Pattern Selection and Super-Patterns in the Bounded Confidence Model

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EB and A. Scheel, EPL **112**, 18002 (2015)

Talk, papers available from: http://cnls.lanl.gov/~ebn

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Deffuant & Weisbuch 02
Hegselmann & Krausse 02
EBN, Krapivsky, Redner 03

The Bounded Confidence Model

- Opinion measured by a <u>discrete</u> variable

$$1 \leq n \leq N$$

1. **Compromise**: reached by pairwise interactions

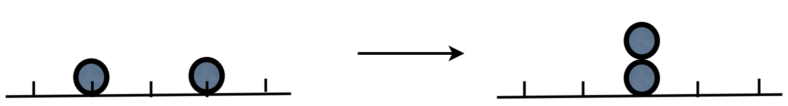
$$(n_1, n_2) \to \left(\tfrac{n_1+n_2}{2}, \tfrac{n_1+n_2}{2}\right)$$

2. **Conviction**: restricted interaction range

$$|n_1 - n_2| \leq \sigma$$

- Only <u>next-nearest neighbors</u> interact $(\sigma = 2)$

$$(n-1, n+1) \to (n, n)$$

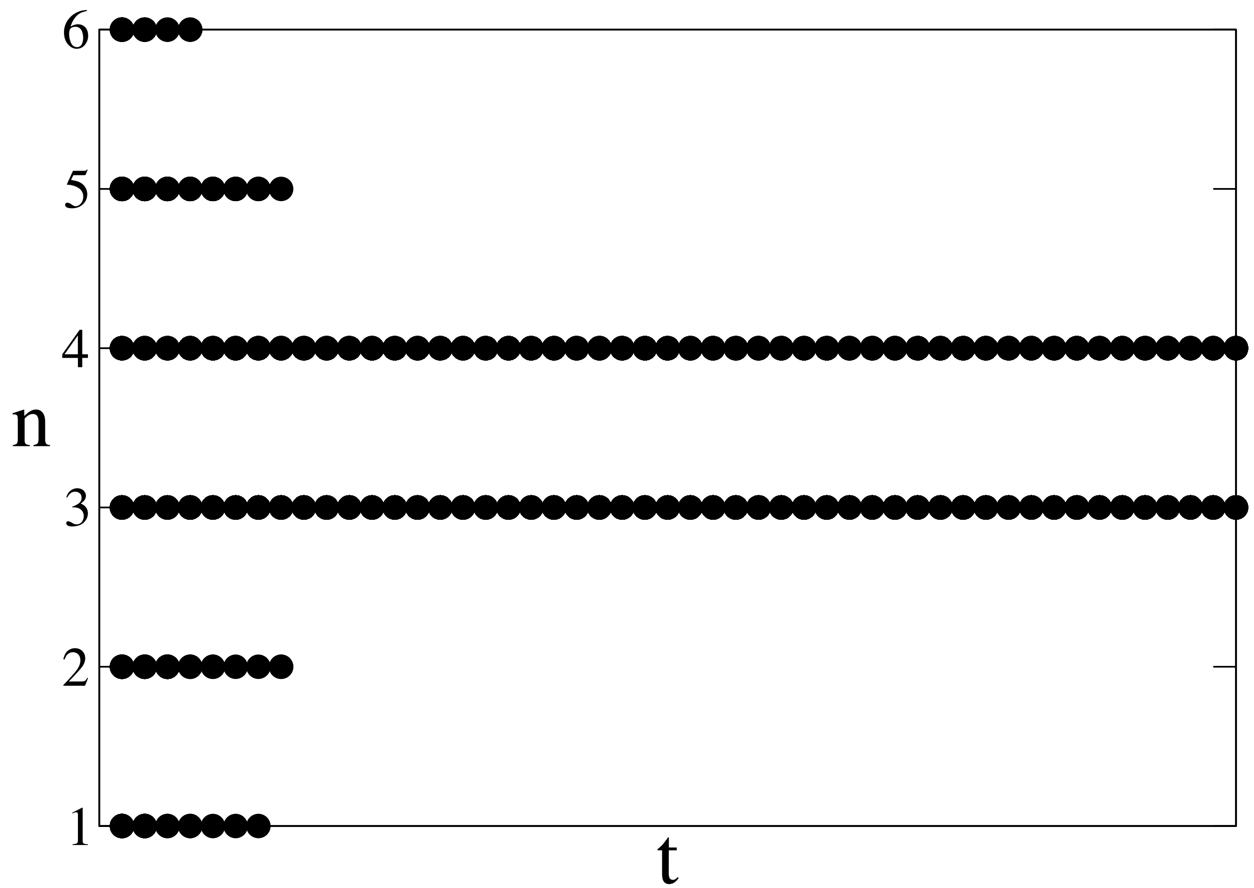


Minimal, parameter-free model
Mimics competition between compromise and conviction

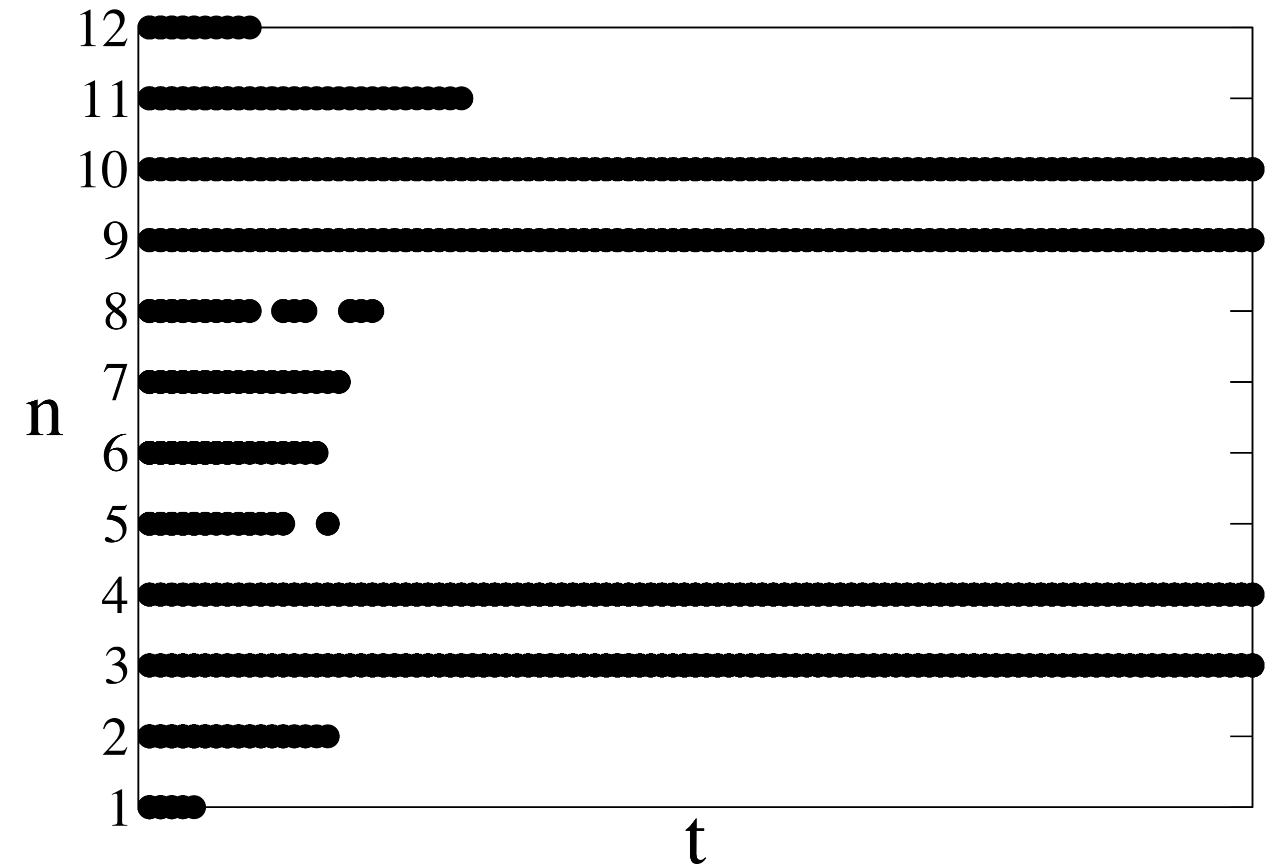
Consensus vs Discord

Monte Carlo simulations (100 agents)

$N = 6$



$N = 12$

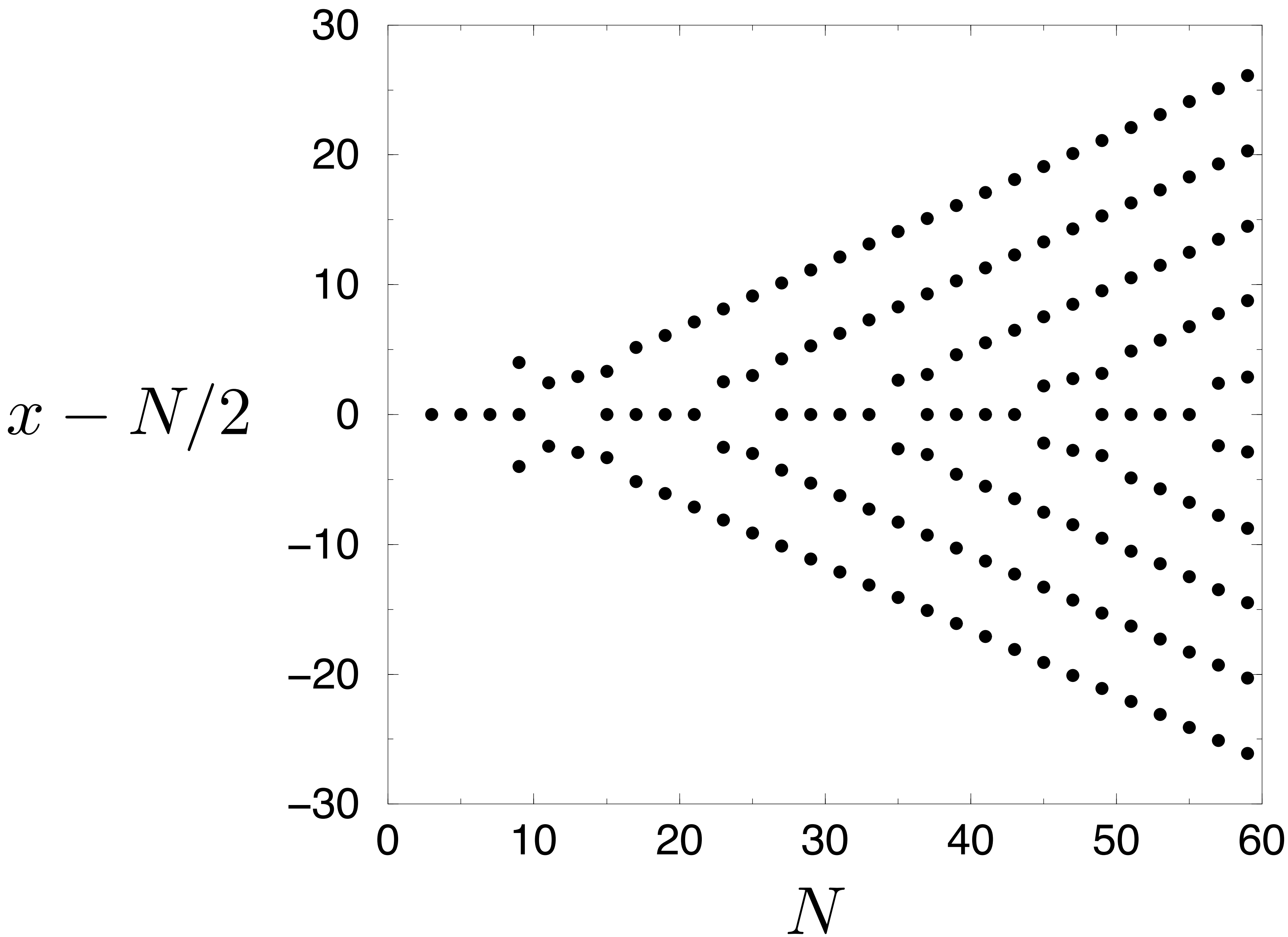


System evolves toward frozen state

Consensus when opinion spectrum is small

Generall, multiple opinion clusters (=political parties)

Periodic Pattern of Clusters



$\mathcal{N} \simeq N/L \qquad \text{with} \qquad L = 5.67$

What is the period L?

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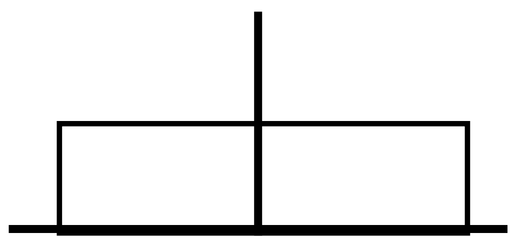
Problem set-up

- Given uniform initial (un-normalized) distribution

$$P_n(0) = \begin{cases} 0 & n < 1, \\ 1 & 1 \le n \le N, \\ 0 & N < n. \end{cases}$$

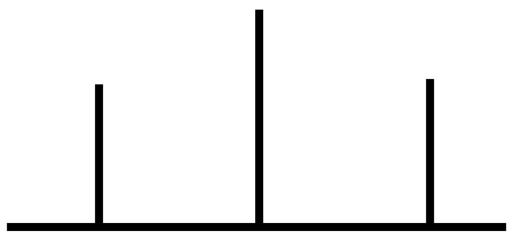


- Find final distribution

$$P_n(\infty)$$

- Multitude of final steady-states

$$P_{n-1}(\infty) P_{n+1}(\infty) = 0$$



- Dynamics selects one (deterministically!)

Multiple localized clusters
separation > interaction range

Master Equation

- Compromise process

$$(n-1, n+1) \to (n, n)$$

- Master equation (infinite population limit)

$$\frac{dP_n}{dt} = 2P_{n-1}P_{n+1} - P_n(P_{n-2} + P_{n+2})$$

- Two conservation laws: population, opinion

$$\sum_n p_n = \text{const.} \qquad \sum_n nP_n = \text{const.}$$

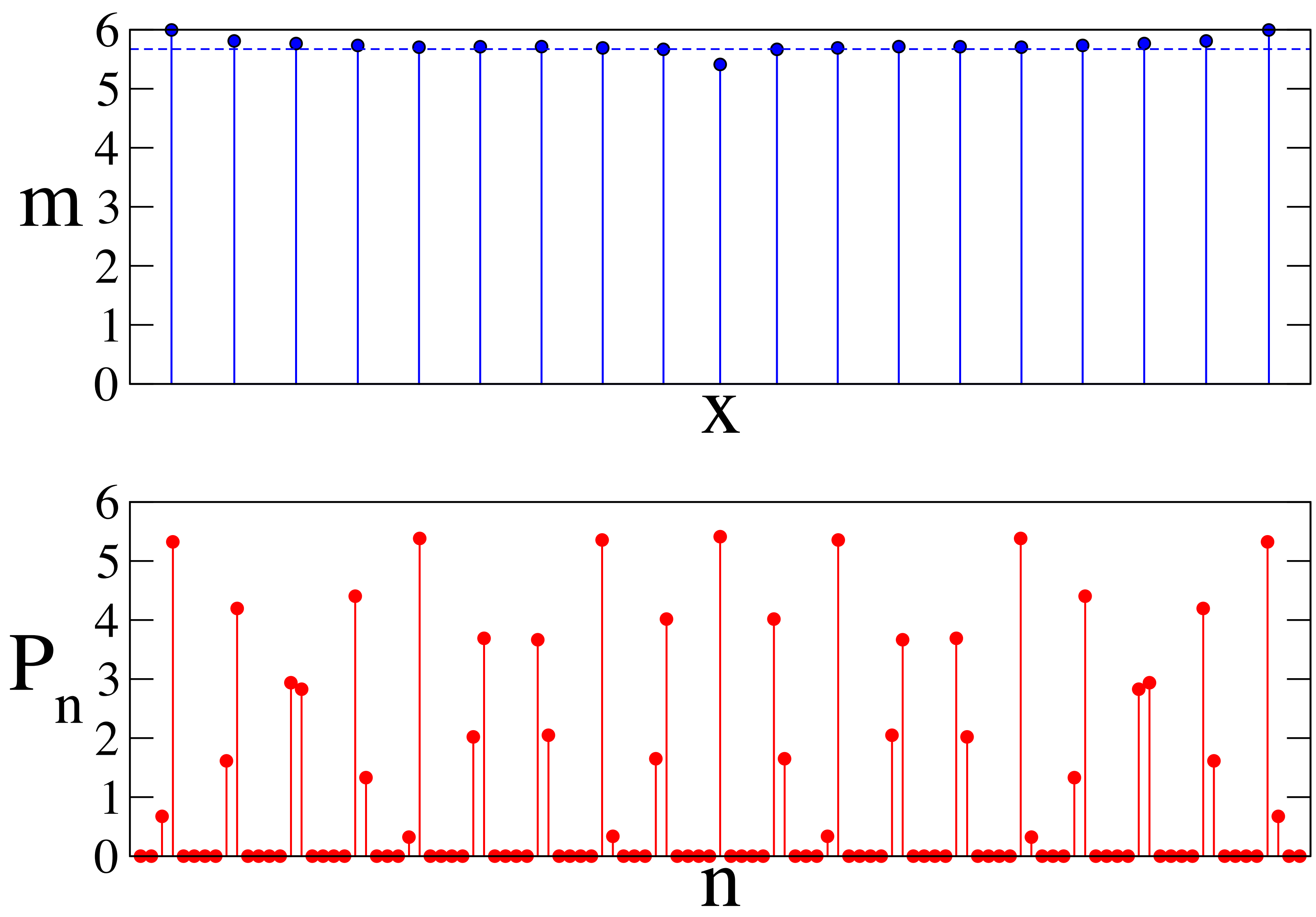
- Characterize cluster by mass and location (opinion)

$$m = P_n(\infty) + P_{n+1}(\infty) \qquad x = \frac{nP_n(\infty) + (n+1)P_{n+1}(\infty)}{m}$$

- Goal: find average cluster mass (=average spacing)

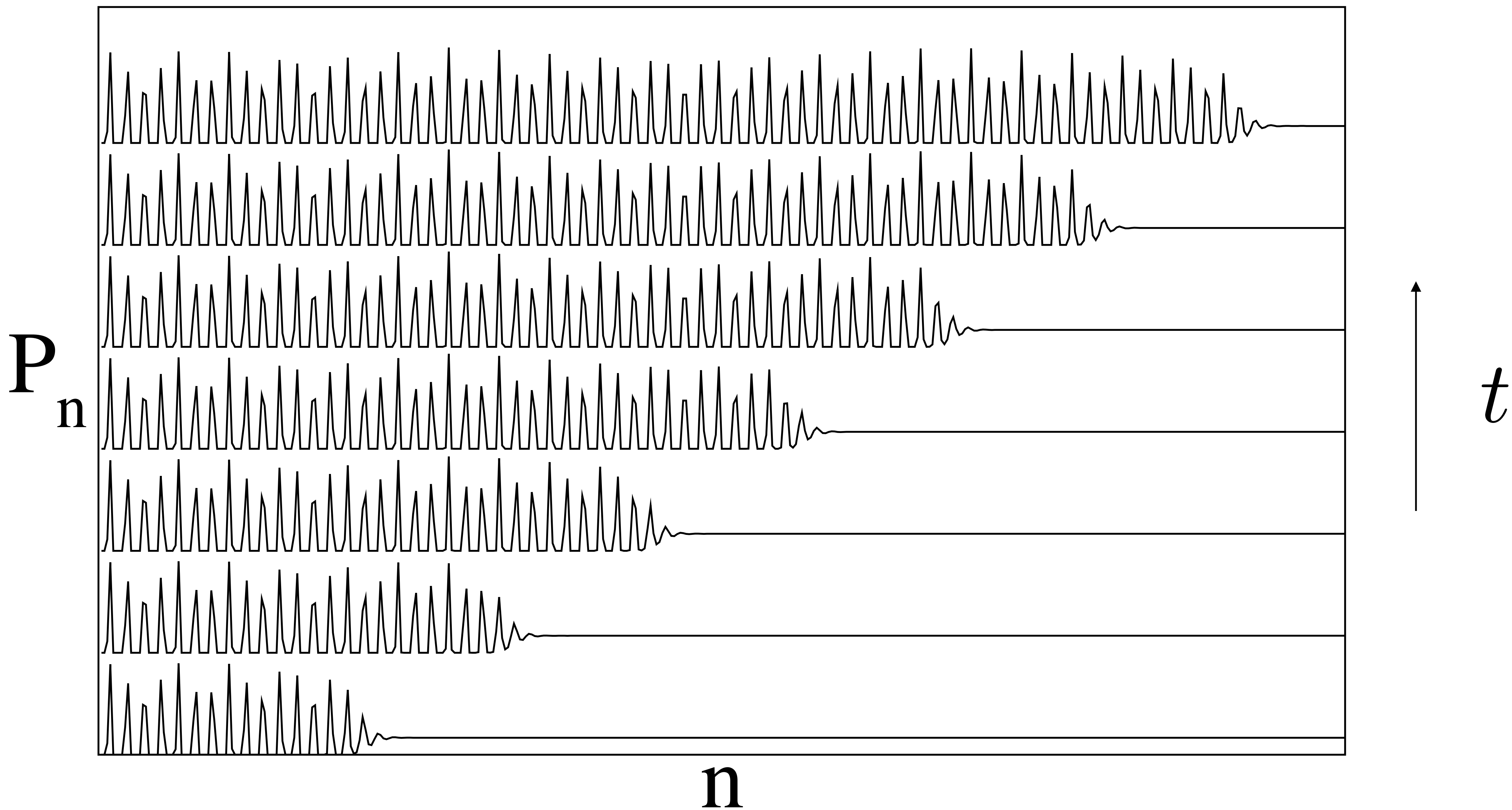
$$L = \lim_{N\to\infty} \langle m \rangle$$

The final state



Probability density is not periodic
Cluster masses are (nearly) identical
Clusters are (nearly) equally spaced

Traveling Wave



Traveling wave nucleates at domain boundary
Propagates into unstable uniform state
Leaves in its wake frozen clusters

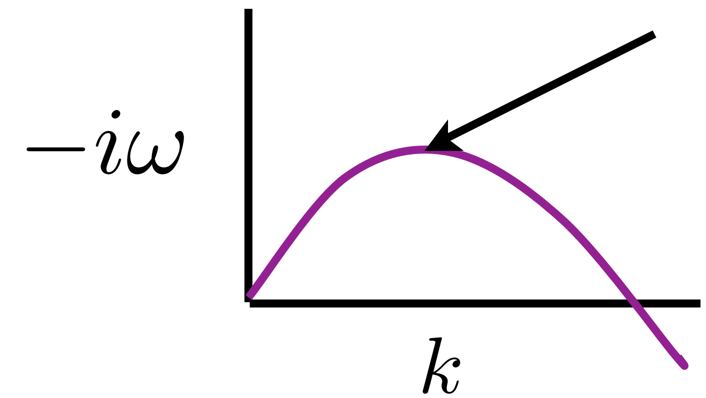
Pattern Selection

Dee & Langer 83
van Saarloos 03
Scheel 10

- Linear stability analysis

$$P_n(t) - 1 \propto e^{i(kn-\omega t)} \quad \Longrightarrow \quad \omega = 2i(2\cos k - \cos 2k - 1)$$

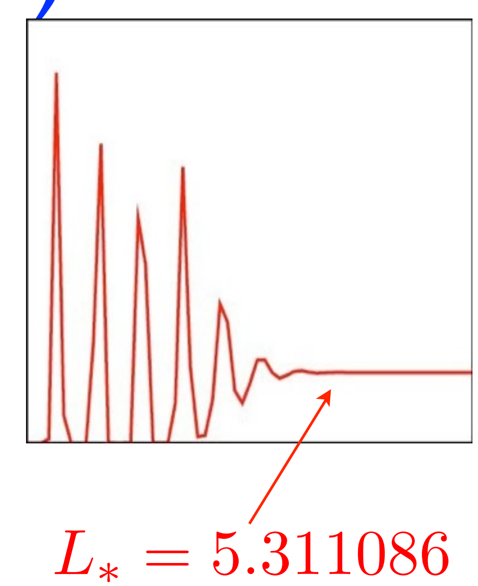
- Fastest growing mode



$$\frac{dw}{dk} \quad \Longrightarrow \quad L = \frac{2\pi}{k} = 6$$

- Traveling wave (FKPP saddle point analysis)

$$v = \frac{dw}{dk} = \frac{\mathrm{Im}[w]}{\mathrm{Im}[k]} \quad \Longrightarrow \quad k_{\text{select}} = k_* - \frac{w_*}{v}$$



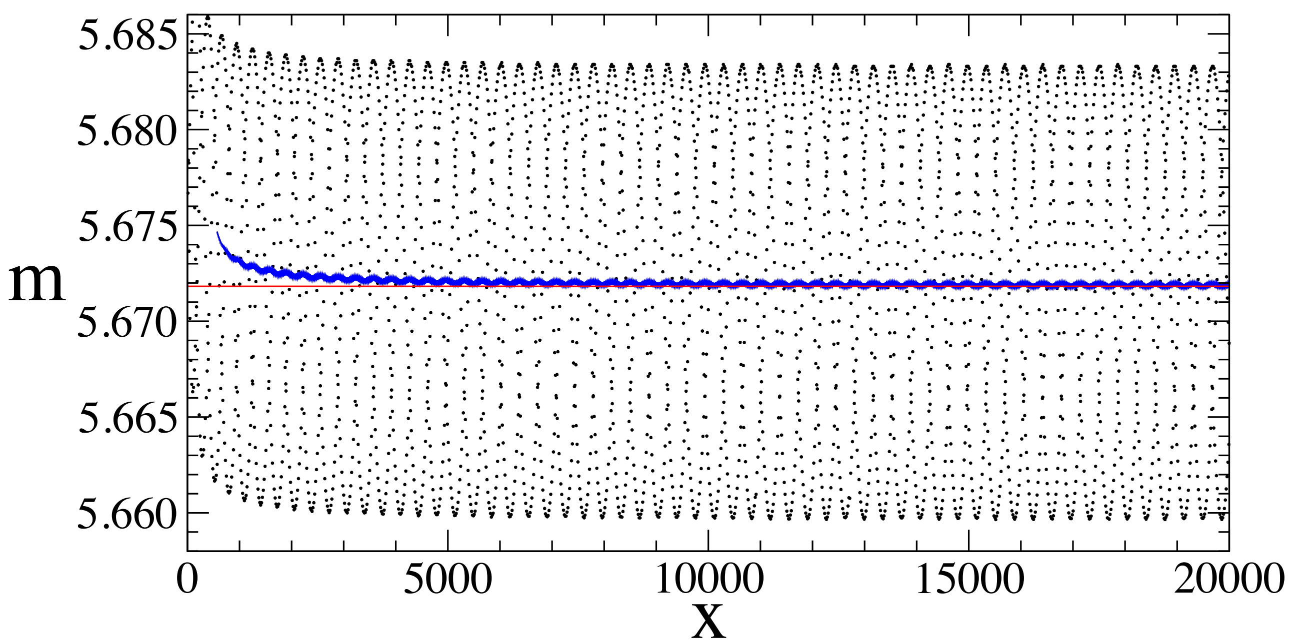
Patterns induced by wave propagation from boundary
Doppler-like shift in wavenumber, wavelength

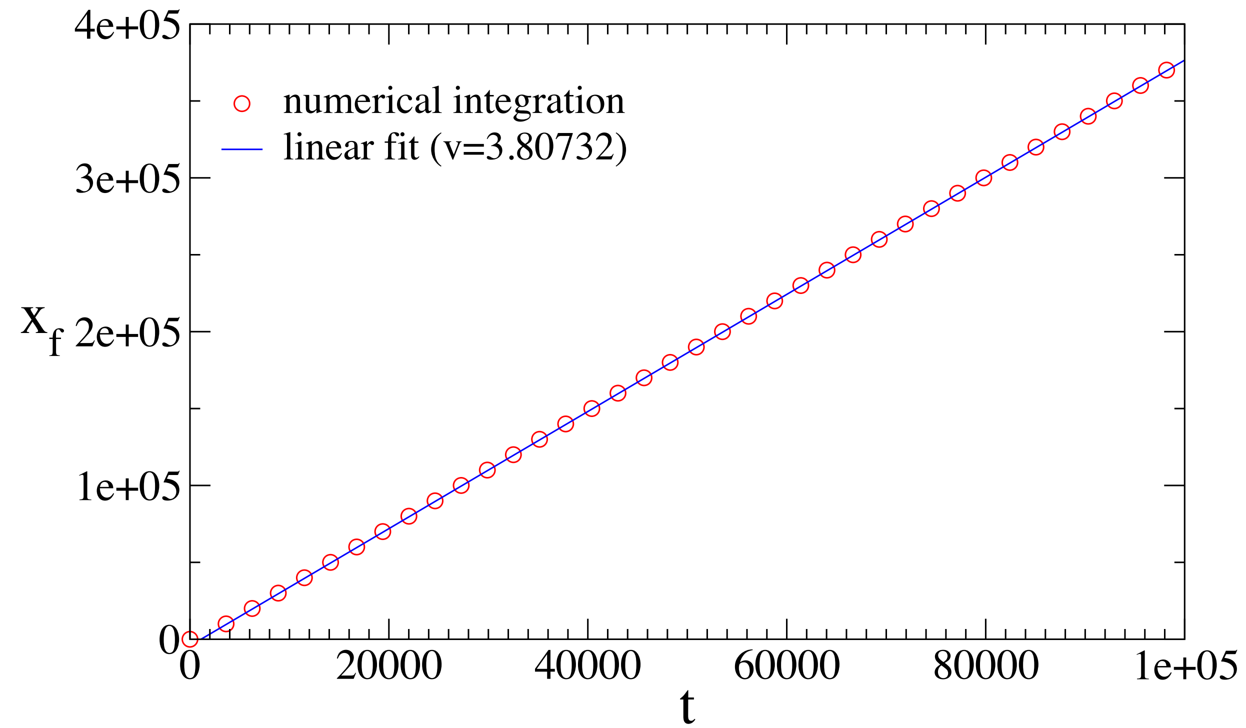
$$v = 3.807397 \qquad L_{\text{select}} = \frac{2\pi}{k_{\text{select}}} = 2.148644$$

Wavelength obtained analytically!

Numerical Verification

Numerical integration of coupled ODEs: Runge-Kutta (4,5)





$$L_{\mathrm{numeric}} = 5.67185$$
$$L_{\mathrm{theory}} = 5.67182$$

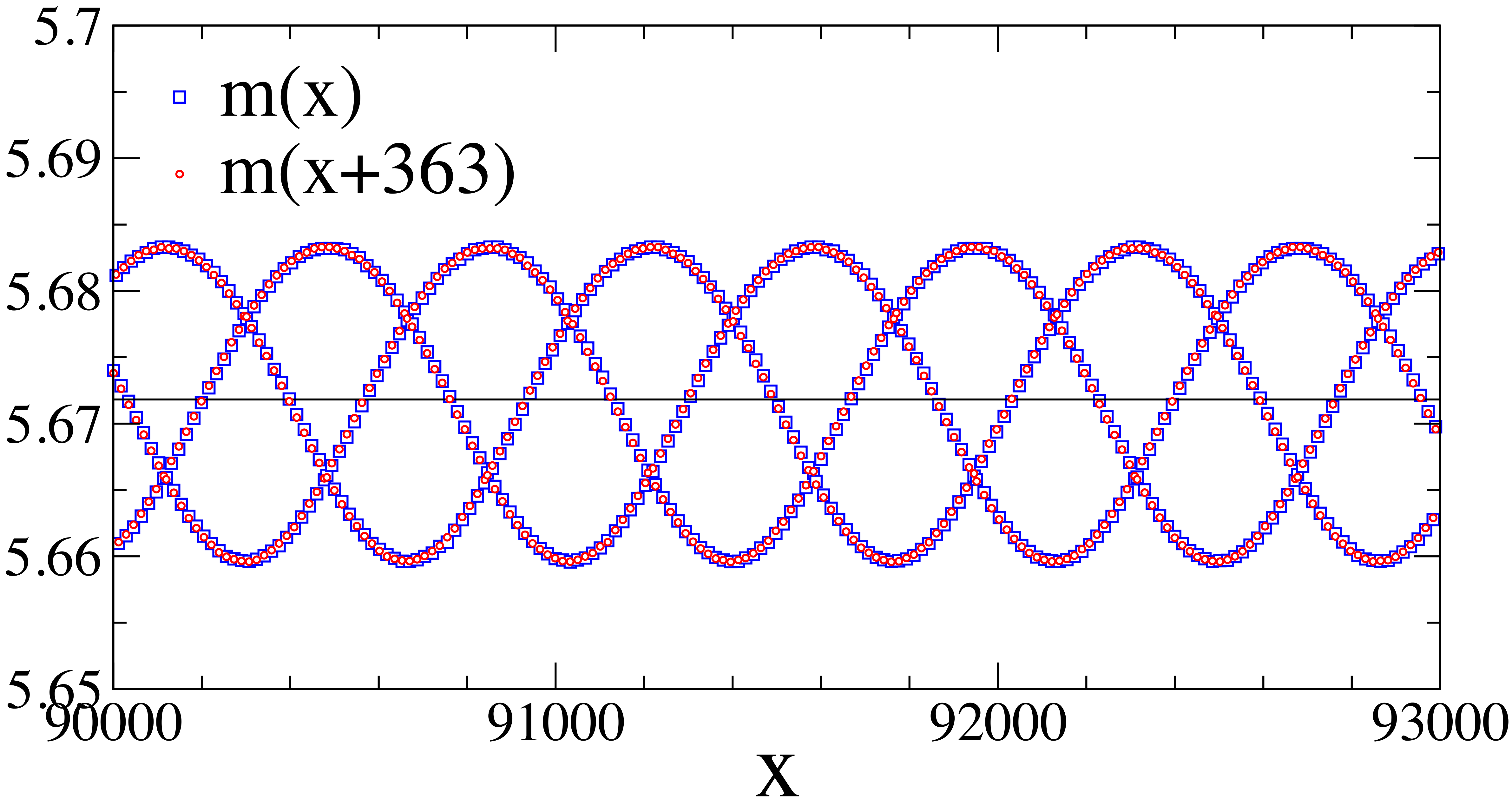
Excellent agreement with theoretical predictions!

$$v_{\mathrm{numeric}} = 3.80732$$
$$v_{\mathrm{theory}} = 3.80739$$

$$\Delta m/m \approx 10^{-3}$$

Small variations in cluster mass
Pattern is quasi-periodic
Clusters arrangement shows intricate patterns

Cluster masses are periodic!



Tiny undulations in cluster mass are periodic
Period of 363 is huge compared with selected wavelength of ~5.67

Super-patterns

- Wavelength is non-integer
- Incommensurate with unit lattice spacing
- Continued-fraction expansion of wavelength

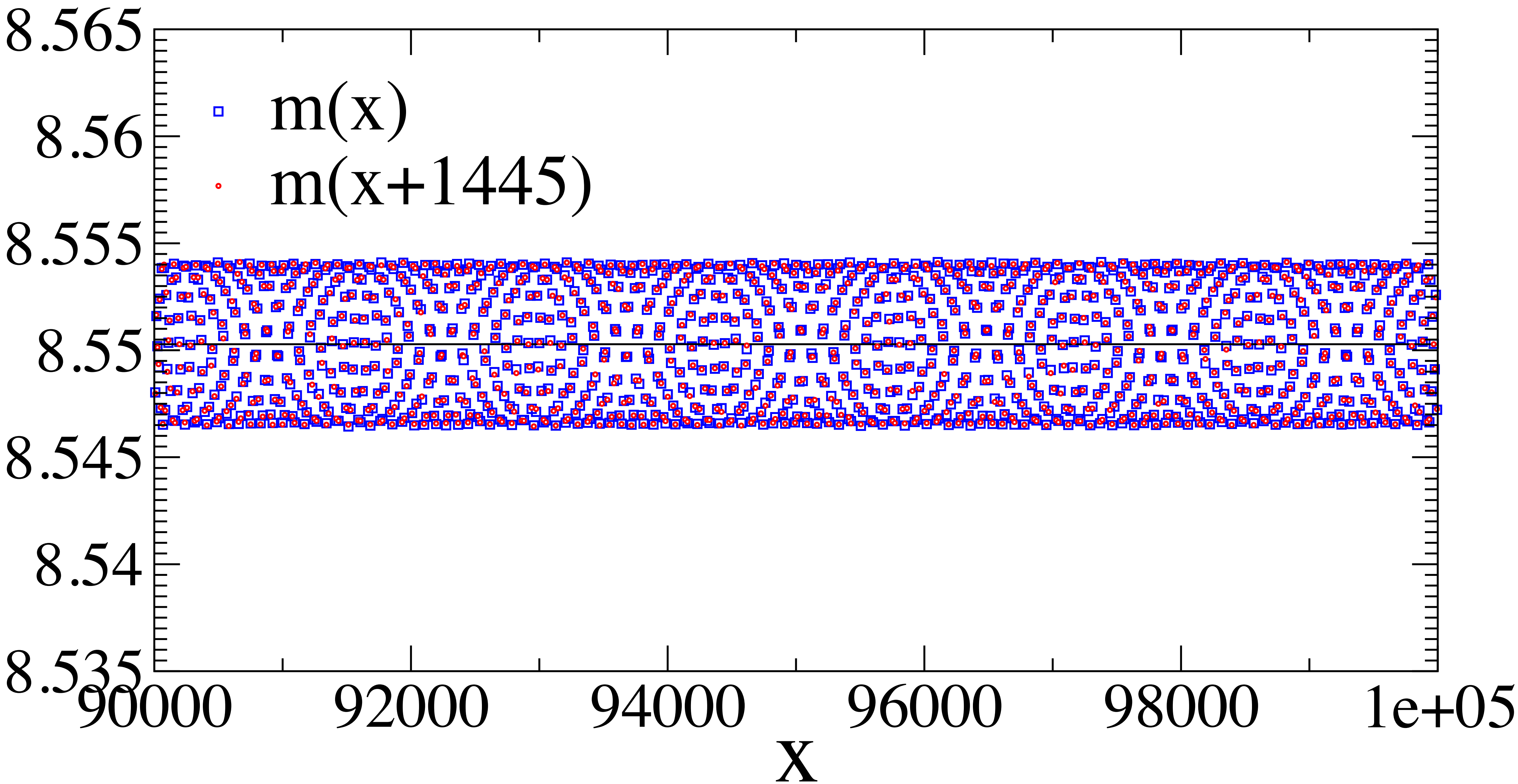
$$L = 5 + \cfrac{1}{1 + \frac{1}{2+\frac{1}{21+\frac{1}{4+\dots}}}} = 6, \frac{17}{3}, \frac{363}{64}, \frac{1469}{259}, \dots$$

- Hierarchy of patterns

1. A pattern of 3 clusters with period 17
2. A pattern of 64 clusters with period 363
3. A pattern of 259 clusters with period 1469?

Hierarchy of patterns with increasing complexity

Next-next nearest neighbor interact



$$L = 8.5502770 = 8 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cdots}}}} \equiv 9, \frac{17}{2}, \frac{77}{9}, \frac{171}{20}, \frac{1445}{169} \cdots$$

Period & pattern complexity increase

Continuous opinions

- Compromise process

$$(x_1, x_2) \to \left(\tfrac{x_1+x_2}{2}, \tfrac{x_1+x_2}{2}\right) \quad \text{if} \quad |x_1 - x_2| < 1.$$

- Master equation

$$\frac{\partial P(x,t)}{\partial t} = \iint_{|x_1 - x_2| < 1} dx_1 dx_2 P(x_1,t) P(x_2,t) \left[\delta\left(x - \frac{x_1 + x_2}{2}\right) - \delta(x - x_1\right]$$

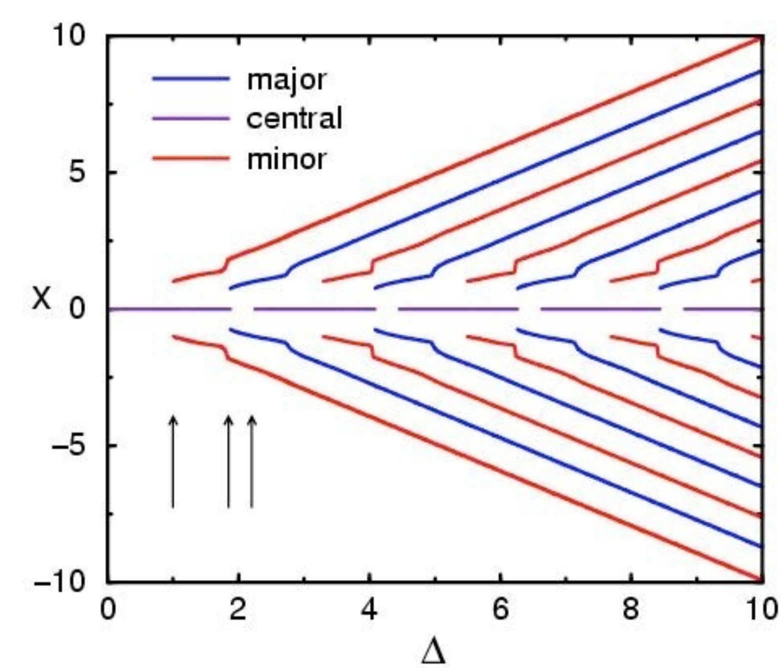
- Linear Stability & dispersion relation

$$P - 1 \propto e^{i(kx+wt)} \quad \Longrightarrow \quad w(k) = \frac{8}{k}\sin\frac{k}{2} - \frac{2}{k}\sin k - 2$$

- Selected wavelength

$$L_{\text{select}} = \frac{2\pi}{k_{\text{select}}} = 2.148644$$

$$L_{\text{numeric}} = 2.155$$



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Conclusions

- Bounded confidence model studied using pattern formation techniques
- Clusters are quasi-periodic, wavelength obtained analytically
- Wavelength incommensurate with lattice
- Superpatterns: integer number of clusters with integer period
- Intricate features can not be detected by Monte Carlo, require sophisticated numerical integration techniques

Outlook

- Two dimensions: opinions on two separate political issues
- Selection mechanism for super-patterns? all integers realized?