative-, zero-, and positive-sequence voltage

$$VUF_n = |\mathbf{V}_n| / |\mathbf{V}_p|,$$
$$VUF_0 = |\mathbf{V}_0| / |\mathbf{V}_p|,$$

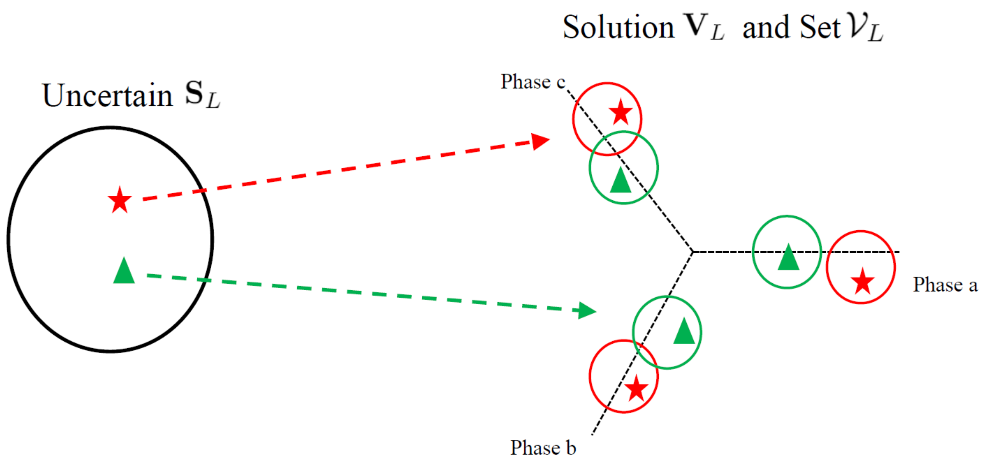
Solvability Condition

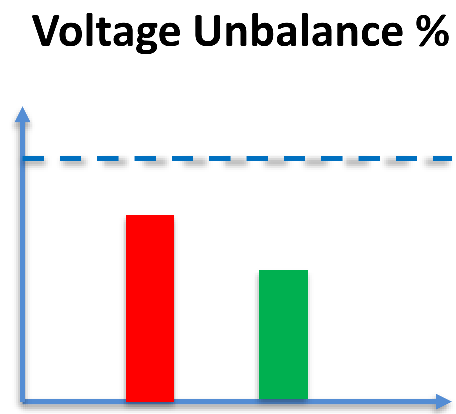
(D. Lee, et al. 2019, K. Dvijotham, et al. 2019, A. Bernstein, et al. 2018, etc)

- Use existing secure criteria that guarantees the existence and uniqueness of power flow solution
  - Given a nominal operating point, define an efficient mathematical condition on uncertain power injection without solving the power flow equations
  - Directly compute a region that contains the power flow solution out of a disturbed power injection realization

- In distribution network, we extend the solvability condition to the balancibility condition to incorporate the voltage balance requirements

Balancibility: Secure Criteria

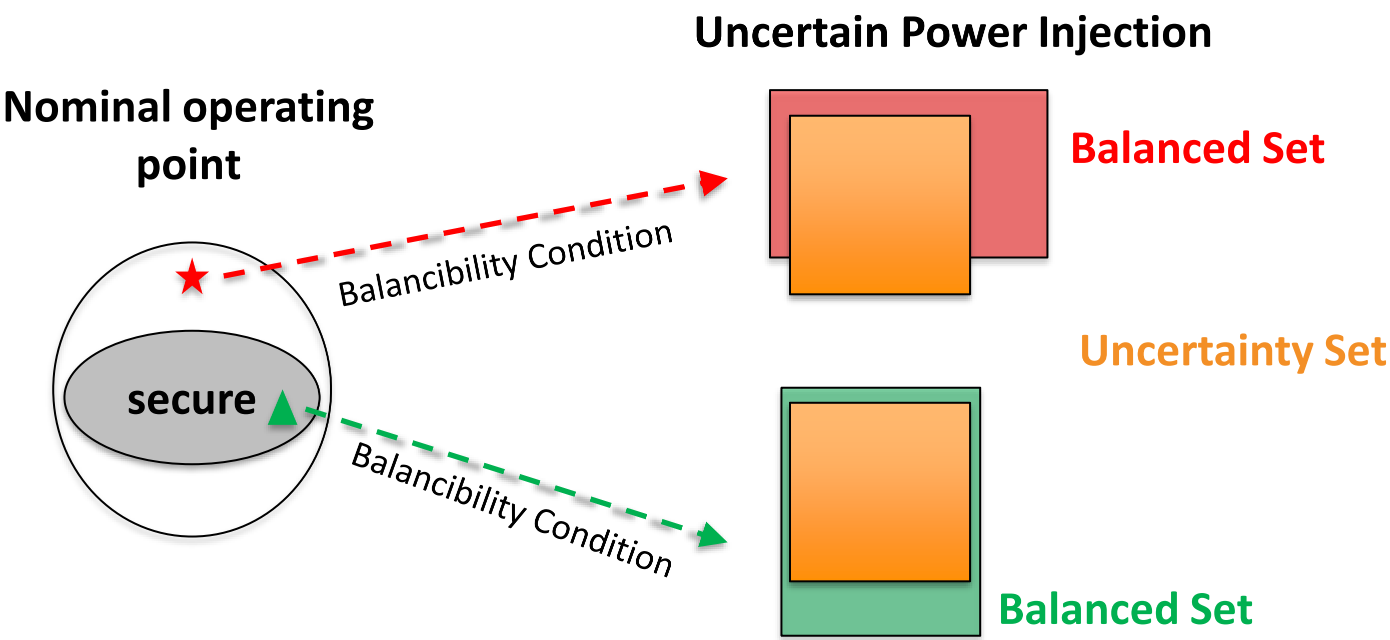




Given a nominal operating point, derive a **balancibility condition** that rigorously quantifies a region of uncertainty **balanced set** that:

- A unique power flow solutions exist
- The voltage balance requirements are always satisfied

Balancibility: Secure Criteria



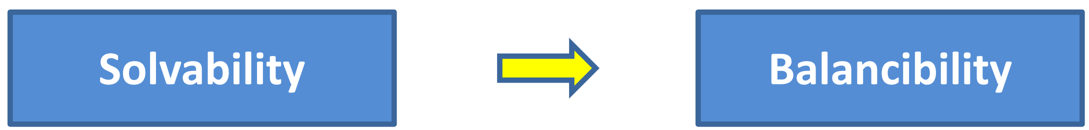
The **balancibility condition** defines the requirements on **uncertain power injection** and **nominal operating point** such that there exist "balanced" power flow solutions.

Derivation

- Output from the **Solvability Condition** (B. Cui and X.A. Sun, 2019)
  - generic topology, single slack bus and wye-connected PQ buses

$$|\mathbf{V}_a - \mathbf{C}_a| \leq r_a, \ |\mathbf{V}_b - \mathbf{C}_b| \leq r_b, \ |\mathbf{V}_c - \mathbf{C}_c| \leq r_c$$

- Properties
  - Chain of guarantees



Existence & Uniqueness → **Existence & Uniqueness + Phase Balance**

  - Quality dependence — **chain of conservativeness**
  - Conceptual generality to other network assumptions

Derivation

- PVUR

$$\begin{bmatrix} \epsilon+2 & \epsilon-1 & \epsilon-1 \\ \epsilon-1 & \epsilon+2 & \epsilon-1 \\ \epsilon-1 & \epsilon-1 & \epsilon+2 \\ \epsilon-2 & \epsilon+1 & \epsilon+1 \\ \epsilon+1 & \epsilon-2 & \epsilon+1 \\ \epsilon+1 & \epsilon+1 & \epsilon-2 \end{bmatrix} \begin{bmatrix} |\mathbf{V}_a| \\ |\mathbf{V}_b| \\ |\mathbf{V}_c| \end{bmatrix} \geq \mathbf{0}$$

Combine with the output of solvability condition

$$\min_{V_{abc} \in \mathcal{V}_{in}} \{(\epsilon+2)|\mathbf{V}_a| + (\epsilon-1)|\mathbf{V}_b| + (\epsilon-1)|\mathbf{V}_c|\} \geq 0$$

- LVUR can be handled similarly

Derivation

- VUF

$$|\mathbf{V}_n|/|\mathbf{V}_p| \leq \epsilon \quad \Leftrightarrow \quad \mathbf{V}_n\bar{\mathbf{V}}_n - \epsilon^2\mathbf{V}_p\bar{\mathbf{V}}_p \leq 0$$

Combine with the output of solvability condition

$$\left\{\max_{\mathbf{V}_{abc}\in\mathbf{U}_{in}} \mathbf{V}_n\bar{\mathbf{V}}_n - \epsilon^2\mathbf{V}_p\bar{\mathbf{V}}_p\right\} \leq 0$$

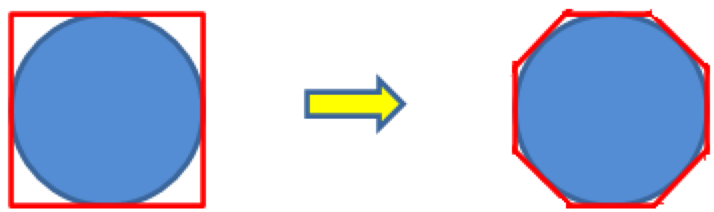
The left-hand side is nonconvex QCQP with multiple convex constraints

- First method: approximation by bound

$$\left\{\max_{\mathbf{V}_{abc}\in\mathbf{U}_{in}} 9\mathbf{V}_n\bar{\mathbf{V}}_n - \epsilon^2\min_{\mathbf{V}_{abc}\in\mathbf{U}_{in}} 9\mathbf{V}_p\bar{\mathbf{V}}_p\right\} \leq 0$$

Derivation

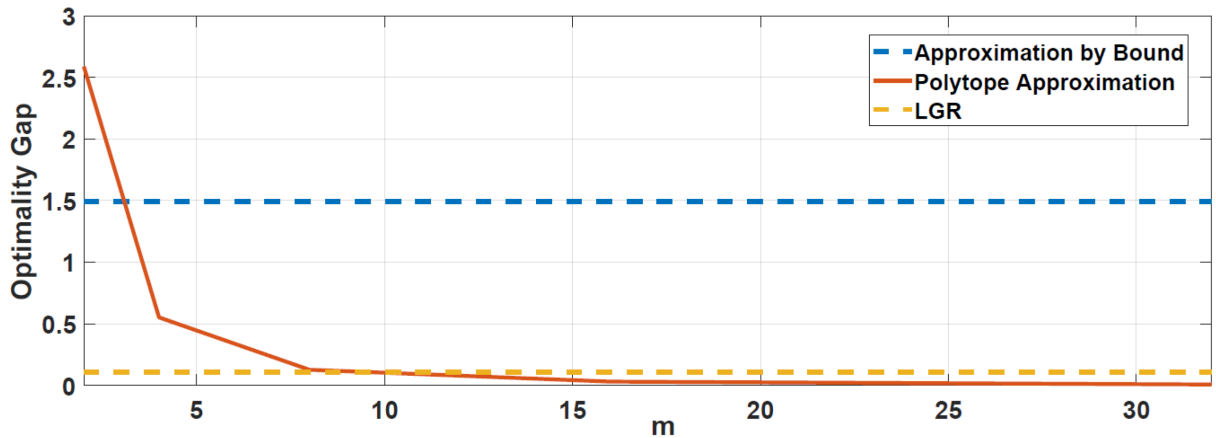
- Second method: Polytope approximation
  - By approximating disks with outer polygon, we achieve a tractable conservative approximation
  - By increasing the number of edges, the conservative approximation converge to the original nonconvex QCQP



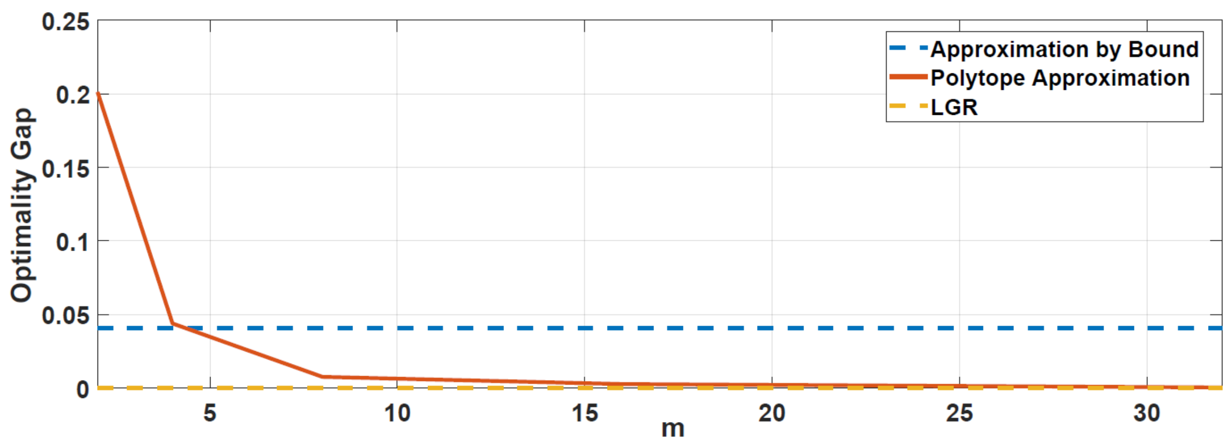
- Third method: Strong relaxations
  - The Lagrangian relaxation and semidefinite relaxation
  - The two relaxations give the same optimal value
  - Under certain conditions, each relaxation gives the same optimal value to the original nonconvex QCQP

Exactness

Non-zero optimality gap



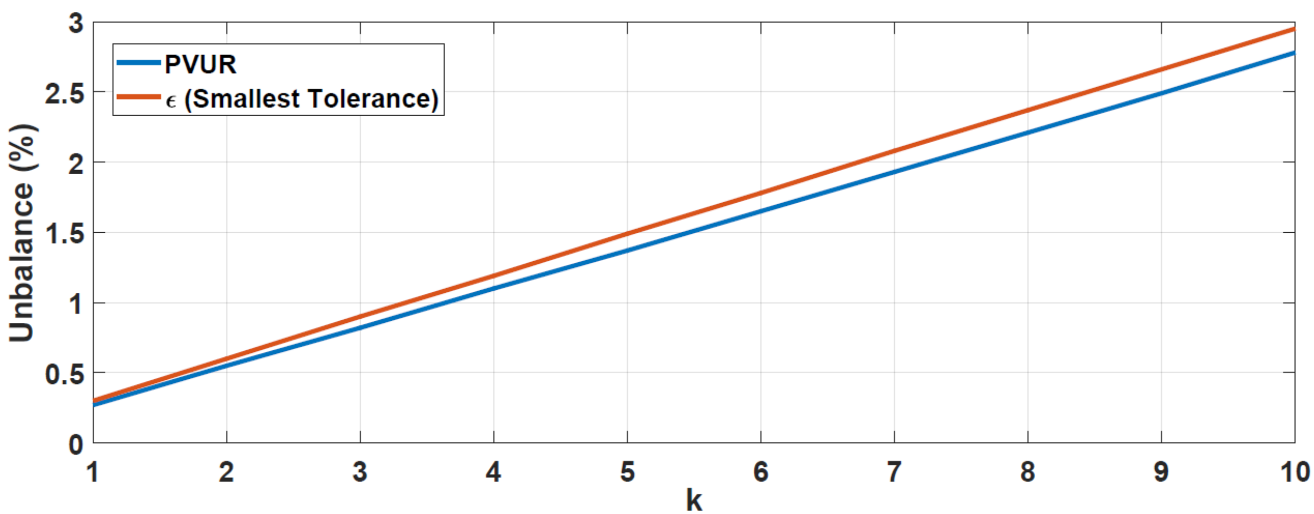
Zero optimality gap



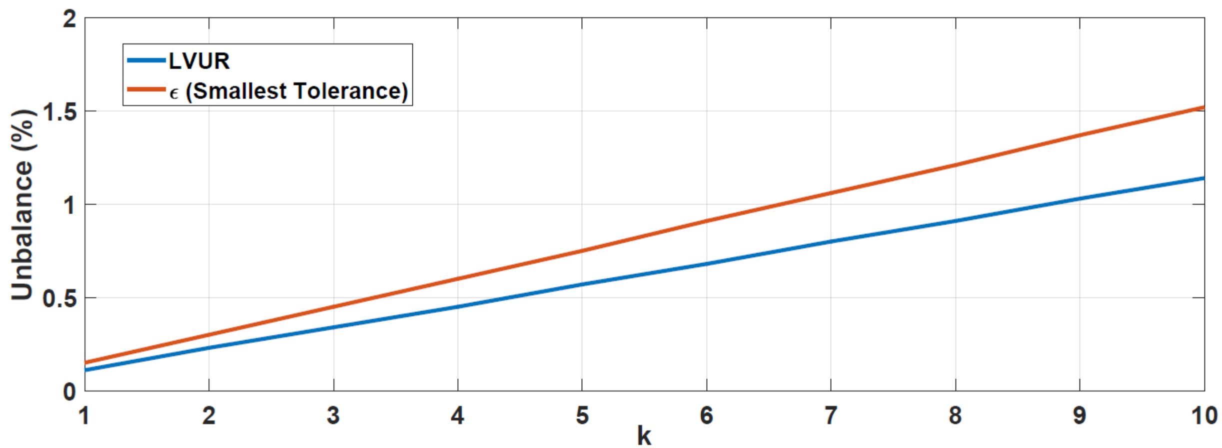
Case Study

- Setup: five-bus system with wye connected PQ loads
  - bus 4, the critical node with respect to voltage balance
  - bus 5, unbalanced load with proportional load variation

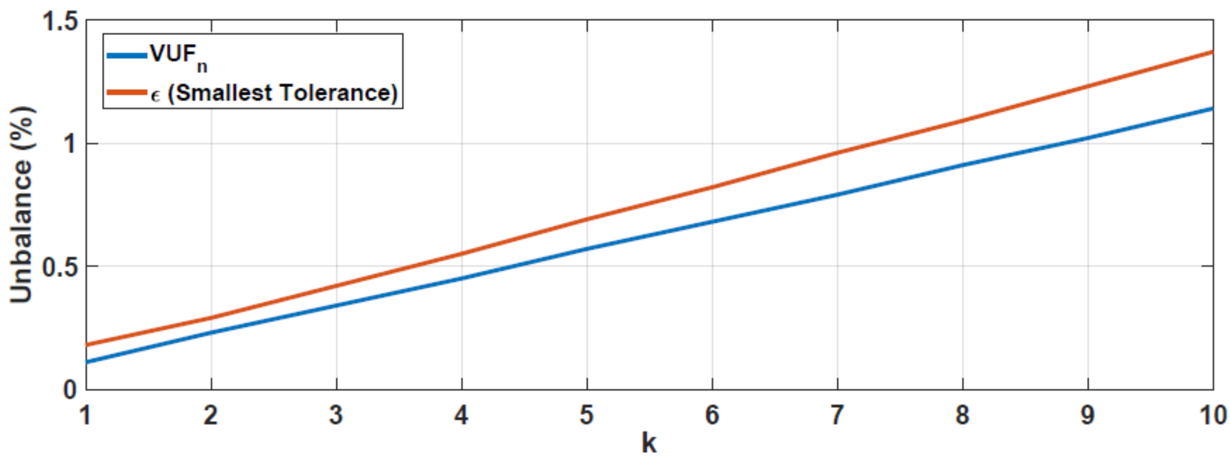
- Results on conservativeness



Case Study



LVUR
ϵ (Smallest Tolerance)
Unbalance (%)
k
2
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10



VUF_n
ϵ (Smallest Tolerance)
Unbalance (%)
k
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10

Conclusions

- Derivation and evaluation of the **balancibility condition**
  - Use multiple voltage unbalance definition
  - Strong theoretical guarantees with less conservative performance

- Future work
  - Test on larger network with high variability
  - Extend to general network assumption

- Potential Application
  - Efficient evaluation on solution existence, uniqueness, and voltage balance
  - Worst-case scenario identification regarding voltage balance limits
  - Robust ACOPF