Energy Distributions in Granular Media

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Talk, papers available from: http://cnls.lanl.gov/~ebn

Collaborators

Experiment

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Theory

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- Katja Lindenberg
  San Diego

Simulation

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  Los Alamos
- Shiyi Chen
  Los Alamos
- Ben Machta
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- Alex Rosas
  San Diego

“Frozen” granular gases

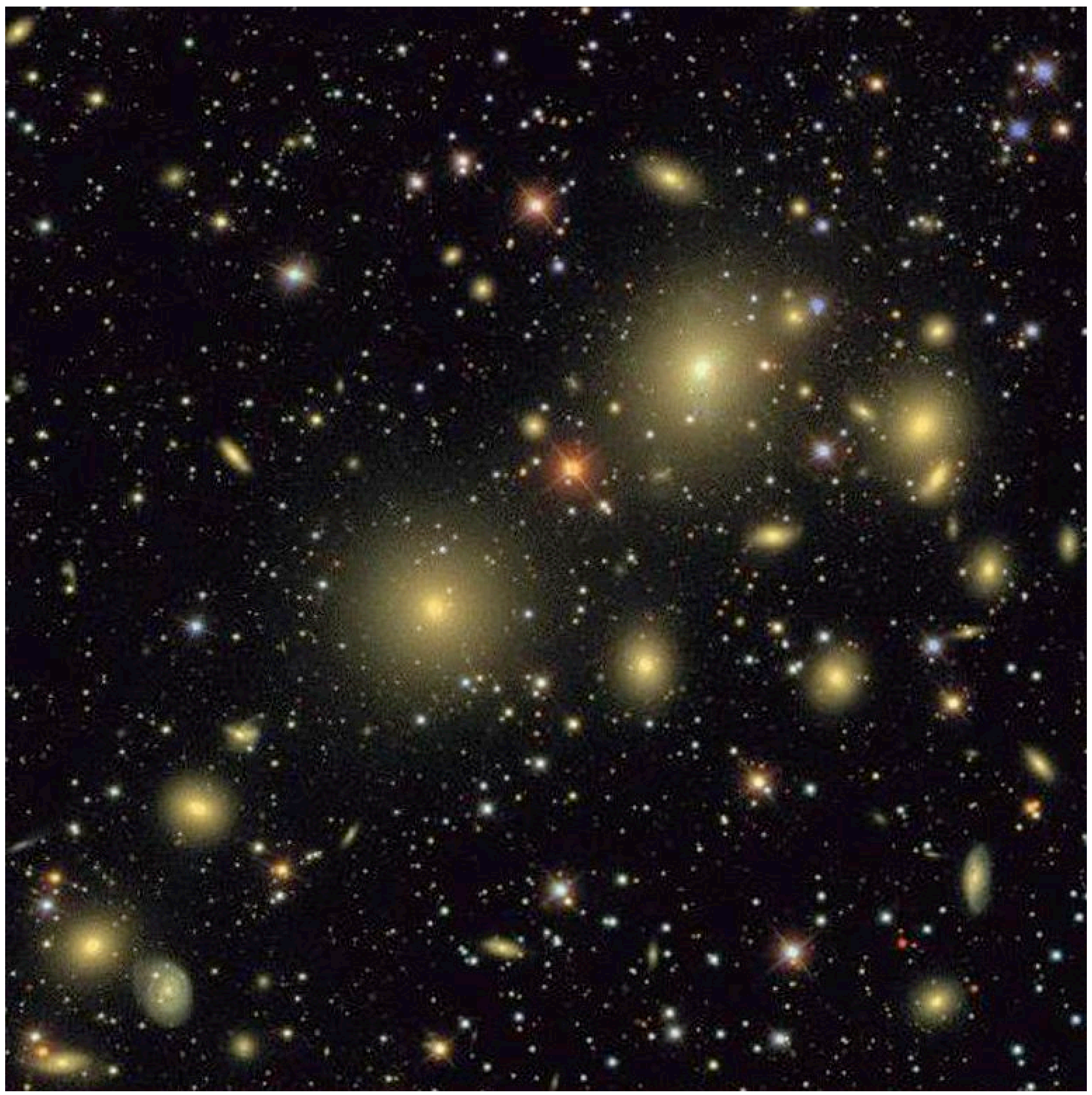
Saturn’s rings

Christoph Horman, artist

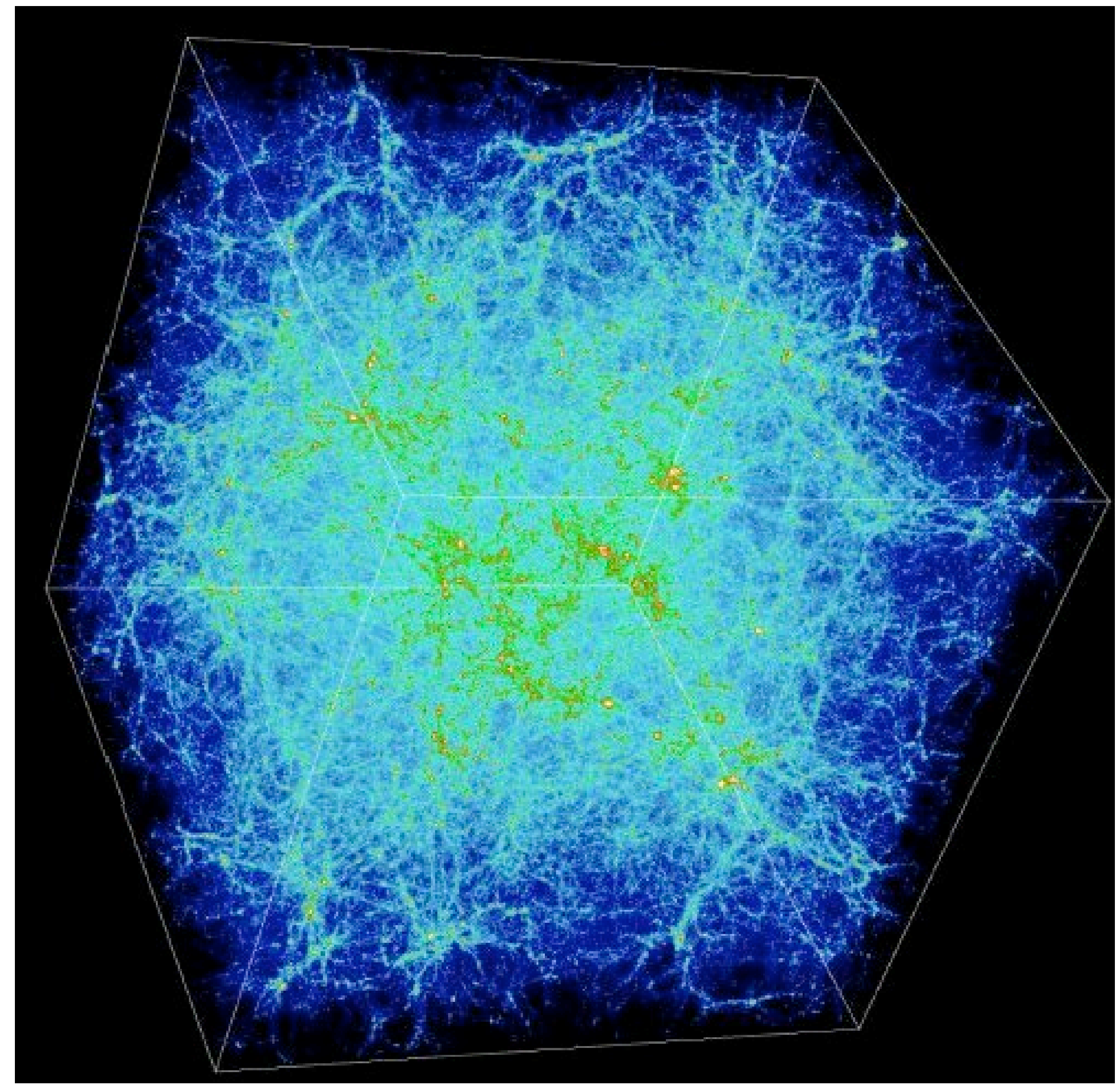
Snow avalanche

swiss institute for snow and avalanch research

Large scale formation of matter in the universe

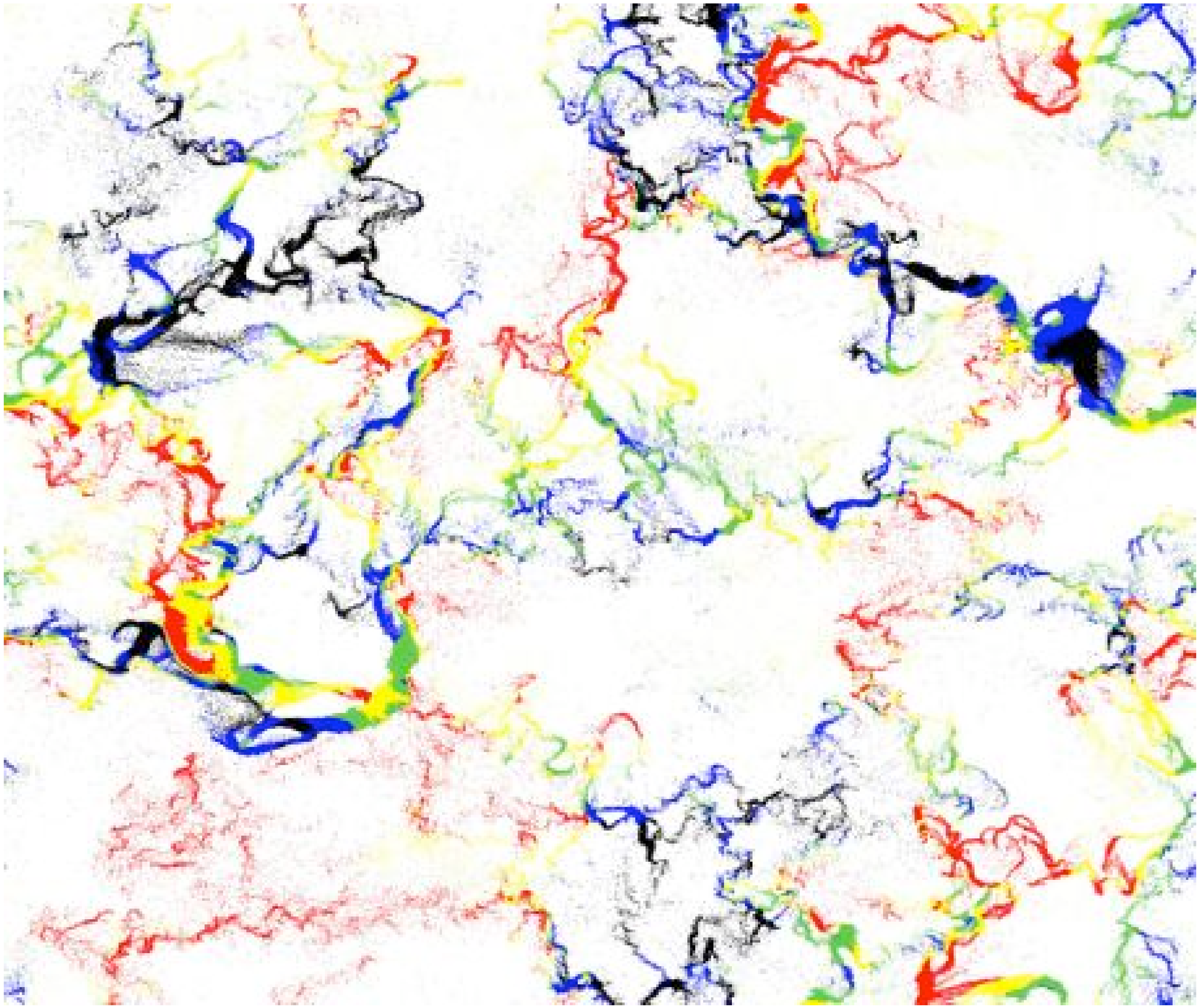


Sloan digital survey



Mike warren, Los Alamos

Filaments in a granular gas

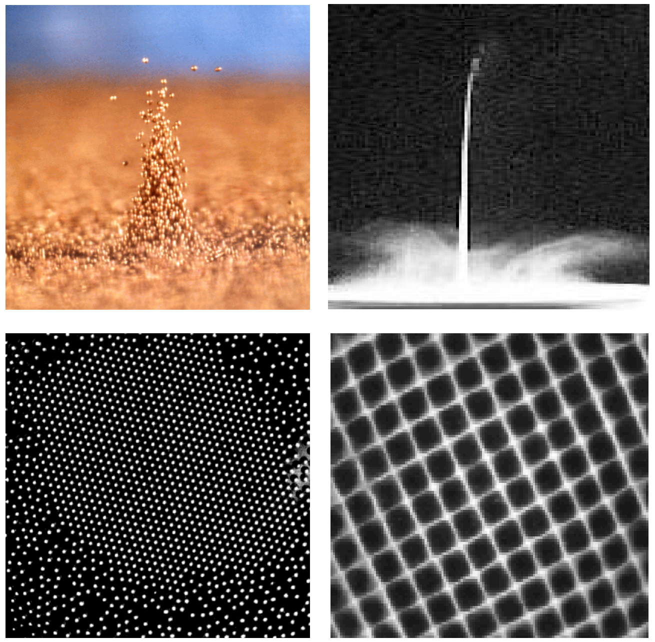


eb, s. chen, x. nie PRL 02

zeldovich & shandarin 89

Energy dissipation in granular media

- Responsible for collective phenomena
  - ★ Clustering
  - ★ Hydrodynamics instabilities
  - ★ Pattern formation

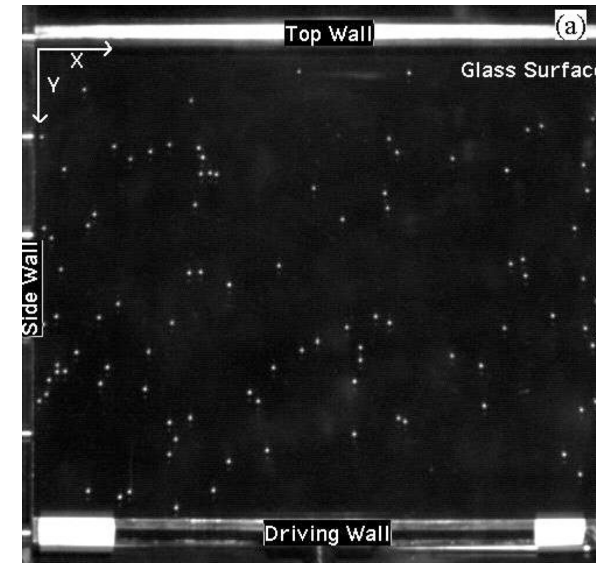


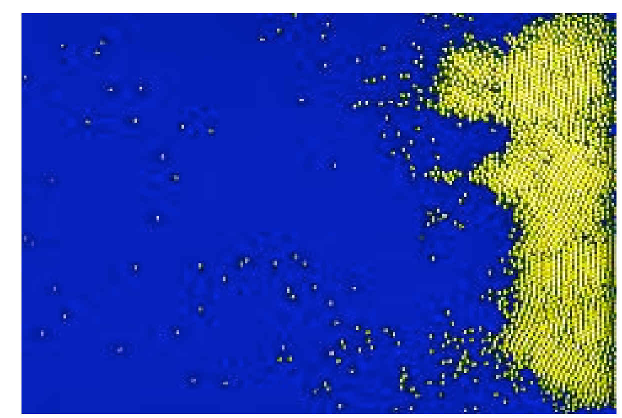
- Anomalous statistical mechanics
  - ★ No energy equipartition
  - ★ Nonequilibrium energy distributions

$$P(E) \neq \exp(-E/kT)$$

Experiments

- Friction Blair & Kudrolli 01
- Rotation Feitosa & Menon 04
- Driving Strength Losert & Gollub 98
- Dimensionality Urbach & Olafsen 98
- Boundary van Zon & Swinney 04
- Fluid drag Kohlstedt, Aronson, eb 05
- Long range interactions Olafsen, Aronson, eb 05
- Substrate Baxter & Olafsen 05





Deviations from Equilibrium Energy Distribution

Nonequilibrium velocity distributions

- Mechanically vibrated beads

F Rouyer & N Menon PRL 00

- Electrostatically driven powders

I Aronson & J Olafsen PRL 05

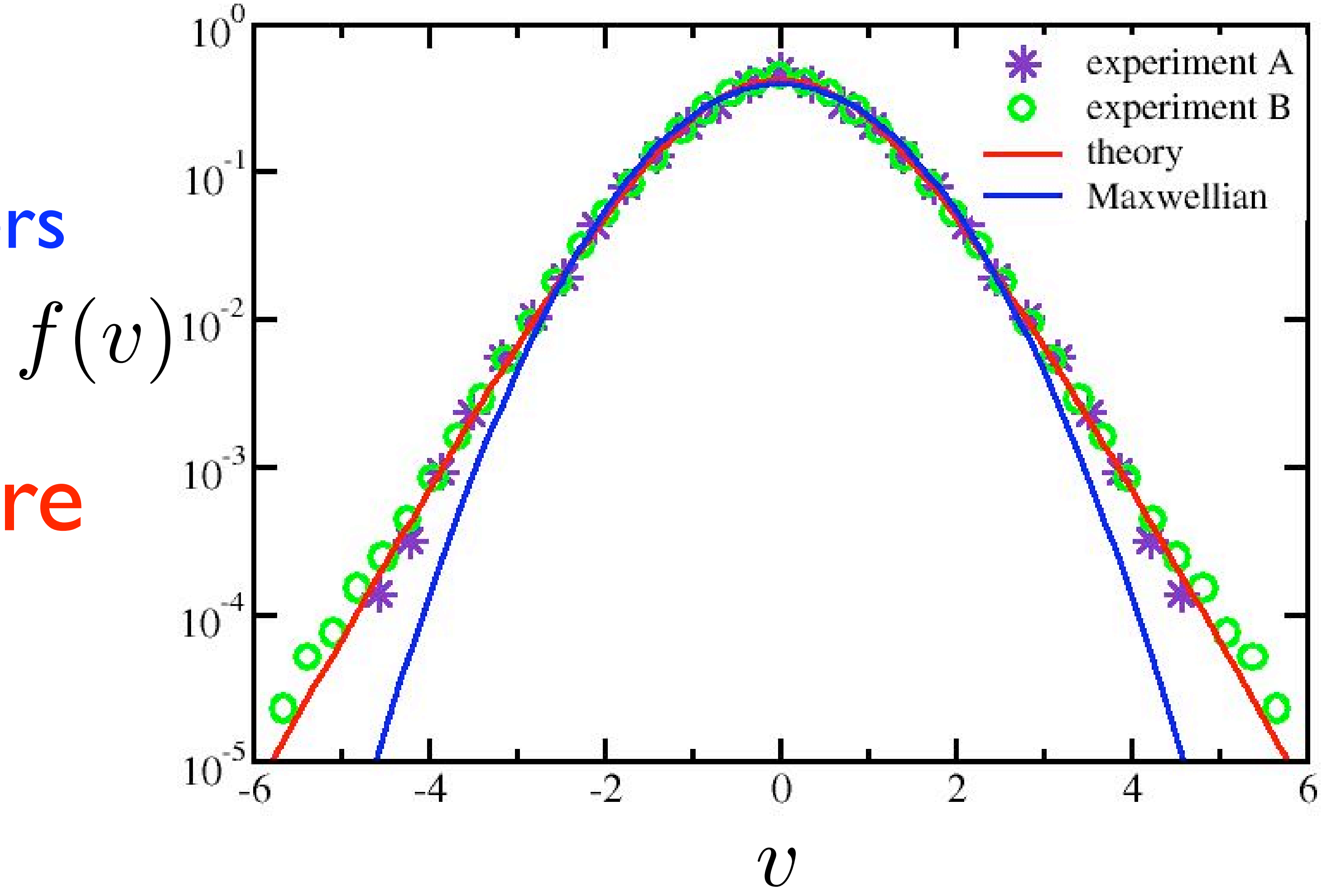
- Maxwell-Boltzmann core

$$f(v) \sim \exp(-v^2)$$

- Overpopulated tails

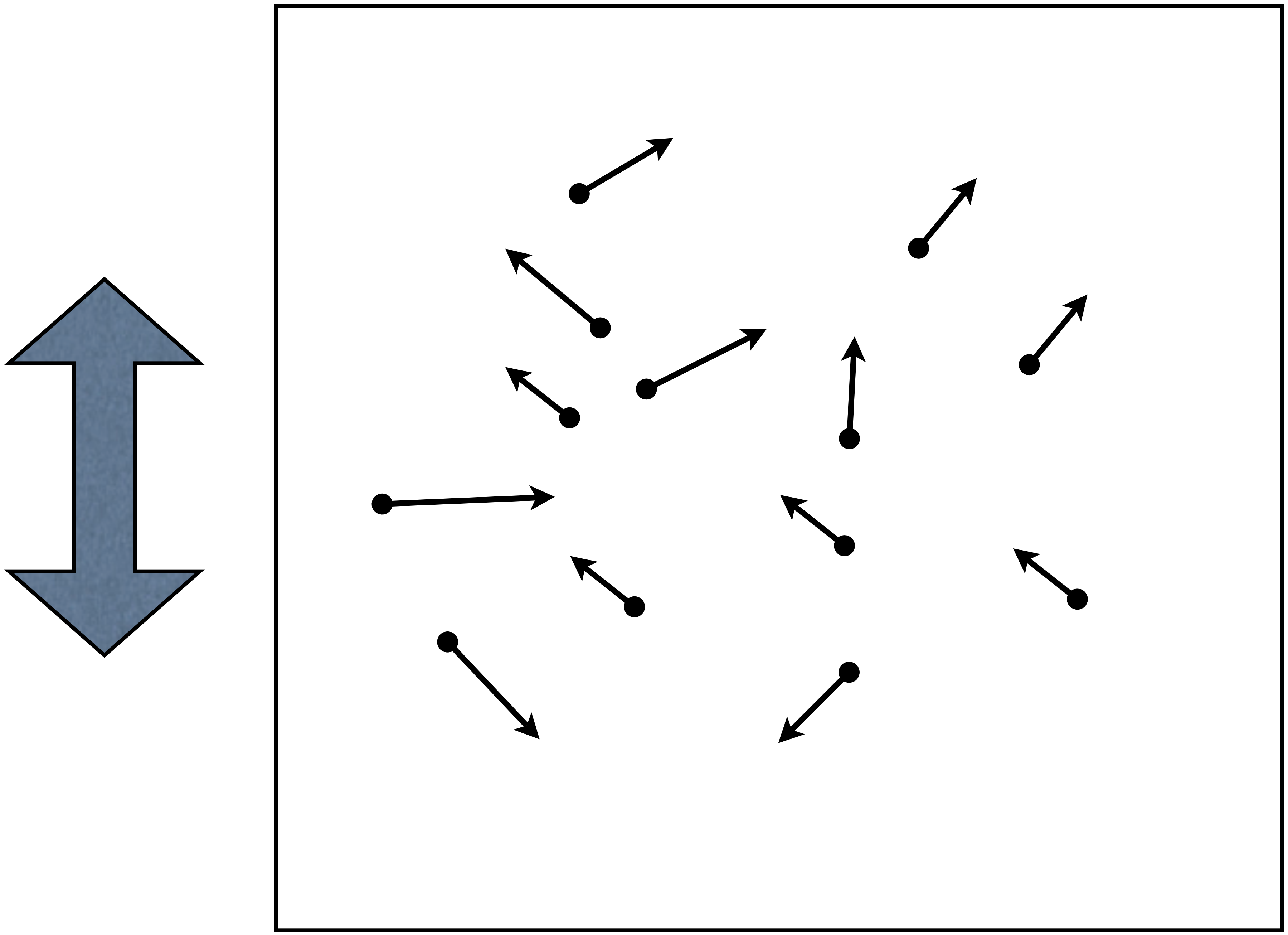
$$f(v) \sim \exp(-|v|^\delta)$$

$$1 \leq \delta \leq 3/2$$



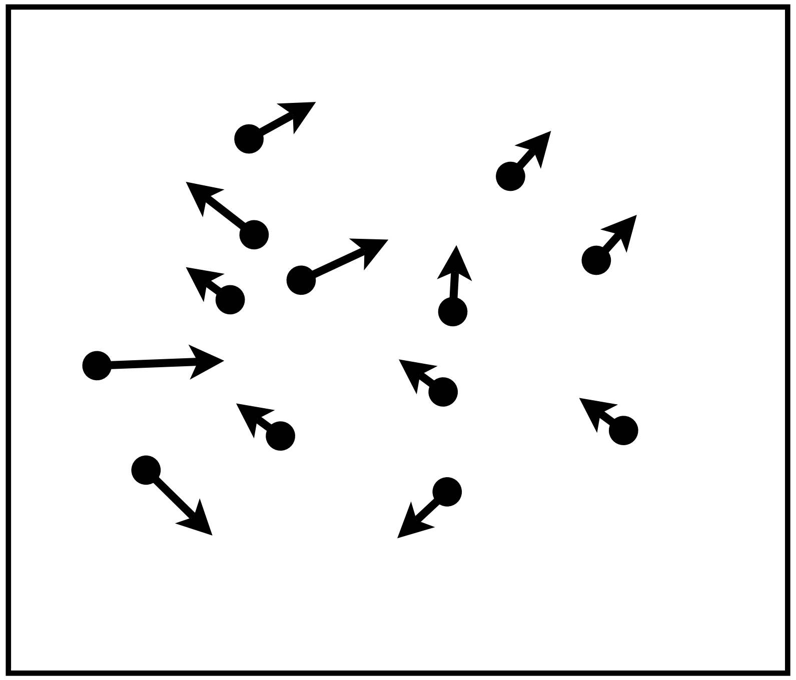
Excellent agreement between theory and experiment

“A shaken box of marbles”



Driven granular gas

- Vigorous driving (gravity irrelevant)
- Spatially uniform system
- Particles undergo binary collisions
- Velocities change due to:
  - ★ Collisions: lose energy
  - ★ Forcing: gain energy
- Time irreversibility, no detailed balance



Nonequilibrium steady state

Inelastic collisions (1D)

- Relative velocity reduced by $0 < r < 1$

$$v_1 - v_2 = -r(u_1 - u_2)$$

- Momentum is conserved

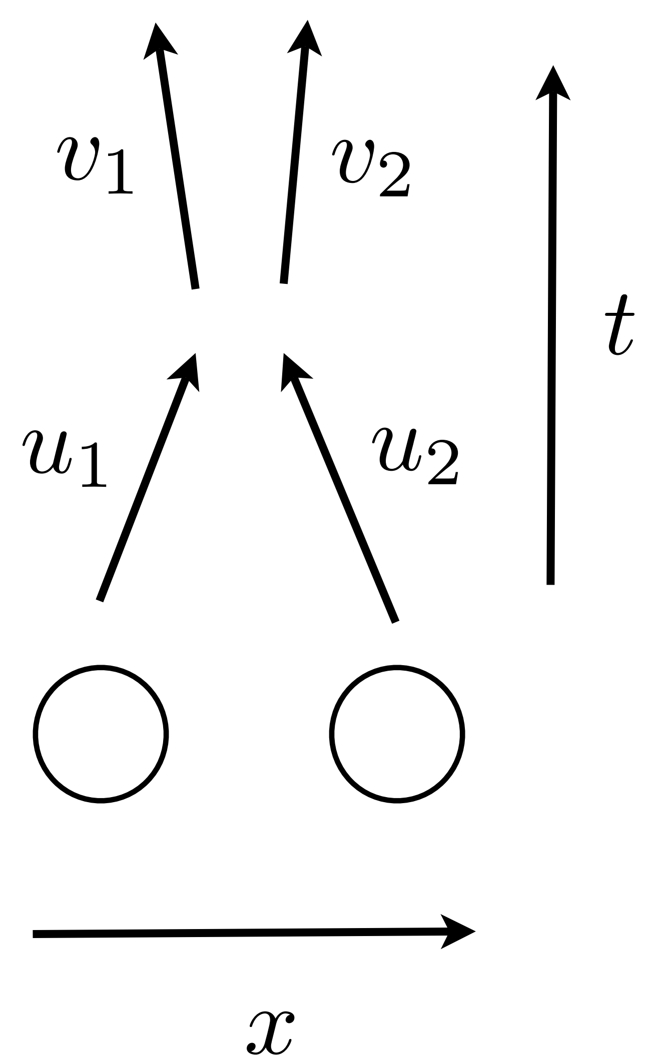
$$v_1 + v_2 = u_1 + u_2$$

- Energy is dissipated

$$\Delta E \propto (u_1 - u_2)^2$$

- Limiting cases

$$r = \begin{cases} 0 & \text{completely inelastic } (\Delta E = \max) \\ 1 & \text{elastic } (\Delta E = 0) \end{cases}$$



Theoretical model

Two independent competing processes

1. Inelastic collisions (nonlinear)

$$(v_1, v_2) \to \left(\frac{v_1 + v_2}{2}, \frac{v_1 + v_2}{2} \right) \qquad r = 0$$

2. Random uncorrelated white noise (linear)

$$\frac{dv_j}{dt} = \eta_j(t) \qquad \langle \eta_j(t) \eta_j(t') \rangle = 2D\delta(t - t')$$

System reaches a nontrivial steady-state
Energy injection balances dissipation

Kinetic theory

- Boltzmann equation

$$\cancel{\frac{\partial P(v)}{\partial t}} = D\frac{\partial^2 P(v)}{\partial v^2} + \iint dv_1 dv_2 P(v_1)P(v_2)\delta\left(v - \frac{v_1+v_2}{2}\right) - P(v)$$

- Fourier transform

$$F(k) = \int dv\, e^{ikv} P(v)$$

- Closed nonlinear and nonlocal equation

$$(1+Dk^2)F(k) = F^2(k/2)$$

- Invariance

$$k \to k\sqrt{D} \qquad \text{or} \qquad v \to v/\sqrt{D}$$

Shape of distribution is independent of forcing strength

Infinite product solution

- Solution by iteration

$$F(k) = \frac{1}{1+Dk^2}F^2(k/2) = \frac{1}{1+Dk^2}\frac{1}{(1+D(k/2)^2)^2}F^4(k/4) = \cdots$$

- Infinite product solution

$$F(k) = \prod_{i=0}^{\infty}\left[1+D(k/2^i)^2\right]^{-2^i}$$

- Exponential tail $v \to \infty$

$$P(v) \propto \exp\left(-|v|/\sqrt{D}\right)$$

$$P(k) \propto \frac{1}{1+Dk^2} \propto \frac{1}{k-i/\sqrt{D}}$$

- Also follows from

$$D\frac{\partial^2 P(v)}{\partial v^2} = -P(v)$$

Ernst 97

Non-Maxwellian distribution/Overpopulated tails

Non-Maxwellian velocity distributions

J C Maxwell 1867

1. Velocity distribution is isotropic

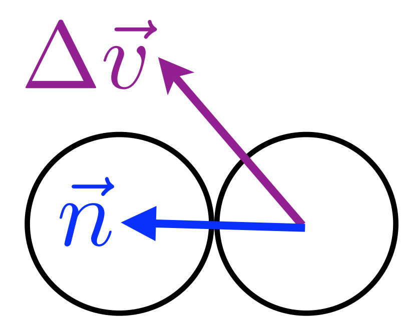
$$f(v_x, v_y, v_z) = f(|v|)$$

2. No correlations between velocity components

$$f(v_x, v_y, v_z) \neq f(v_x) f(v_y) f(v_z)$$

$$\begin{aligned} \vec{n} \cdot \Delta \vec{v}' &= -r_n \, \vec{n} \cdot \Delta \vec{v} \\ \vec{n} \times \Delta \vec{v}' &= -r_t \, \vec{n} \times \Delta \vec{v} \end{aligned}$$

Only possibility is Maxwellian



$$f(v_x, v_y, v_z) \neq C \exp\left(-\frac{v_x^2 + v_y^2 + v_z^2}{2T} \right)$$

Granular media: collisions generate correlations

Deviations from Maxwell-Boltzmnn

- Velocity correlations

$$C_{xy} = \frac{\langle v_x^2 v_y^2 \rangle}{\langle v_x^2 \rangle \langle v_y^2 \rangle}$$

- Exact expression

$$C_{xy} = \frac{6\left(\frac{1-r}{2}\right)^2}{d - \left[1 + 3\left(\frac{1-r}{2}\right)^2\right]}$$

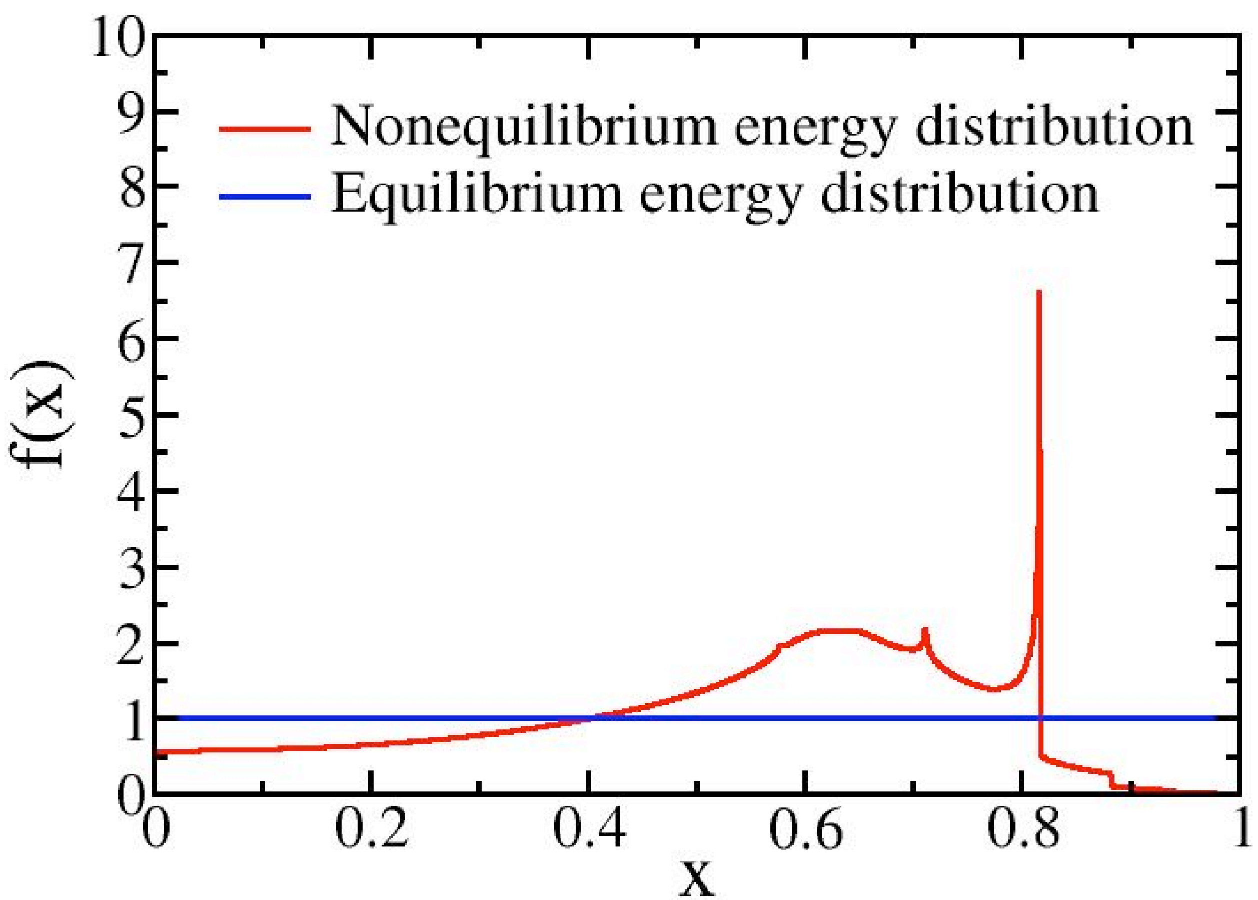
- Inversely proportional to dimension

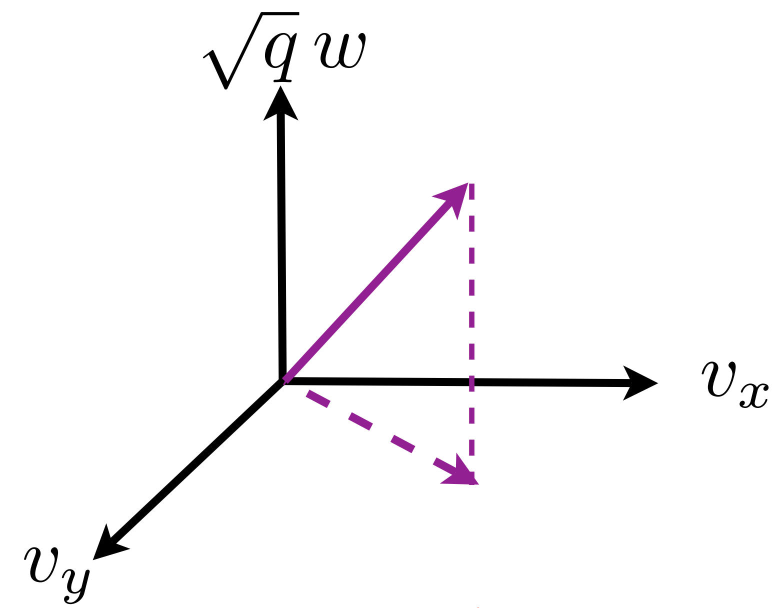
$$C_{xy} \sim d^{-1} \qquad \text{as} \qquad d \to \infty$$

- Vanished in the elastic limit

$$C_{xy} \sim (1-r)^2 \qquad \text{as} \qquad r \to 1$$

Energy equipartition among degrees of freedom





$$x^2 = \frac{\frac{1}{2}Iw^2}{\frac{1}{2}mv^2 + \frac{1}{2}Iw^2}$$

$$f_{\text{equilibrium}}(x) = 1$$

<u>Equilibrium (molecular gases)</u>

translational and rotational energies perfectly equivalent

- ❖ Energy distribution is singular!
- ❖ “Magic” values of rotational energy preferred
- ❖ Energy partitioned unequally among degrees of freedom

Stationary solutions

- Stationary solutions do exist!

$$F(k) = F^2(k/2)$$

- Family of exponential solutions

$$F(k) = \exp(-kv_0)$$

- Lorenz distribution

$$P(v) = \frac{1}{\pi v_0} \frac{1}{1 + (v/v_0)^2}$$

How is a stationary solution consistent with energy dissipation?

Extreme statistics

- Large velocities, cascade process

$$v \to \left(\frac{v}{2}, \frac{v}{2}\right)$$

$$(v_1, v_2) \to \left(\frac{v_1 + v_2}{2}, \frac{v_1 + v_2}{2}\right)$$

- Linear evolution equation

$$\frac{\partial P(v)}{\partial t} = 4P\left(\frac{v}{2}\right) - P(v)$$

- Steady-state: power-law distribution

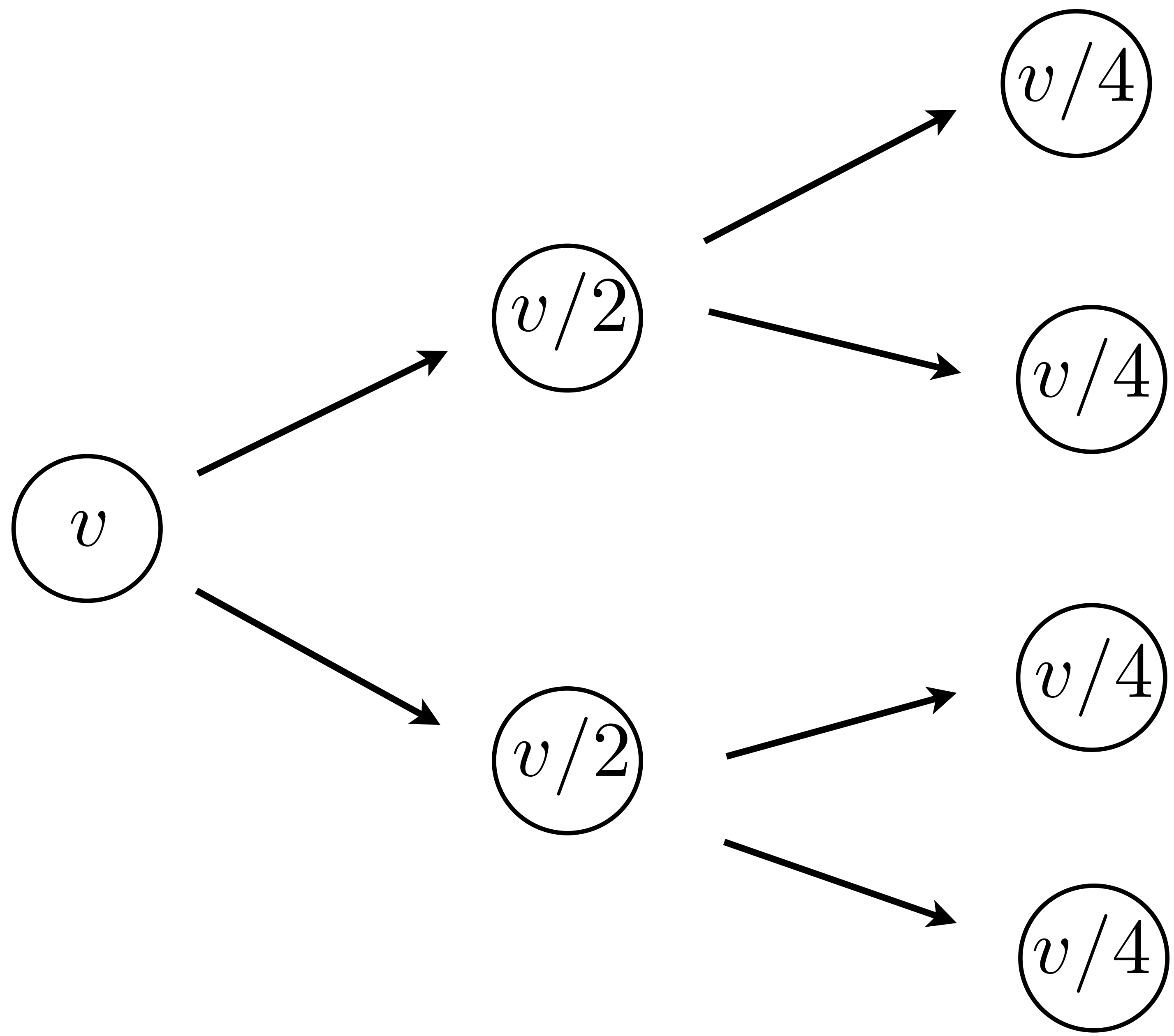
$$P(v) \sim v^{-2}$$

$$4P\left(\frac{v}{2}\right) = P(v)$$

- Divergent energy, divergent dissipation rate

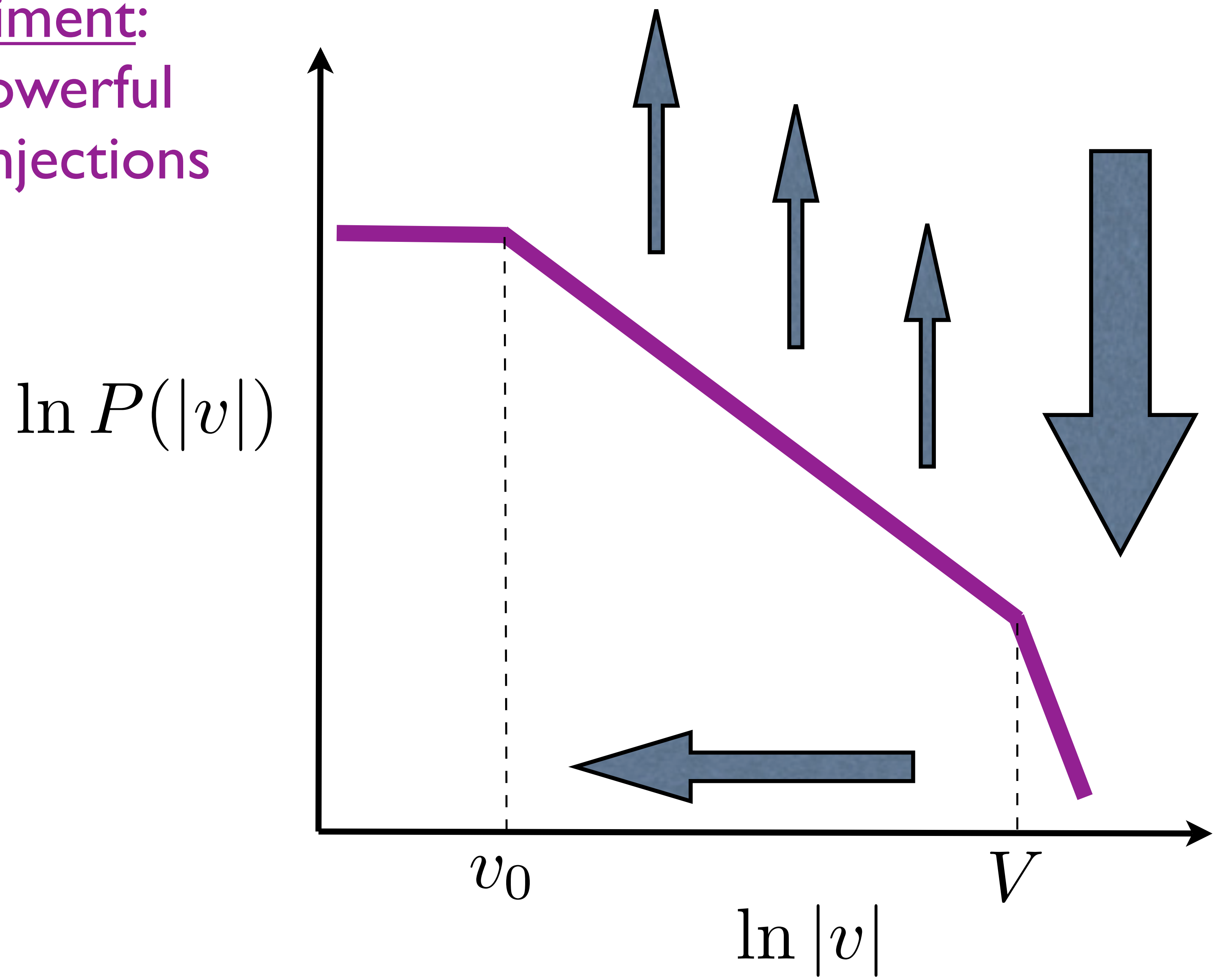
Power-law energy distribution $P(E) \sim E^{-3/2}$

Energy cascade



Injection, Cascade, Dissipation

<u>Experiment:</u>
rare, powerful
energy injections



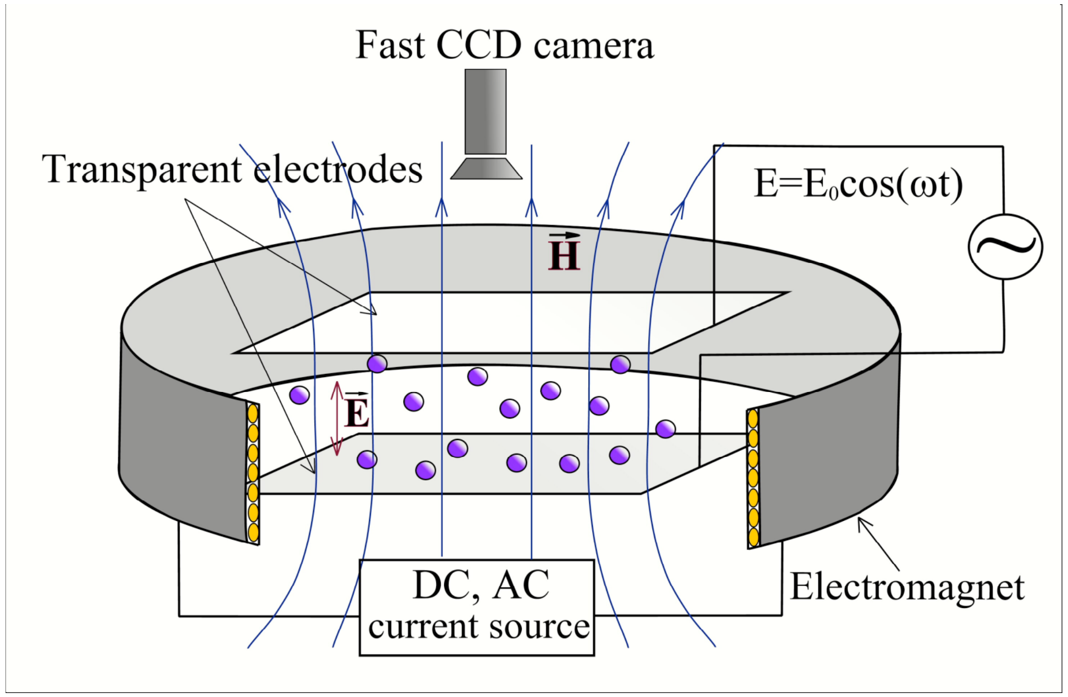
<u>Lottery MC:</u>
award one particle
all dissipated energy

Injection selects the typical scale!

Fluid drag

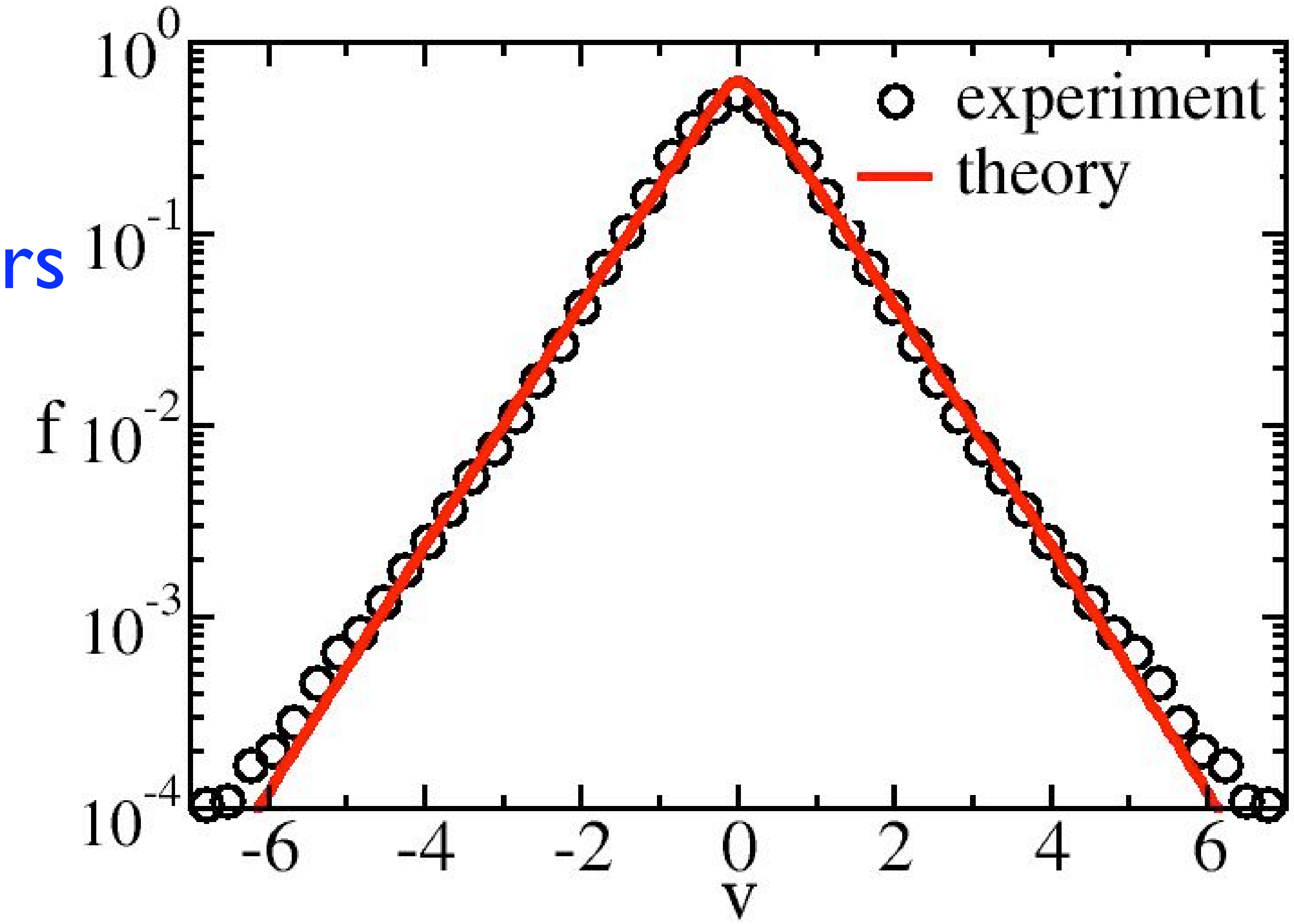
- Electrostatically driven powders

I Aronson & J Olafsen PRL 05



Exponential distribution

$$f(v) \sim \exp(-|v|)$$



Theoretical model

- Discrete fluid drag

$$v \to \gamma v$$

- White noise forcing

$$\frac{dv_j}{dt} = \eta_j(t)$$

Conclusions

- Nonequilibrium steady-states
- Energy pumped and dissipated by different mechanisms
- Overpopulation of high-energy tail with respect to equilibrium distribution
- Energy partitioned unevenly between translational and rotational degrees of freedom
- Cascade of energy from high to low energies