Reexamining the Distributed Slack Bus

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"Largest" generator ☒

Representative of "rest of system" ☒

Angle reference ☒

Reexamining the Distributed Slack Bus

Sairaj V. Dhople, *Member, IEEE*, Yu Christine Chen, *Member, IEEE*,
Abdullah Al-Digs, *Student Member, IEEE*, and Alejandro Domínguez-García, *Senior Member, IEEE*

***Abstract*—Power flow formulated with a distributed slack bus involves modeling the active-power output of each generator with three elements: a *nominal injection* modulated by a fraction of the *net-load imbalance* allocated via a *participation factor*. This setup acknowledges generator dynamics and system operations, but it has long been plagued by ambiguous and inconsistent interpretations of its constituent elemental quantities. In this paper, we establish that, with the: i) nominal active-power injections set to be the economic dispatch setpoints, ii) participation factors fixed to be the ones used in automatic generation control, and iii) net-load imbalance considered to be the total**

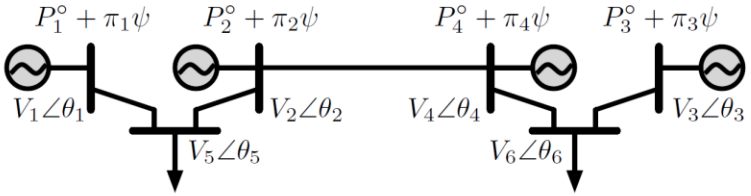
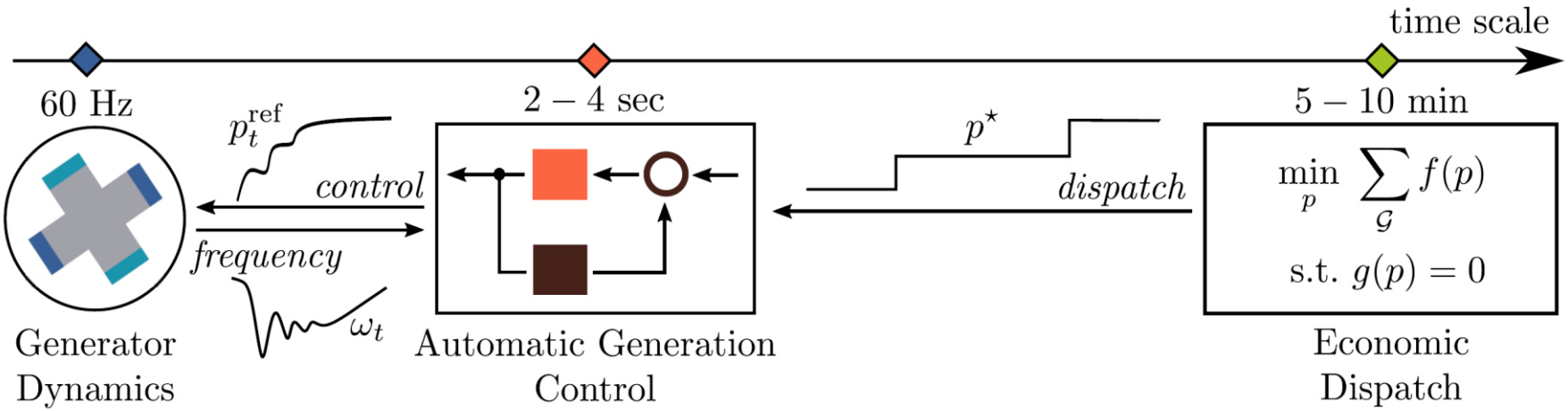
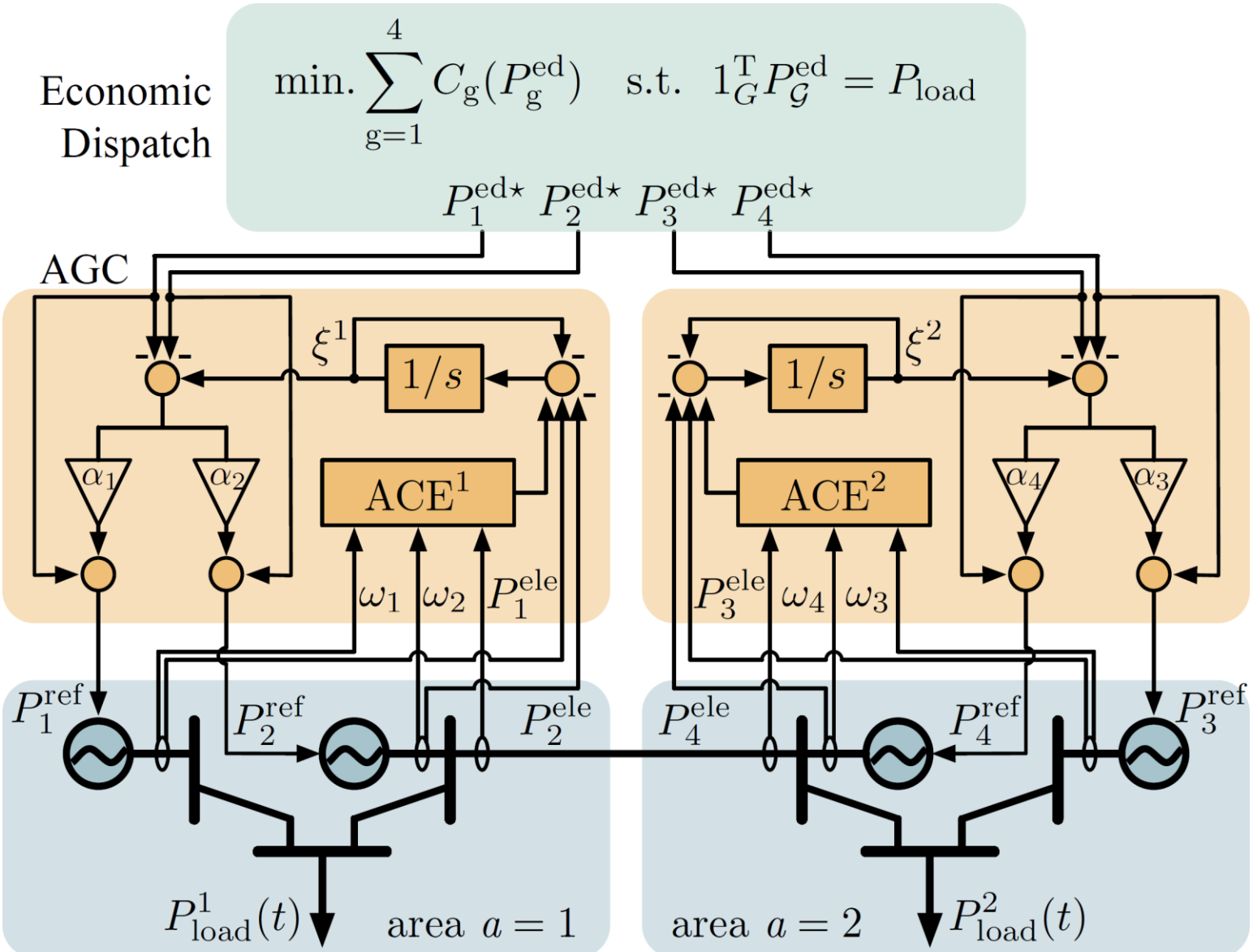


Fig. 1: Generator active-power injections are modeled as $P_g^\circ + \pi_g\psi$ in the distributed slack bus formulation. This paper uncovers appropriate values for P_g°, π_g, and ψ, so that bus voltage magnitudes and phase angles solved from the power flow match steady-state results from a dynamic simulation.

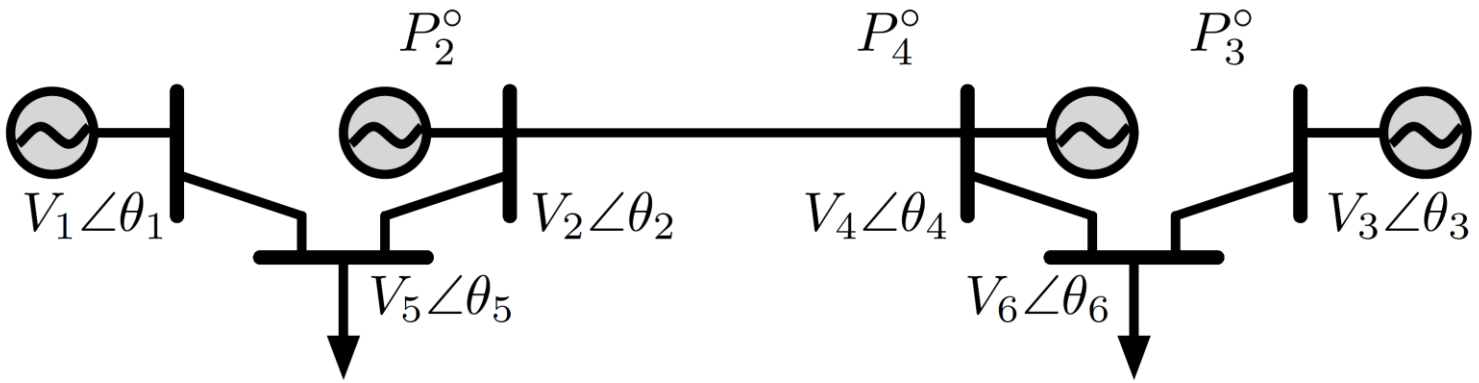
Grid architecture: time



Grid architecture: space



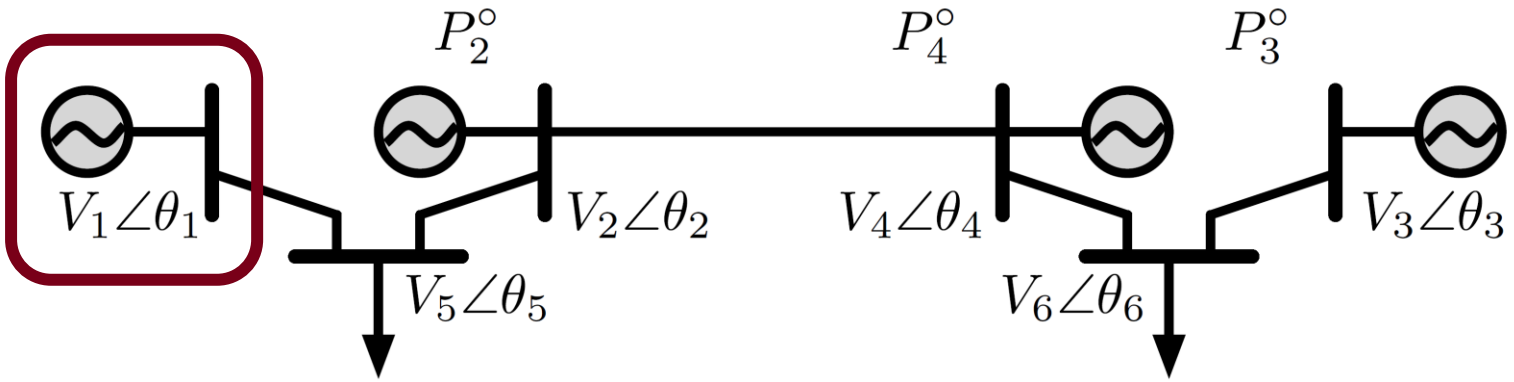
Power-flow formulation



$$P_k = V_k \sum V_j (B_{kj} \sin\theta_{kj} + G_{kj} \cos\theta_{kj})$$

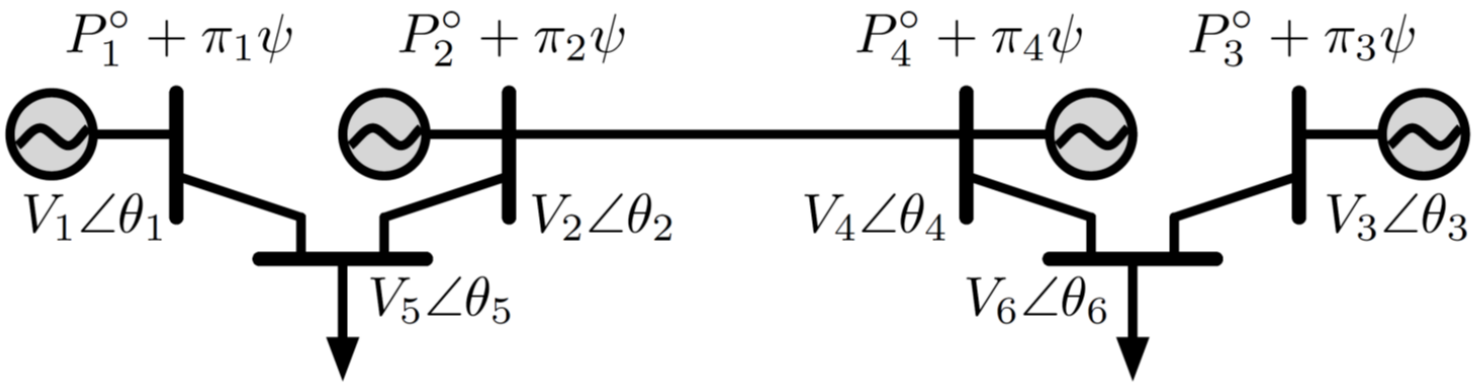
Slack bus

- Network losses are unknown
- Cannot specify the power at all generator buses
- Power at one generator is left unspecified



$$P_k = V_k \sum V_j(B_{kj}\sin\theta_{kj} + G_{kj}\cos\theta_{kj})$$

Distributed slack formulation



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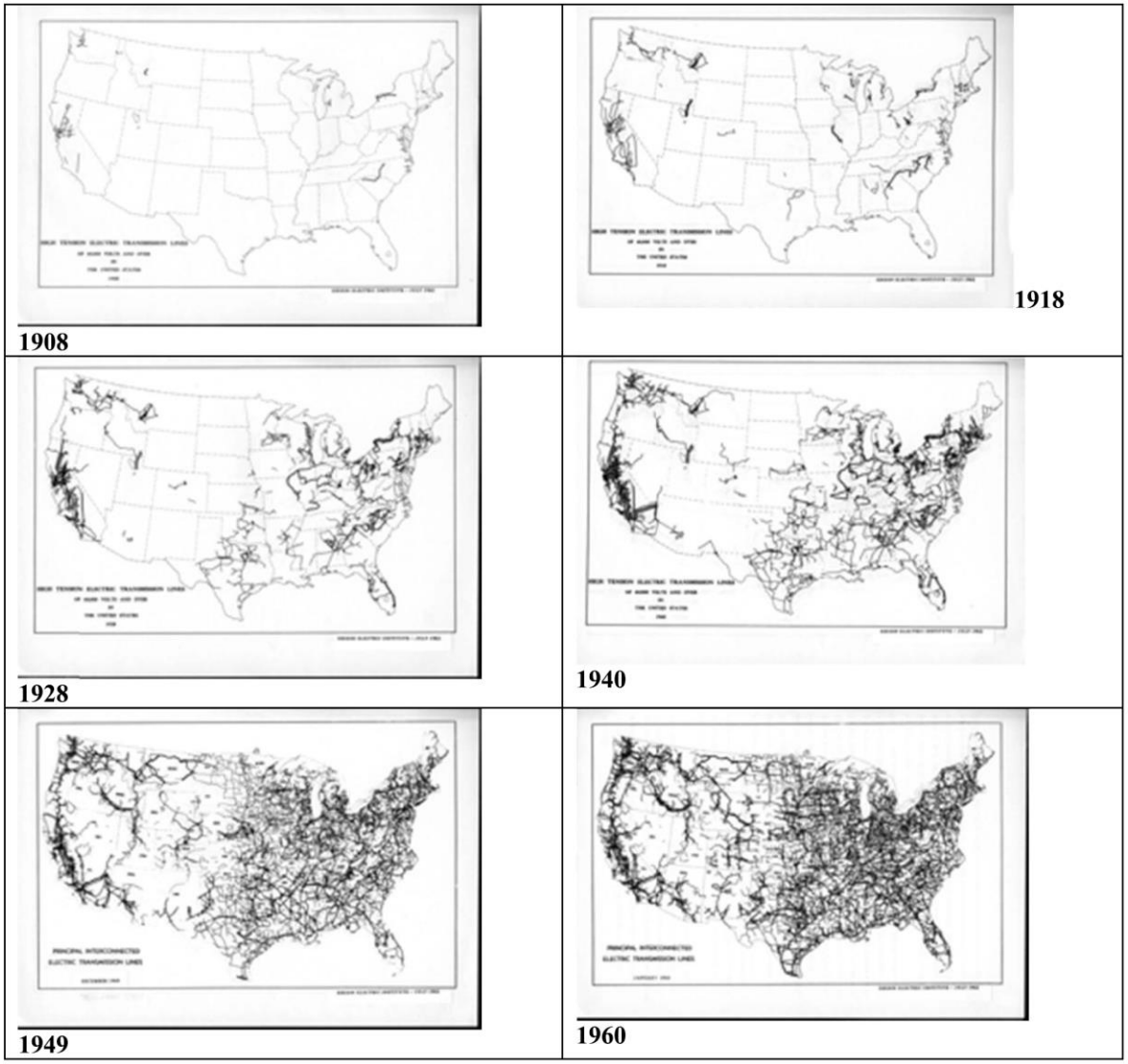
A brief history

1940s — 1956 — 1971



A brief history

1940s

Cohn

- Worked for 48 years with L&N
- US NAE (1969); IEEE Edison Medal (1982)
- Introduced net-interchange tie-line bias control
- United Pool: Iowa, Illinois, Kansas, Missouri

A brief history

1940s

[…] relatively little control theory. Simulation as practiced in recent years was not available for control experimentation. It was not, however, especially missed…Experimentation on the best of all simulators, power systems themselves, was feasible, and was practiced.

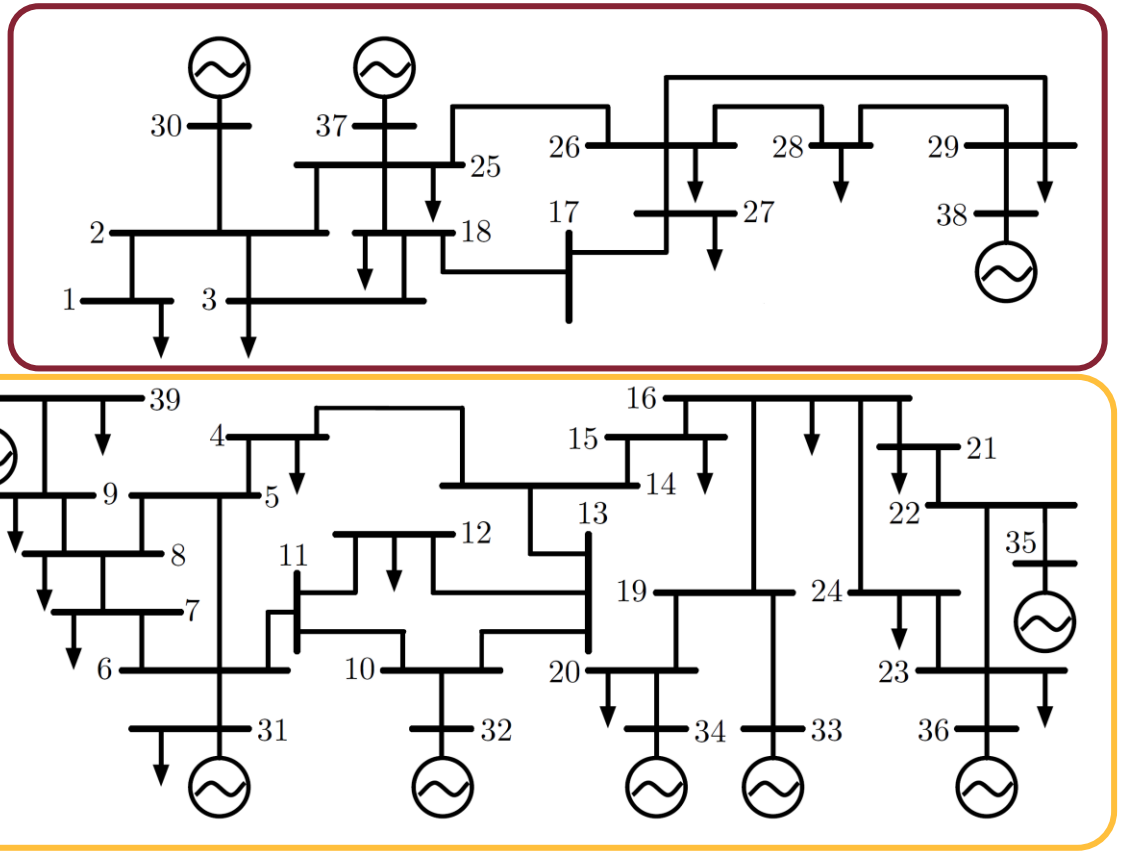
A brief history

1940s

Operations were fully in accord with expectations.

A brief history

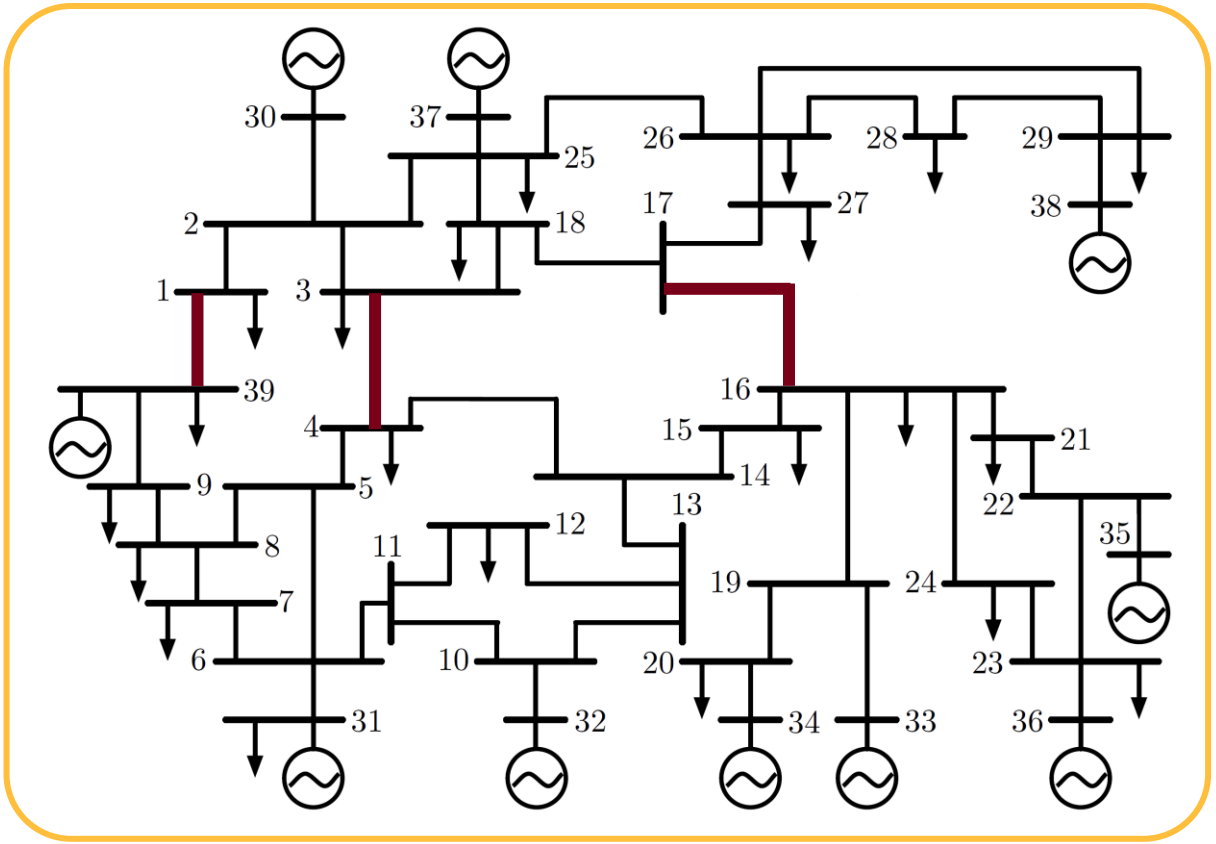
<1940s



Limited attempts at and examples of interconnected operation across the US

A brief history

>1940s



Multiple generators began to regulate frequency across balancing areas …

A brief history

Ward & Hale

Digital Computer Solution of Power-Flow Problems

J. B. WARD
MEMBER AIEE

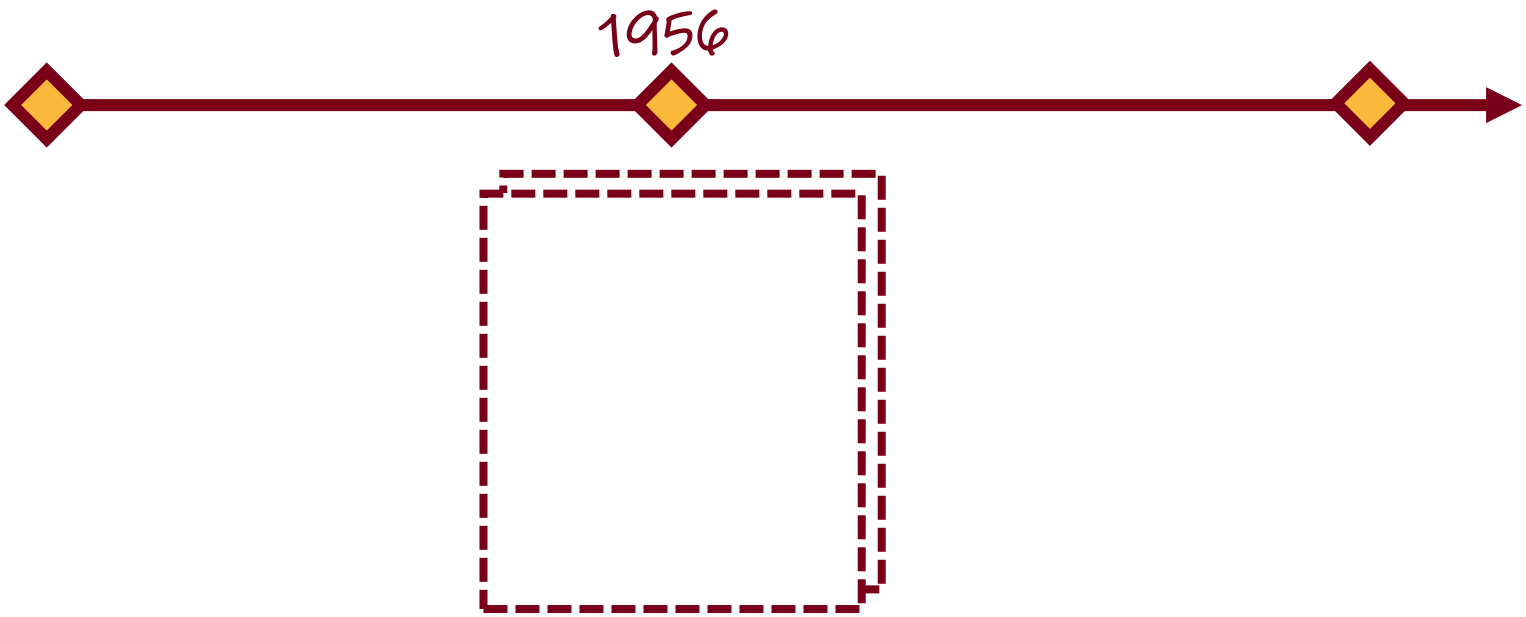
H. W. HALE
ASSOCIATE MEMBER AIEE

THE capabilities of automatic digital power input and voltage magnitude, or

parameters: The node basis of establishing a mathematical description of the network is used, and off-nominal transformer turns-ratio are treated rigorously. A part of the process in this first step is done manually, such as labelling the diagram, listing impedances, and forming a connection matrix. The major part of numerical computation can be mechanized on the computer.

2. Iterative solution for terminal voltages which satisfy the prescribed terminal conditions: This can be completely automa-

A brief history



Discussion

G. W. Stagg (American Gas and Electric Service Corporation, New York, N. Y.): The authors are to be complimented not only for their technical contribution but also for focussing attention again on a very new and important phase of power system engineering. In our opinion, the authors have appraised the situation correctly in both of their conclusions. However, it should be noted that the disadvantages of the digital computer solution exist in terms of the computers and techniques now available. It seems likely that the use of a faster and larger computer can reduce the cost of digital power-flow studies considerably. Moreover, it is apparent that rapid development of new techniques and computers will make the digital method

plans for future facilities for any one of its subtransmission systems. Today, studies of less than 4 to 8 weeks are rare. This has come about because of the need to extend studies to cover upward of 15 to 20 years to insure not only that our present-day investments in new facilities will tie in economically with our future needs but also to insure that we have adequate rights-of-way and station sites. Also, because of the large investment in system facilities now being required to meet load growth, a greater emphasis on economic analysis has been necessary. This requires the complete study of various alternate schemes before a final decision.

It seems imperative then that, if the electrical utility industry is to keep abreast of this increasingly complex problem, particularly in system planning work, new techniques and new tools must be developed

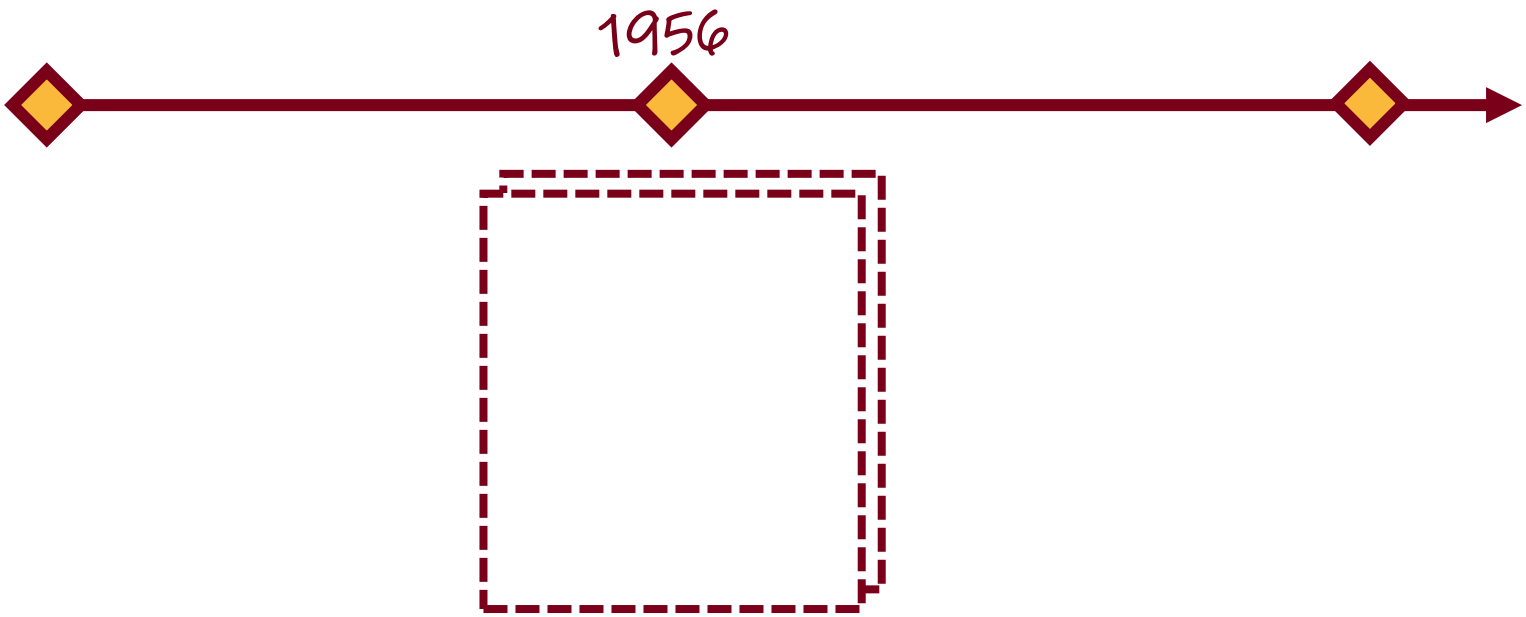
REFERENCE

1. APPLICATIONS OF DIGITAL ANALYSIS TO POWER SYSTEM PROBLEMS. *Proceedings*, American Power Conference, Chicago, Ill., 1954.

E. E. George (Ebasco Services, Inc., New York, N. Y.): In view of the rapid recent developments in computer solution of problems of power system analysis, this paper is particularly welcome because it provides the following features which are apparently new or have never been published:

1. The procedure takes advantage of the high speed and large memory or storage capabilities of one of the new types of medium-size digital computers.
2. The method described requires no matrix inversion, thus eliminating the con-

A brief history



For those who have not followed digital computer developments heretofore, two or three of the technical terms taken from network theory as developed by mathematicians and communication engineers should perhaps be explained:

1. A node is the same as a bus.
2. A branch is a line, transformer, or other series impedance between two busses.
3. The slack machine is the regulating generator which controls frequency or tie-line loading, and which cannot be scheduled in megawatt output until the difference between total generation and total load plus loss is calculated, or measured by telemeters, or balanced by a frequency controller.

nificant figures. Bus load estimates are indicated to only two significant figures. These are normal accuracies because of the uncertainties involved. Impedances are affected by conductor temperature and number of wire transpositions. Estimating future bus loads is chancy, particularly if reactive loads are involved. Even today's bus loads are probably not known to better than two significant figures.

There is some justification for carrying along four or even five figures in the calculations to avoid stray errors and the need for rounding-off operations. However, to spend computer time to get a degree of precision in the solution better than 1 per cent is difficult to justify. Certainly to get better than four significant figures can ultimately be achieved otherwise with less time and expense, then the analyzer may well be displaced.

The questions of accuracy of initial data and precision of results brought out by Mr. MacArthur are certainly pertinent to any engineering computation. In displaying results in the paper to five or six decimal places we do not imply that the voltages or flows are this accurate or this near to the true values that would appear on the actual system. In an extensive iterative computation such as this it is necessary to carry intermediate calculations to a much greater precision than contained in the input data, particularly if the result sought is a comparison of losses. Since calculation of line flows and source loadings

A brief history

The slack machine is the regulating generator which controls frequency or tie-line loading, and which cannot be scheduled in megawatt output until the difference between total generation and total load plus loss is calculated, or measured by telemeters, or balanced by a frequency controller.

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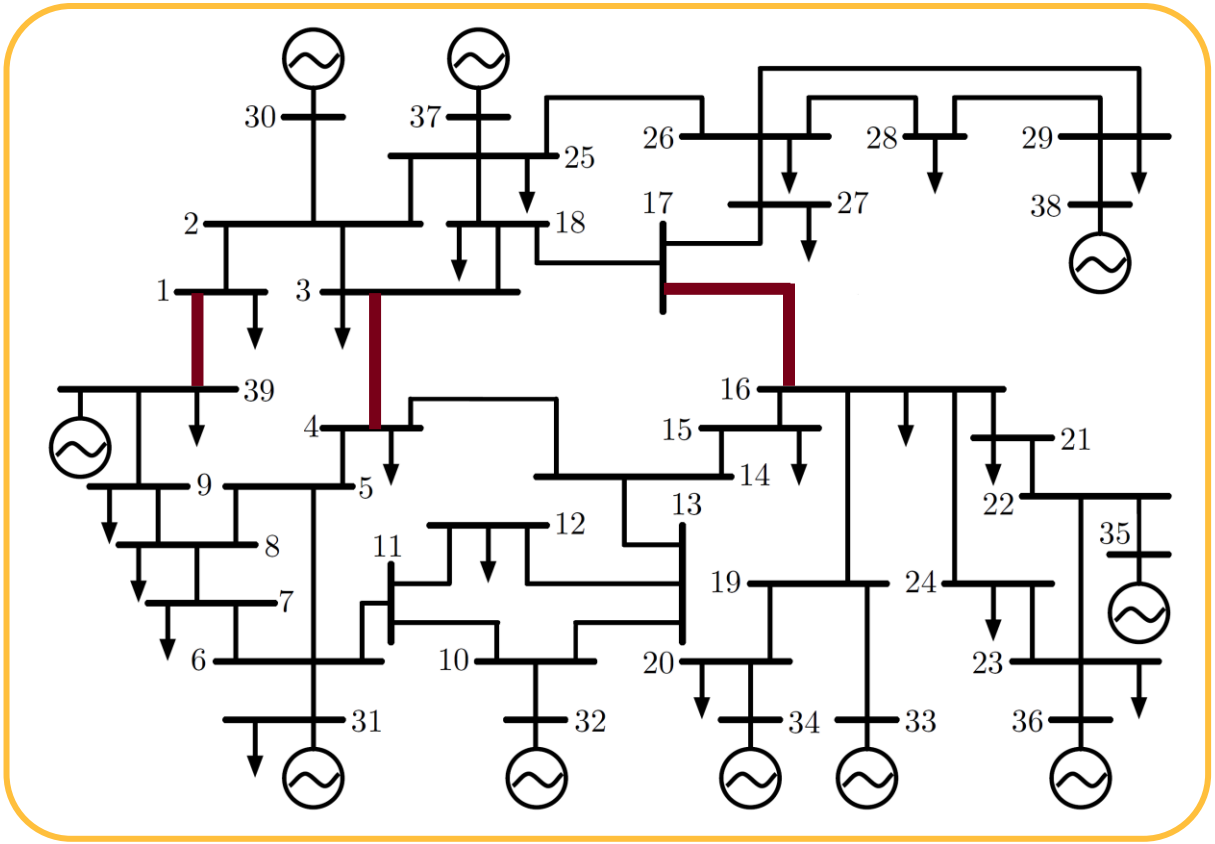
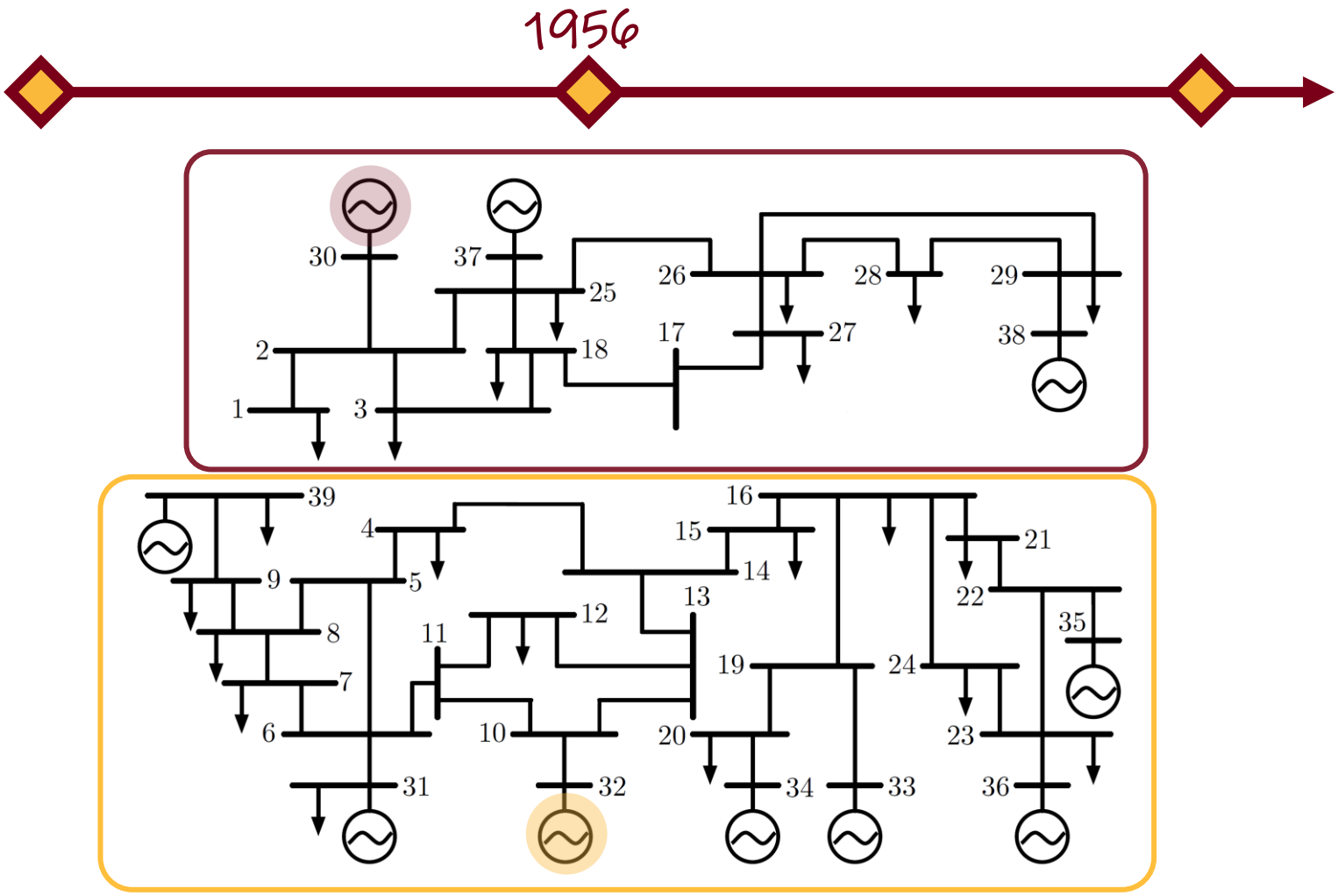
A brief history



Multiple generators were regulating frequency across balancing areas …

A brief history



Formulations aligned with control schemes where a single generator provides frequency regulation!

A brief history

1971

NEW METHODS FOR LOAD FLOW CALCULATION
WITHOUT ANY SWING BUS

by

KATSUMI YAMANE

B.E., University of Tokyo
(1965)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1971

Signature of Author Signature redacted
Department of Electrical Engineering, May 14, 1971
Certified by Signature redacted
Thesis Supervisor
Accepted by Signature redacted
Chairman, Departmental Committee on Graduate Students

Archives
MASS. INST. TECH
JUN 28 1971
LIBRARIES

Schweppe

A brief history

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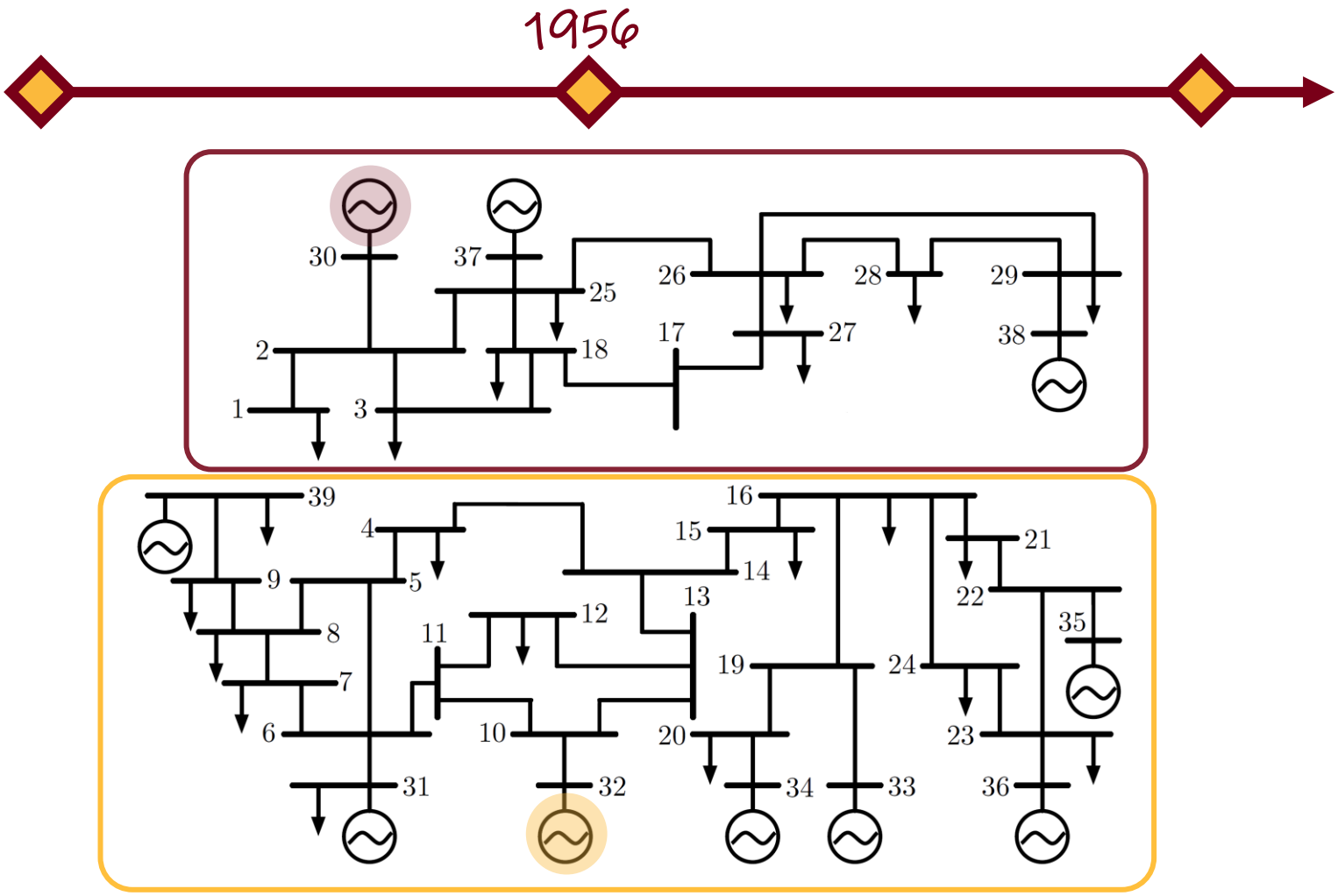
New Methods for Load Flow Calculation Without any Swing Bus

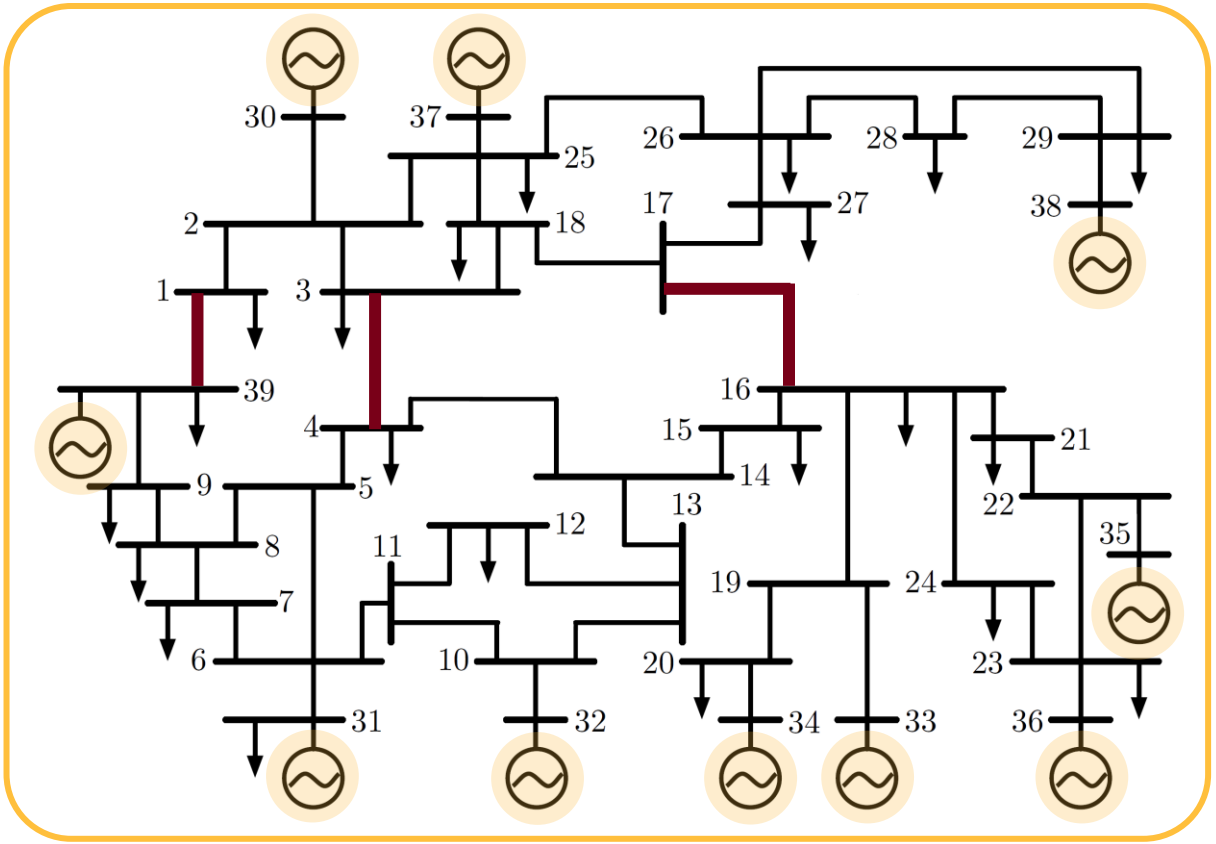
A brief history



Formulations aligned with control schemes where a single generator provides frequency regulation

A brief history

1971



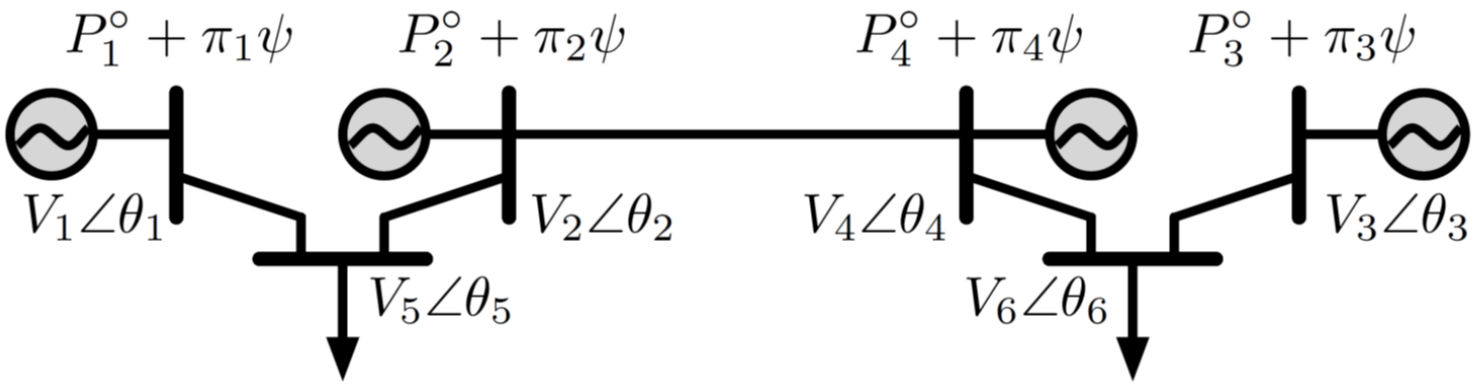
Updated power flow with distributed slack bus …

but never formalized …

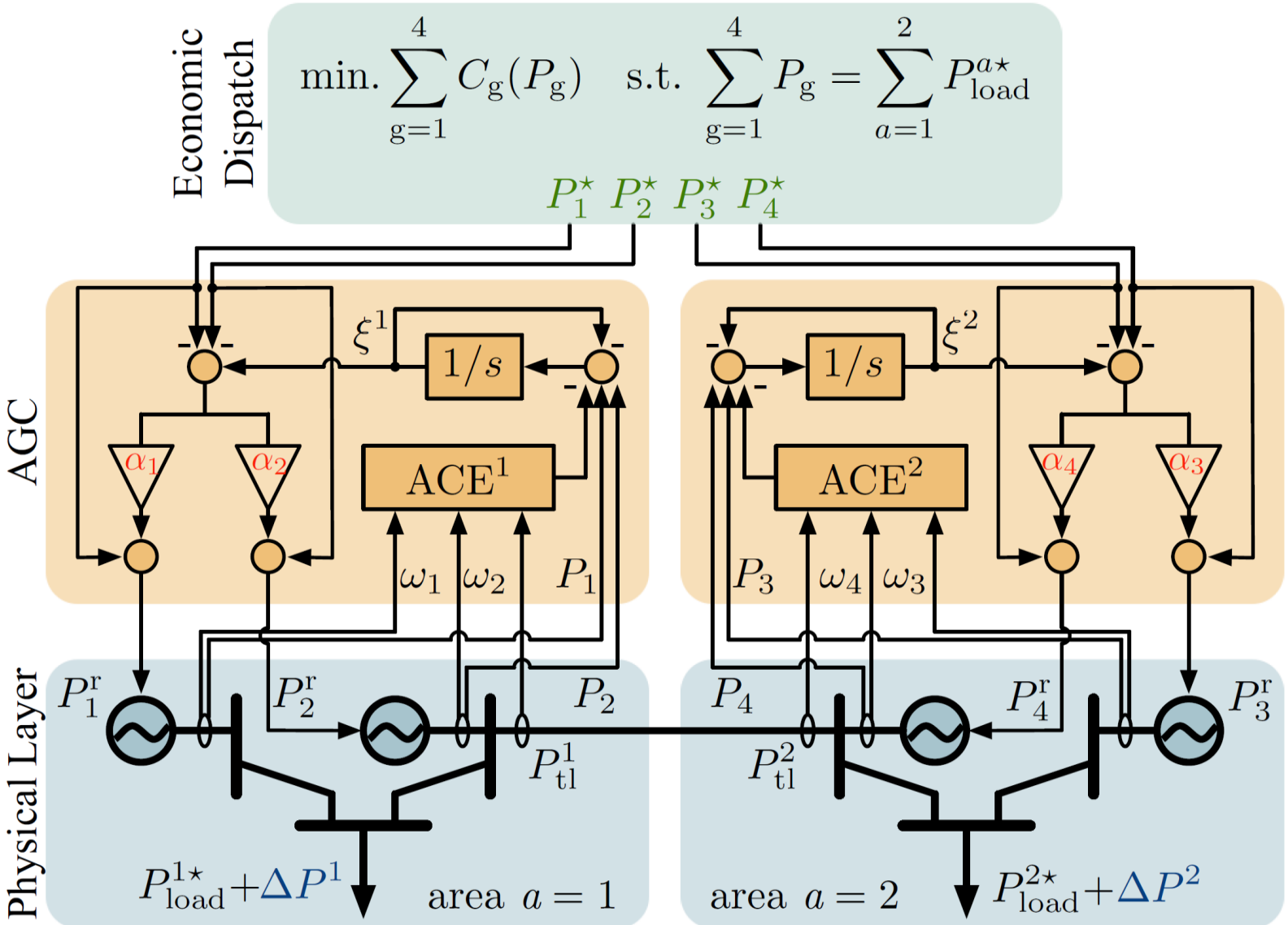
A brief history

Applications of these new methods are suggested to Tie Line Control and Economic Dispatch of a power system.

Distributed slack formulation



Recall system architecture



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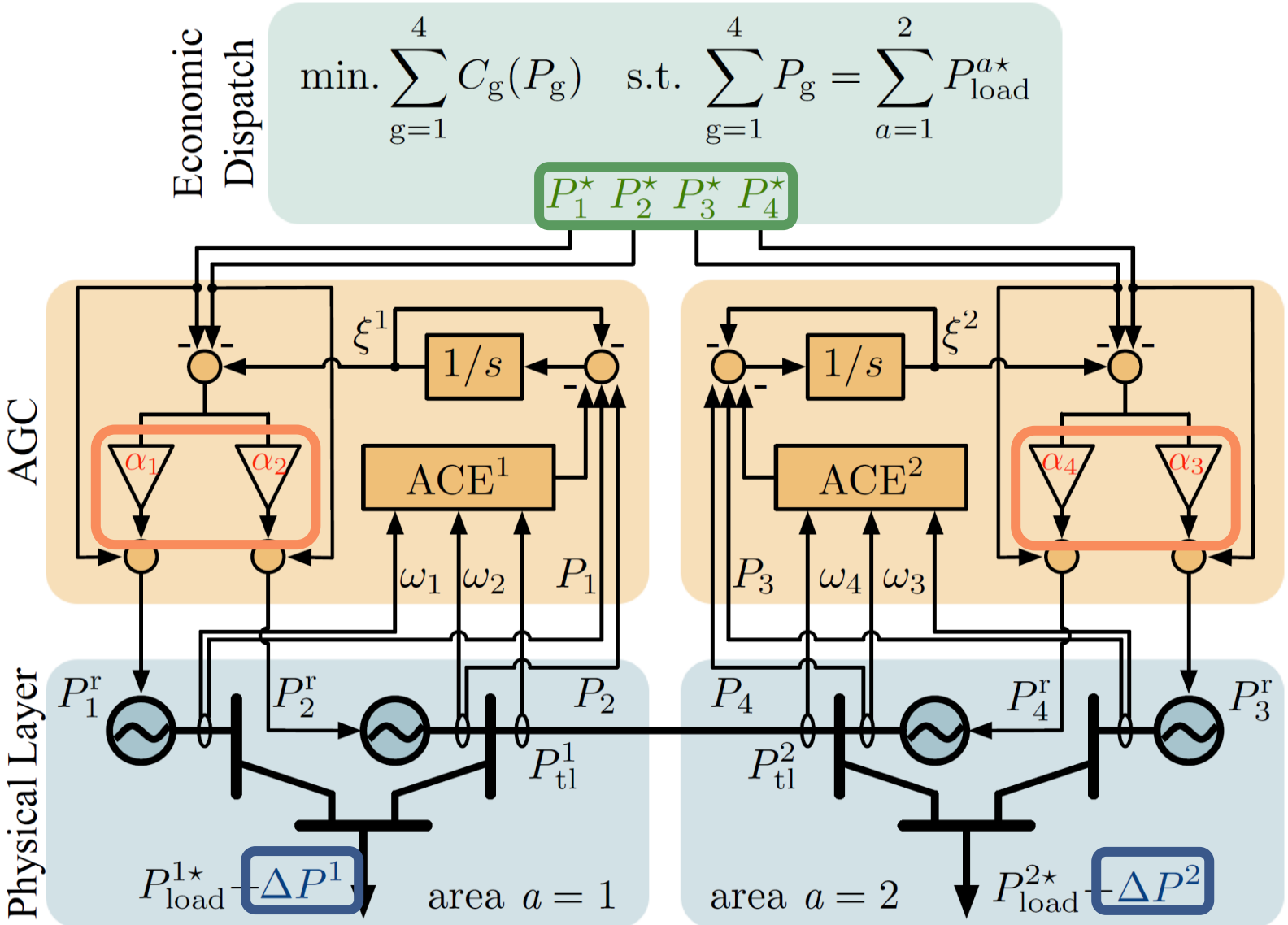
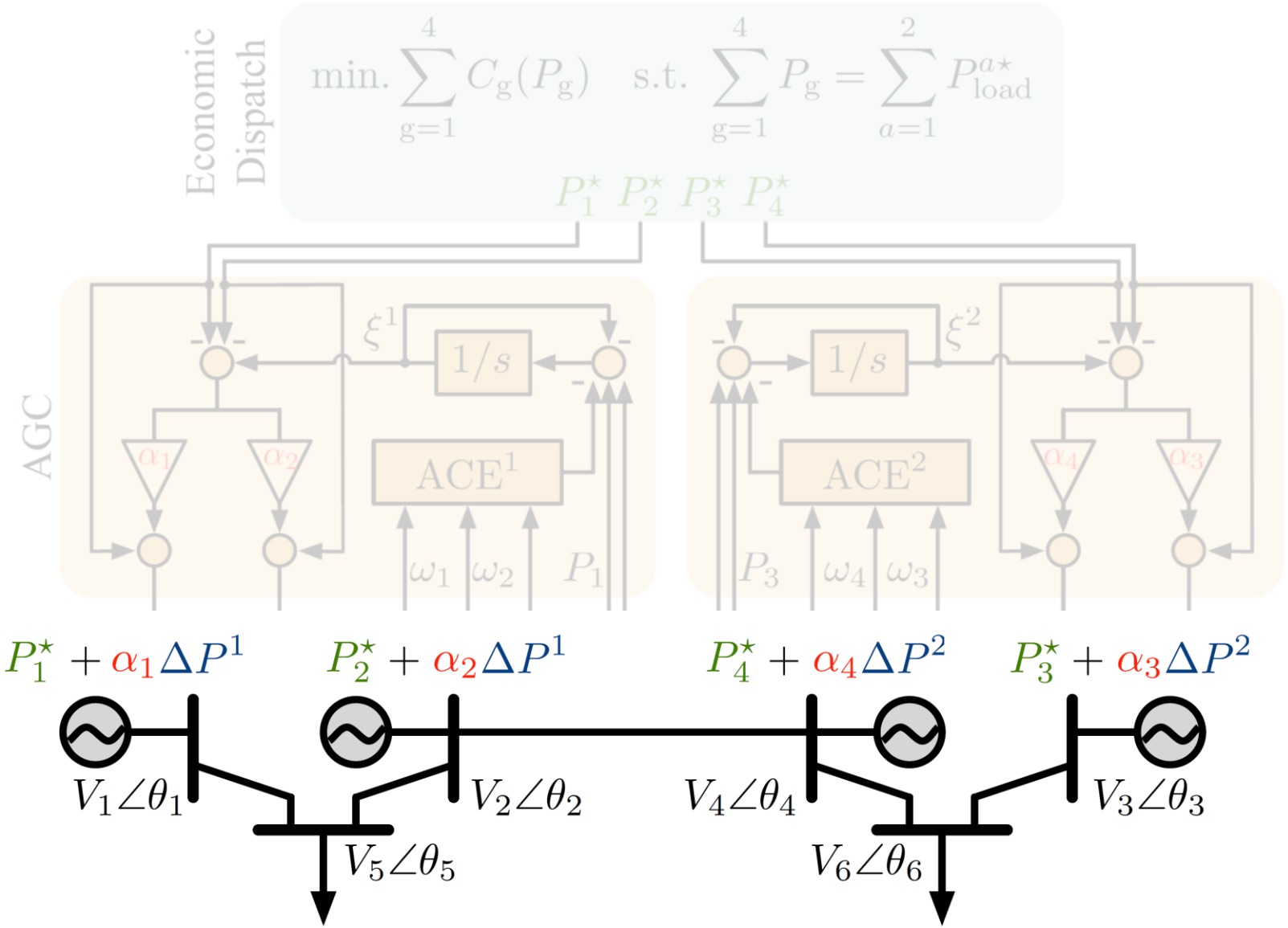
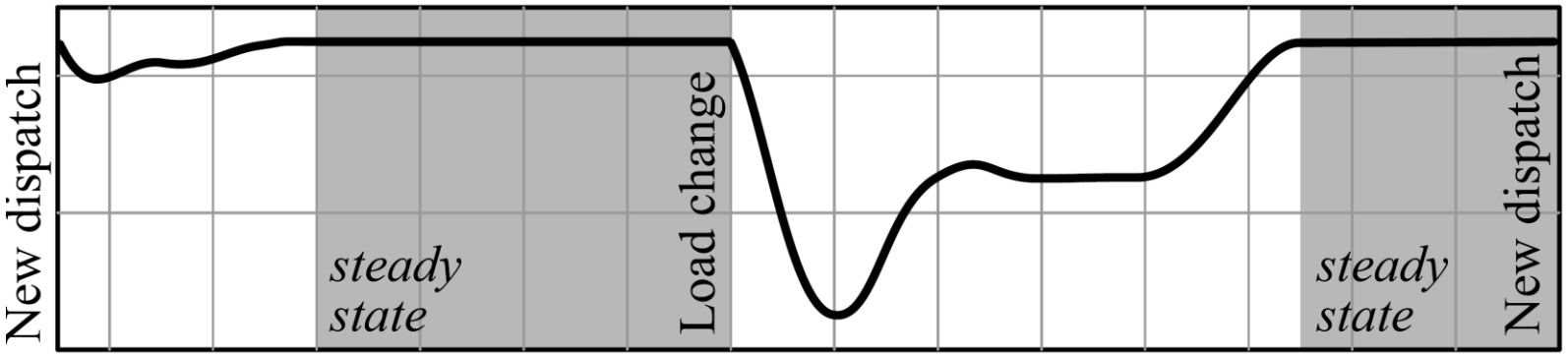


"Correct" formulation

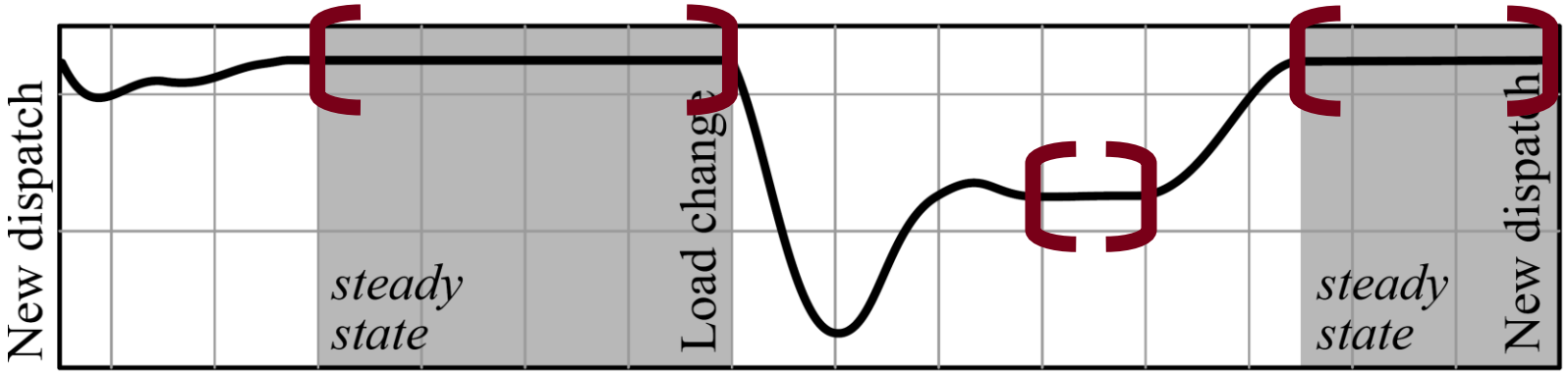


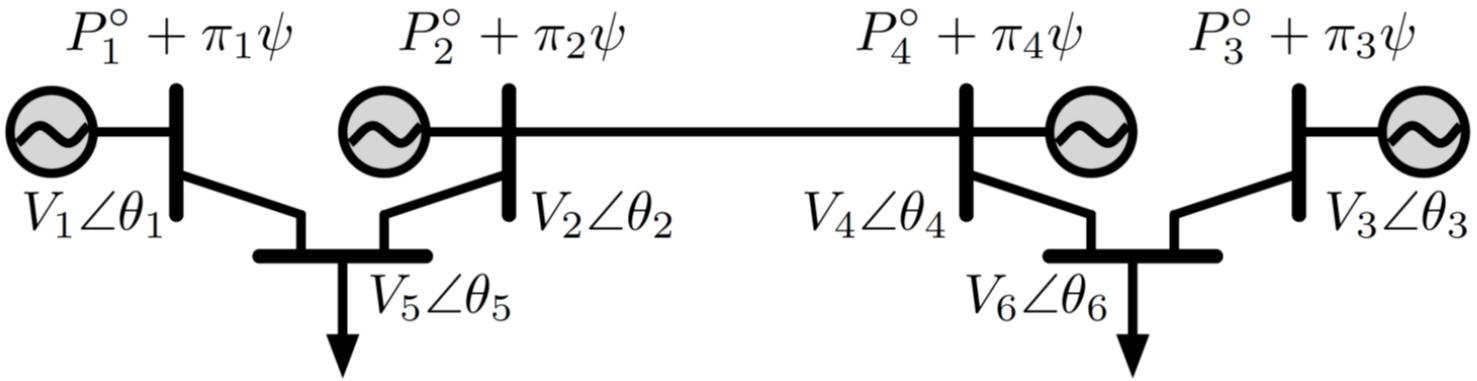
When is this valid?



Steady state

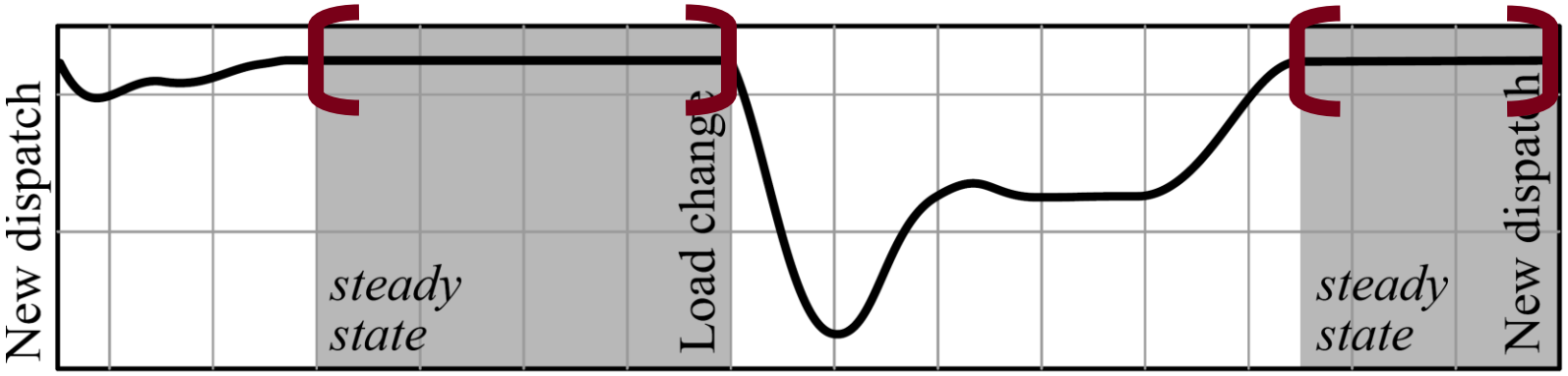
$$P_g^\circ + \pi_g \psi = V_g \sum_{j \in \mathcal{N}^a} V_j (B_{gj} \sin \theta_{gj} + G_{gj} \cos \theta_{gj})$$

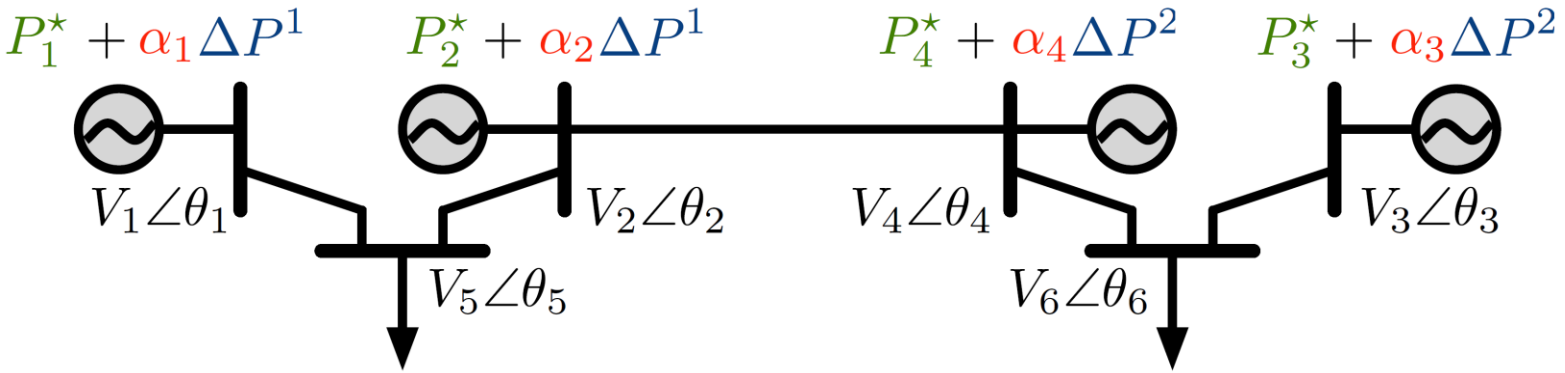




Synchronous steady state

$$P_g^\circ + \pi_g \psi = V_g \sum_{j \in \mathcal{N}^a} V_j (B_{gj} \sin \theta_{gj} + G_{gj} \cos \theta_{gj})$$

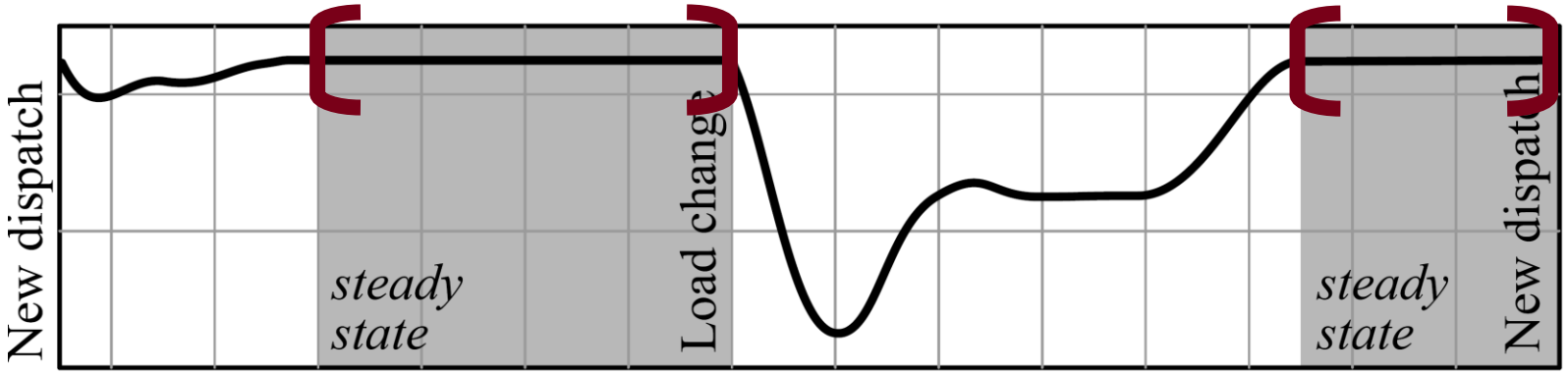


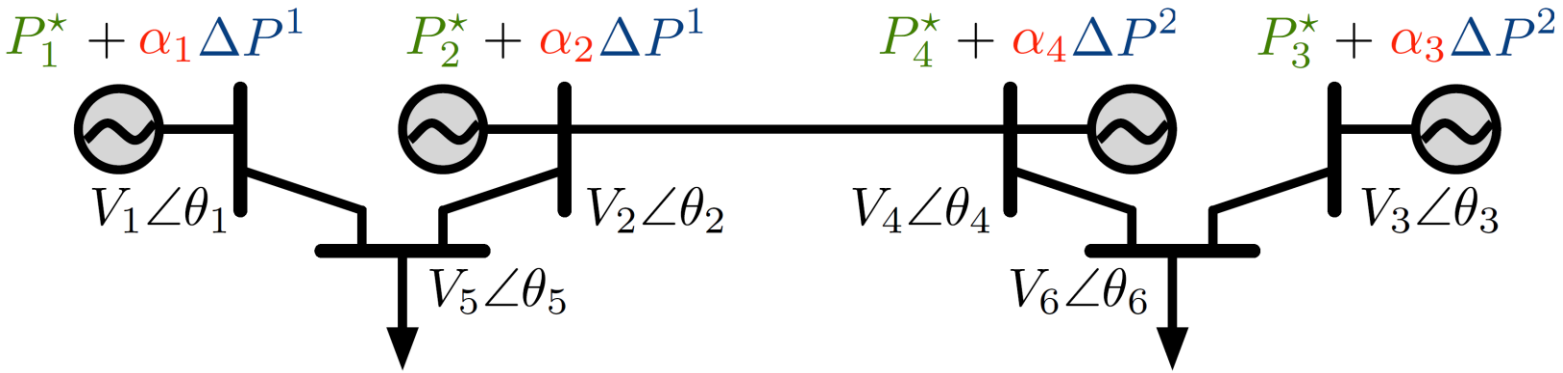


Tertiary control

$$P_g^\circ + \pi_g \psi = V_g \sum_{j \in \mathcal{N}^a} V_j (B_{gj} \sin \theta_{gj} + G_{gj} \cos \theta_{gj})$$

$$P_g^\circ = P_g^\star$$

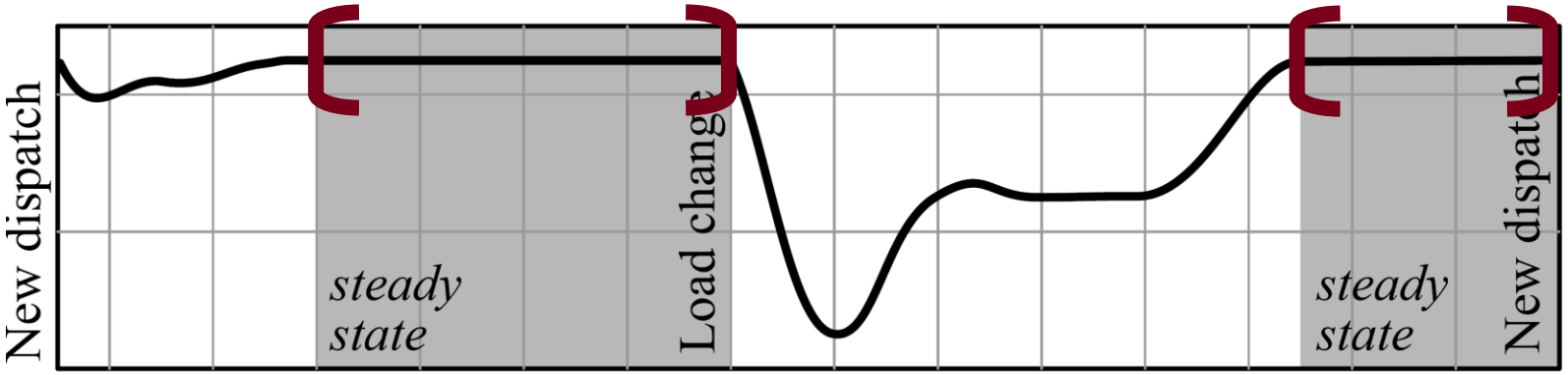


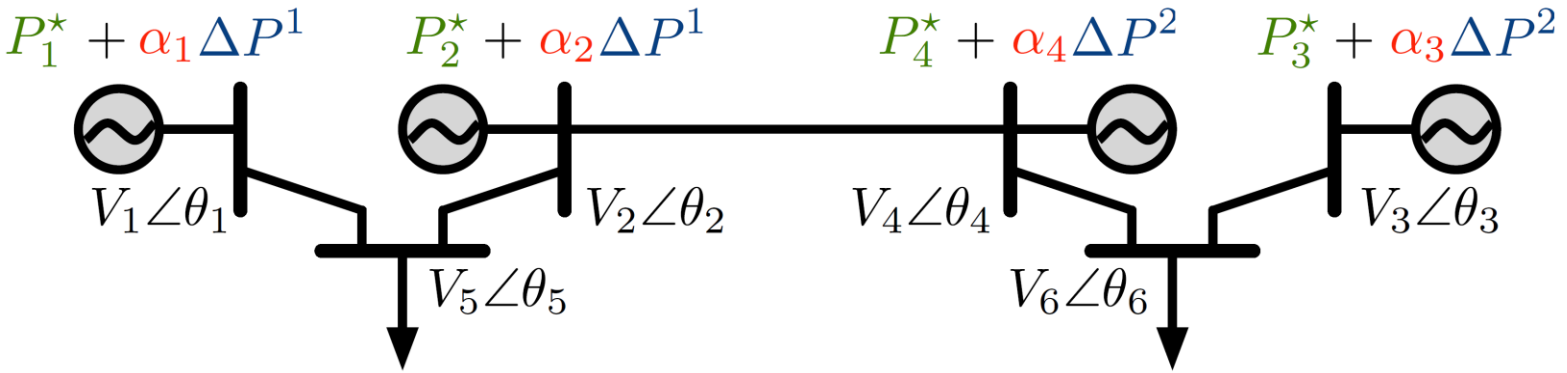


Secondary control

$$P_g^\circ + \pi_g \omega = V_g \sum_{j \in \mathcal{N}^a} V_j (B_{gj} \sin\theta_{gj} + G_{gj} \cos\theta_{gj})$$

$\pi_g = \alpha_g$

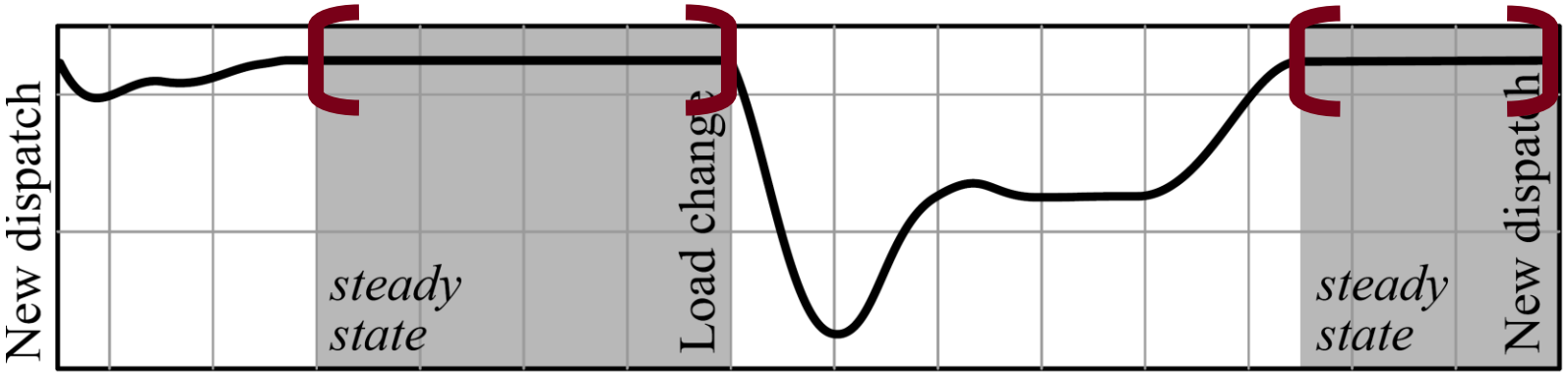


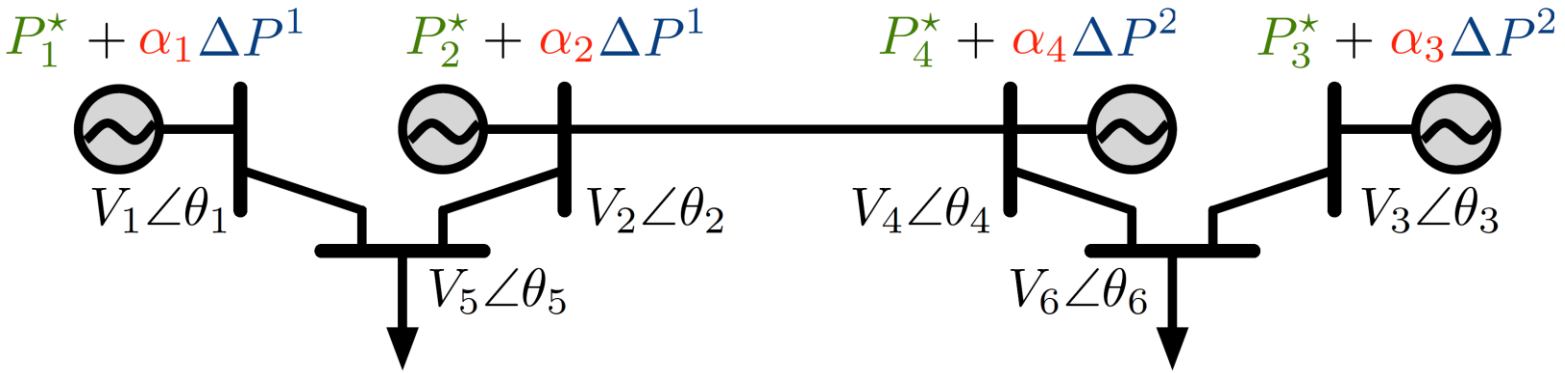


Architecture

$$P_g^\circ + \pi_g \psi = V_g \sum_{j \in \mathcal{N}^a} V_j (B_{gj} \sin\theta_{gj} + G_{gj} \cos\theta_{gj})$$

$$\psi = \Delta P^a$$

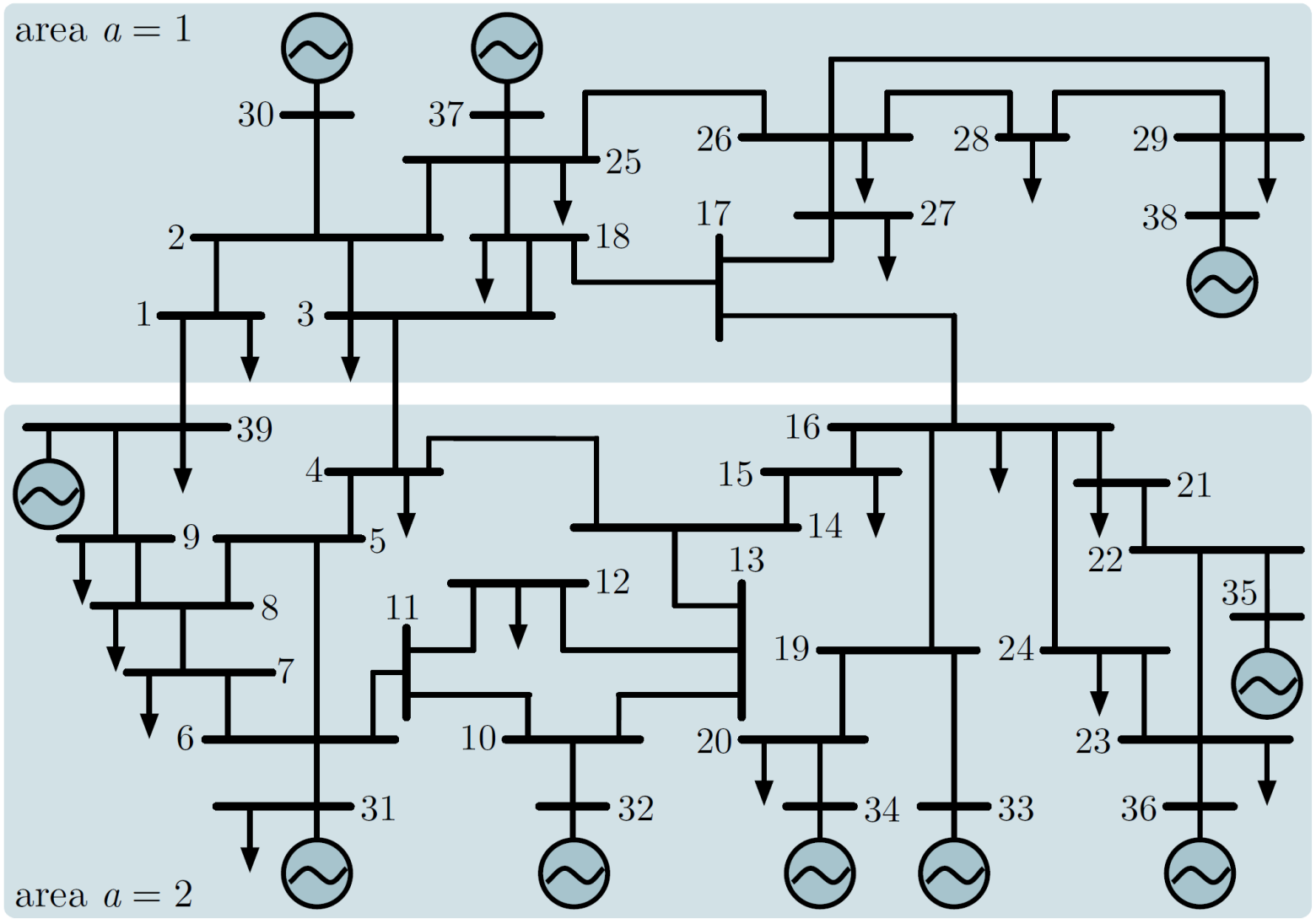




Extensions & corollaries

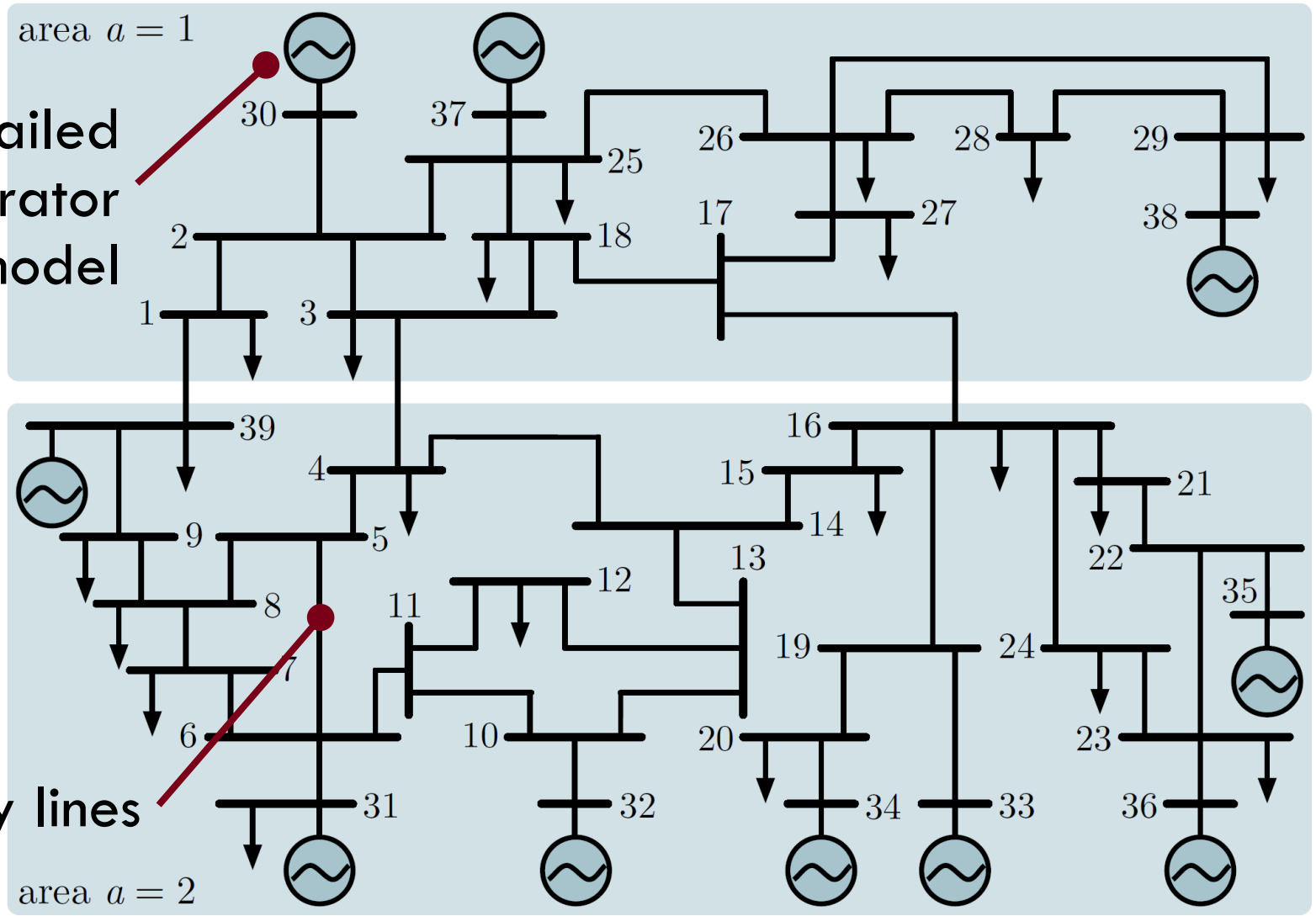
- Subset of generators participate in AGC
  - participation factors for the remainder set to zero
- Only primary control (no AGC)
  - participation factors are based on governor droop slopes
  - slack variable is system wide and not area specific
- DC power flow
  - similar form as conventional setup
  - additionally yields net-load imbalance as a solution

Simulation setup



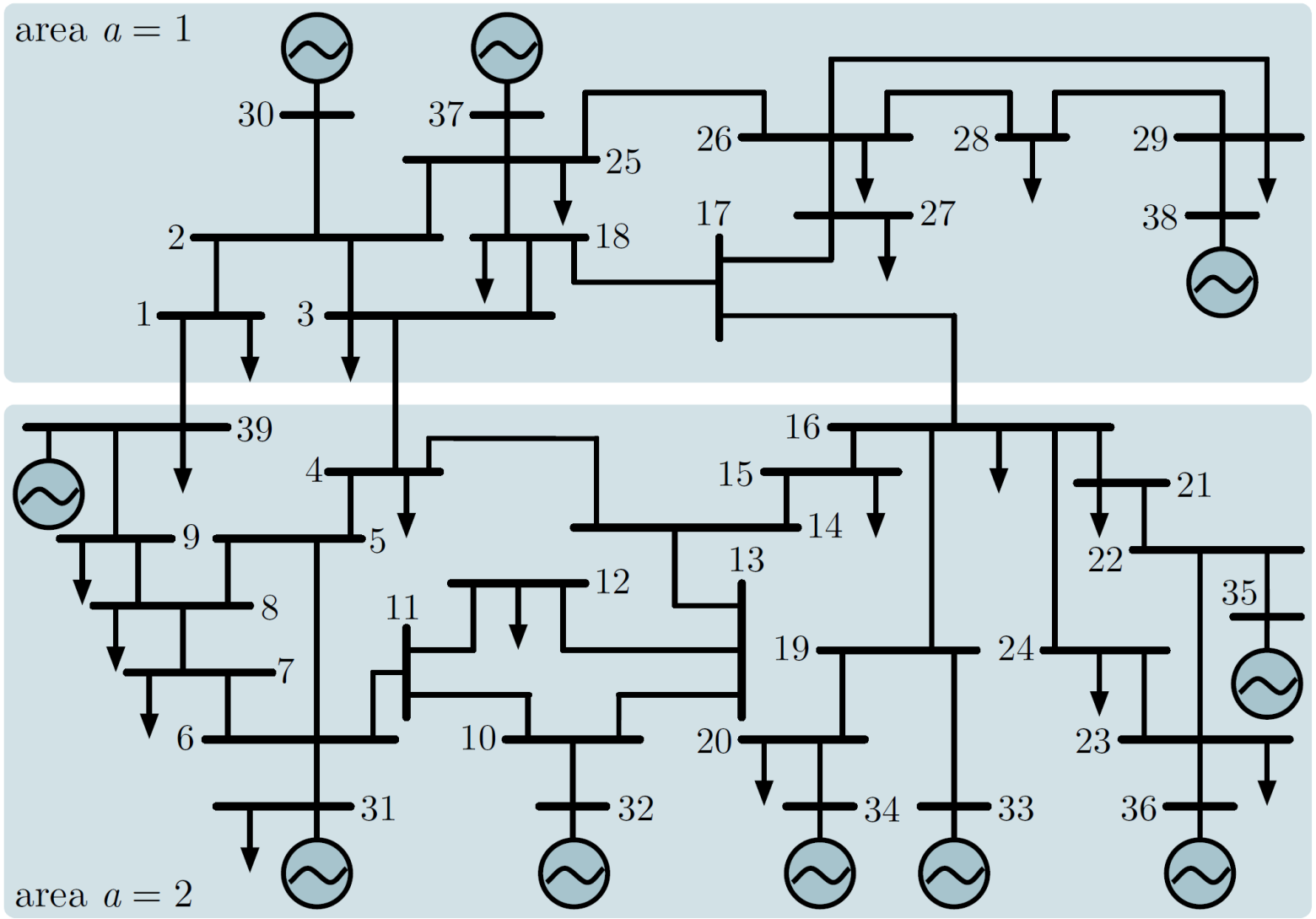
Ground truth

PSAT simulation of system DAE model



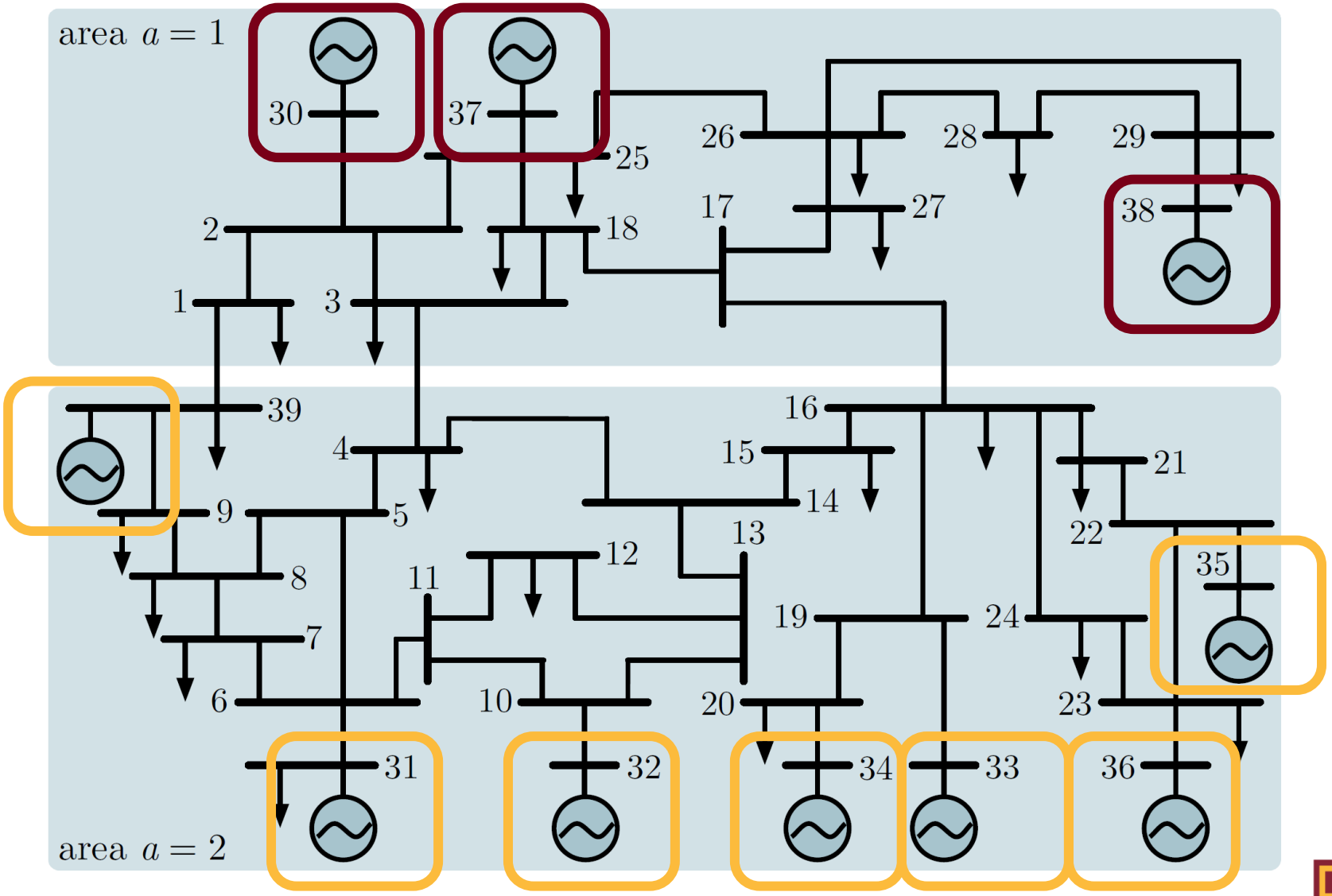
Compare

Distributed slack power flow

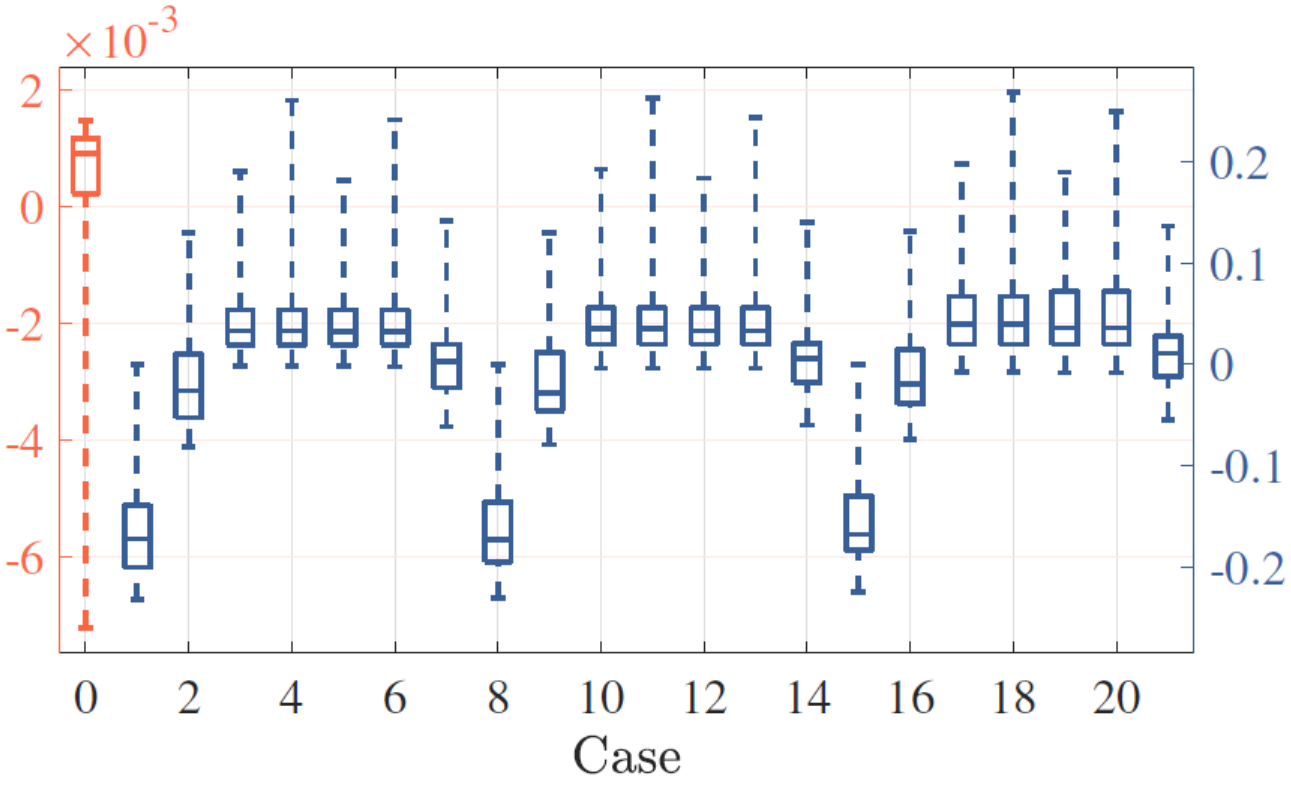


To

Single slack power flow

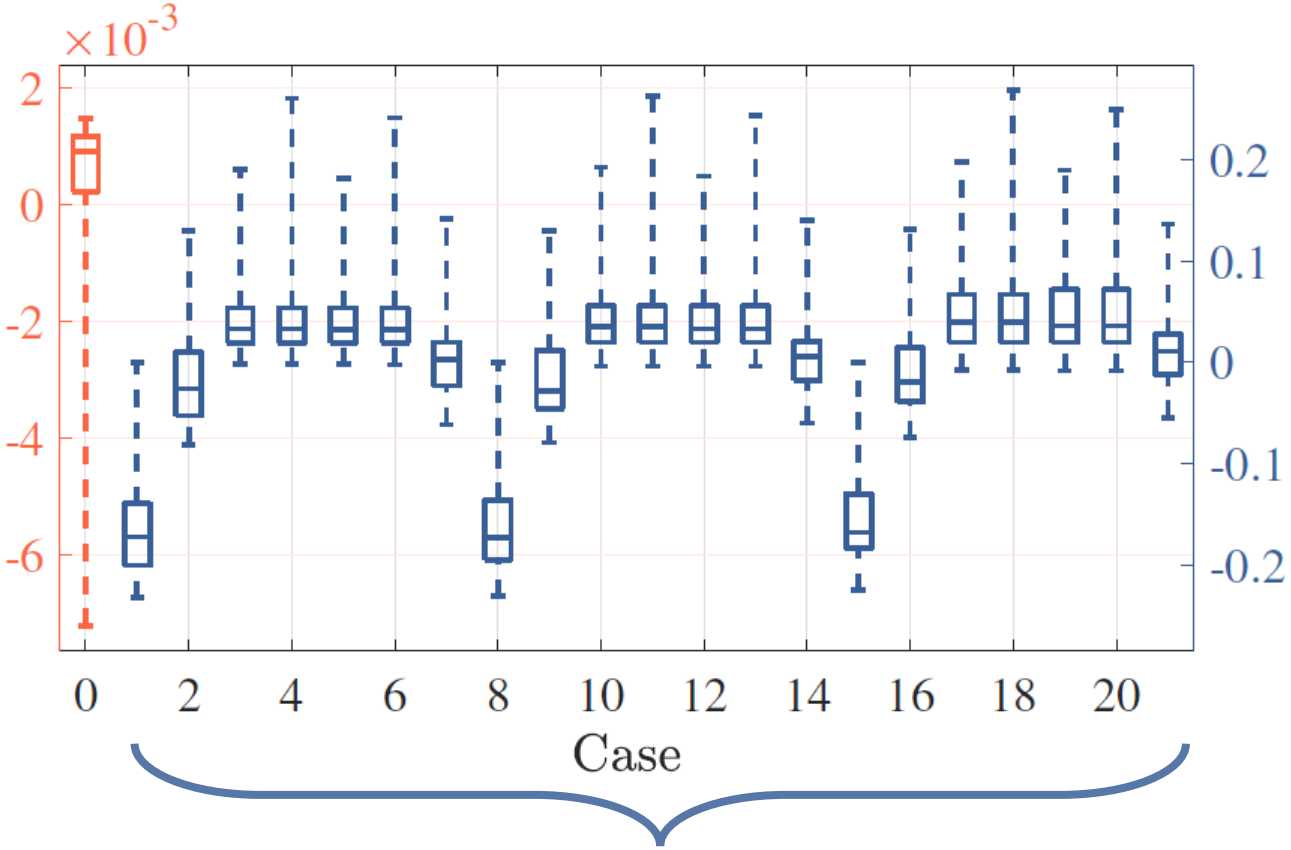


Some results



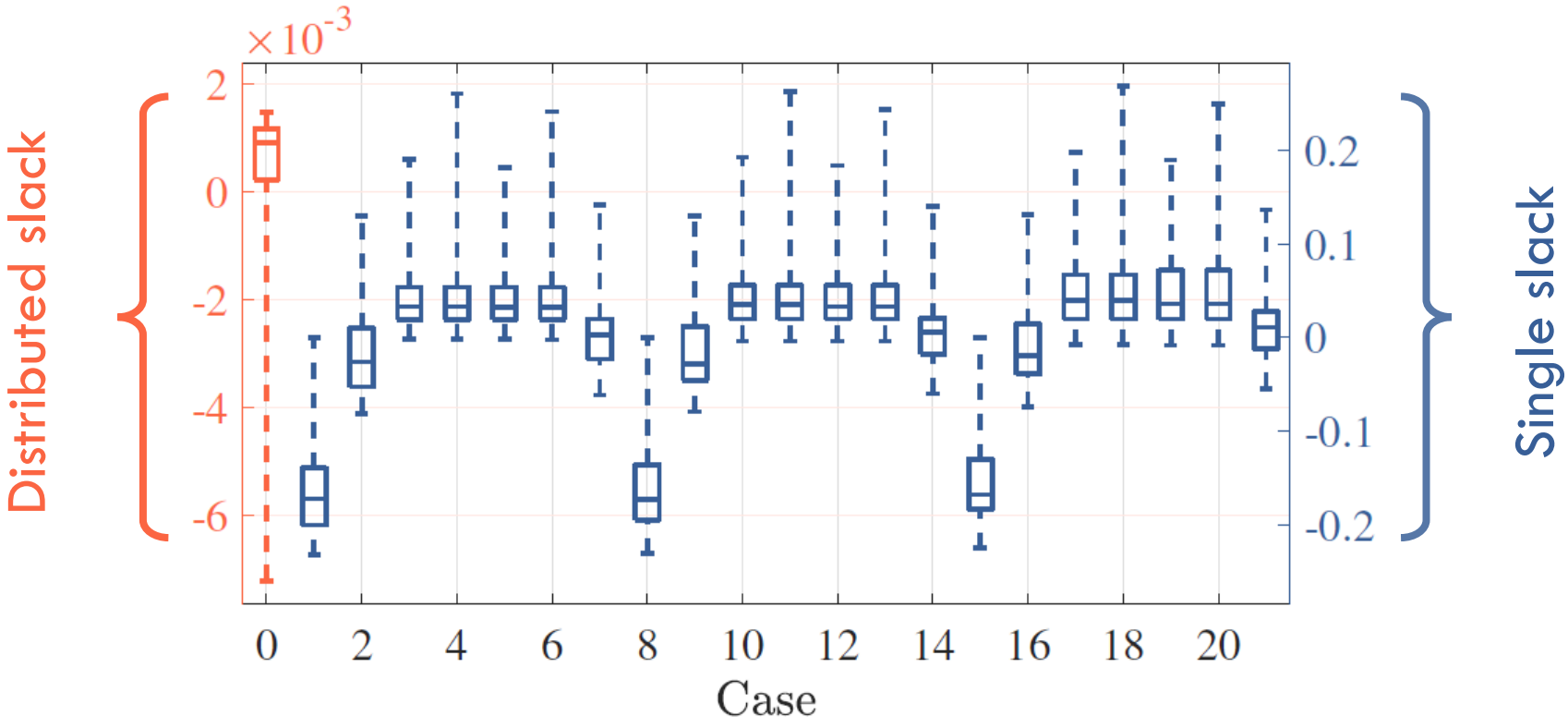
Cases

Distributed slack [Case: 0]

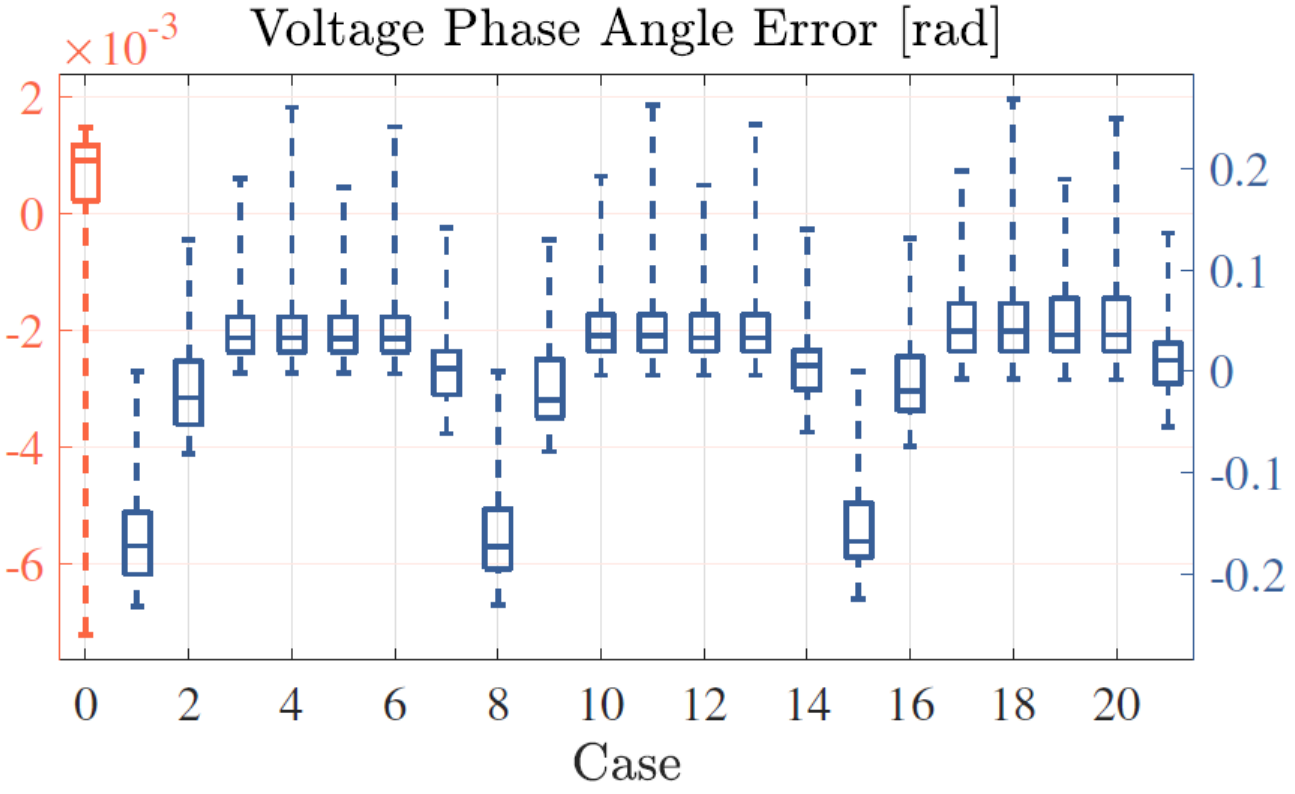


Single slack [Cases: 1-21 (3 x 7)]

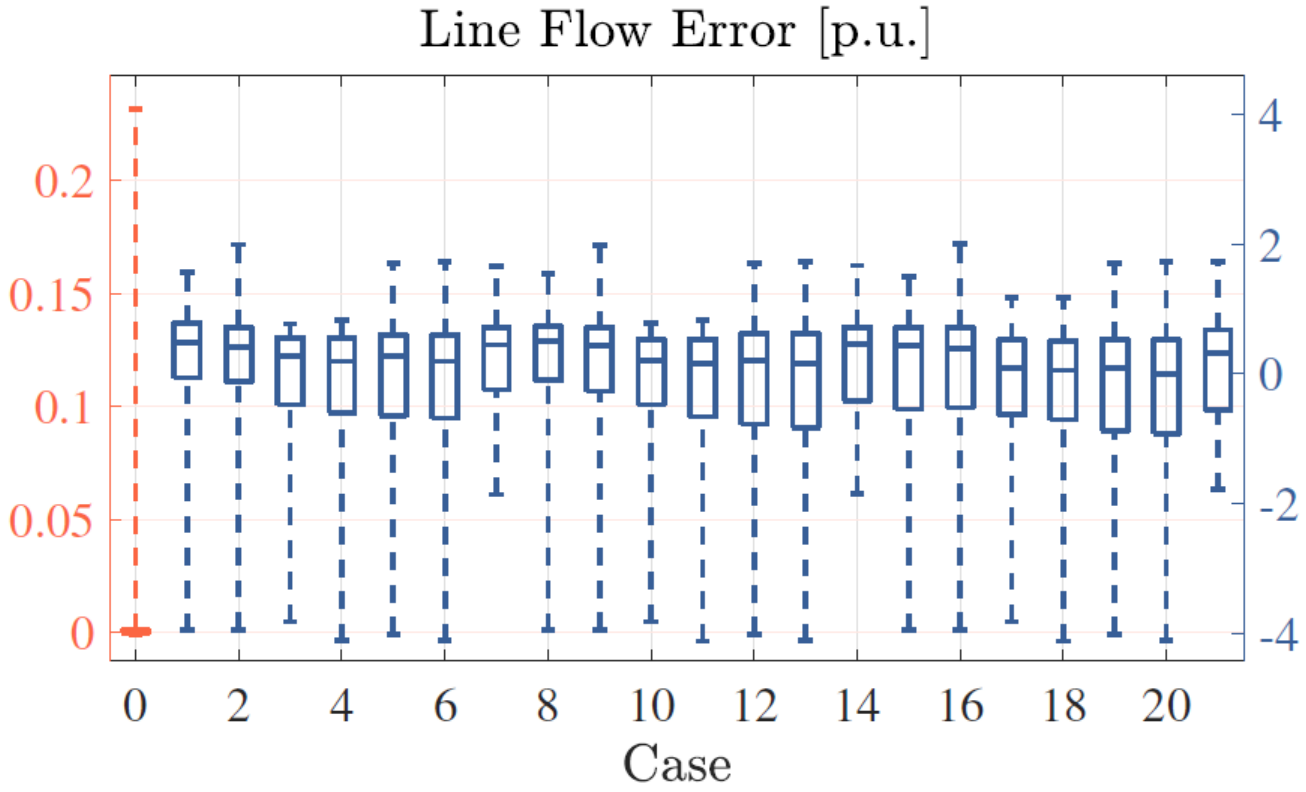
Axes


Distributed slack
×10^-3
2
0
-2
-4
-6
0.2
0.1
0
-0.1
-0.2
Single slack
0
2
4
6
8
10
12
14
16
18
20
Case

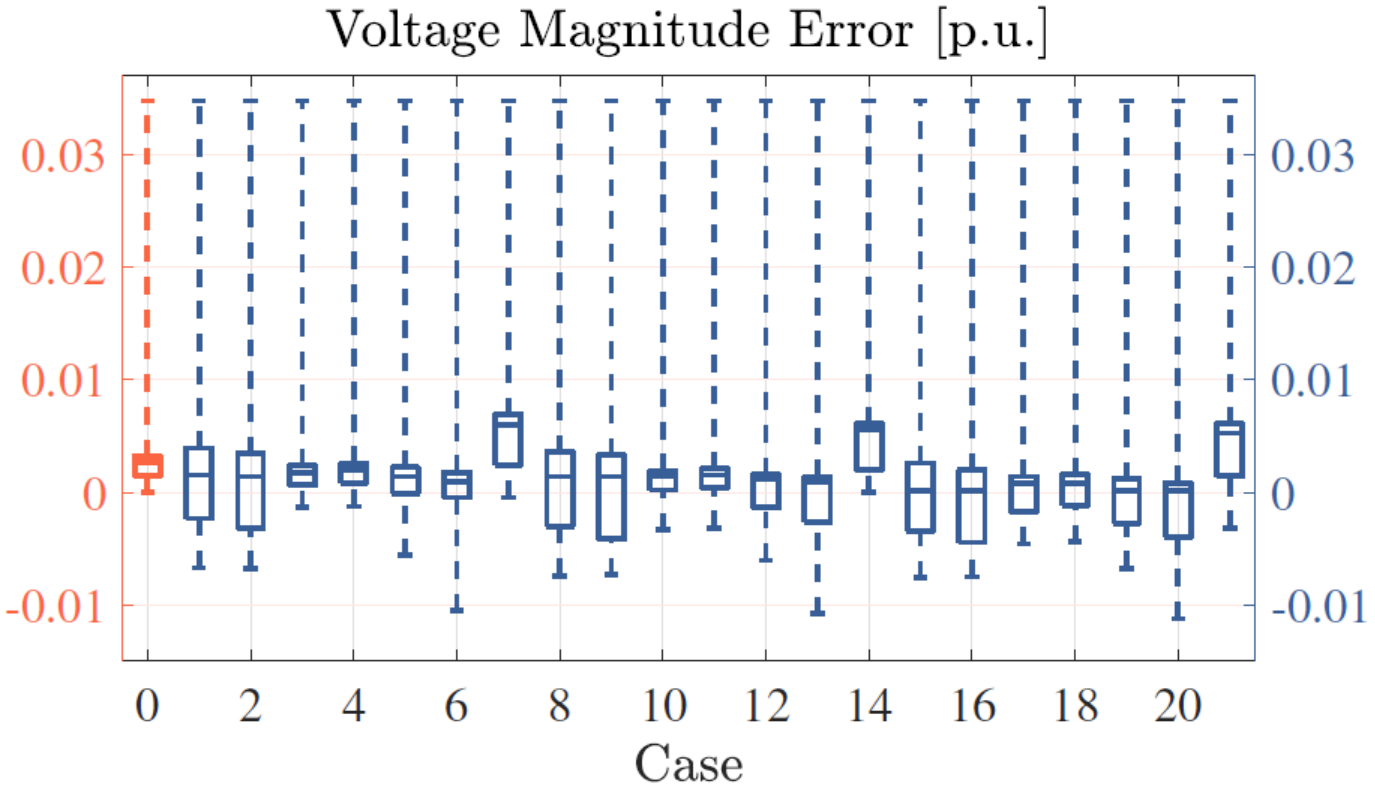
Phase angles



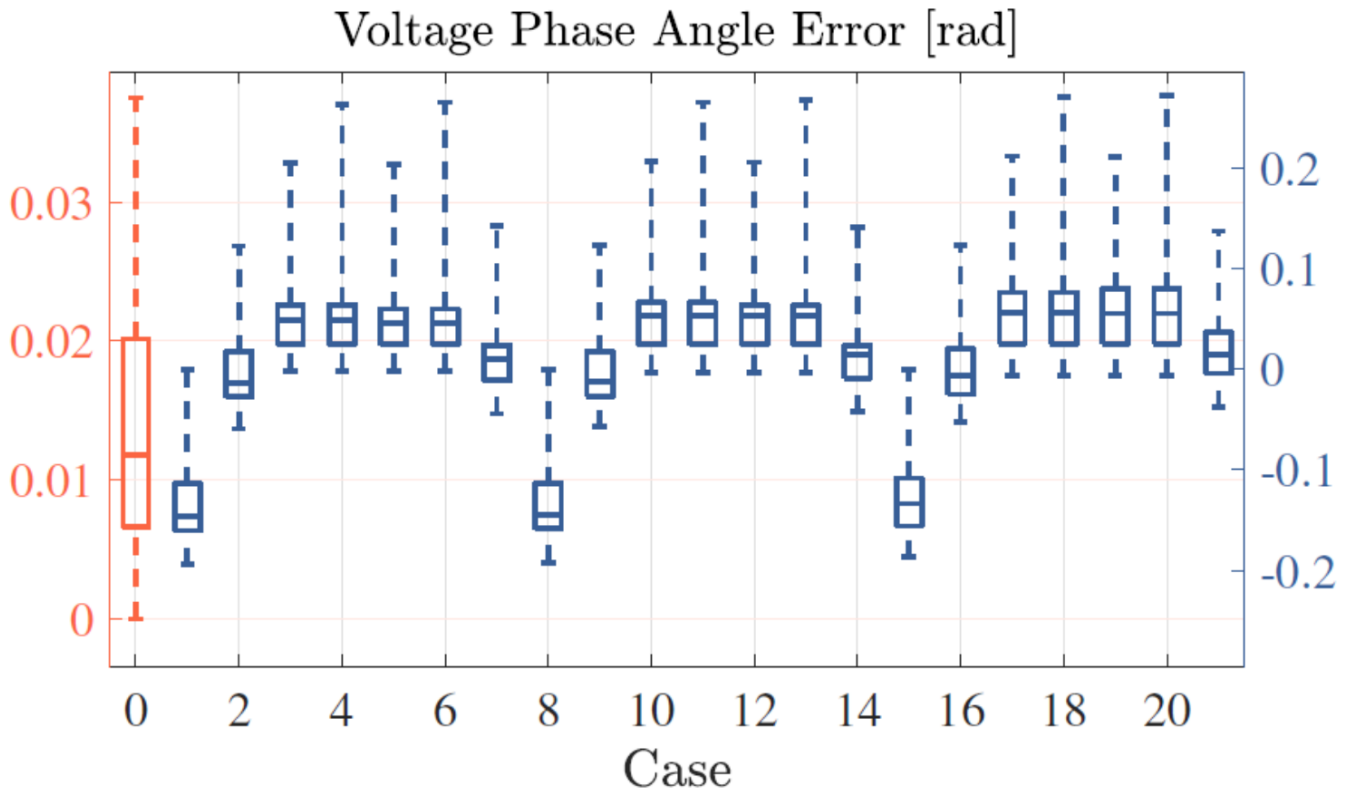
Active-power flows



Terminal voltages

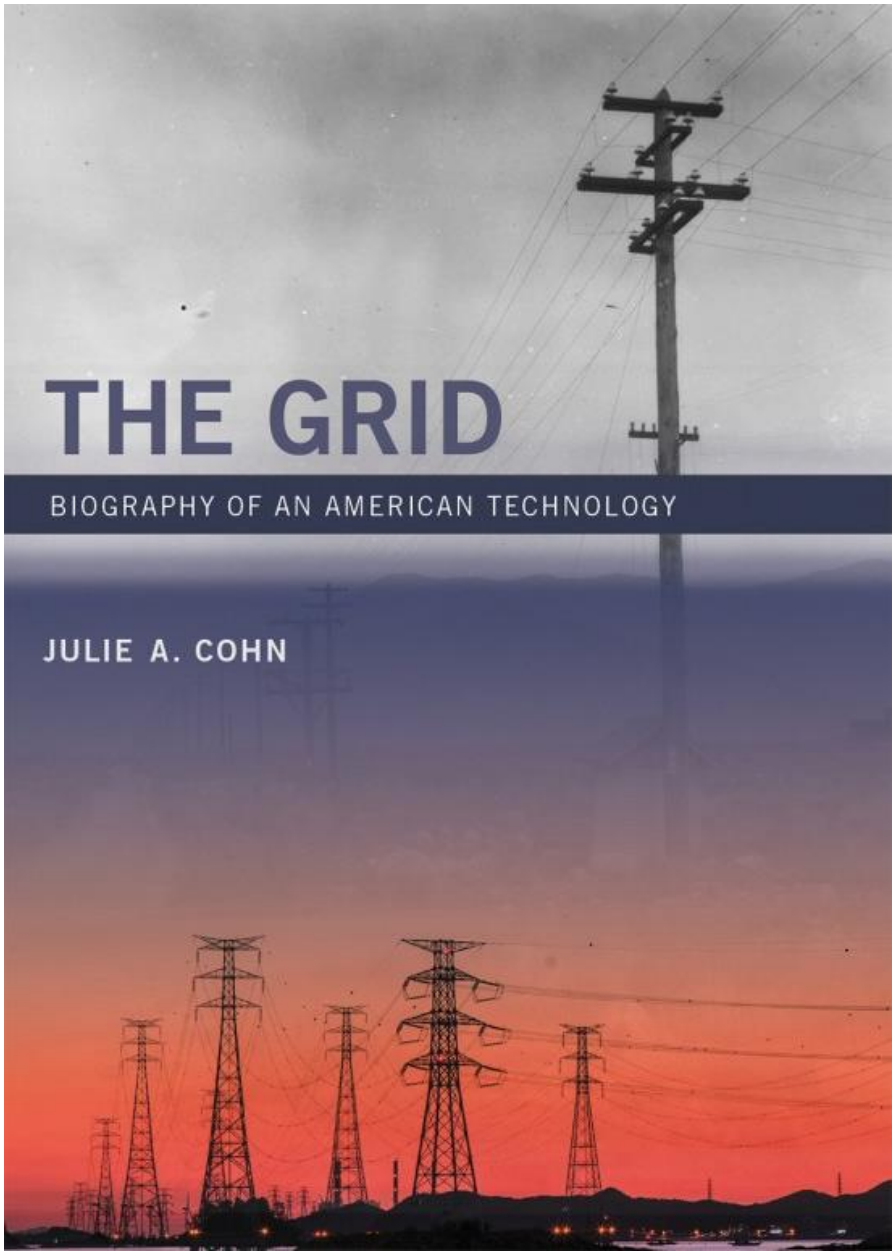


DC Power flow



Back to the future

- Julie Cohn
- Historian at University of Houston
- (Nathan Cohn's daughter)



Questions?