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Empirical Validation of the Relationship Between **Survey Propagation and Covers** in Random 3-SAT

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Introduction

$$F = (\neg x \vee y \vee z) \wedge (x \vee \neg y \vee z) \wedge (x \vee y \vee \neg z)$$

What are covers?

- Generalized {0,1,*} assignments such that

  i. Every clause has a satisfying literal or ≥ 2 *s

  ii. Every non-* variable has a certifying clause in which all other literals are false

  e.g. F has covers (***) and (000)

- Relation to satisfying assignments:
  - True covers = generalize some satisfying assignment
  - False covers = do not generalize any sat. assignment

Introduction

Why are covers interesting?

- SP is Belief Propagation on the Cover Problem
  [Braunstein, Zecchina '03;
  Maneva, Mossel, Wainwright '04]

SP must compute a loopy approximation to cover marginals

- Covers provably exist in k-SAT for k≥9
  [Achlioptas, Ricci-Tersenghi '06]
- Believed *not* to exist in random 3-SAT

Could SP be computing something else on random 3-SAT?

Preview of Our Results

1) Empirical evidence that covers do exist in large random 3-SAT formulas

2) SP computes cover marginals remarkably well

3) Cover marginals correlate well with solution marginals

4) BP/SP provides useful information even on structured formulas

Properties of Covers I

- Covers represent clusters of solutions
  - * generalizes both 0 and 1
  - Solutions that differ in one bit are represented by the same cover

1	*	0	*
1	0	0	0
1	0	0	1
1	1	0	0
1	1	0	1

- Every formula (sat or unsat) without unit clauses has the *trivial cover*, ***
- Tree formulas without unit clauses have *only* the trivial cover

⇒ *SP does not compute anything interesting on tree formulas!*

Properties of Covers II

- Unlike finding solutions, finding covers is not a self-reducible problem

  $\Rightarrow$ covers cannot be computed by simple decimation

  e.g. if we guess that in some cover $x=0$, and use decimation:

$$F=(\neg x \vee y \vee z)\wedge(x \vee \neg y \vee z)\wedge(x \vee y \vee \neg z)$$

$$F'=(\neg y \vee z)\wedge(y \vee \neg z)$$

  (11) is a cover for F'

  but (011) is *not* a cover for F

Searching for Covers

- Using an appropriate SAT encoding
  - Create a new formula whose solutions represent covers of the original formula
  - Can enumerate all covers
  - Not scalable (up to N~100 variables)

- Using local search on the original formula
  - Scales well (can find true covers for N=20K)
  - Algorithm inspired by the “peeling-procedure” [Maneva, Mossel, Wainwright '04] :

    (a) Sample a solution using SampleSat

    (b) *-propagate to a cover (turn every uncertified 0 or 1 into a * until no such variable)

Part 1

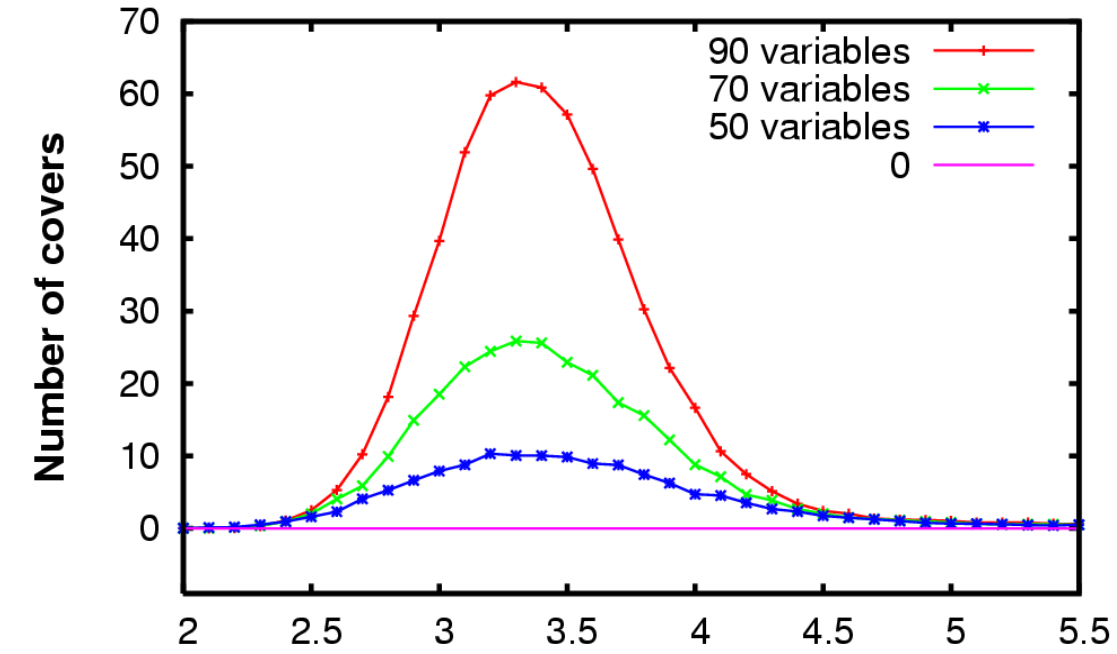
Do non-trivial covers exist in random 3-SAT?

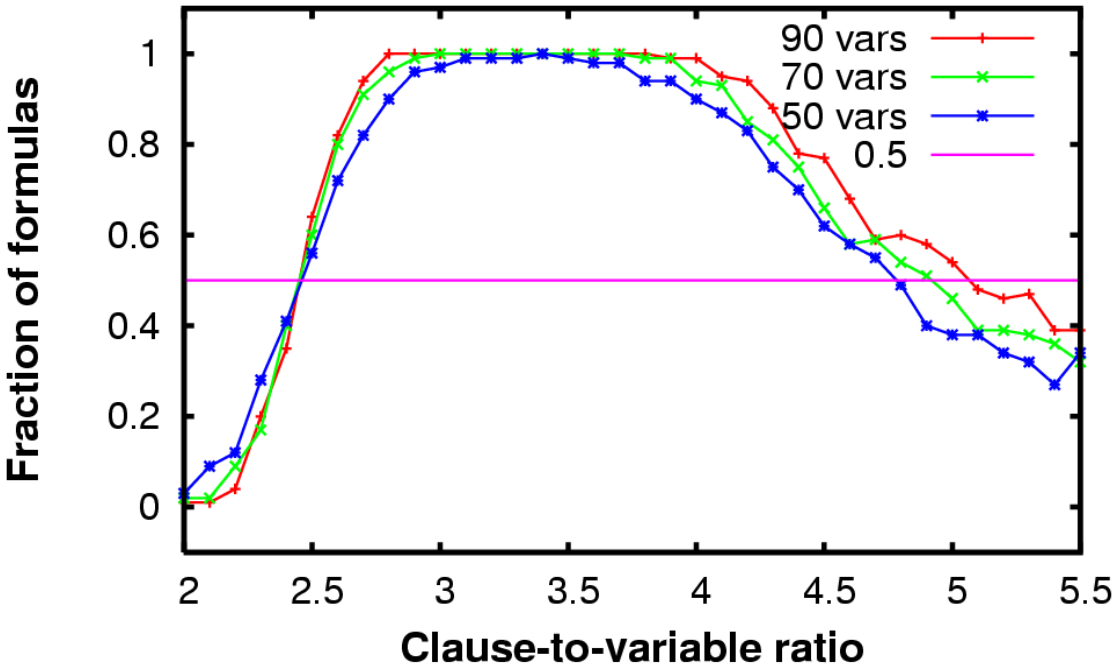
Can SP compute cover marginals?

How do cover marginals relate to solutions?

Can BP/SP be used on non-random instances?

SAT Encoding of Covers

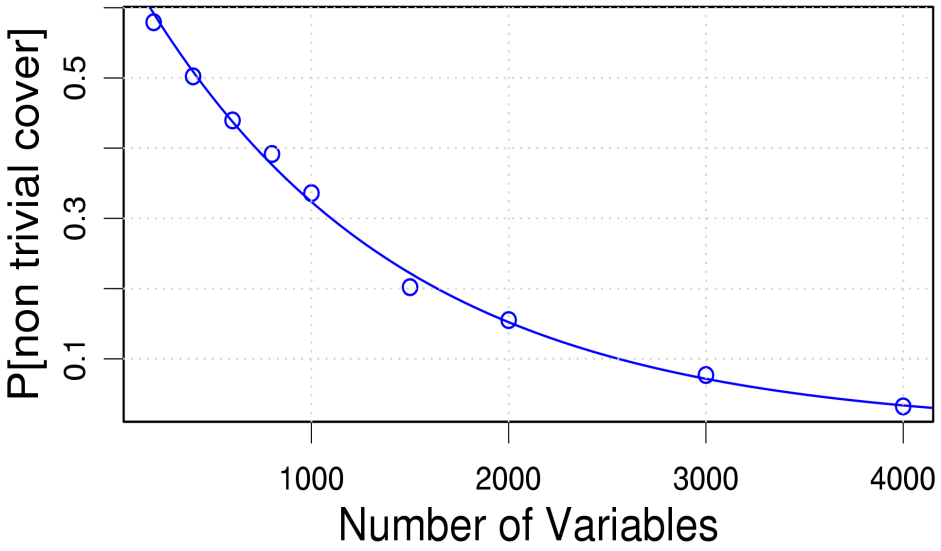


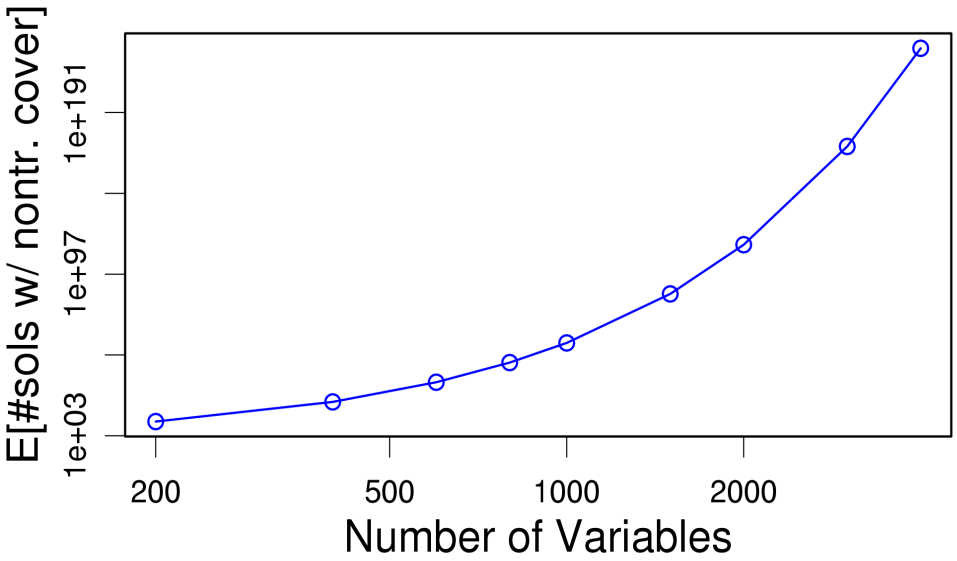


- Number of covers grows with N
- Covers are relatively few
  e.g. ~10 covers vs.
  150K solutions
  for N=90 at α=4.2
- Phase transition near $\alpha = 2.5$
- For larger N, covers exist for a broader range of α

Local Search for Covers

*How often do solutions *-propagate to non-trivial covers?*





Fit $\propto 2^{-\frac{N}{1000}}$ vs. number of solutions $\propto 2^{\frac{N}{5}}$ (α=4.2)

$\Rightarrow$ Expected no. of solutions *-propagating to a non-trivial cover increases exponentially with N

Part 2

Covers do exist in random 3-SAT

Can SP compute cover marginals?

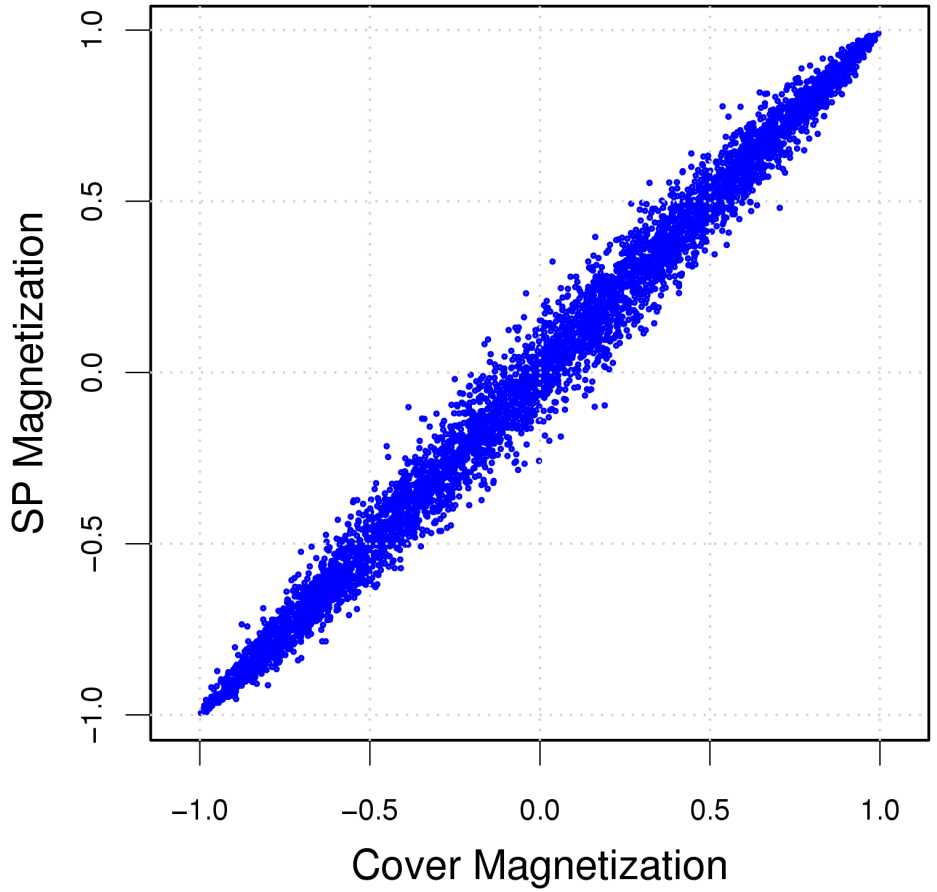
How do cover marginals relate to solutions?

Can BP/SP be used on non-random instances?

Covers vs. SP

Experiment:

1. sample many covers using local search
2. compute cover magnetization from samples (x-axis)
3. compare with SP magnetization (y-axis)



5,000 variables

α=4.2

Part 3

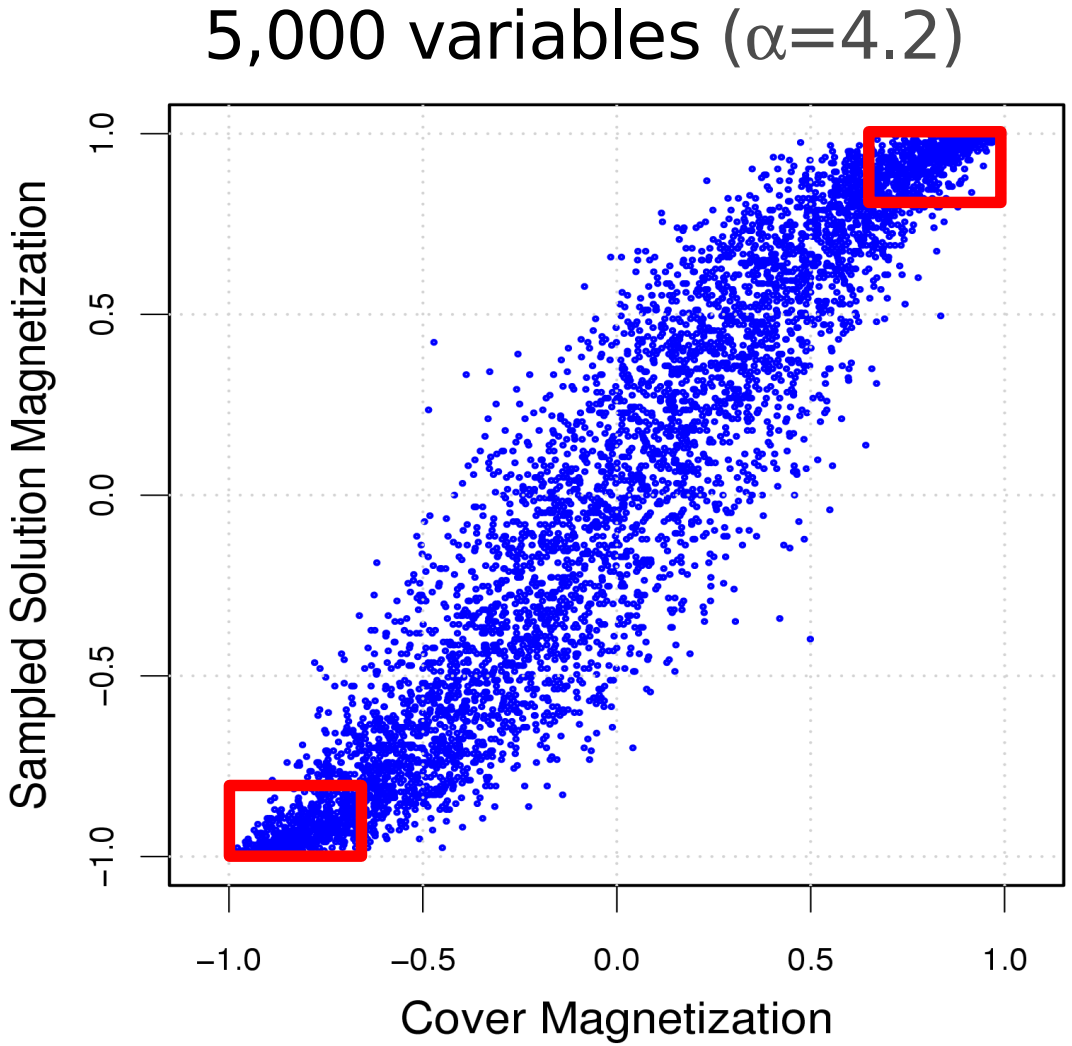
Covers do exist in random 3-SAT

SP is good at computing cover marginals

How do cover marginals relate to solutions?

Can BP/SP be used on non-random instances?

Covers vs. Solutions



Cover marginals appear to be more conservative than (sampled) solution marginals

Part 4

Covers exist in random 3-SAT

SP is good at computing cover marginals

Cover marginals correlate well with solutions

Can BP/SP be used on non-random instances?

BP/SP on Non-random Formulas

- SAT solving by decimation relies heavily on marginals
  - Mistakes can be fatal
  - SP does not work on anything but random formulas

$\Rightarrow$ More natural application:

Counting number of solutions

Counting With BP

- BPcount = marginal estimation + solution search
  - Quality of marginals ∝ Quality of the count
  - (damped) BP gives reasonable estimates
- Results

Problem	Exact Count	**BPcount**	Random margs.
2bitmax	10^29	**10^28**	10^26
LatinSquare8	10^11	**10^11**	10^7
Langford15	10^7	**10^6**	10^3

⇒ BP provides useful info about marginals

Conclusions

- Empirical evidence for the existence of covers in large random 3-SAT formulas
- SP is remarkably accurate in computing marginals over these covers
- Marginals over covers closely correlate with sampled solution marginals
- BP/SP provides useful information even on structured (non-random) formulas