Bifurcations and Patterns in Opinion Dynamics

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Thanks

Paul Krapivsky and Sidney Redner *(Boston)*

http://cnls.lanl.gov/~ebn

Plan

1. Motivation: modeling social systems
2. Continuous opinions: simulations, scaling
3. Discrete opinions: general features, theory
4. Pattern selection: linear stability analysis
5. Extensions: initial conditions, 2D, noise

Modeling social dynamics

- Goal: predictive models of human opinions
- Relevance: politics, economics, consumer, sports

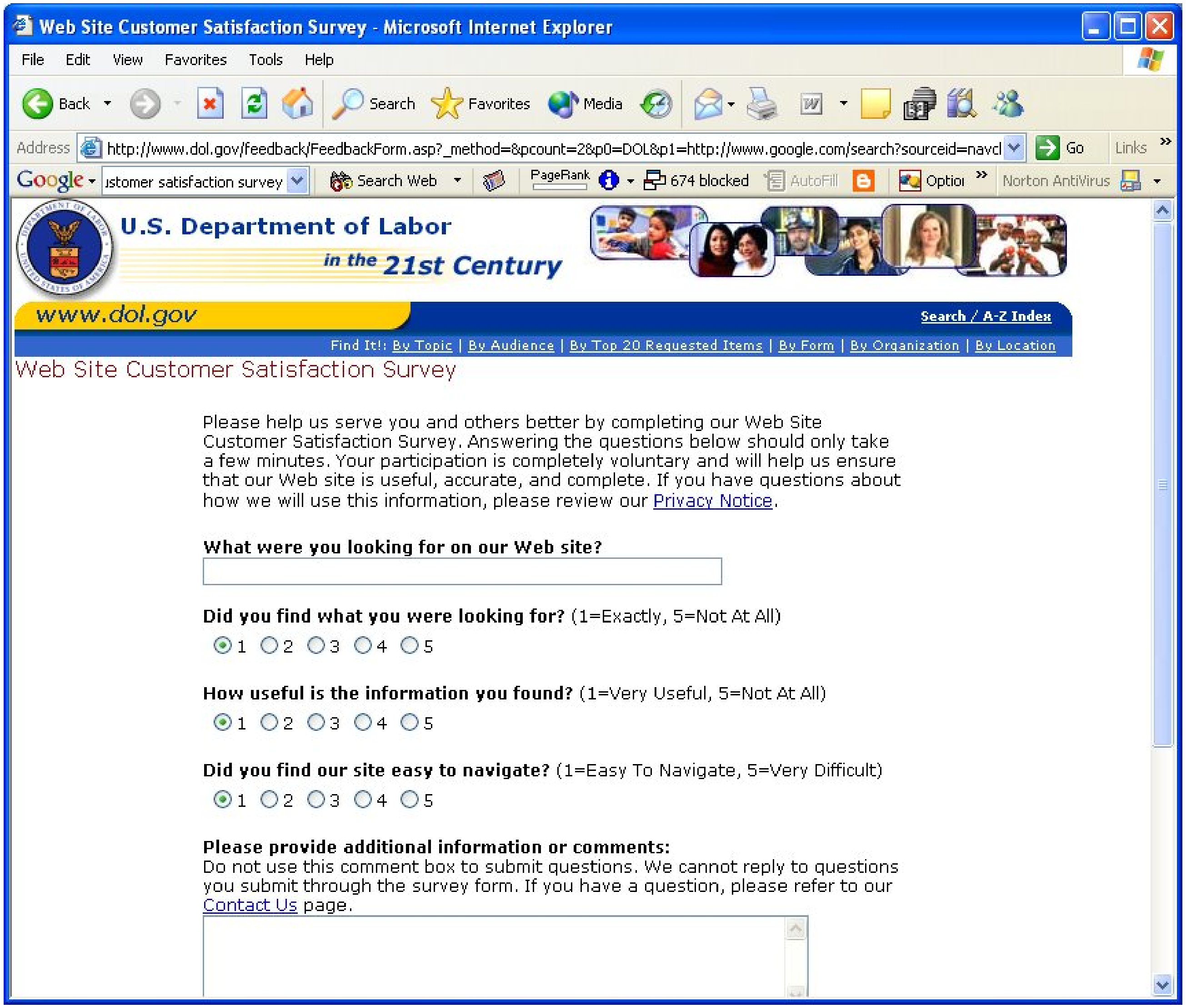
Questions

- Are “physics concepts useful?
- Are human interactions predictable?

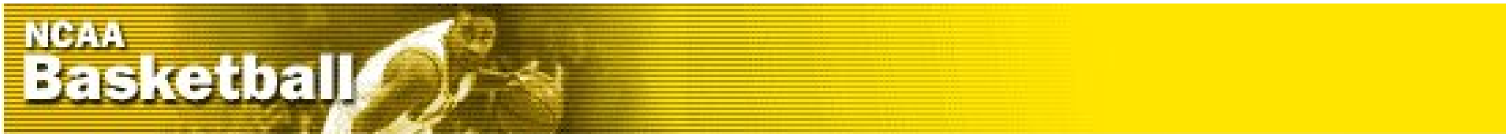
This should help

- Large data sets available
- Large number of humans $N \sim 10^9$
- Human opinions are quantitative

Quantifying opinions



Humans interact, opinions evolve



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Rankings

Division I Polls

AP Top 25		USA Today/ESPN	
1.	Connecticut (37)	1.	Connecticut (18)
2.	Duke (5)	2.	Duke (6)
3.	Georgia Tech (21)	3.	Georgia Tech (5)
4.	Arizona (4)	4.	Arizona
5.	Stanford (3)	5.	Wake Forest (1)
6.	Wake Forest (1)	6.	Kentucky
7.	Oklahoma	7.	Stanford
8.	Kentucky	8.	Oklahoma
9.	North Carolina	9.	North Carolina
10.	St. Joseph's (PA)	10.	St. Joseph's (PA) (1)

tendency to reach consensus?

The Compromise Process

- **Opinion measured by continuum variable**

$$-\Delta < x < \Delta$$

- **Compromise: reached via pairwise interactions**

$$(x_1, x_2) \rightarrow \left(\frac{x_1 + x_2}{2}, \frac{x_1 + x_2}{2} \right)$$

- **Conviction: restricted interaction range**

$$|x_1 - x_2| < 1$$

- **Minimal, one parameter model**
- **Mimics competition between compromise and conviction**

Weisbuch 2001

Problem

- **Given initial distribution**

$$P_0(x) = \begin{cases} 1 & |x| < \Delta \\ 0 & |x| > \Delta \end{cases}$$

- **Find final distribution**

$$P_\infty(x) = ?$$

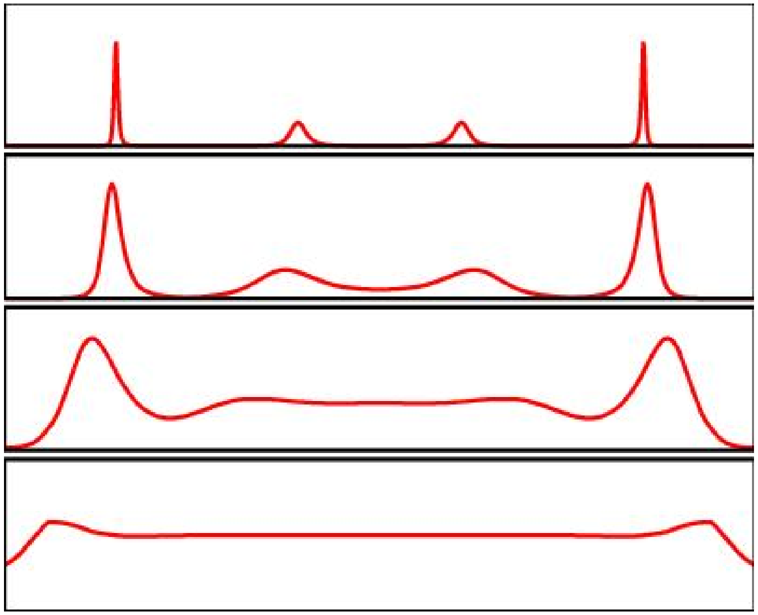
- **Multitude of final states**

$$P_\infty(x) = \sum_{i=1}^{N} m_i \delta(x - x_i) \qquad |x_i - x_j| > 1$$

- **Dynamics selects one (deterministically)**

Multiple localized clusters (parties)

Numerical methods, kinetic theory



☑Numerical integration of probability distribution

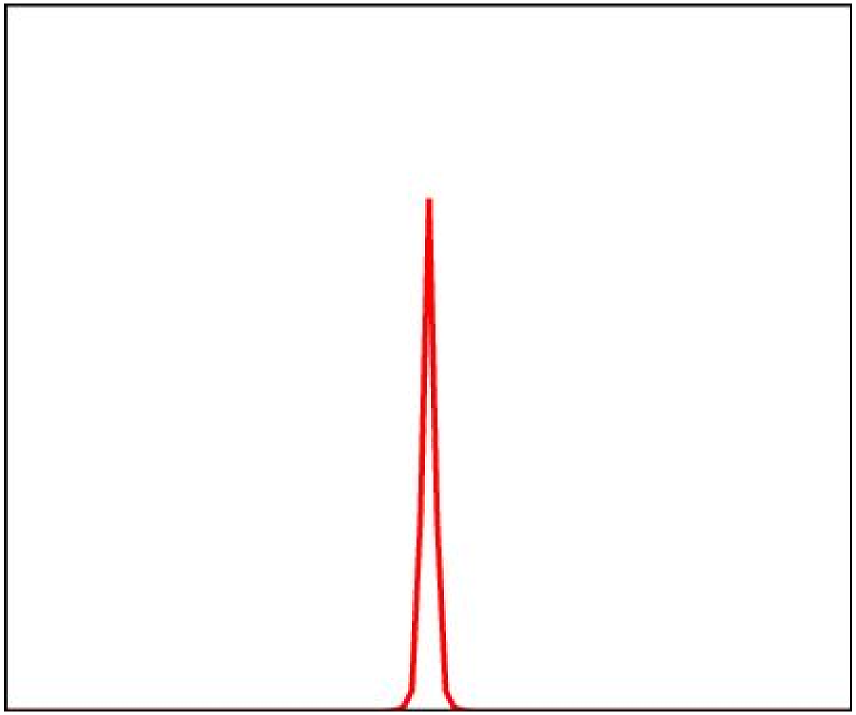
Kinetic theory: nonlinear rate equations

$$\frac{\partial}{\partial t}P(x,t) = \iint_{|x_1-x_2|<1} dx_1 dx_2 P(x_1,t)P(x_2,t)\left[2\delta\left(x-(x_1+x_2)/2\right)-\delta(x-x_1)-\delta(x-x_2)\right]$$

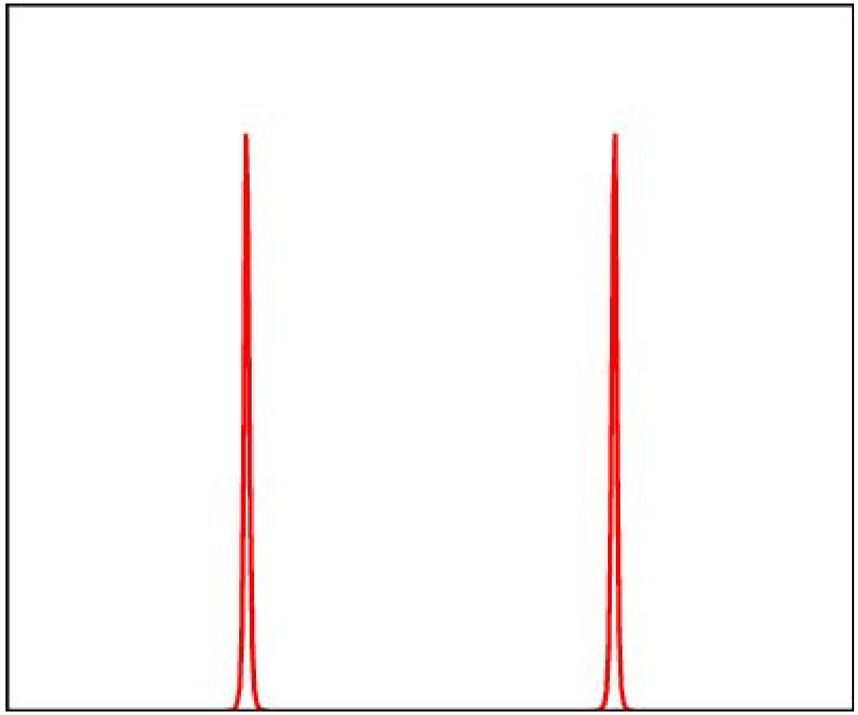
☒Direct simulation of stochastic process

Rise and fall of central party

$0 < \Delta < 1.871$



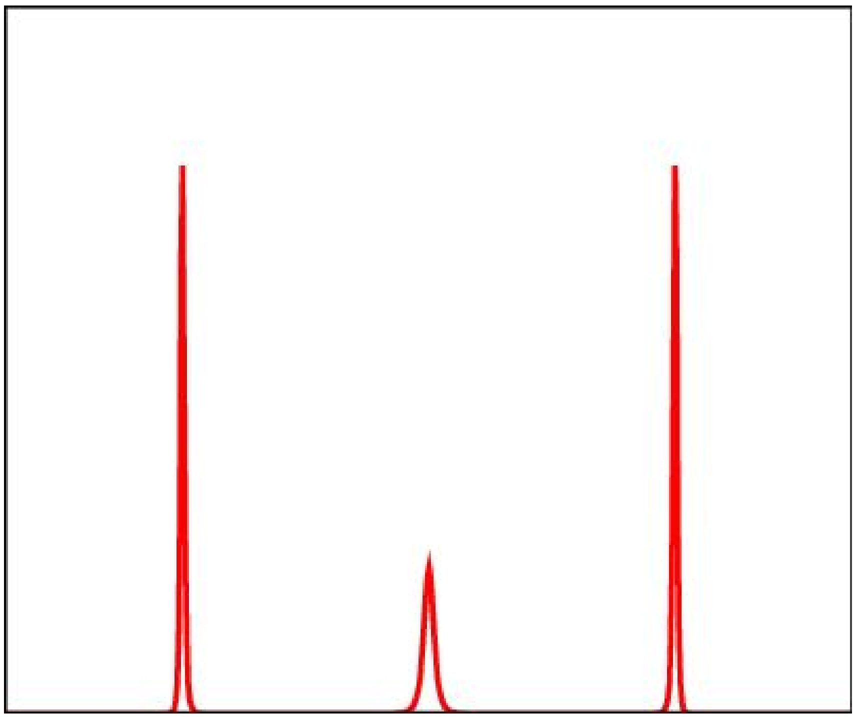
$1.871 < \Delta < 2.724$



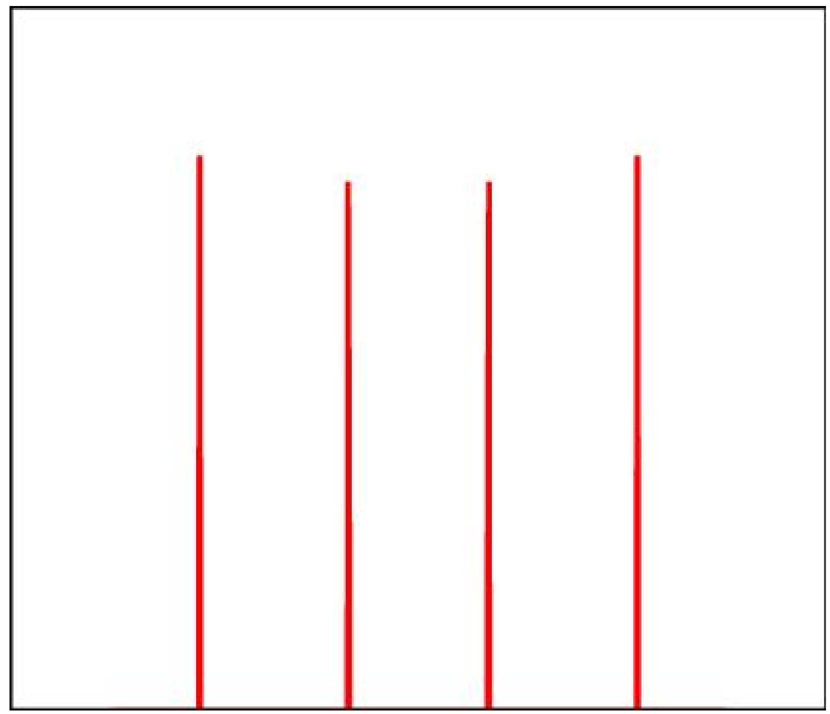
Central party may or may not exist!

Reemergence of central party

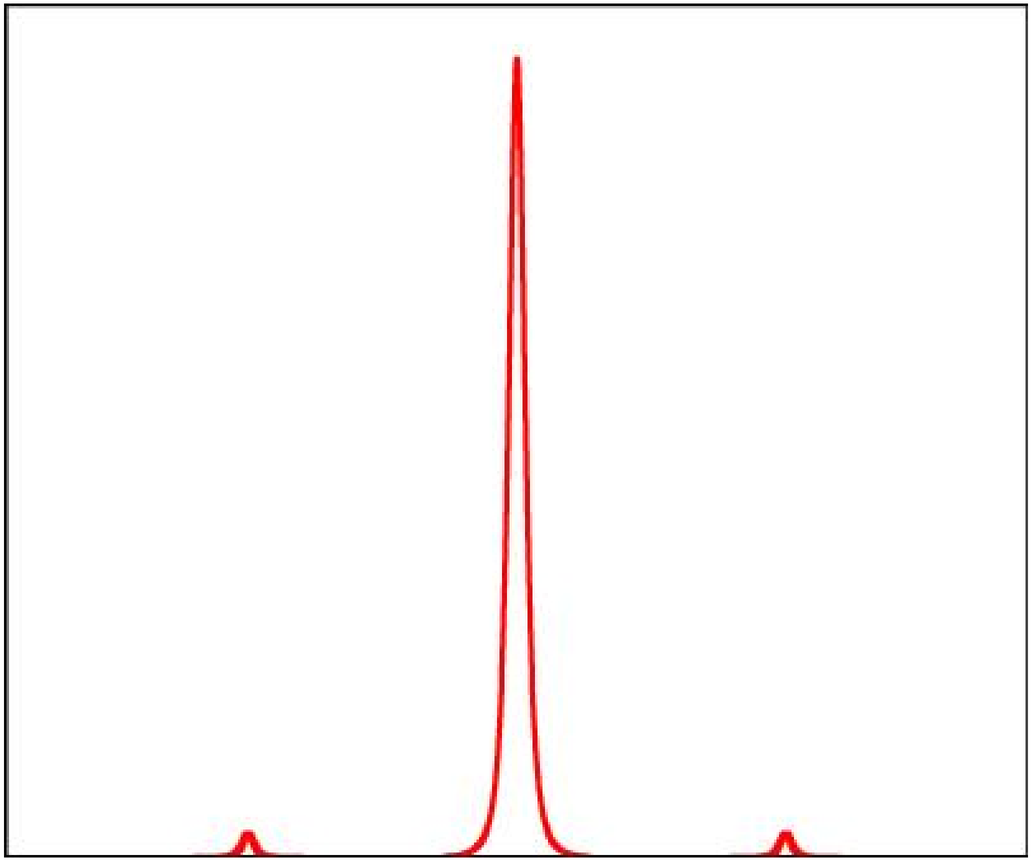
$2.724 < \Delta < 4.079$



$4.079 < \Delta < 4.956$

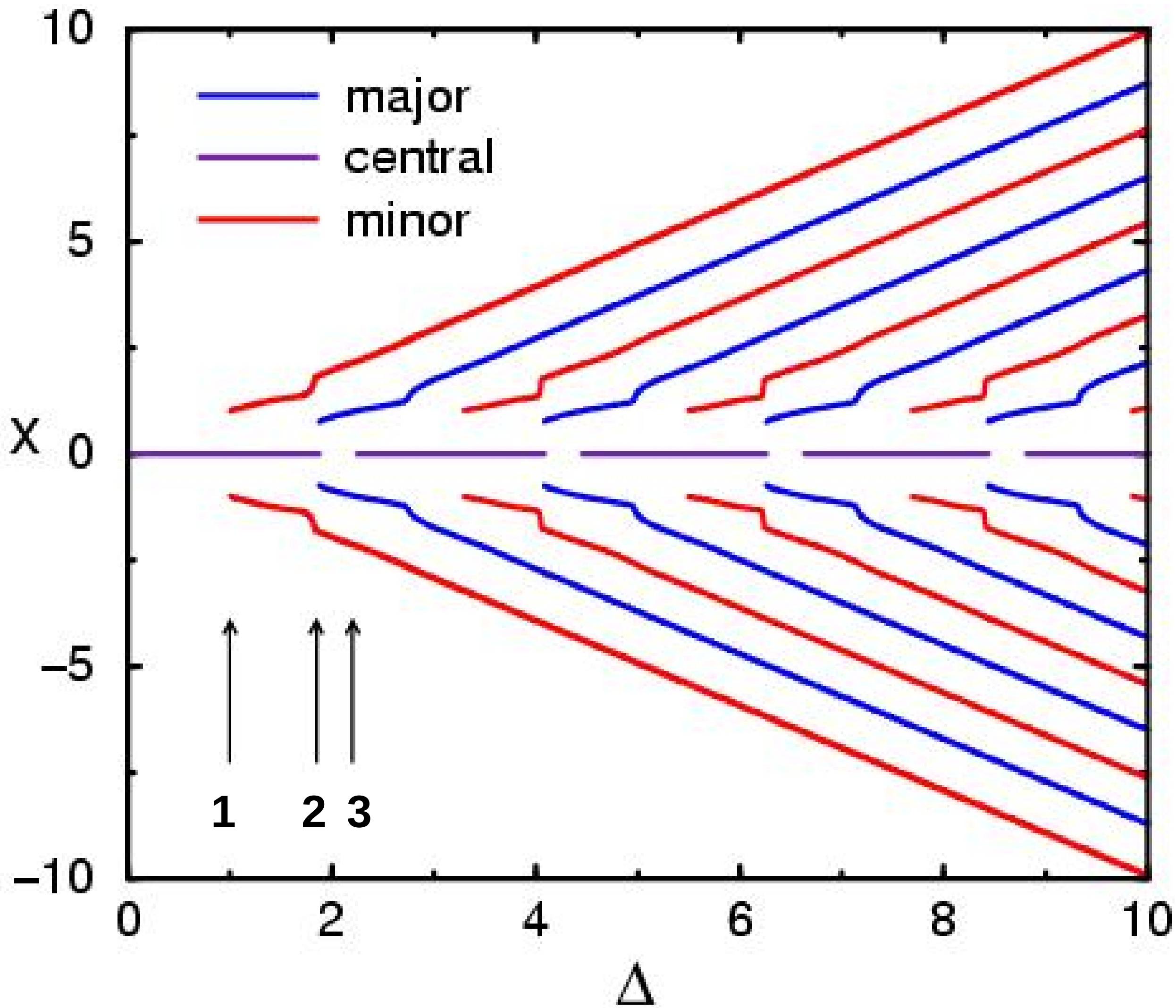


Hidden clusters



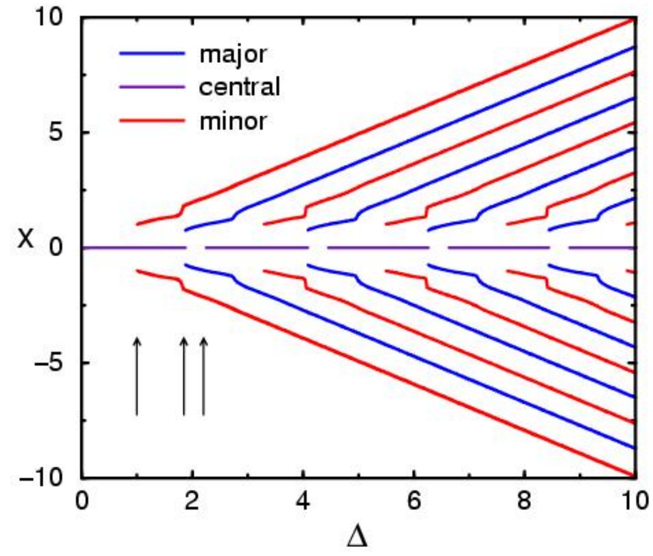
Tiny parties (mass <10^{-3}), extremists

Bifurcations and Patterns



Self-similar structure, universality

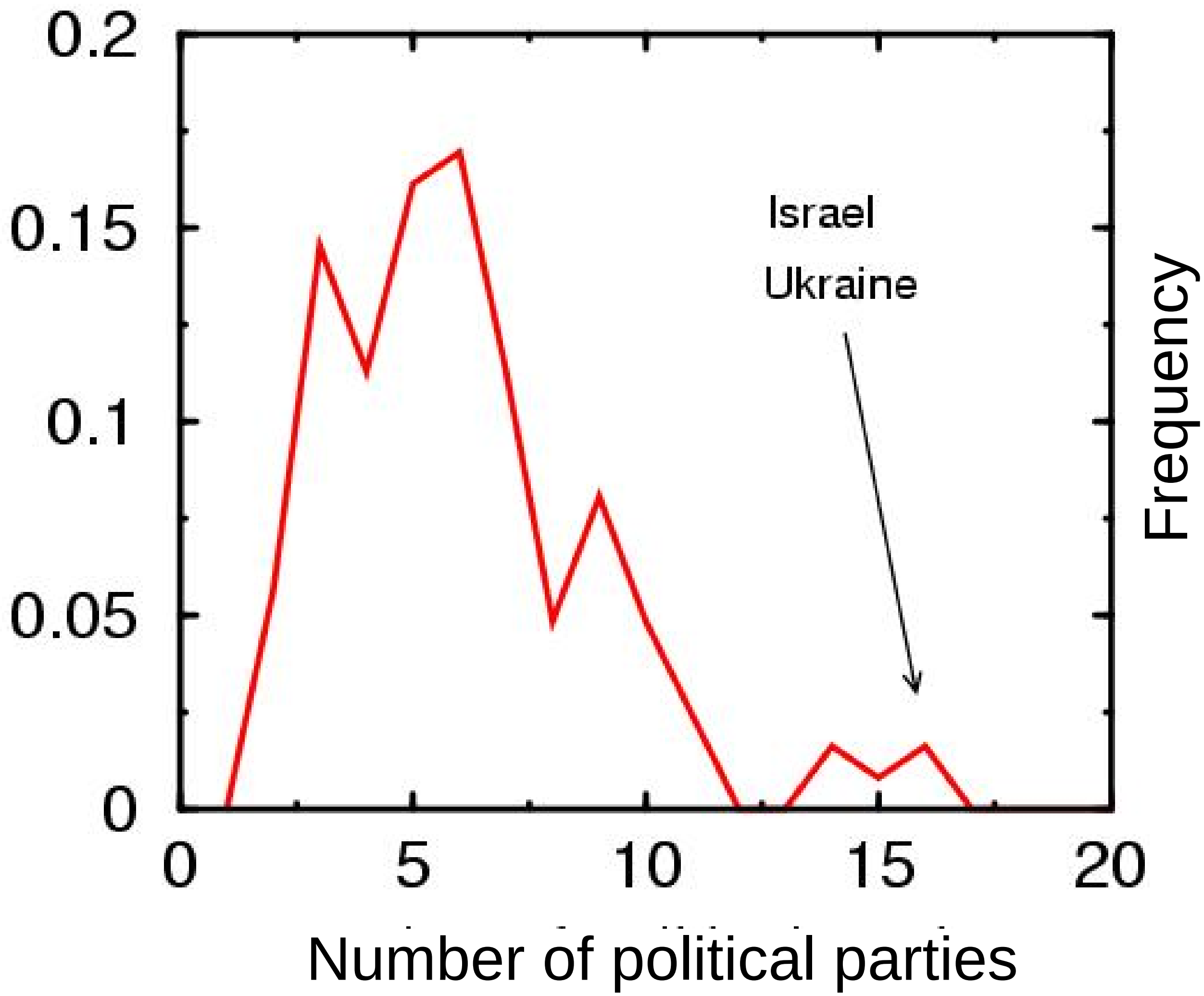
- **Periodic sequence of bifurcations**
  1. **Nucleation of minor cluster branch**
  2. **Nucleation of major cluster branch**
  3. **Nucleation of central cluster**
- **Alternating major-minor pattern**
- **Clusters are equally spaced**
- **Period gives major cluster mass, separation**



$$x(\Delta) = x(\Delta + L) \qquad L = 2.155$$

How many political parties?

- Data: CIA world factbook 2002
- 120 countries with multi-patry parliaments
- Average=5.8 standard deviation=2.9



Cluster mass

- **Masses are periodic**

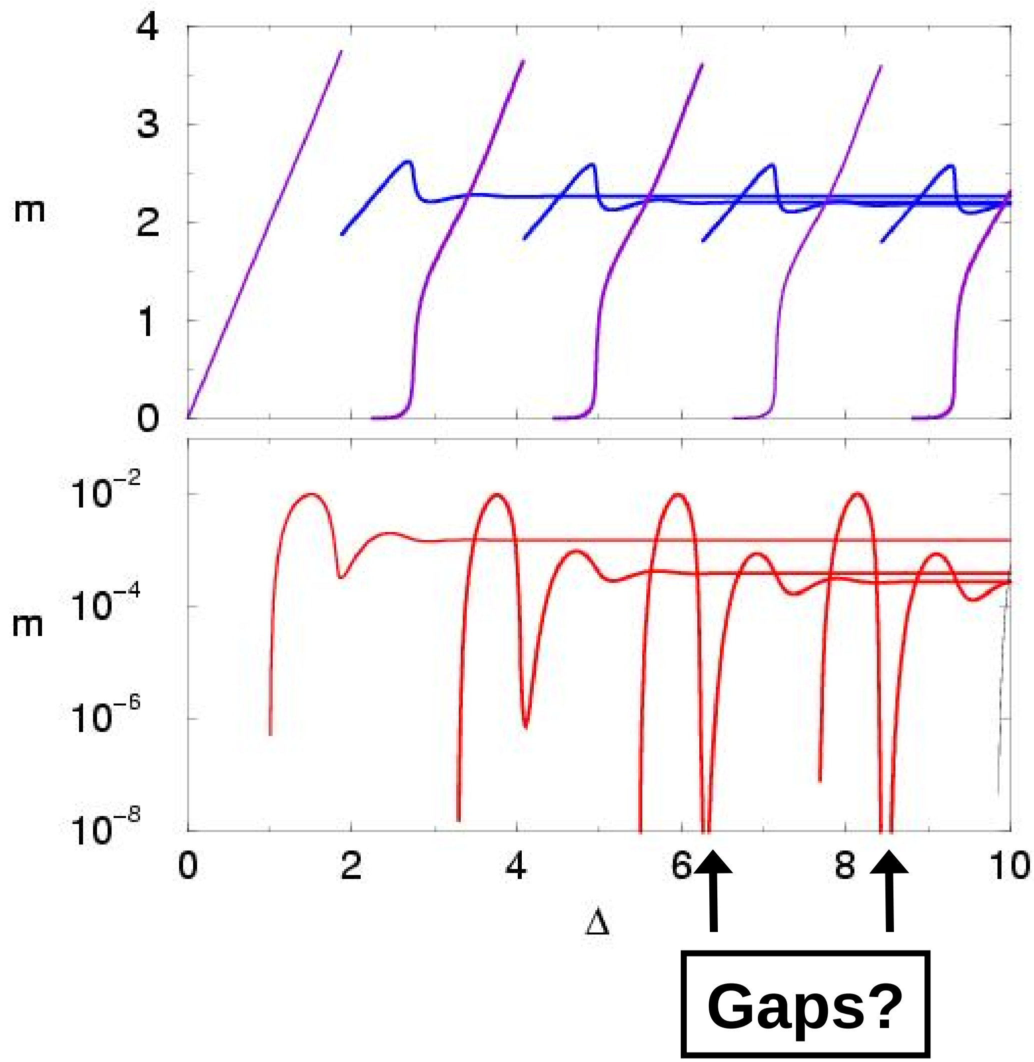
$$m(\Delta) = m(\Delta + L)$$

- **Major mass**

$$M \to L = 2.155$$

- **Minor mass**

$$m \to 3\times10^{-4}$$



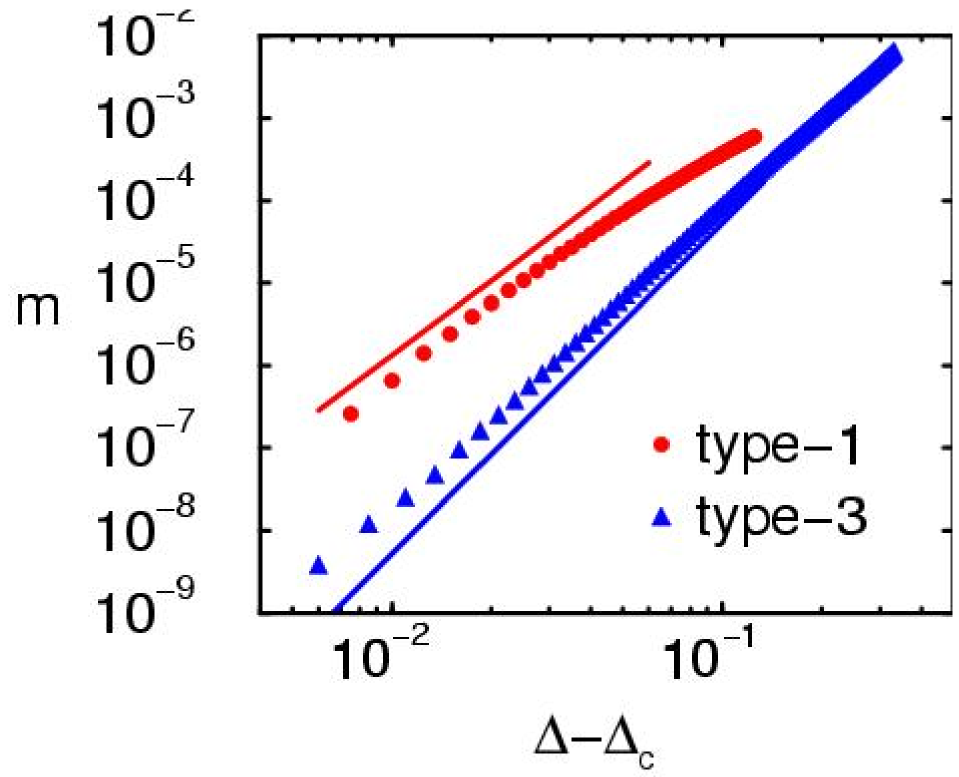
Scaling near bifurcation points

- **Minor mass vanishes**

$$m \sim (\Delta - \Delta_c)^\alpha$$

- **Universal exponents**

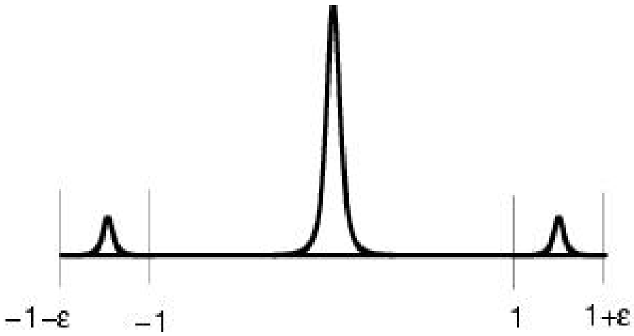
$$\alpha = \begin{cases} 3 & \text{type}\,1 \\ 4 & \text{type}\,3 \end{cases}$$



L-2 is the small parameter
explains small saturation mass

Heuristic derivation of exponents

- **Perturbation theory** $\Delta = 1 + \varepsilon$
- **Central cluster** $x(\infty) = 0$
- **Extremist minor cluster** $x(\infty) = 1 + \varepsilon / 2$



◆ **Rate of transfer from minor cluster to major cluster**

$$dm / dt = -mM \quad \rightarrow \quad m(t) \sim \varepsilon e^{-t}$$

◆ **Process stops when**

$$x \sim e^{-t_f / 2} \sim \varepsilon$$

◆ **Final minor cluster mass**

$$m(\infty) \sim m(t_f) \sim \varepsilon^3$$

Consensus

- **Integrable for** $\Delta < 1/2$

$$\langle x^2(t)\rangle = \langle x^2(0)\rangle e^{-\Delta t}$$

- **Final state: localized**

$$P_\infty(x) = 2\Delta\delta(x)$$

- **Rate equations in Fourier space**

$$P_t(k) + P(k) = P^2(k/2)$$

- **Self-similar collapse dynamics**

$$\Phi(z) \propto \left(1+z^2\right)^{-2} \qquad z = \frac{x}{\sqrt{\langle x^2(t)\rangle}}$$

The Inelastic Maxwell Model, Ben-Naim & Krapivsky, Lecture Notes in Physics **624,** 65 (2003)

Discrete opinions

- **Compromise process**

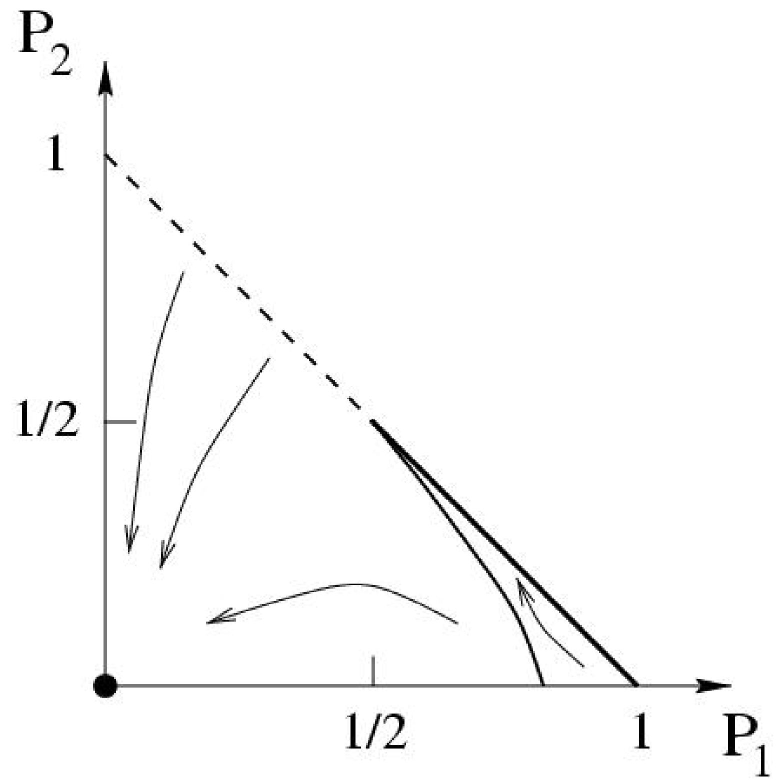
$$(i-1, i+1) \to (i, i)$$

- **Master equation**

$$dP_i / dt = 2P_{i-1}P_{i+1} - P_i(P_{i-2} + P_{i+2})$$

- **Example: 6 states**
- **Symmetry + normalization:**

  two-dimensional problem

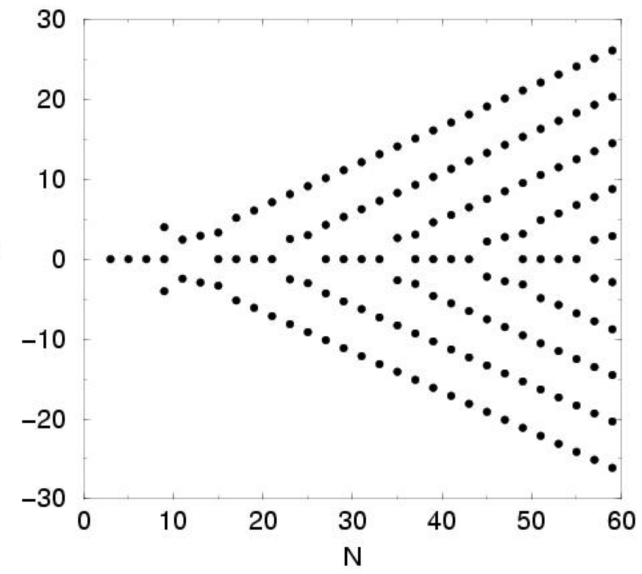


Initial conditions determine final state

Isolated fixed points, lines of fixed points

General features

- Dissipative system, volume contracts
- Energy (Lyapunov) function exists: <x^2>
- No cycles or strange attractors
- Uniform state is unstable (Cahn-Hilliard)



$$P_i = 1 + \phi_i \qquad \phi_t + \left(\phi + a\phi_{xx} + b\phi^2\right)_{xx} = 0$$

Discrete case yields useful insights

Pattern selection

- **Linear stability analysis**

$$P-1 \propto e^{i(kx+\omega t)} \quad \Rightarrow \quad \omega(k)=\frac{8}{k}\sin\frac{k}{2}-\frac{2}{k}\sin k-2$$

- **Fastest growing mode**

$$d\omega/dk=0 \quad \Rightarrow \quad L=2\pi/k=2.2515$$

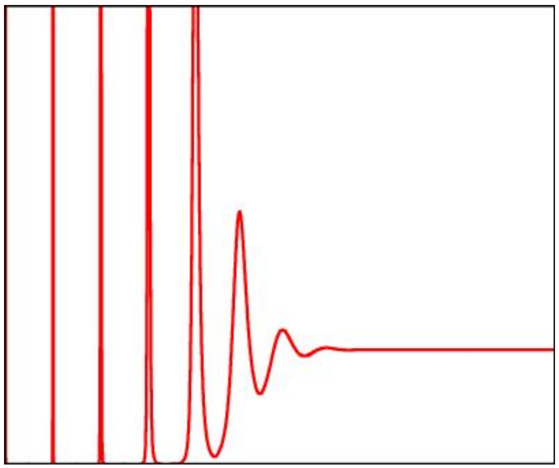
- **Traveling wave (FKPP extremal selection)**

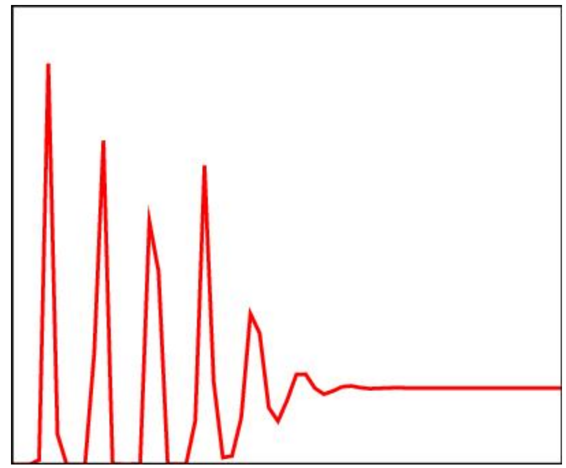
$$d\omega/dk=\mathrm{Im}(\omega)/\mathrm{Im}(k) \quad \Rightarrow \quad L=2\pi/k=2.0375$$

Patterns induced by wave propagating from boundary.
However, emerging period is different L=2.155!

Pattern selection intrinsically nonlinear

Traveling waves



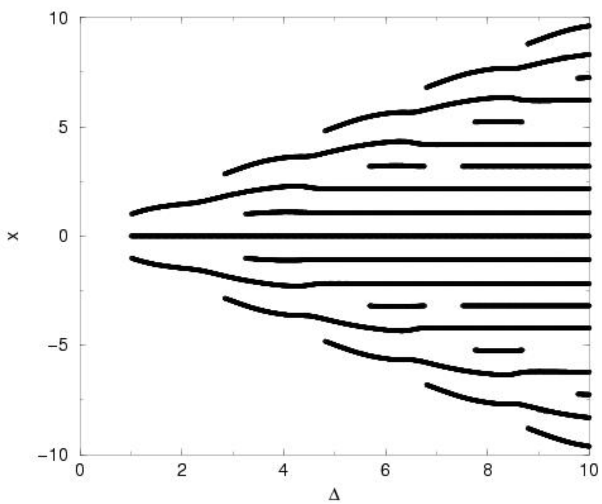


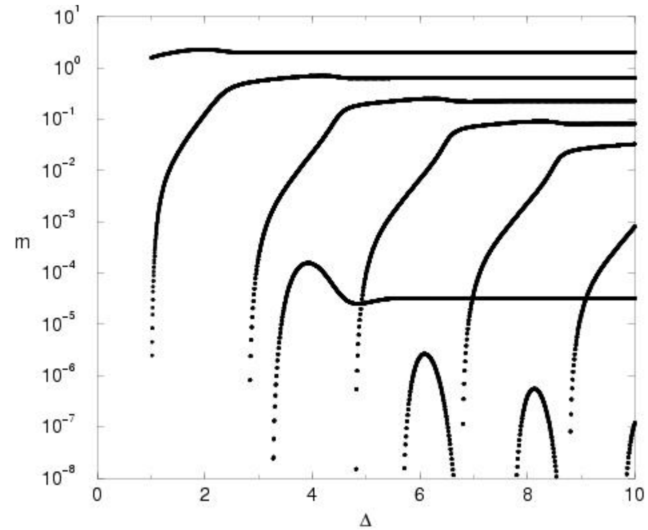
$$P-1 \propto \exp\left[-\lambda(x-vt)+i(kx+wt)\right]$$

Discrete case

$$L_{\max} = 6 \qquad L = 5.67 \qquad L_{\text{trav wave}} = 5.31$$

Exponential initial conditions

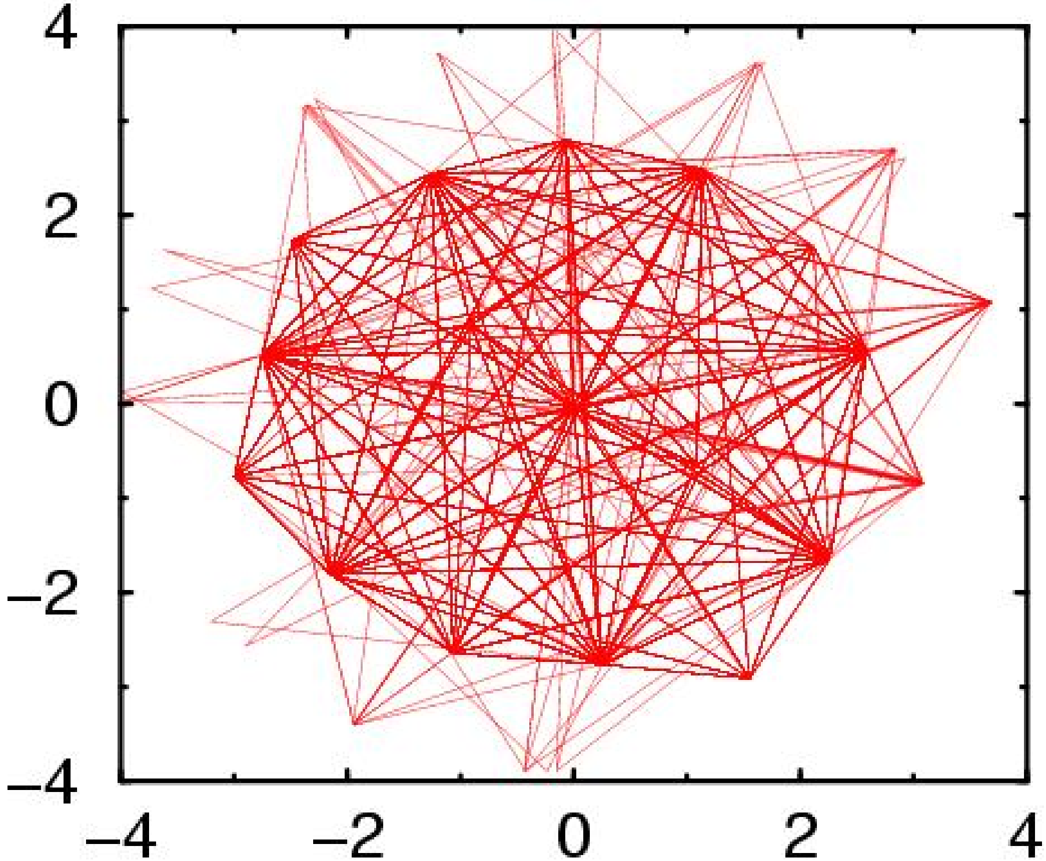




- **Bifurcations induced at the boundary**
- **Periodic structure, nontrivial period**
- **Two types of bifurcations**
  1. **Nucleation of major branch**
  2. **Nucleation of minor branch**

Central cluster is stable

Two kinds of opinions



symmetry breaking, packing

Noisy dynamics

- **Opinions change due to interaction with environment (news, events, editorials)**

$$i \to i \pm 1 \qquad \text{with rate } D$$

- **Add diffusion term**

$$\lambda \to \lambda - Dk^2 \quad \Rightarrow \quad \lambda \cong (D_c - D)k^2 + \cdots$$

- **Sub-critical noise: slow coarsening (single party)**

$$D_m = D/m, \quad m \sim (D_m t)^{1/2} \quad \Rightarrow \quad m \sim (Dt)^{1/3}$$

- **Super-critical noise: flat state stable (no parties)**

Conclusions

- **Clusters form via bifurcations**
- **Periodic structure**
- **Alternating minor-major pattern**
- **Central party not always exists**
- **Power-law behavior near transitions**

E. Ben-Naim, P.L. Krapivsky , and S. Redner, Physica D **183**, 190 (2003)

Outlook

- Pattern selection criteria
- Gaps
- Role of initial conditions, classification
- Role of spatial dimension, correlations
- Disorder, noise, inhomogeneities
- Tiling/Packing in 2D
- Discord dynamics (seceder model, Halpin-Heally 03)

Many open questions