Los Alamos NATIONAL LABORATORY EST. 1943

Center for Nonlinear Studies

New Results on Glassy aspect of Random Optimization Problems

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in collaboration with Florent Krzakala (ParisTech), see next talk ....

Outline

I. Connection between glasses and optimization

II. The glassy landscapes

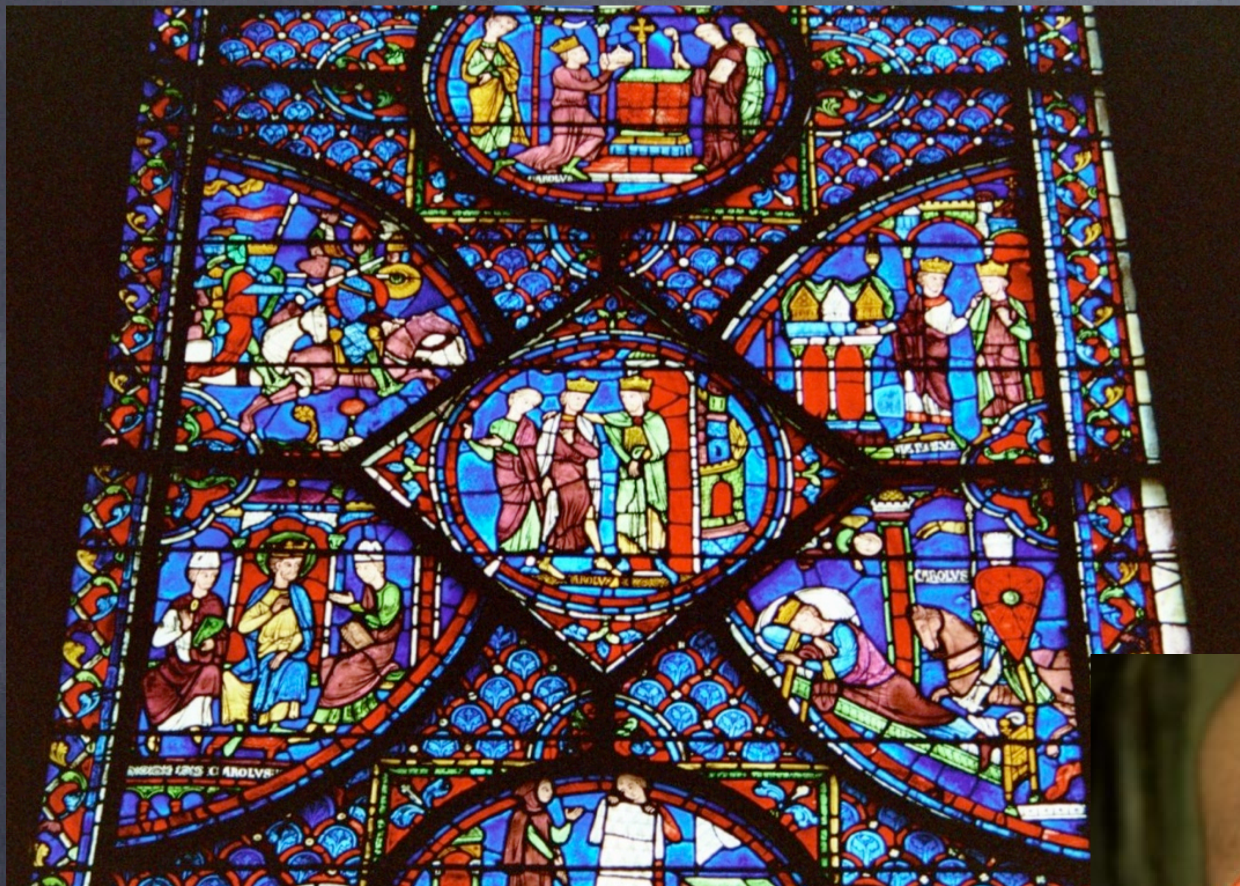
III. Our new method to describe the landscape

IV. Result I: How good is infinitely slow annealing ("analytical" results)

V. Result II: When is it hard to find solutions? Canyons versus Valleys.

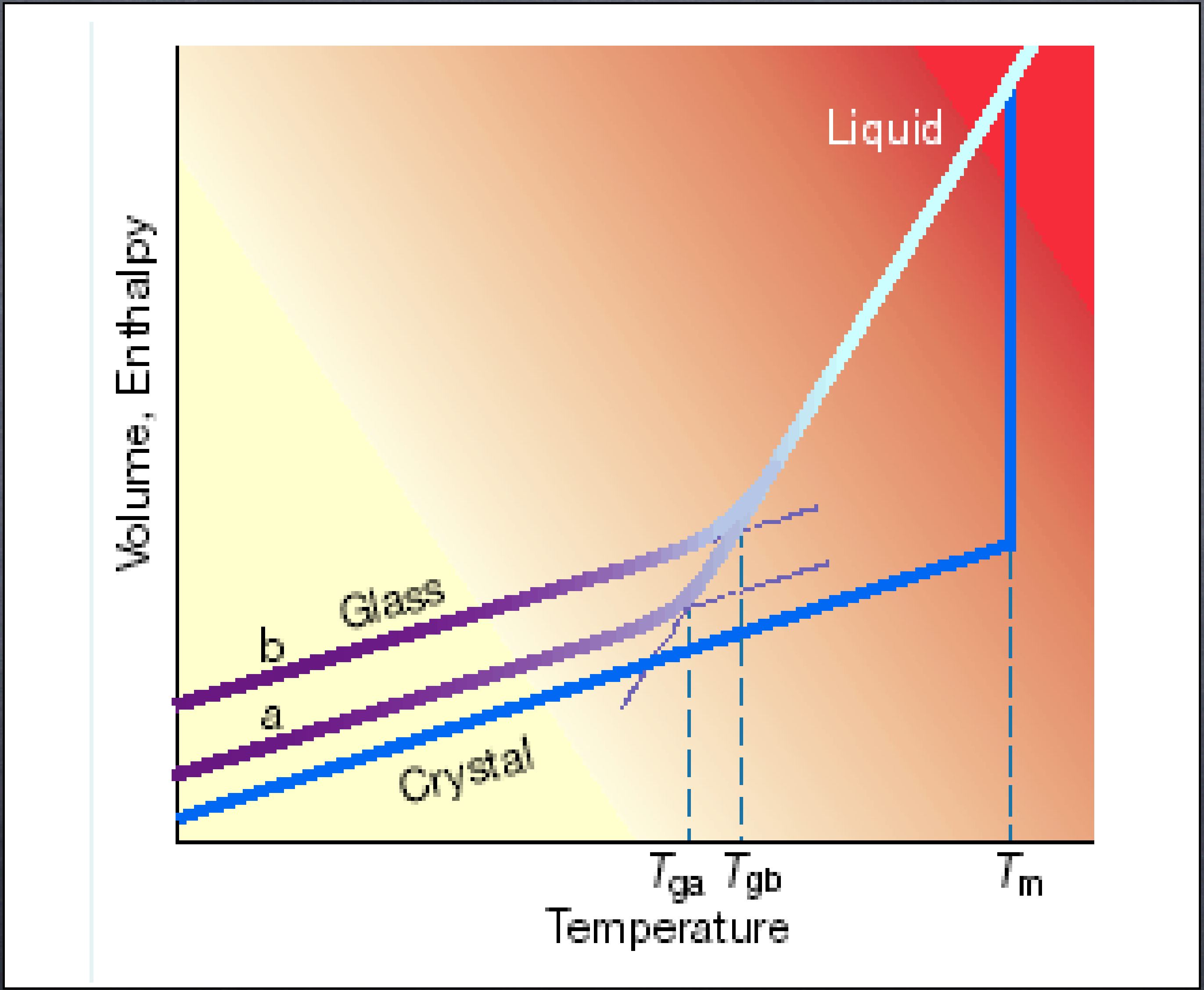
VI. Result III-x: Next talk by Florent Krzakala

Glasses



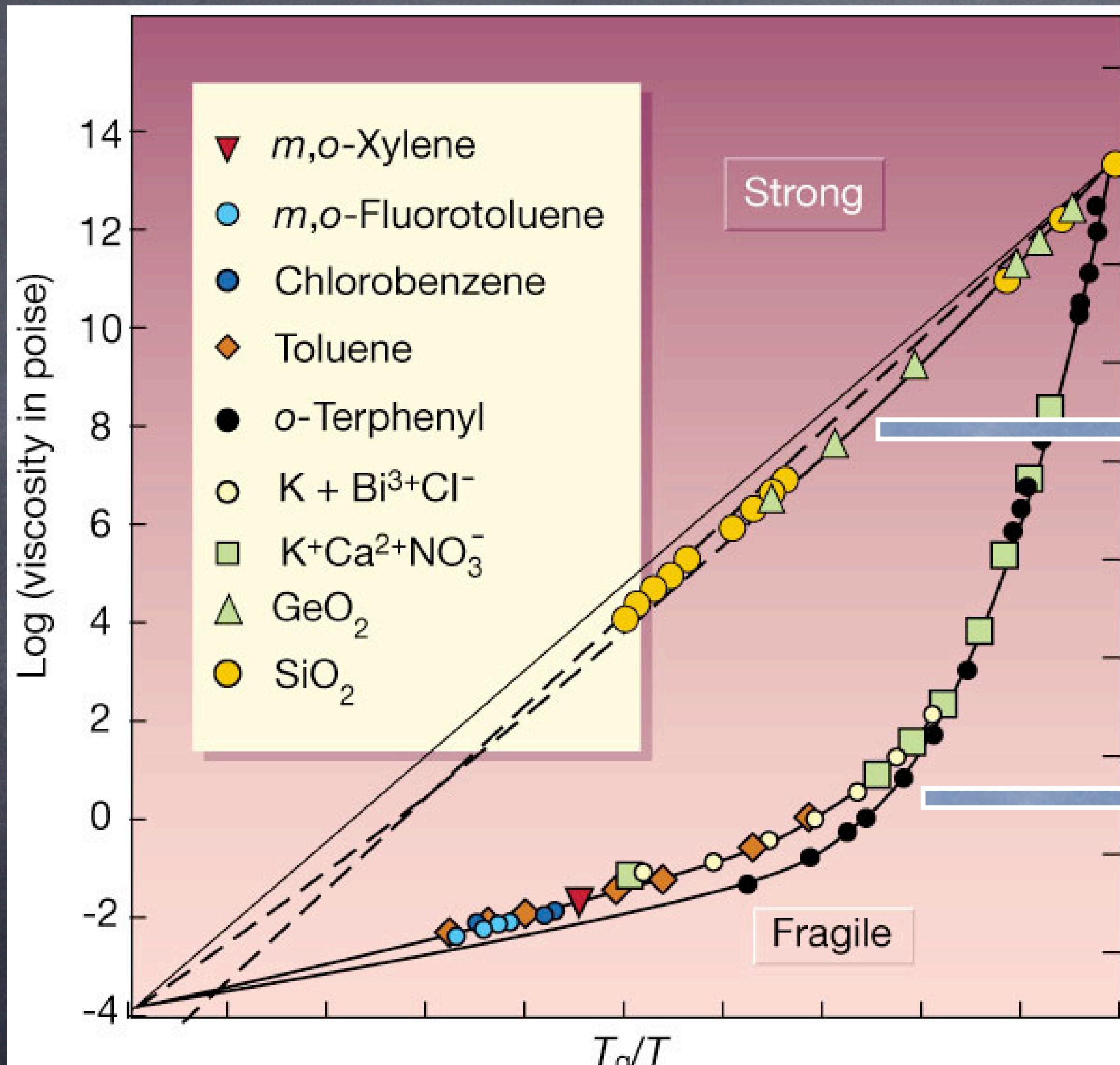
Glass transition

"Almost any liquid when quenched fast enough undergoes a glass transition."



Angell's plot

log(viscosity)



$$\eta \approx e^{\frac{\Delta}{T}}$$

$$\eta \approx e^{\frac{\Delta(T)}{T-T_K}}$$

inverse temperature

Mean Field Theory of Glass transition

models of glasses = common optimization problems

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p-spin glass: $$H = -\sum_{(ij)} J_{ij} \prod_{i=1}^{p} S_i$$

$S_i \in \{-1, +1\}$

XOR-SAT

$J_{ij} = -1$

Mean Field Theory of Glass transition

models of glasses = common optimization problems

p-spin glass: $$H = -\sum_{(ij)} J_{ij} \prod_{i=1}^{p} S_i$$ $\boxed{\text{XOR-SAT}}$

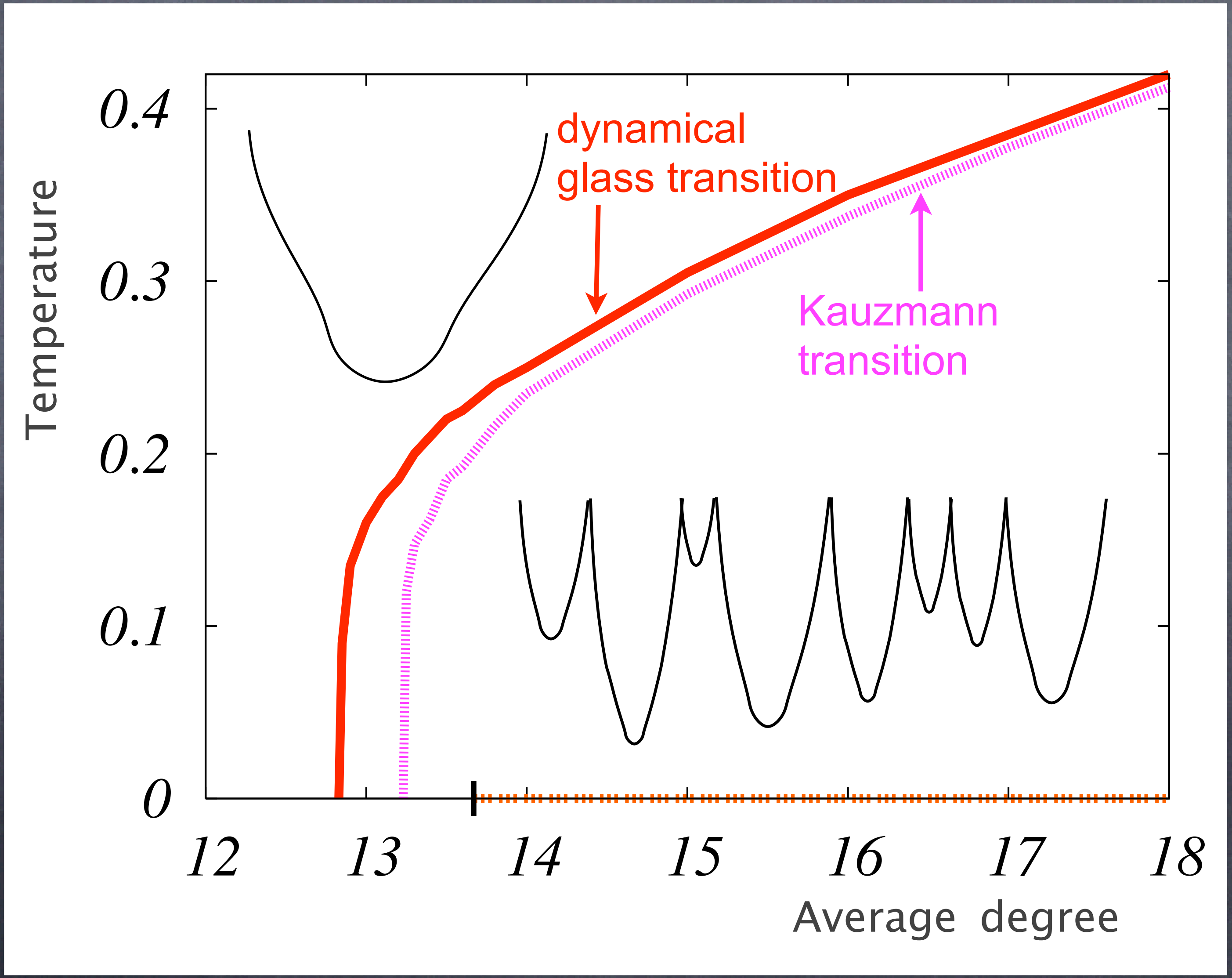
$S_i \in \{-1, +1\}$ $J_{ij} = -1$

Potts glass: $$H = \sum_{(ij)} \delta_{S_i, S_j}$$ $\boxed{\text{graph coloring}}$

$S_i \in \{1, \ldots, q\}$

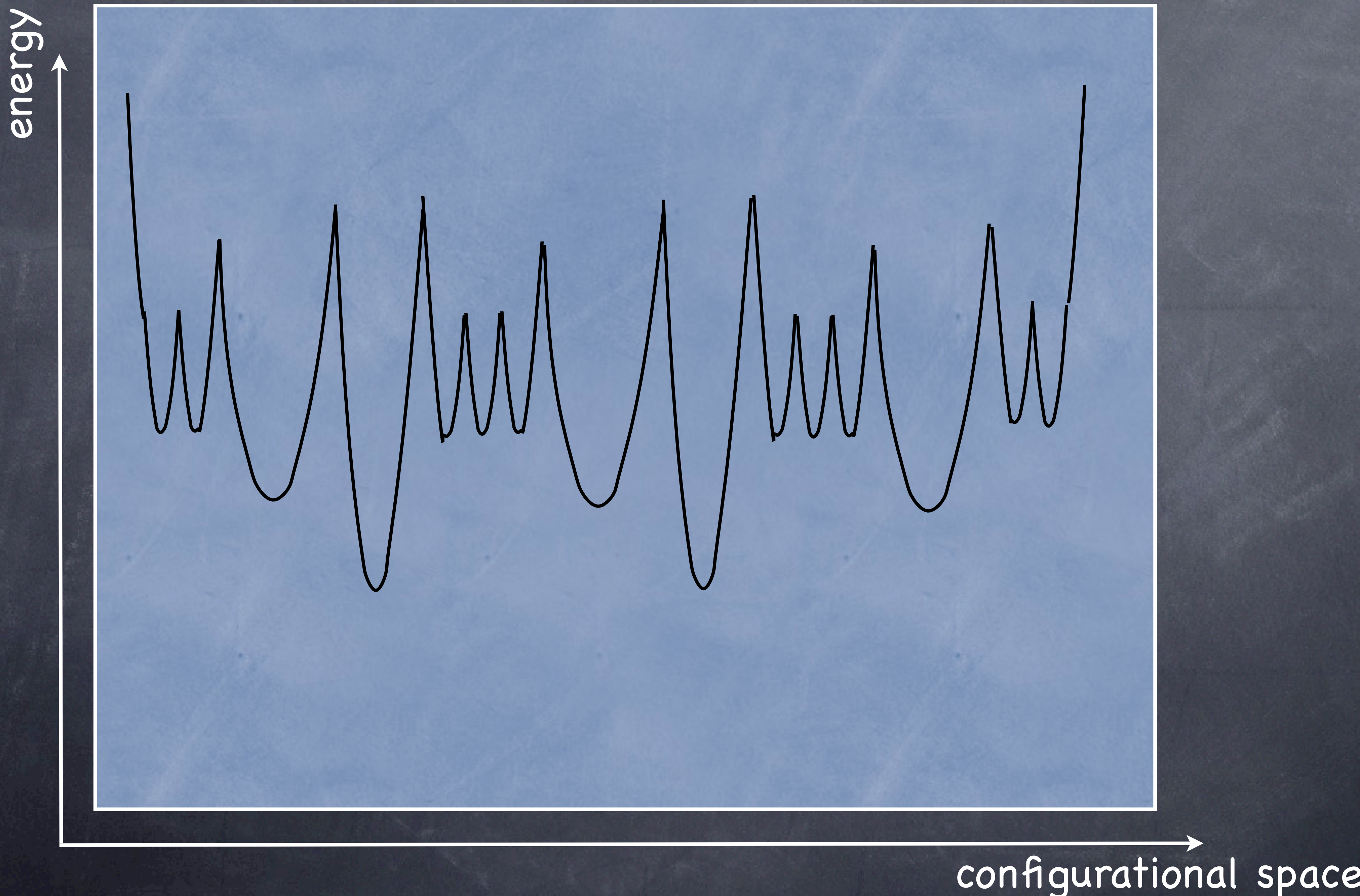
The phase diagram

- Ideal Glasses & Hard Optimization Problems



5-coloring of random graphs

Glassy Energy Landscape



Cavity Method

(Mezard, Parisi'01)

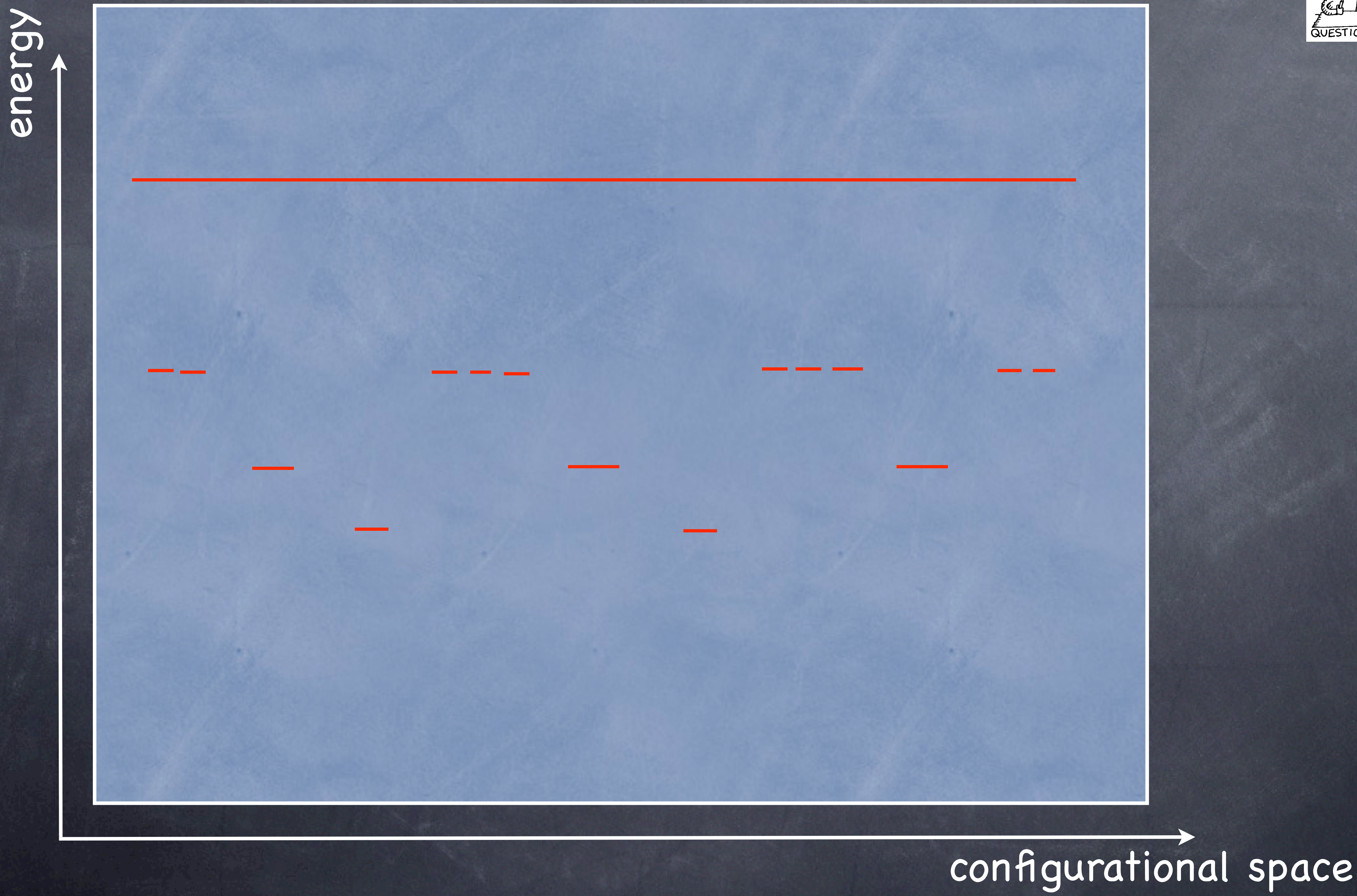
Computational method giving properties of the energy landscape:

- Total energy, entropy, temperature $T \equiv \frac{\partial E}{\partial S}$
- Properties of states - their number

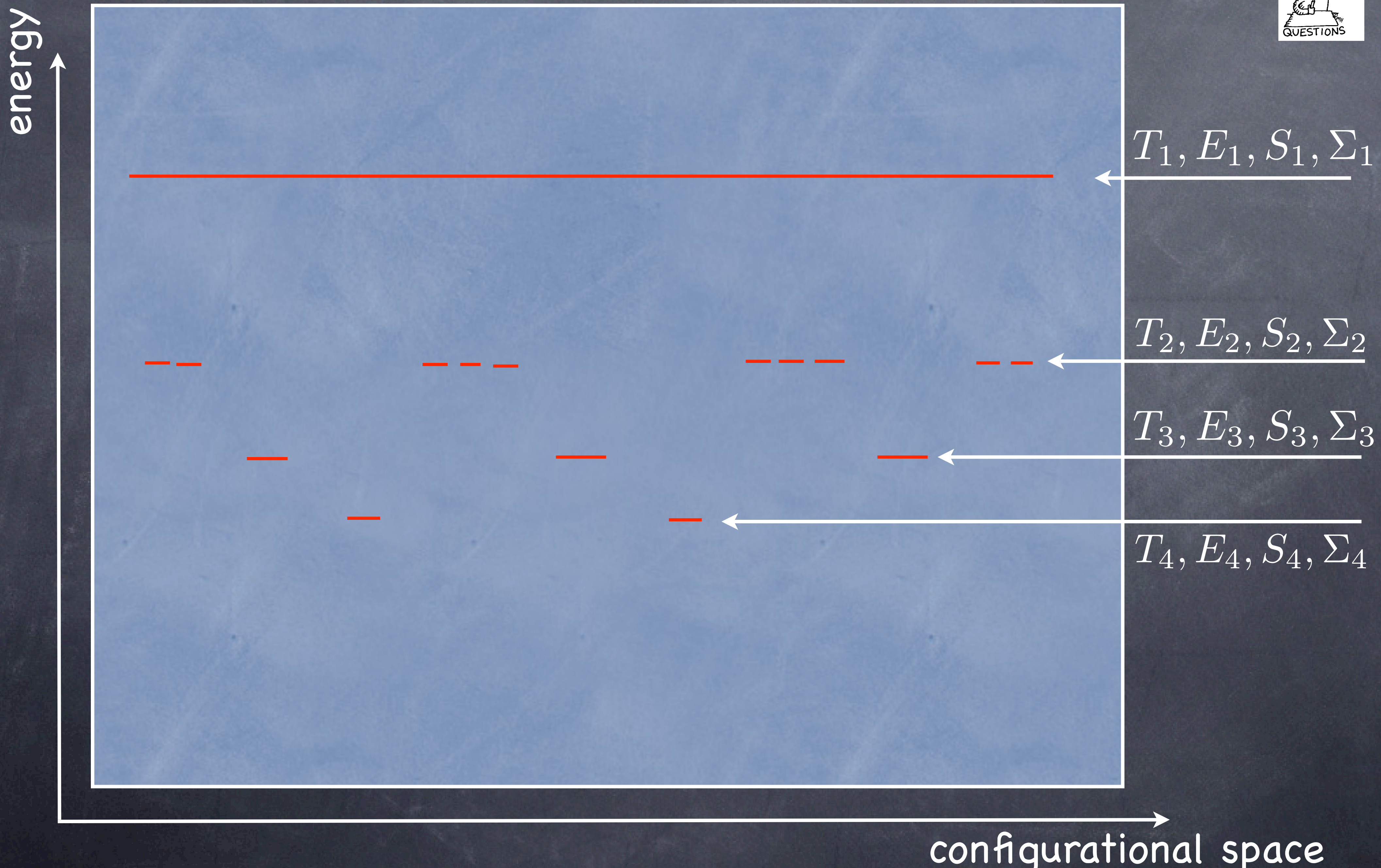
$$\mathcal{N}_{\text{states}} = e^{N\Sigma}$$

- Overlaps between and within states etc.

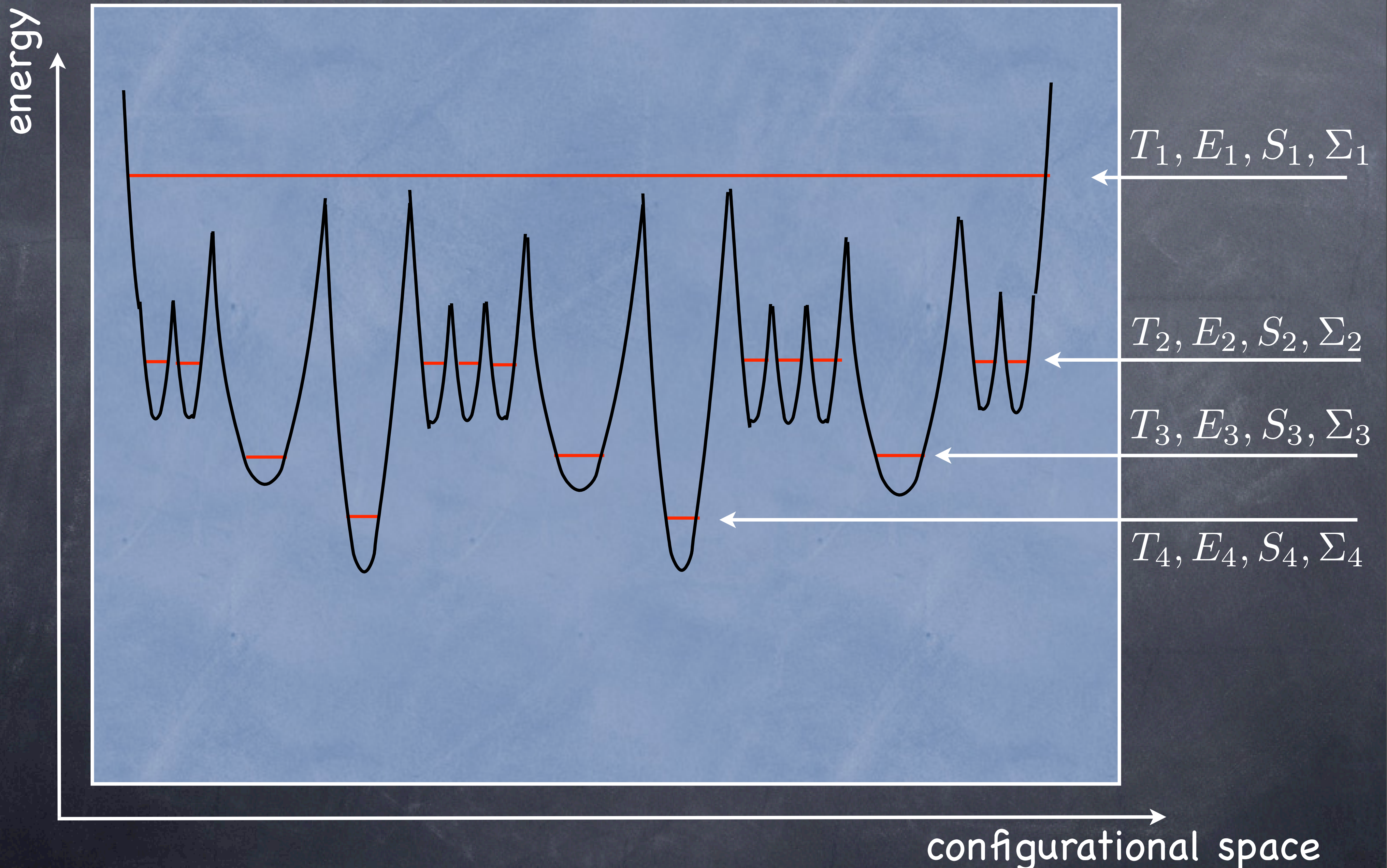
Cavity Method



Cavity Method



Following states



Following states

- New Computational Method
- Generalization of the cavity method

How does that work?

(1) "Take" a random configuration in the state of interest.

(2) Initialize belief propagation in that configuration and change the temperature parameter.

How does that work?

In some problems this can be done via planting

See the next talk by Florent Krzakala

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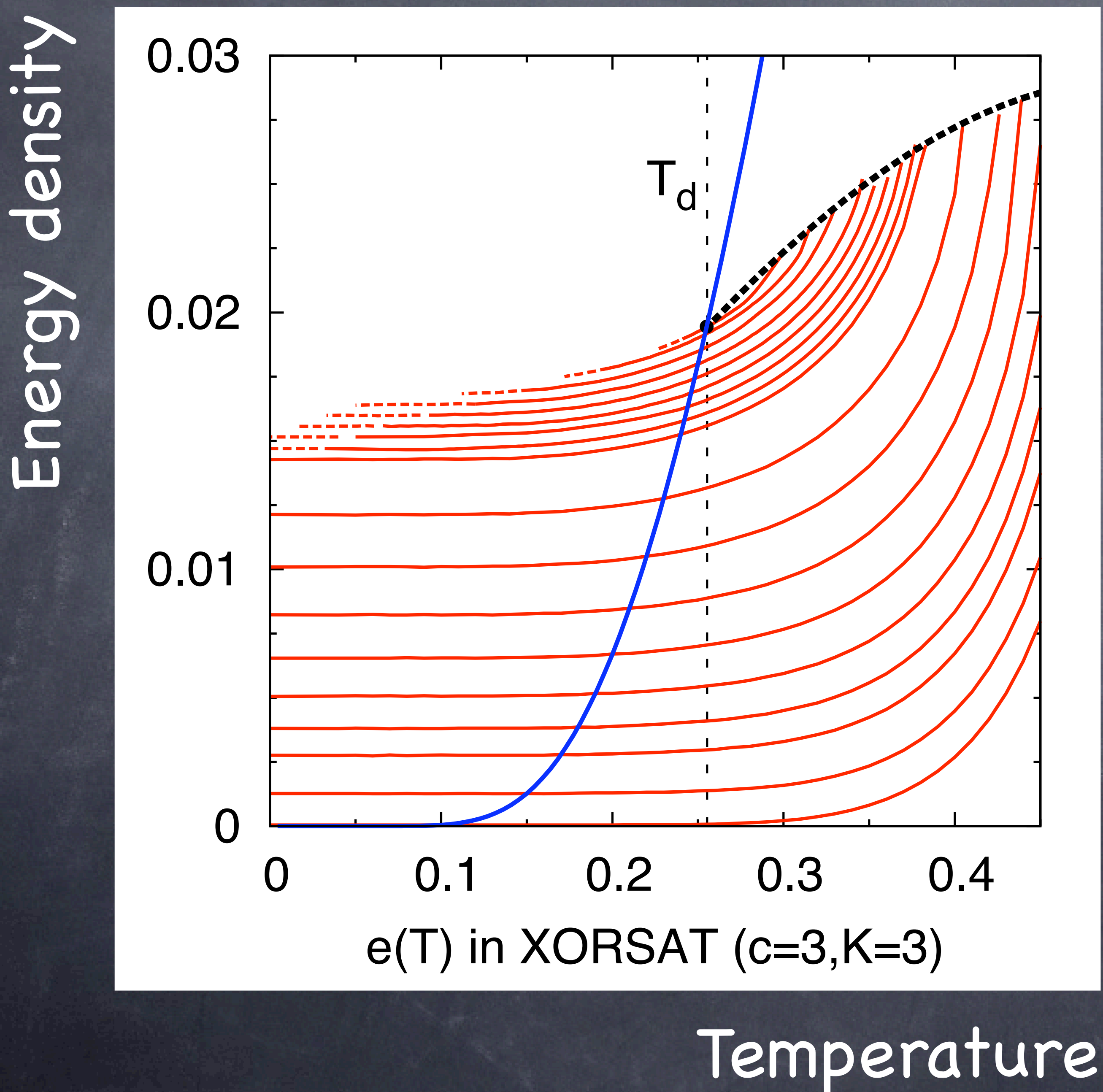
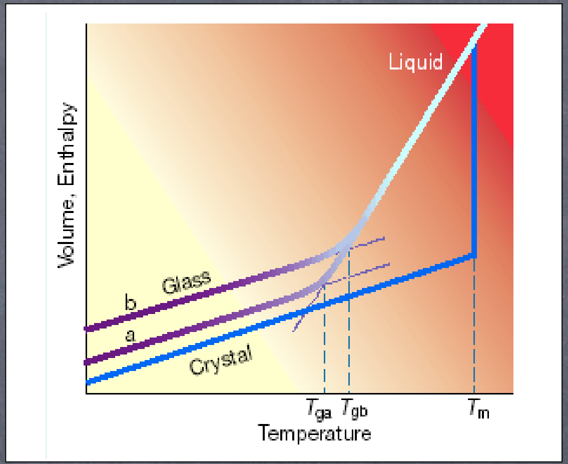
In general only on the level of cavity equations

$$P^{a\to i}(\psi^{a\to i}) = \frac{1}{\mathcal{Z}^{a\to i}(\beta)} \int \prod_{j\in\partial a\backslash i} \prod_{b\in\partial j\backslash a} \mathrm{d}P^{b\to j}(\psi^{b\to j}) \boxed{\left[Z^{a\to i}(\{\psi^{b\to j}\},\beta)\right]^m} \delta[\psi^{a\to i} - \mathcal{F}(\{\psi^{b\to j}\},\beta)]$$

$$\tilde{P}^{a\to i}(\tilde{\psi}^{a\to i}) = \frac{1}{\tilde{\mathcal{Z}}^{a\to i}(\tilde{\beta})} \int \prod_{j\in\partial a\backslash i} \prod_{b\in\partial j\backslash a} \mathrm{d}\tilde{P}^{b\to j}(\tilde{\psi}^{b\to j}) \boxed{\left[Z^{a\to i}(\{\psi^{b\to j}\},\beta)\right]^m} \delta[\tilde{\psi}^{a\to i} - \mathcal{F}(\{\tilde{\psi}^{b\to j}\},\tilde{\beta})]$$

Following states: Results





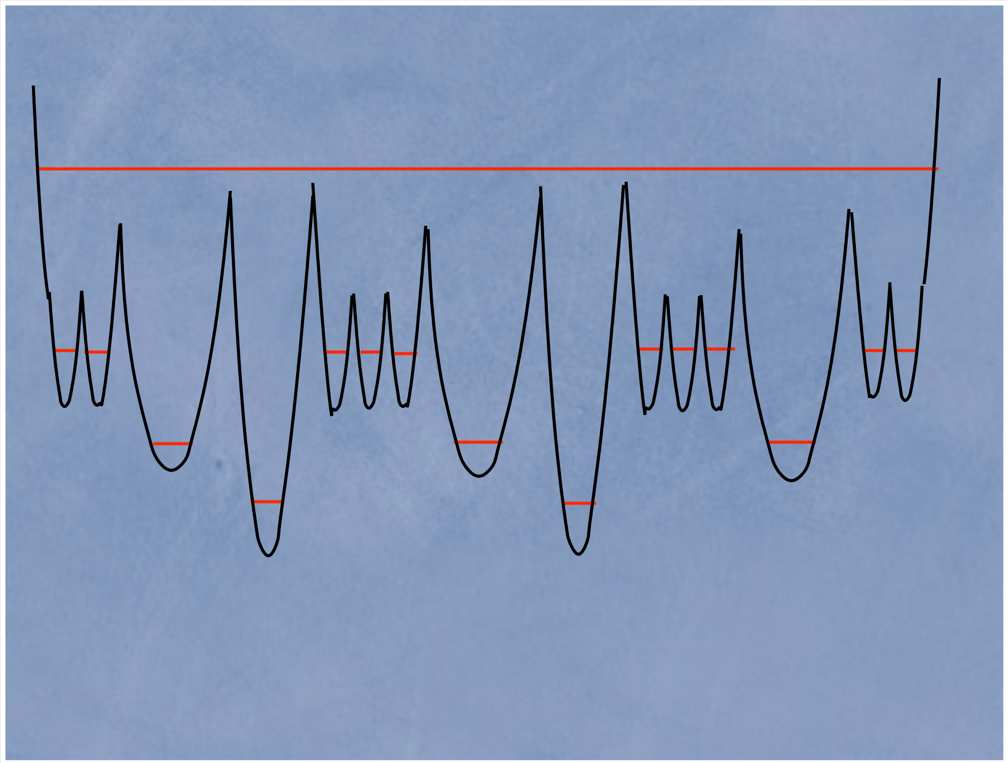
Blue line: The statics

$$Z(\beta) = \sum_{\{s_i\}} e^{-\beta H(\{s_i\})} = \int \mathrm{d}e \, e^{N s(e) - N\beta e} = e^{N[s(e)-\beta e]}$$

$$\boxed{\frac{\partial s}{\partial e} = \beta}$$

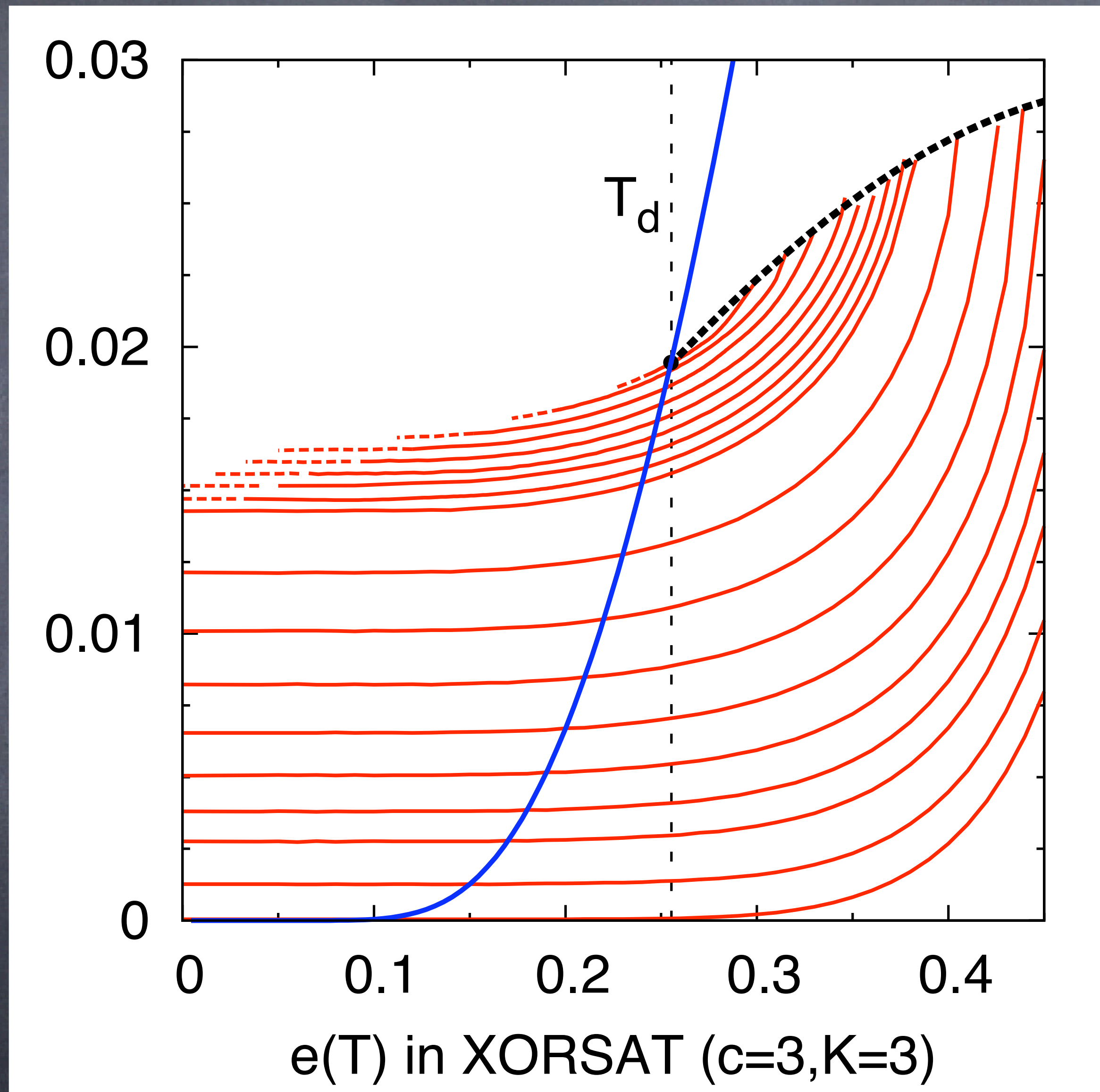
The set of configurations of energy e is split into exponentially many Gibbs states (clusters)

$$\mathcal{N}_{\text{states}} = e^{N\Sigma}$$

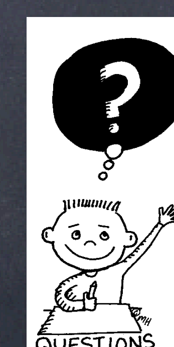


Following states: Results

Energy density



Temperature



What can be done with that?

Result n. 1

Analysis of Simulated annealing

(What energy does it achieve?)

Result n. 1

Central question: How good is certain algorithm?

Simulated annealing

Finds ground state if temperature is decreased exponentially slowly

$$T = \frac{cN}{\log t}$$

(Geman, Geman'84)

But physics seeks $T = T_0 - \frac{ct}{N}$

with first $N \to \infty$ and then $c \to 0$

(we call this: Infinitely slow annealing)

SA in non-glassy systems
(energy landscape with one state)

Infinitely slow annealing finds the ground state

SA in glassy models

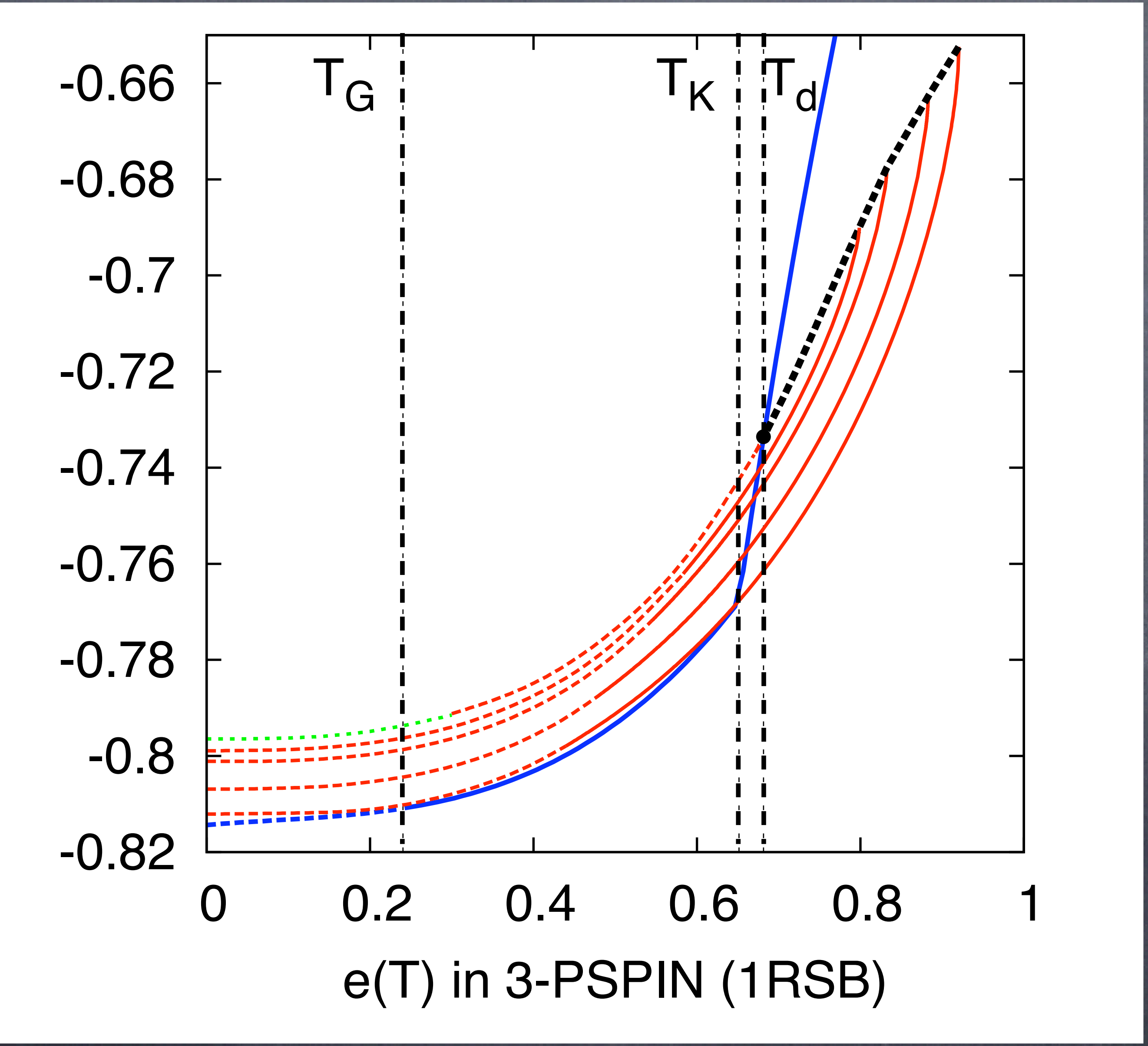
Assumption based on the knowledge of the system:

Infinitely slow annealing equilibrates down to the glass transition (Montanari, Semerjian'06), then it is stacked in one of the Gibbs states and goes to the bottom of that state.

The method of following states computes the bottoms of states

(more precisely lower bounds - 1RSB versus FRSB)

Example for fully connected p-spin



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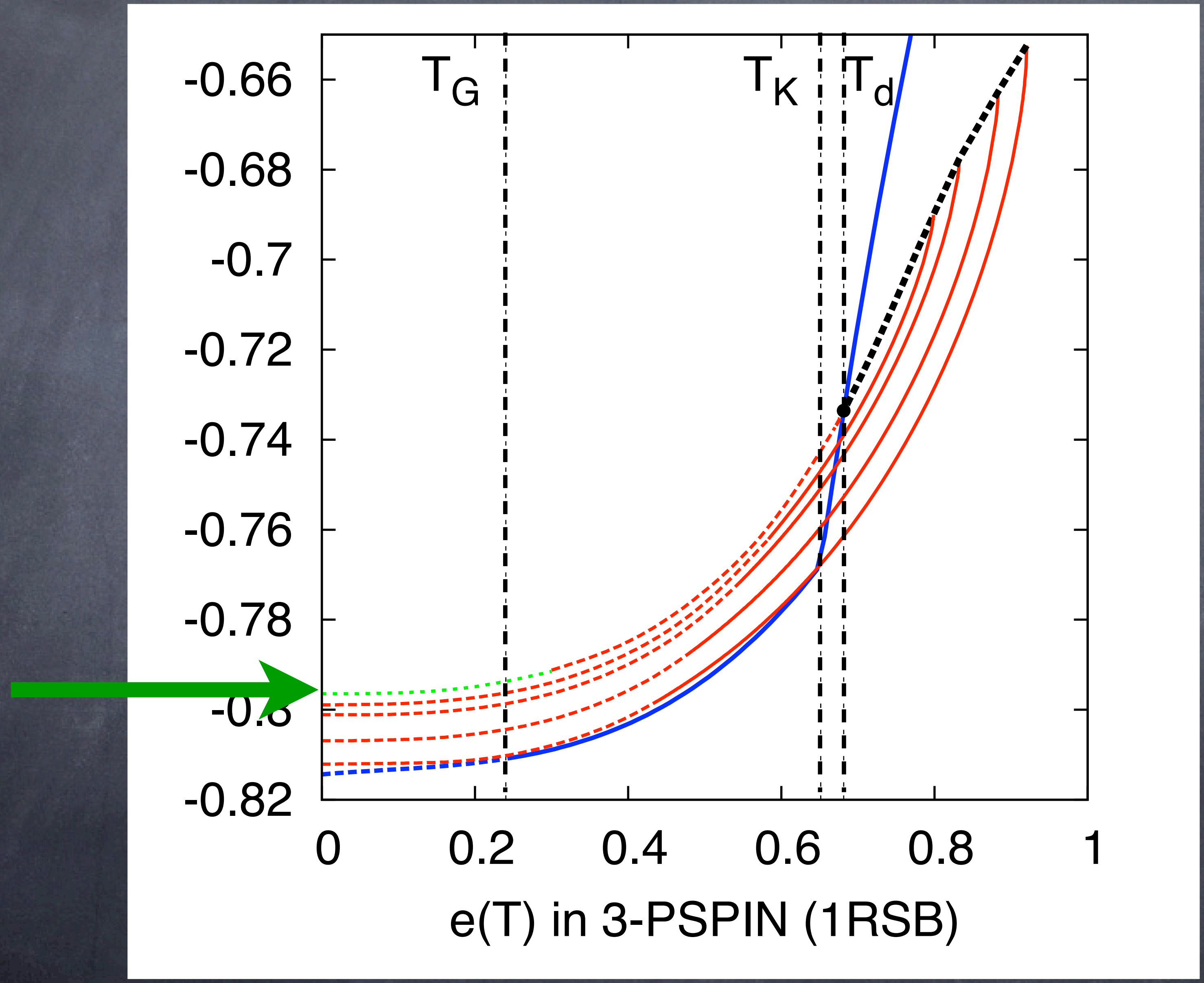

Result n. 2

Canyons versus Valleys

Where the really hard problems are?

- Random K-satisfiability
- Random graph coloring

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- Random graph coloring

Answer 1: Around the satisfiability threshold

(Cheeseman, Kanefsky, Taylor'91; Mitchell, Selman, Levesque'92)

Answer 2: Glassiness makes problems hard

(Mezard, Parisi, Zecchina'02)

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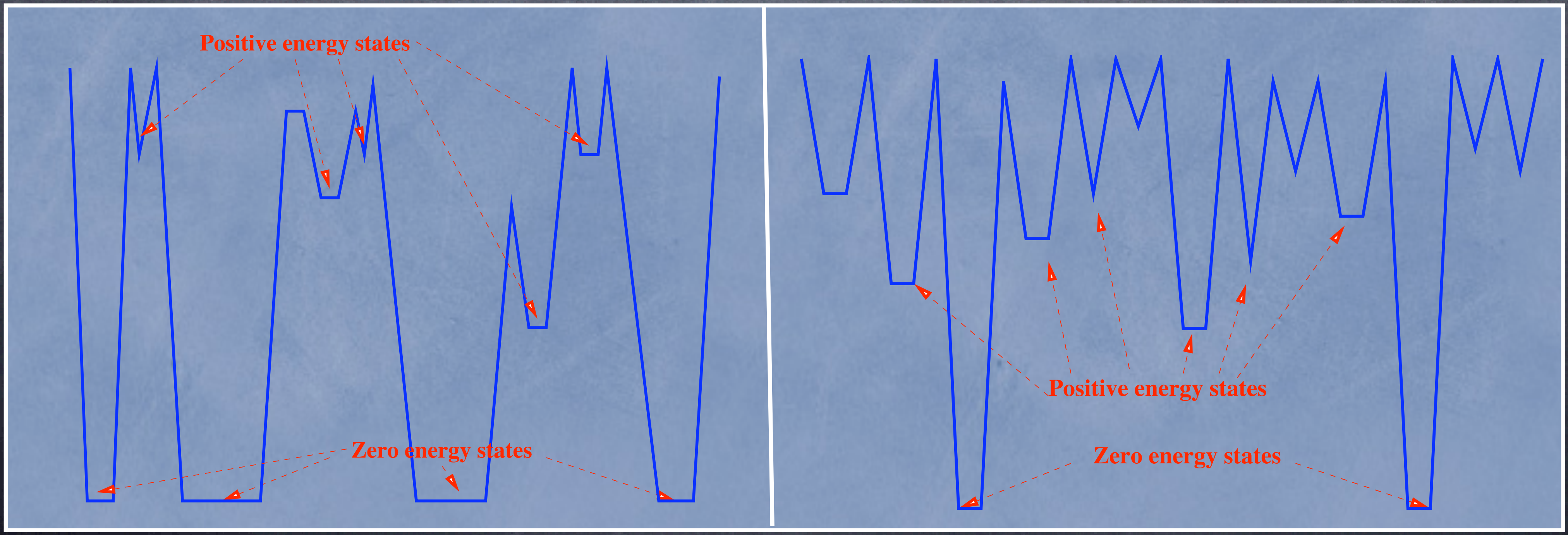
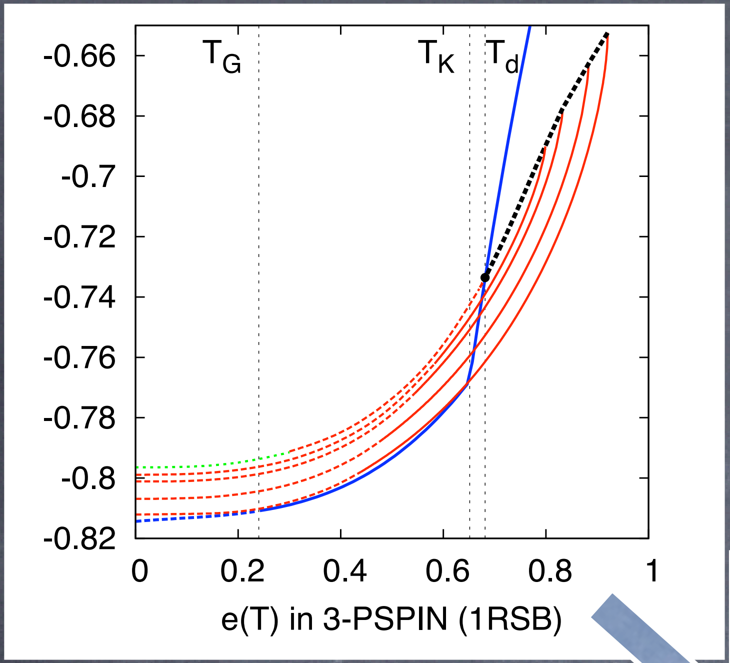
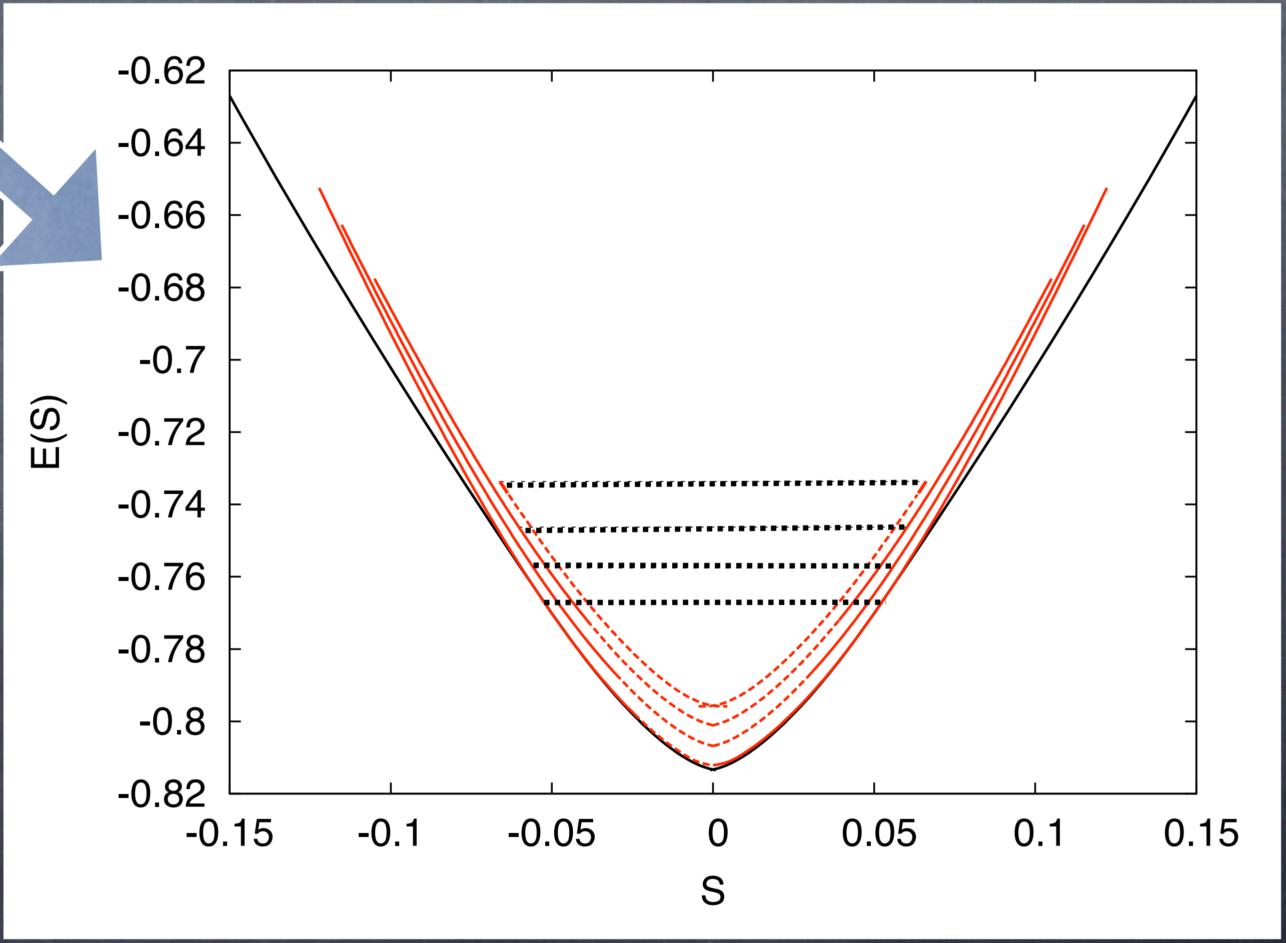
Canyon dominated vs. Valley dominated



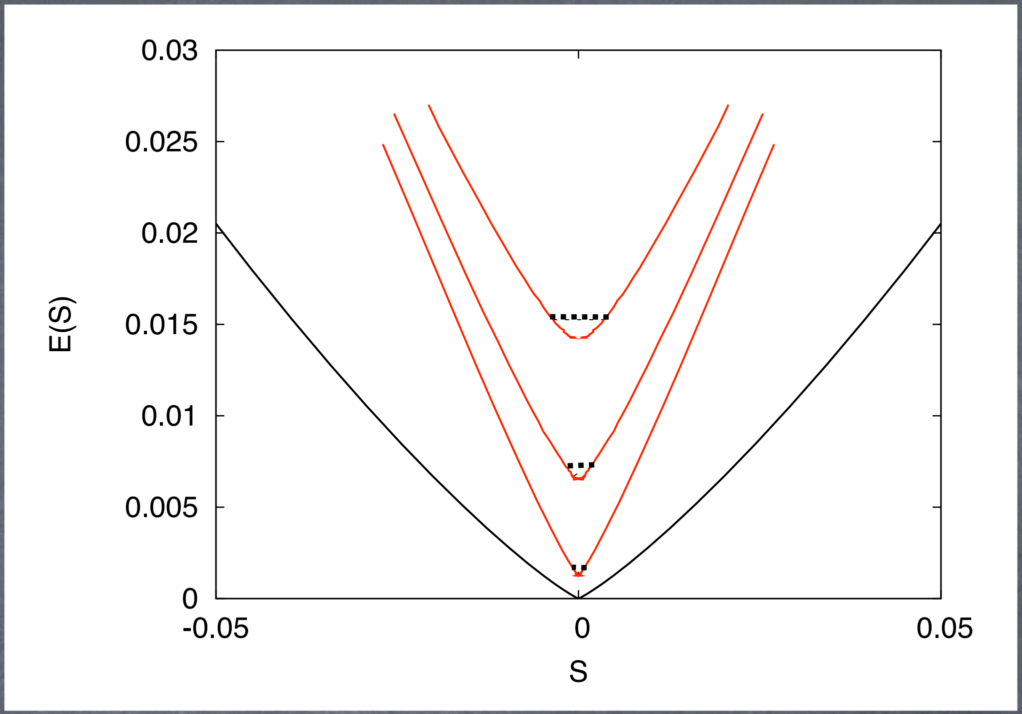
viewing the landscapes





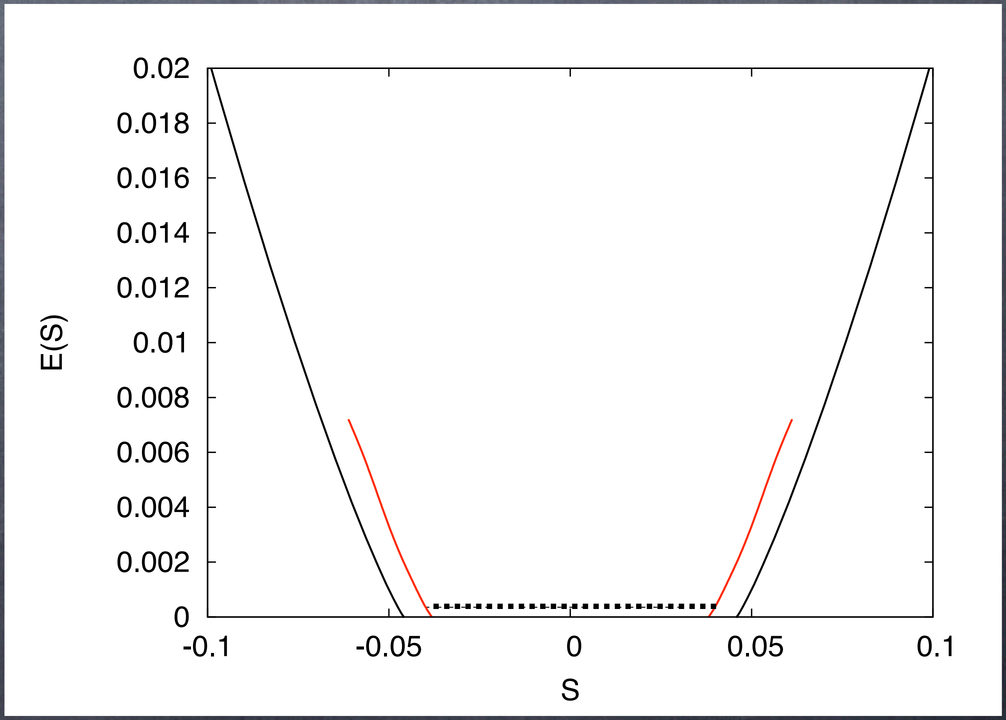
Valleys

3-XOR-SAT with L=3
solvable only by Gauss



Canyons

4-coloring of 9-regular random graphs
solvable by reinforced belief propagation



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Do frozen variables in clusters have some connection to valleys or canyon?

Yes: Frozen variables imply valleys.

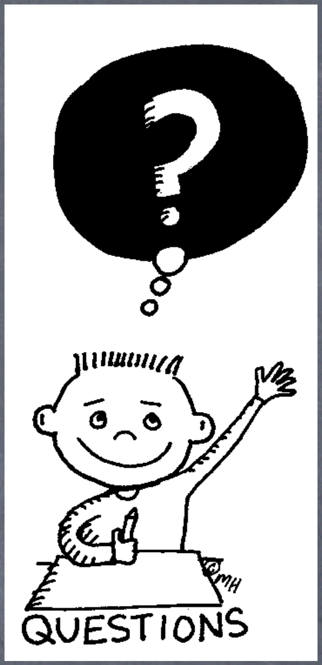
Conclusions

- New Method for describing evolution of glassy states

Results:

- Analysis of infinitely slow simulated annealing
- Canyons versus Valleys picture - implications for algorithmic hardness
- Some more in next talk by Florent Krzakala

References



- 3 papers in preparation

Related papers on planting:

- F. Krzakala, L. Zdeborová; Phys. Rev. Lett., 102, 238701 (2009).
- L. Zdeborová, F. Krzakala; submitted to SIAM J. of Discrete Math