Lattice Boltzmann Simulations of Oscillating Flow and Heat Transfer under Linearization Assumption

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Abstract The analytical solution of temperature distributions for an incompressible oscillating laminar flow between two parallel plates with constant wall temperature is derived, where the viscous dissipation effect is considered. A thermal lattice Boltzmann method (LBM) is then adopted to simulate the incompressible and compressible oscillating flow and heat transfer for a broad range of Stokes number and Reynolds number. Both the velocity and temperature simulation results for incompressible flow show good agreements with the analytical solutions. For the compressible flow, the simulation results show that all fluid particles only oscillating within a respective narrow distance, and the fluid temperature changes with fluid expanding or deflating.

Keywords: Oscillating flow, heat transfer, lattice Boltzmann method, viscous dissipation

INTRODUCTION

Oscillating flow exists in extensive engineering applications, such as biomedical engineering [1], MEMS and regenerative heat engines [2]. In oscillating flow, the flow direction changes periodically and the net mass flow rate is zero in each period, therefore analyses of oscillating flow and its heat transfer characteristics are more difficult than that of unidirectional flow.

A literature survey on oscillating flow and heat transfer shows that the heat transfer investigation in such flows is almost undeveloped due to its complexity though the oscillating flow behavior has been studied greatly both experimentally and theoretically. Xiao [3] presented the analytical solutions for temperature distributions in compressible oscillating flows under linearization assumption. However in many cases, for example, high Prandtl number oscillating flow, the effect of viscous dissipation on the temperature distribution in oscillating flow can not be neglected. The analytical solution of temperature distribution in an incompressible laminar oscillating flow with viscous dissipation has not been developed up to now.

Most numerical simulations of oscillating flow and heat transfer were based on conventional CFD methods [4, 5]. Recently, the lattice Boltzmann method has been rapidly developed, which provides an alternative promising numerical approach for flow and heat transfer studies. Cosgrove et al. [6] adopted LBM to study the flow transition from laminar to turbulent in oscillating channel flow. Additionally, the LBM has also successfully modeled sound waves [7] and acoustic streaming [8].

This paper investigates the oscillating flow and heat transfer theoretically and numerically in consideration with the viscous dissipation effect.

1. ANALYTICAL SOLUTION OF TEMPERATURE DISTRIBUTION

Considering the incompressible, laminar oscillating flow between two parallel plates with a constant wall temperature T_w, as shown in Fig. 1, the fluid is driven only by a sinusoidal pressure gradient along x-direction.

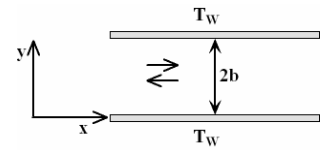

Fig. 1 Schematic diagram of the problem

$$-dp/dx = p_0 e^{i\omega t} \quad , \tag{1}$$

where ω is the oscillating angular frequency. Under linearization assumption, the velocity distribution was given by Pozrikidis [9],

$$u_x = Real\left\{ \frac{p_0}{i\omega\rho}[1-\frac{\cosh[(1+i)\frac{y-b}{\delta_\mu}]}{\cosh[(1+i)b/\delta_\mu]}]e^{i\omega t} \right\}, \tag{2}$$

where *Real* denotes the real part of the expression, $\delta_\mu = \sqrt{2\nu/\omega}$ is the viscous penetration depth. The Stokes number $\Lambda=b/\delta_\mu$ and the Reynolds number $Re_\delta=u_{xmax}\delta_\mu/\nu$, based on the viscous penetration depth δ_μ are used for characterizing the dominant features of the oscillating flow. u_{xmax} is the velocity amplitude of oscillating flow.

The energy equation for a periodically oscillating flow under linearization assumption can be written as

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial^2 y} + \frac{\mu}{\rho c_p}(\frac{\partial u_x}{\partial y})^2 \cdot \tag{3}$$

The temperature solution can be expressed in the form of truncated infinite series if the initial temperature of fluid is equal to the wall temperature. The temperature distribution of the incompressible laminar oscillating flow in consideration with viscous dissipation is derived as,

$$T(y,t) = T_w + \frac{\mu}{\rho c_p}(\frac{p_0}{\omega\rho_0})^2 \frac{1}{b} \cdot \sum_{n=1}^{\infty}\{\int_0^t\int_0^{2b}[\frac{\sin\omega(t-\tau)}{C}\frac{dA_1}{dy} + \frac{\cos\omega(t-\tau)}{C}\frac{dA_2}{dy}]^2 \sin(\frac{n\pi y}{2b})\exp(-\frac{n^2\pi^2 a^2}{4b^2}\tau)dyd\tau\} \tag{4}$$

where

$$A_1 = \cosh\frac{b}{\delta_\mu}\cos\frac{b}{\delta_\mu}\cosh\frac{y-b}{\delta_\mu}\cos\frac{y-b}{\delta_\mu} + \sinh\frac{b}{\delta_\mu}\sin\frac{b}{\delta_\mu}\sinh\frac{y-b}{\delta_\mu}\sin\frac{y-b}{\delta_\mu}$$

$$A_2 = \cosh\frac{b}{\delta_\mu}\cos\frac{b}{\delta_\mu}\sinh\frac{y-b}{\delta_\mu}\sin\frac{y-b}{\delta_\mu} - \sinh\frac{b}{\delta_\mu}\sin\frac{b}{\delta_\mu}\cosh\frac{y-b}{\delta_\mu}\cos\frac{y-b}{\delta_\mu}$$

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$$C = (\cosh\Lambda\cos\Lambda)^2 + (\sinh\Lambda\sin\Lambda)^2 .$$

The detailed deriving process is not given here due to the space limitation.

2. LATTICE BOLTZMANN MODELS

The lattice Boltzmann model implemented in the current investigation is the double-species LBE model proposed by He et al. [10]. The macroscopic density and velocity fields are simulated using the density distribution function, and the temperature field is simulated using the thermal energy distribution function. The evolution equation for density distribution function is

$$f_i'(\boldsymbol{r}+\boldsymbol{c}_i\delta t, t+\delta t) - f_i'(\boldsymbol{r},t) = -\frac{\delta t}{\tau_v + 0.5\delta t}[f_i'(\boldsymbol{r},t) - f_i^{eq}(\boldsymbol{r},t)] + \frac{\tau_v F_i \delta t}{\tau_v + 0.5\delta t} . \quad (5)$$

The evolution equation for the thermal energy distribution function is

$$g_i'(\boldsymbol{r}+\boldsymbol{c}_i\delta t, t+\delta t) - g_i'(\boldsymbol{r},t) = -\frac{\delta t}{\tau_c + 0.5\delta t}[g_i'(\boldsymbol{r},t) - g_i^{eq}(\boldsymbol{r},t)] - \frac{\tau_c f_i(\boldsymbol{r},t) q_i \delta t}{\tau_c + 0.5\delta t} \quad (6)$$

where

$$f_i' = f_i + \frac{\delta t}{2\tau_v}(f_i - f_i^{eq}) - 0.5F_i\delta t , \quad (7)$$

$$g_i' = g_i + \frac{\delta t}{2\tau_v}(g_i - g_i^{eq}) + 0.5 f_i q_i \delta t , \quad (8)$$

$$F_i = \frac{\boldsymbol{G}\cdot(\boldsymbol{c}_i - \boldsymbol{u})}{RT} f_i^{eq} , \quad (9)$$

$$q_i = (\boldsymbol{c}_i - \boldsymbol{u})\cdot[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{c}_i\cdot\nabla)\boldsymbol{u}] . \quad (10)$$

$\boldsymbol{G}$ is the external force acting on unit mass. q_i represents the effects of viscous dissipation. In the simulation, the macroscopic density ρ, velocity $\boldsymbol{u}$, internal energy ε, heat flux $\boldsymbol{\varphi}$, kinematic viscosity ν and thermal diffusivity ζ are

$$\rho = \sum_i f_i' \quad (11)$$

$$\rho\boldsymbol{u} = \sum_i c_i f_i' + \frac{\rho\boldsymbol{G}\delta t}{2}, \quad (12)$$

$$\rho\varepsilon = \rho\frac{DRT}{2} = \sum_i g_i' - \frac{\delta t}{2}\sum_i f_i q_i, \quad (13)$$

$$\boldsymbol{\varphi} = (\sum_i \boldsymbol{c}_i g_i' - \rho\varepsilon\boldsymbol{u} - \frac{\delta t}{2}\sum_i \boldsymbol{c}_i f_i q_i)\frac{\tau_c}{\tau_c + 0.5\delta t}, \quad (14)$$

$$\nu = \tau_v RT_m , \quad (15)$$

$$\zeta = 2\tau_c RT_m . \quad (16)$$

In Eq. (13), D is the dimension number. The D2Q9 model is adopted, and the lattice velocity is $c = \sqrt{3RT_m}$, here T_m is the fluid average temperature. The equilibrium distribution f_i^{eq} and g_i^{eq} expressions can be found in Ref. [10]. Using the Chapman–Enskog expansion, the Boltzmann equation recovers the correct continuity, momentum and energy equations.

The non-slip boundary condition with second-order accuracy for the velocity boundary on the walls proposed by Inamuro et al. [11] and the thermal boundary conditions by D'Orazio and Succi [12] are adopted. The periodic boundary conditions are used at inlet and outlet boundaries.

3. RESULTS AND DISCUSSION

3.1 Incompressible Flow

Considering that the pressure gradient is uniform in the domain but changes with time at a cosine function, as Eq. (1). The fluid average temperature $T_m=313K$, the average pressure $p_m=101kPa$ and the average density $\rho_m=1.13kg/m^3$, the kinetic viscosity ν is $1.69\times10^{-5}m^2/s$ and the thermal diffusivity ζ is $2.41\times10^{-5}m^2/s$. Two cases listed in Table 1 are investigated.

TABLE 1 parameters for the simulated cases

case number	1	2
pressure gradient amplitude(Pa/m)	10^4	10^4
angular frequency(rad/s)	400π	400π
space step(m)	4.976×10^{-6}	1.244×10^{-6}
time step(s)	9.591×10^{-9}	2.398×10^{-8}
Grid number	5×59	5×224
Stokes number	0.8941	8.4866

3.1.1 Velocity and Temperature Distributions

The reference velocity u_n and reference instantaneous temperature T_b are defined respectively as

$$u_n = p_0 b^2 / 2\mu . \quad (17)$$

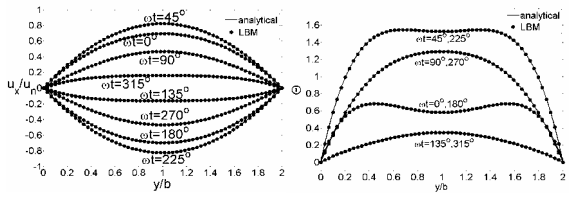
$$T_b(x,t) = \frac{\int_0^{2b} |u_x(x,y,t)|\, T(x,y,t)dy}{\int_0^{2b} |u_x(x,y,t)|\, dy} , \quad (18)$$

Therefore, the time-average bulk temperature,

$$\bar{T}_b(x,t)=\frac{\int_0^{\frac{2\pi}{\omega}}\int_0^{2b}|u_x(x,y,t)|\,T(x,y,t)dydt}{\int_0^{\frac{2\pi}{\omega}}\int_0^{2b}|u_x(x,y,t)|\,dydt}. \tag{19}$$

The dimensionless temperature Θ is defined as

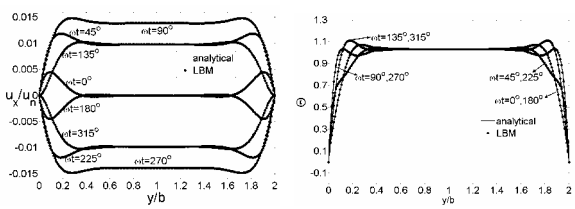
$$\Theta(y,t)=\frac{T(y,t)-T_w}{\bar{T}_b-T_w}. \tag{20}$$



(a) (b)

Fig. 2 Analytical and LBM solutions of velocity (a) and temperature (b) distribution across the channel of at different time, case 1, $\Lambda=0.8941$

Figs. 2 and 3 show good agreements between the analytical solutions given by Eq. (2) and Eq. (4) and LBM simulation results of velocity and temperature distributions. Both velocity and temperature oscillate periodically, and the oscillating period of temperature is half of that of velocity. The fluid temperature is higher than wall temperature all the times since the viscous dissipation always heats the fluid.



(a) (b)

Fig. 3 Analytical and LBM solutions of velocity (a) and temperature (b) distributions across the channel of at different time, case 2, $\Lambda=8.4866$

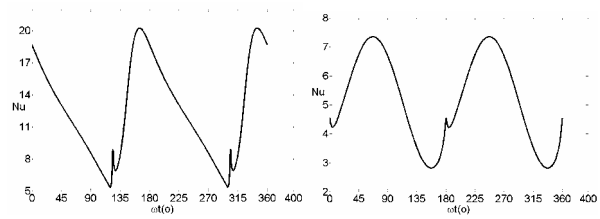
The maximum temperature locates at certain place near the walls. Such phenomenon is called "annular effect", which is caused by the velocity annular effect via viscous dissipation. Another factor influencing the annular effect is the Stokes number Λ. As illustrated in Fig. 2, when $\Lambda<1$, the annular velocity effect can not be observed obviously but temperature annular effect obviously exists. The velocity annular effect is less sensitive to Λ than that of temperature. The velocity and temperature variationa along the y-direction mainly locate near the walls for the current linearization cases. For the case of large Λ, as shown in Fig. 3, the flow is in slug flow near the center region and the effect of viscous dissipation on temperature in the central region can be neglected.

3.1.2 Heat Transfer Characteristics

The instantaneous local Nusselt number, Nu, is defined as

$$Nu=\frac{\varphi_w(x,t)}{T_b(x,t)-T_w}\cdot\frac{4b}{k}, \tag{21}$$

where $4b$ is the hydraulics diameter of the parallel channel and φ_w is the wall heat flux. The variations of the instantaneous local Nusselt numbers with time for cases 1 and 2 are shown in Fig. 4. It is seen that oscillating period of Nusselt number is half of that of velocity. For small Λ case, such as case 1, the Nu varies more sharply and the value is larger than that of case 2, the reason is that, for $\Lambda<1$, the two viscous penetration layers on the upper and bottom walls disturb each other, therefore, the thermal resistance is reduced.



(a) (b)

Fig.4 The instantaneous local Nusselt number of case 1 (a) and case 2 (b)

3.2 Compressible Flow

The pressure gradient is a function of time and position,

$$-dp/dx=p_0\cos(2\pi x/\lambda)\cos(\omega t), \tag{22}$$

When the plate length L is equal to one wavelength, the velocity and pressure at inlet are same as those at outlet. Therefore, the periodic boundary conditions can be applied at inlet and outlet. In the simulation, the aspect ratio of the channel is $L/2b=15.5$ and p_0 is $10^6 Pa/m$, $f=3.61MHz$. The high frequency aims to satisfy the linearization assumption and to decrease the computing grids number.

3.2.1 Velocity Field

The velocity profiles at different time and positions are shown in Fig. 5. For the case of high frequency and low pressure oscillating amplitude, fluid particles only oscillates back and forth within a very narrow region in x-direction. At a certain time, the velocities at different x-position may have different direction and values. At the standing points,

which locate at $x=L/4$ and $x=3L/4$, the fluid particles keep static all the time.

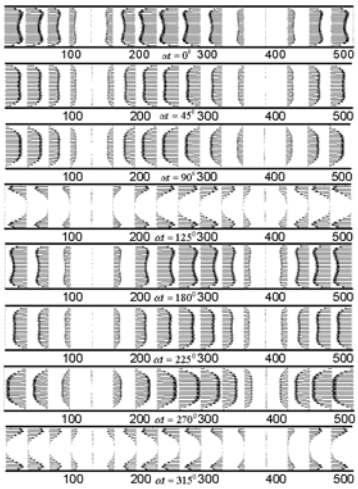


Fig. 5 velocity vectors field across the whole channel at different times

3.2.2 Temperature Field

The dimensionless temperature defined by Eq. (19) at certain x-position also oscillates with the same frequency as that of the pressure gradient, as shown in Fig. 6. The temperature oscillation is unlike that of the incompressible flow where the fluid temperature is always higher than the wall temperature affected by viscous dissipation. This temperature oscillating phenomenon is caused by compressibility effect.

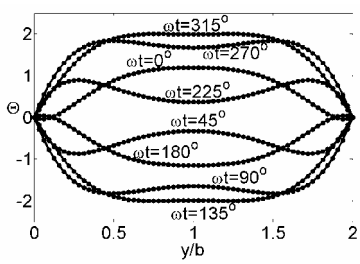


Fig.6 The temperature distribution at x=449 at different time

4. CONCLUSION

The analytical solution for the temperature distribution in laminar incompressible oscillating flow between two parallel constant-wall-temperature plates with viscous dissipation is derived under complete linearization assumption. The thermal lattice Boltzmann method is used to simulate oscillating flow and heat transfer for a broad range of Λ and Re_δ. Good agreements have been obtained between the analytical and numerical solutions. The simulation results show that the Stokes number Λ is an important factor affecting the Nu. For the compressible flow case, the simulation results show that all fluid particles only oscillating within a respective narrow distance. The fluid temperature changes with fluid expanding or deflating. The current simulation showed that the thermal LBM is suitable for simulating compressible oscillating flow and heat transfer.

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