How well do Zeeman measurements reflect the turbulent solar magnetic field?

Jonathan Pietarila Graham, Sanja Danilovic, and Manfred Schüssler

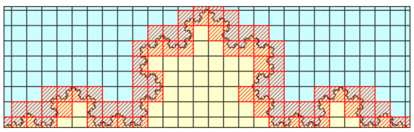
Max Planck Institute for Solar System Research, 37191 Katlenburg-Lindau, Germany

ABSTRACT

We employ the fractal self-similar geometry of the turbulent solar magnetic field to derive two estimates of the true mean unsigned vertical flux density. The estimates from *Hinode* Stokes V observations and from MURaM simulations are in good agreement. Our estimates significantly reduce the order-of-magnitude discrepancy between Zeeman- and Hanle-based estimates.

1. Fractality, self-similarity, and power-laws

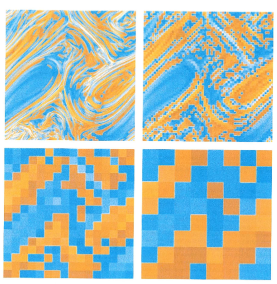
A power-law expresses that a physical process is the same at different scales (i.e., self-similar = fractal). This connection can be illustrated with a simple fractal where the number of boxes, $N(l)$, of edge length l covering the fractal set scales as $N(l) \propto l^{-D}$ where D is the fractal dimension.



Koch Fractal - A fractal is self-similar, portions look similar to the whole.

2. The cancellation function, $\chi(l)$

When a fractal field (such as net vertical magnetic flux, Lawrence et al. 1993) possesses both magnitude and sign, instead of counting boxes we quantify the cancellation statistics (Ott et al. 1992).



The cancellation function, $\chi(l)$, measures the portion of flux remaining after averaging over boxes of edge length l. Its scaling exponent, κ, is called the cancellation exponent.

$$\chi(l) \equiv \frac{\sum_i \left| \int_{\mathcal{A}_i(l)} \mathbf{B_z} \mathbf{da} \right|}{\int_{\mathcal{A}} |\mathbf{B_z}| \mathbf{da}}$$

$$\chi(l) \propto l^{-\kappa}$$

The domain, $\mathcal{A}$, is partitioned into boxes $\mathcal{A}_i$ of edge length l.

$\chi(l) = 1$ when l < size of smallest feature

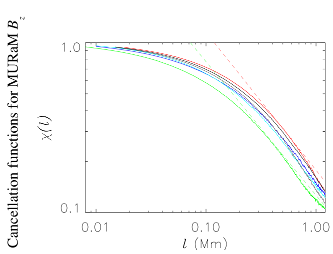
3. Measuring $\chi(l)$ for MURaM simulations

MURaM makes realistic simulations of solar magneto-convection including radiative transfer and partial ionization. (Vögler et al. 2004)

Horizontal extent 4.86×4.86 Mm^2, depth 1.4 Mm

Simulation	Grid	Horizontal Res.	Re_M
Run E	540 x 540 x 140	9 km	≈2000
Run C	648 x 648 x 140	7.5 km	≈2600
NGrey	648 x 648 x 140	7.5 km	≈2600
Prandtl	972 x 972 x 200	5 km	≈2600
Ultra	972 x 972 x 200	5 km	≈5200
Hyper	1215 x 1215 x 350	4 km	≈8100

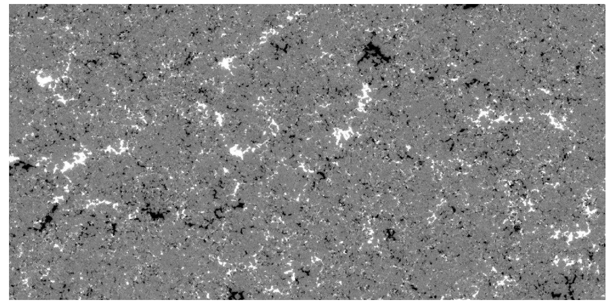
Ngrey differs from Run C only by the inclusion of non-grey radiative tranfer. Prandtl is computed with the same effective Reynolds number, *Re*, as Ultra but with the magnetic Reynolds, Re_M, as Run C. Hence at a lower magnetic Prandtl number, $P_M = Re_M/Re$.



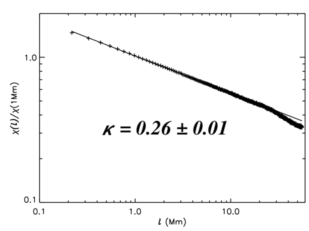
Note the **turnover in the smallest decade of scales** and that no power-law scaling is evident. This turnover is due to dissipation. The lack of a power-law can be due to insufficient scale separation between the box size and the dissipation.

$\chi(l)$ for vertical field, B_z, and for synthetic magnetogram signal, B^L_{app} (see Lites et al. 2008), were found to be equivalent suggesting *Hinode* $\chi(l)$ for B^L_{app} may be taken as a proxy for that of B_z.

4. What *Hinode* $\chi(l)$ means for the dissipative length scale



Hinode 327" x 164" magnetogram (B^L_{app}, Lites et al. 2008) thresholded at $|B^L_{app}| = 60$G (12G mean) resolution = 0."3 ≈ 200km



Cancellation function for Hinode B^L_{app}. **Self similarity extends over 2 decades of length scales (including the granulation scale)** down to the resolution limit, *200*km. Since we do not see a turnover, the observations indicate the magnetic dissipation scale is 20km or smaller.

5. What *Hinode* $\chi(l)$ means for the mean vertical flux density

True mean unsigned vertical flux density: $\Phi_u \equiv \dfrac{\int_{\mathcal{A}} |\mathbf{B_z}| \mathbf{da}}{\int_{\mathcal{A}} \mathbf{da}}$

Mean absolute value of vertical flux density at resolution l: $\Phi_{u,l} \equiv \dfrac{\sum_i \left| \int_{\mathcal{A}_i(l)} \mathbf{B_z} \mathbf{da} \right|}{\int_{\mathcal{A}} \mathbf{da}} = \chi(l) \cdot \Phi_u$

No cancellation at magnetic dissipation scale, l_η: $\chi(l_\eta) = 1$

$$\boxed{\Phi_u = \Phi_{u,l} \cdot \frac{\chi(l_\eta)}{\chi(l)} = 12\mathrm{G} \cdot \left(\frac{100\mathrm{km}}{l_\eta}\right)^{0.26}}$$

Kolmogorov (K41) theory predicts $l_\eta \sim L Re_M^{-3/4}$

L = 1Mm (granular scale)

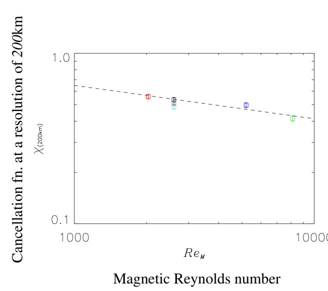
$Re_M = L \cdot v_{rms}/\eta \sim$ 1Mm•3kms^{-1}/10^8cm^{2}s^{-1} ~ $3 \cdot 10^5$

$\Rightarrow l_\eta \sim 80$m

For conservative lower bound consider strong turnover for smallest decade of scales & take $l_\eta \sim 800$m

$$\boxed{\Rightarrow \Phi_u \geq 40\mathrm{G}}$$

6. What MURaM suggests for the mean vertical flux density



The flux remaining at l = 200km decreases with magnetic Reynolds number. A power-law scaling is apparent and can be extrapolated to solar values, ~3×10^5, for which we estimate $\chi(200\mathrm{km}) = 0.2$. That is, **80% of vertical unsigned flux density is canceled at 200km resolution**.

CONCLUSIONS

Our estimates reduce the disparity between ~*10*G Zeeman-based estimates and ~*100*G Hanle-based estimates (Trujillo Bueno et al. 2004):

- Our estimate from *Hinode* observations ≥ *40*G
- MURaM simulations suggest ×5 at *200*km resolution ⇒ ~*50*G

Furthermore, our analysis provides observational evidence that the scales of magnetic structuring extend at least down to *20*km.

REFERENCES

Danilovic et al. 2008, *A&A*, **484**, L17
Lawrence et al. 1993, *ApJ*, **417**, 805
Ott et al. 1992, *Phys. Rev. Lett.*, **69**, 2654
Trujillo Bueno et al. 2004, *Nature*, **430**, 326
Kovitya & Cram, *Sol. Phys.*, **84**, 45
Lites et al. 2008, *ApJ*, **672**, 1237
Sorriso-Valvo et al. 2002, *Phys. Plasmas*, **9**, 89
Vögler et al. 2005, *A&A*, **429**, 335