Forward message passing detector for probe storage.

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Plan.

- Thermomechanical probe storage.
- Detection/decoding schemes for probe storage devices.
- Channel model for probe storage.
- Soft output detection: forward message passing (FMP).
- FMP detector for probe storage.
- Performance analysis: mutual information and BER
- Performance analysis: SER
- Conclusions
Storage density trends.

- Magnetic storage density is expected to reach $1 \, Tb/in^2$
- Further growth is limited by the superparamagnetic effect.

- Probe storage has already demonstrated $1 \, Tb/in^2$ with $4 \, Tb/in^2$ demonstrators being developed.
IBM’s Thermomechanical Probe Storage Concept

- Thin polymer medium is positioned under the array of $64 \times 64$ atomic force probes.
- Each probe operates in its own field of size $100\mu m \times 100\mu m$. Tip radius $\sim 10\ nm$.
- Encoded data are stored as pits on the surface of the medium.
Read/Write

- Writing: the probe’s tip is heated and pressed into the softened polymer film
- Reading: The probe heated to a smaller $T$ follows the landscape of the polymer surface
- A probe inserted into a pit is cooler than the probe whose tip touches the surface. These variations are captured using a thermo-resistive sensor
Non-linear Inter-symbol Interference (ISI)

\[ I_k(x_{k-1}, x_k, x_{k+1}) = x_k + (\alpha - 1)x_k x_{k+1} + \beta x_k x_{k-1} \]

- Ideal readout at the \( k \)-th sampling point
- A signal due to an isolated pit at \( k \)
- Reduction in the signal strength due to plastic displaced from the \((k + 1)\)-st pit
- Signal enhancement due to plastic displaced into the \((k - 1)\)-st pit

- For experiments at \( 1 \text{ Tb/in}^2 \), \( \alpha \approx 0.8 \), \( \beta \approx 0.1 \)
- \( \alpha, \beta \) depend on write parameters, tip shape and medium material properties
Position Jitter

- Jitter = positioning error
- $J << \text{pit width}$
- $\Delta I_k \sim J_k^2$
- Over 40% of total noise power is due to data-dependent position jitter

$$r_k \approx I_k \cdot \left(1 - \left(\frac{\sigma_j}{h}W_k\right)^2\right) + \sigma_e N_k,$$

$W_k, N_k$ are independent normal r. v.'s, $h$ is the pit's radius of curvature; $\sigma_j, \sigma_e$ are the strengths of jitter and electronics noise correspondingly
Statistics of Signal Distortion

- Let $\eta_k = \frac{r_k - I_k}{I_k}$
- Let $\epsilon = \frac{\sigma_j}{h}$, $\delta = \frac{\sigma_e}{I_k}$.
- $\rho(\eta) \sim \frac{{\text{Const}}^-}{|\eta|^{1/2}} e^{-\frac{\eta^2}{2\epsilon^2}}$, $\eta << -\epsilon^2$
- $\rho(\eta) \sim \frac{{\text{Const}}^+}{\eta^{1/2}} e^{-\frac{\eta^2}{2\delta^2}}$, $\eta >> \delta$
- Signal distortion is non-Gaussian
Detection/decoding
The currently employed scheme

- Read channel: hard output threshold detector
- ECC: Reed-Solomon code

- HDD read channel: a sector of data is detected as the *most likely* binary string given the digitised received string using Viterbi algorithm.

- A significant increase of recording density beyond 1 Tb/in² would require a significantly more advanced detection decoding scheme.
**Desired scheme**

- Read channel: soft output data detector
- ECC: Soft input decoder for $LDPC$, $LDPC \circ RS$, $SPC \circ RS$, etc. code
- MAP detector per probe is too complex
- Any easy ways to generate soft outputs?
Soft detection via forward message passing
Soft threshold detector.

\[ LLR_k \overset{\text{def}}{=} \ln \frac{Pr(x_k=1|r_k)}{Pr(x_k=0|r_k)} \overset{\text{Bayes}}{=} \ln \frac{Pr(r_k|x_k=1)}{Pr(r_k|x_k=0)} \]

- Threshold bit estimate: \( \hat{x}_k = \text{sign} LLR_k \)
- Information contained in \( r_{k'}: k' \neq k \) is not used in the computation of \( LLR_k \).
Forward message passing detector.

- Assume that $r_k$ depends on $x_k, x_{k\pm 1}$ only.

- Let $LLR_k = \ln \left( \frac{Pr(x_k=1|\vec{r}_{k+1})}{Pr(x_k=0|\vec{r}_{k+1})} \right)$, where $
vec{r}_k = \ldots r_{k-3}r_{k-2}r_{k-1}r_k$. Then

$$Pr(\vec{r}_k \mid x_{k+1}, x_k, x_{k-1}) = \frac{1}{2} Pr(r_k \mid x_{k+1}, x_k, x_{k-1}) \sum_{\sum x_{k-2}=0}^1 Pr(\vec{r}_{k-1} \mid x_k, x_{k-1}, x_{k-2})$$

- Message is an 8-dimensional vector of probabilities $Pr(\vec{r}_k \mid x_{k+1}, x_k, x_{k-1})$ propagated left-to-right using transfer matrix built out of conditional probabilities $Pr(r_k \mid x_{k+1}, x_k, x_{k-1})$. 
Transition matrix

\[
T = \begin{pmatrix}
\alpha_i & \alpha_i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_i & \alpha_i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_i^{(1)} & \beta_i^{(1)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_i^{(2)} & \beta_i^{(2)} \\
\alpha_i & \alpha_i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_i & \alpha_i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_i^{(3)} & \beta_i^{(3)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_i^{(4)} & \beta_i^{(4)}
\end{pmatrix}
\]

where \( \alpha \)'s and \( \beta \)'s are conditional probabilities.

- \( T_k \) is time-dependent. But, there are 4 time independent right null vectors and 2 time independent left null vectors.

- Forward recursion can be reduced to a \( 3 \times 3 \) recursion
Reduced recursion.
Performance analysis
Mutual information

- Mutual info between data and output LLR’s:
  \[ I(X, L) = \mathbf{E}_X \left( \int_{-\infty}^{\infty} dl \rho(l | x) \log_2 \left( \frac{\rho(l | x)}{\rho(l)} \right) \right) \]

- FMP detector resolves the asymmetry of LLR’s. Channel capacity is increased by about 5% compared to THD channel
The performance of FMP (green curve) matches the performance of Viterbi detector (black curve).
Sector error rate: large deviations

**Outer code:** RS$(w, \tau, nN)$. **Inner code:** block size is $nw + t$ bits. Symbol error counts for different IC blocks are independent identically distributed random variables. Let $\vec{p} = \{p_0, p_1, \ldots, p_n\}$ be the probability distribution of symbol error count $\xi$ in an IC block such that $E(\xi) < \tau$. Then

$$\frac{\ln(P_{se})}{N} \rightarrow -D_{KL}(\vec{q}||\vec{p}), \text{ as } N \rightarrow \infty.$$ 

where $\vec{q}$ is the effective probability distribution given by

$$q_k = \frac{p_k \mu^k}{\sum_{m=0}^{n} p_m \mu^m}, \quad k = 0, 1, \ldots, n$$

and $\mu$ is the unique positive solution of the critical point equation,

$$\sum_{k=0}^{n} (k - n\tau) p_k \mu^k = 0; \quad D_{KL} \text{ is relative entropy}$$
Sector error rate comparison

Non-Linear PS Channel, Jitter=0.05, 4000 Probes, R = 0.8

- RS
- SPC_{11}
- SPC_{21}
- SPC_{31}
Conclusions-I

- Probe storage DSP is challenging to the extreme: on the one hand we have a very noisy channel, on the other - the allowed complexity of read channel per probe is severely restricted.

- Forward message passing detector allows a generation of soft outputs at the complexity cost of a 3-state Viterbi detector without traceback unit with the performance matching that of the full 4-state Viterbi detector matched to the non-linear thermomechanical channel.

- Large deviations analysis leads to an analytic expression for SER in probe storage, which is useful for sufficiently short inner codes.

- Soft input $SPC + RS$ code outperforms hard input $RS$ code of the same rate by about $1\ dB$ at $SER = 10^{-15}$. 

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Conclusions-II

- A twist in the tale: asymptotically, $RS$ code is better!
- Research supported by PROTEM FP6 European network grant
- The reported results will be published in the proceedings of ICC2008.