Cavity Method for Quantum Spin Glasses

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Outline

• What we’ve studied
• Why is this interesting
• Some results
• Future directions
Ising Spin Glass on Bethe Lattice

• Infinite limit of Cayley (q-regular) trees

\[ H = - \sum_{(ij)} J_{ij} \sigma_i^z \sigma_j^z - B_t \sum_i \sigma_i^x \]

Non-commuting (i.e. Quantum)

\[ P(J_{ij}) = \frac{1}{2} \delta(J_{ij} - J) + \frac{1}{2} \delta(J_{ij} + J) \]
Classical Cavity Method ($B_t = 0$)

Iteration Equations for Cavity Fields

where

\[ \psi_0(\sigma_0) = \frac{1}{Z} \sum_{\sigma_1, \ldots, \sigma_{q-1} = \pm 1} \exp \left( \beta \sum_i J_{0i} \sigma_0 \sigma_i \right) \psi_1(\sigma_1) \ldots \psi_{q-1}(\sigma_{q-1}) \]

\[ h_0 = \frac{1}{\beta} \sum_{i=1}^{q-1} \tanh^{-1}(\tanh(\beta J_{0i}) \tanh(\beta h_i)) \]

\[ \psi_i(\sigma_i) = \frac{e^{\beta h_i \sigma_i}}{2 \cosh(\beta h_i)} \]
Classical Cavity Method (cont’d)

• For the non random case, fixed points of the iteration equation yield cavity fields well away from the outer layer of the tree

• For the random case, consider a distribution of cavity fields which reproduces itself in the interior of the tree (generational average)
Classical Cavity Method (cont’d)

Replica Symmetric Fixed Point for Cavity Field Distribution:

\[ P(h) = \int \prod_{i=1}^{q-1} dh_i P(h_i) \langle \delta(h - U(h_i, J_{0i})) \rangle_J \]

Solution in SG phase; above T_c weight only at h=0
Quantum formulation

• Useful to think in path integral language
• Integrating out ancestor spins generates a cavity effective action for given spin
• At next step combination of “bare” action and cavity action give rise to a functional recursion relation
• For spin glass, study distribution of cavity actions
Why is this interesting?

• Nature of ordering in spin glass unsettled (Catholics vs Protestants)
• Quantum effects in ordered phase could use more microscopic investigation
• Leads to approximate theory of systems with sparse loops, such as random graphs with fixed connectivity
• Corresponding algorithm is belief propagation

QMAC problems
Quantum BP
Details: Trotter Decomposition

- Transverse field $B_t$ generates ferromagnetic coupling $\Gamma$ in imaginary time.
- Disordered within hyperplanes; correlated along imaginary time.
Cavity Actions

Cavity Fields

\[ \psi_i(\sigma_i) = \frac{e^{\beta h_i \sigma_i}}{2 \cosh(\beta h_i)} \]

Cavity Actions

\[ \psi[\sigma(t)] = e^{-S[\sigma]} \]

\[ S[\sigma] = - \log Z - \hbar \Delta t \sum_t \sigma(t) - \sum_{t,t'} \Delta t^2 C^{(2)}(t' - t)\sigma(t)\sigma(t') - \]
\[ - \sum_{t,t',t''} \Delta t^3 C^{(3)}(t' - t, t'' - t')\sigma(t)\sigma(t')\sigma(t'') + ... \]
Quantum Fixed Point

Use population dynamics to generate distribution

\[
P_{FP}[\psi[\sigma(t)]] = \langle \delta [\psi[\sigma(t)] - \psi_0[\sigma(t); \{J_{0i}, \psi_i\}_{i=1}^{q-1}]] \rangle_{J_{0i}, \psi_i}
\]

\[
= \int (\prod_{i=1}^{q-1} D\psi_i P_{FP}[\psi_i]) dJ_{0i} P(J_{0i}) \delta [\psi[\sigma(t)] - \psi_0[\sigma(t); \{J_{0i}, \psi_i\}_{i=1}^{q-1}]]
\]
Elementary treatment

- 6-11 time slices
- 2500 cavity actions
- 2500 * 1000 iterations
- Keep only two spin interactions in effective action, but between all time slices
- Still, nontrivial results, cf Usadel & Co.
Phase diagram: $q = 3$
Fixed point distributions

• In PM phase there is no local magnetic field; spin-spin interaction has unique value (generally, cavity action is unique in PM phase)
• In SG phase field has a distribution – needed to produce EA op - as does the interaction
Imaginary Time Interactions & Correlations

Next Nearest Neighbor Interaction ($N_y = 6, N_z = 1$)

Imaginary Time Correlation Function ($\beta = 4, N_y = 11$)

$\tau_z = \beta$

Units of $\beta = \beta / N_y$
Single Spin von Neumann Entropy
Future directions

• Run the iteration on a fixed instance of a random graph (QBP)
• Continuous time formalism
• MC evaluation of cavity action (MCRG)
• Analytic limits
• What would a truly quantum BP look like - one that would run on a quantum computer? (Fixed point behavior versus unitary evolution, QMAC by QC)