Exchange Driven Growth

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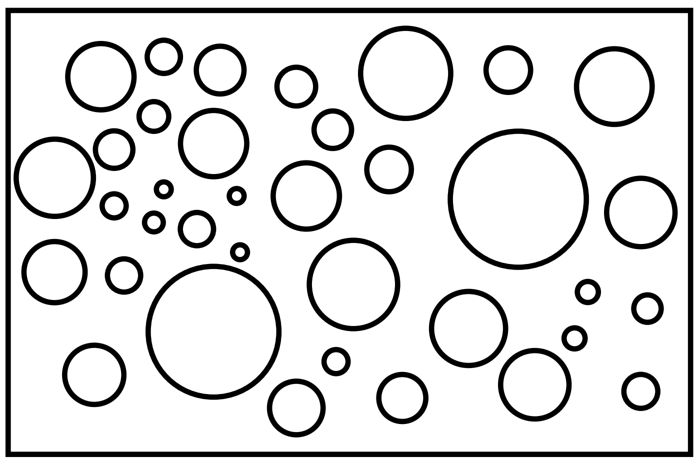


E. Ben-Naim and P. L. Krapivsky, Phys. Rev. E **68**, 031104 (2003). cond-mat/0305154

Exchange Processes

- Cluster consist of individual particles
- Infinitely many clusters
- Random exchange process

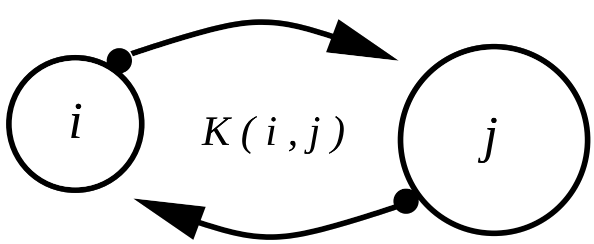
  A particle detaches from one cluster

  It Immediately reattaches to another cluster

$$(i, j) \to (i-1, j+1)$$

- Exchange rate (homogeneous)

$$K(i, j) = (ij)^{\lambda}$$



Clusters disappear when last particle disappears

Kinetic Theory

- Cluster size distribution $A_k(t)$

$$\frac{dA_k}{dt} = \sum_{i,j} A_i A_j K(i,j) \left[\delta_{k,i-1} + \delta_{k,i+1} - 2\delta_{k,i}\right]$$

- Constant rates $K(i,j) = 1$

$$A_k = e^{-2\tau} \left[I_{k-1}(2\tau) - I_{k+1}(2\tau)\right].$$

- Linear rates $K(i,j) = ij$

$$A_k = \frac{t^{k-1}}{(1+t)^{k+1}}$$

- Clusters grow indefinitely

Similarity solutions

I Growth: $\lambda < 3/2$

- Typical cluster size grows algebraically

$$k_* \sim t^\beta \qquad\qquad \beta = \frac{1}{2-\lambda}$$

- Cluster size distribution is self-similar

$$A_k(t) \sim k_*^{-1}\Phi(k/k_*)$$

- Scaling distribution

$$\Phi(x) \propto x^{1-\lambda}\exp[-c\,x^{2-\lambda}]$$

Scaling, self-similar distributions

II Gelation: $3/2 < \lambda < 2$

- Infinite cluster forms in finite time

$$k_* \sim (t_g - t)^{\beta} \qquad\qquad \beta = \frac{1}{3-2\lambda}.$$

- Self-similar growth

Multiscaling: $\lambda = 2$

- Log-normal distribution

$$A_k(t) \sim k^{-3/2} \exp\left[-f(t)(\ln k)^2\right]$$

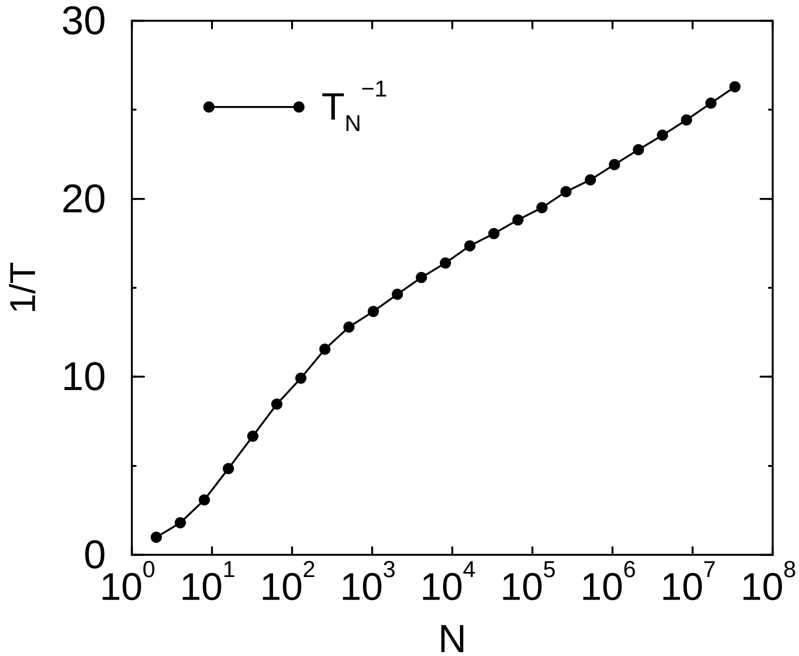
- Multiscaling of the moments

$$\langle k^n \rangle \sim (t_c - t)^{-n(n-1)/4} \qquad n > 1$$

III Instant Gelation $\lambda > 2$

- Infinite cluster forms in no time!
- Study finite system: N particles
- Gelation time vanishes with system size

$$T_g \sim (\ln N)^{-(\lambda-2)}$$



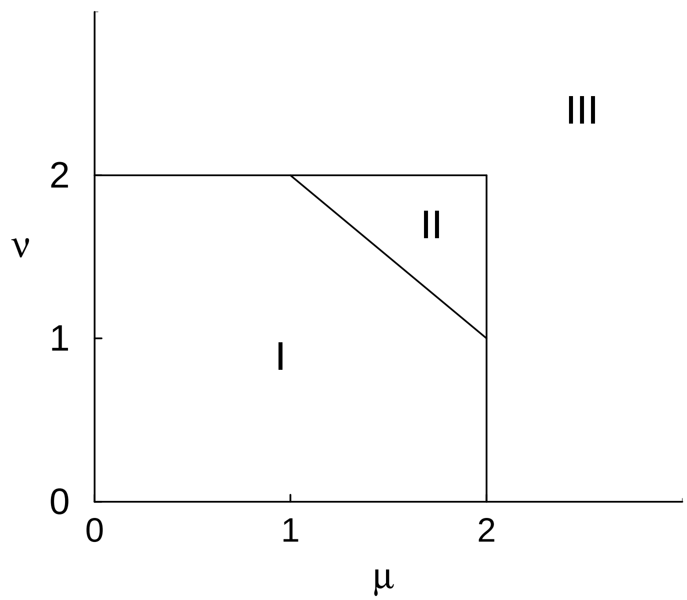
No thermodynamic limit!

Generalized exchange rates

- Two-index exchange rates

$$K(i,j) = i^{\nu} j^{\mu} + i^{\mu} j^{\nu}$$

- Region I: Growth
- Region II: Gelation
- Region III: Instant Gelation
- Marginal case: $\Phi(x) \sim x^{-5}$



Ising Model, Kawasaki Dynamics

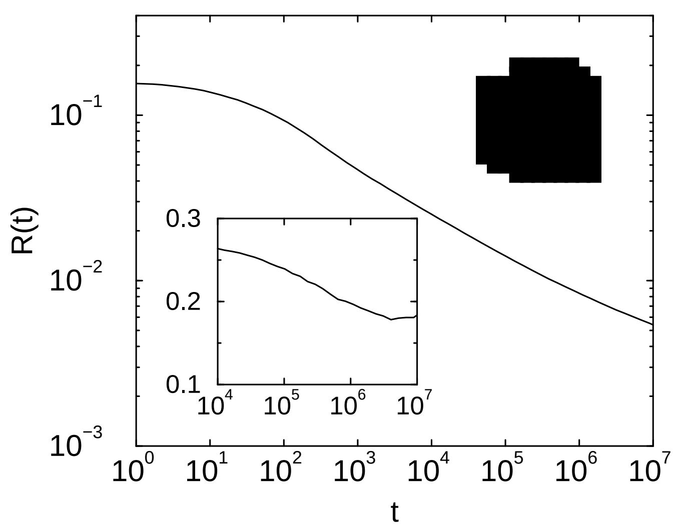
- $T = 0$, infinite range, dilute limit
- Randomly select spins
- Exchange if energy is not lowered

$$\cdots\uparrow\cdots\downarrow\cdots\to\cdots\downarrow\cdots\uparrow\cdots$$

- Exchange rate $K \sim \sigma \qquad \lambda = \frac{d}{d-1}$

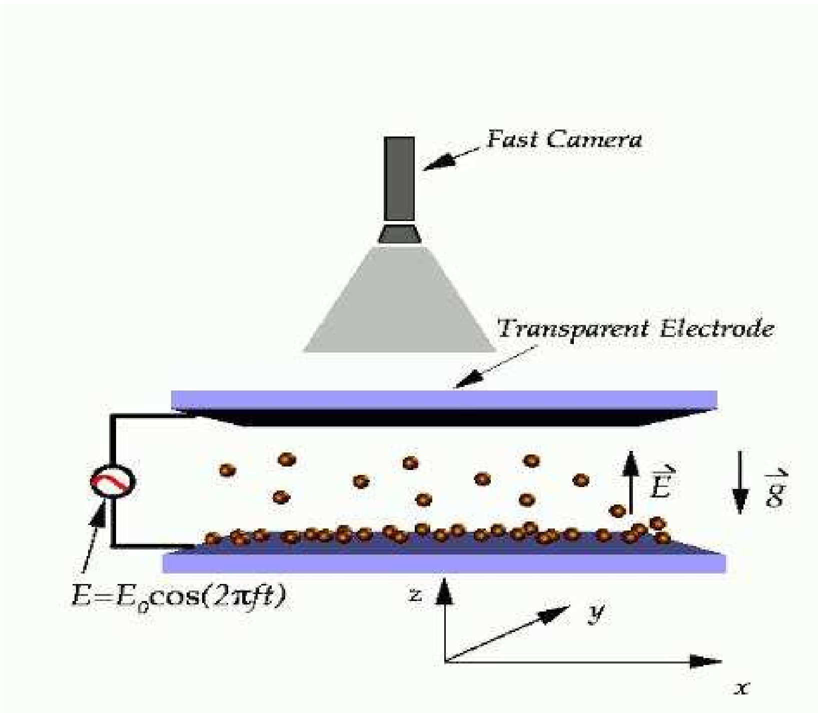
$$R \sim t^z \qquad z = \frac{1}{3d}$$

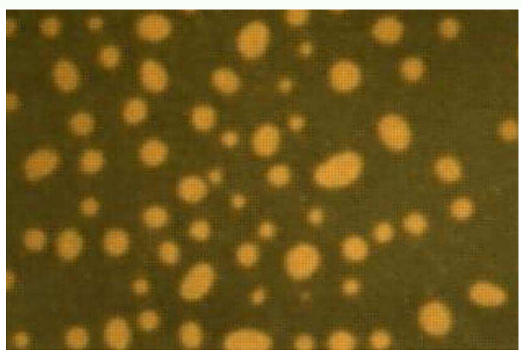
- 1D: Agrees with exact solution $z = 1/3$
- 2D: Consistent with simulations $z = 1/6$



$\boxed{\textbf{Lattice effects important}}$

Coarsening in Granular Monolayers





- Experiment: electrostatically driven layers
- Measurement: cluster size

Igor Aronson (Argonne)

Theory vs. Experiment

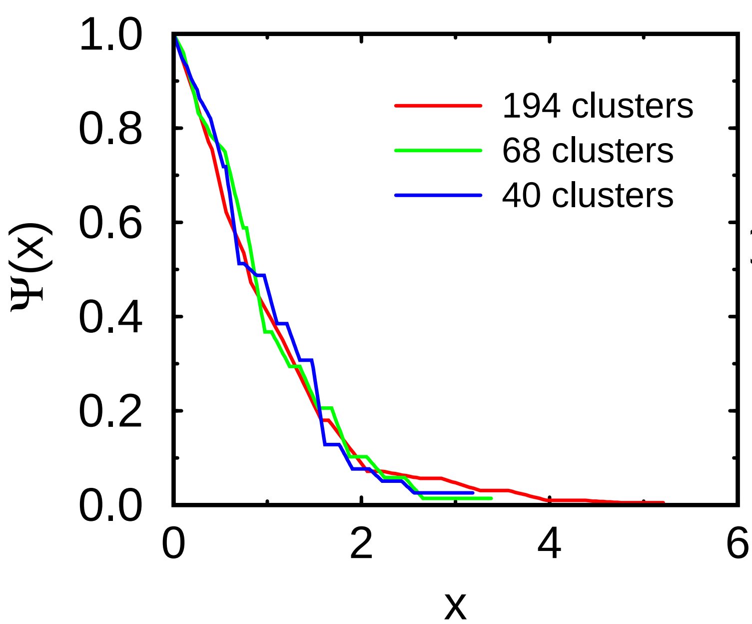
- Theory: $K(i,j) = ij$
- Growth of typical size   Blair & Aronson PRL 99

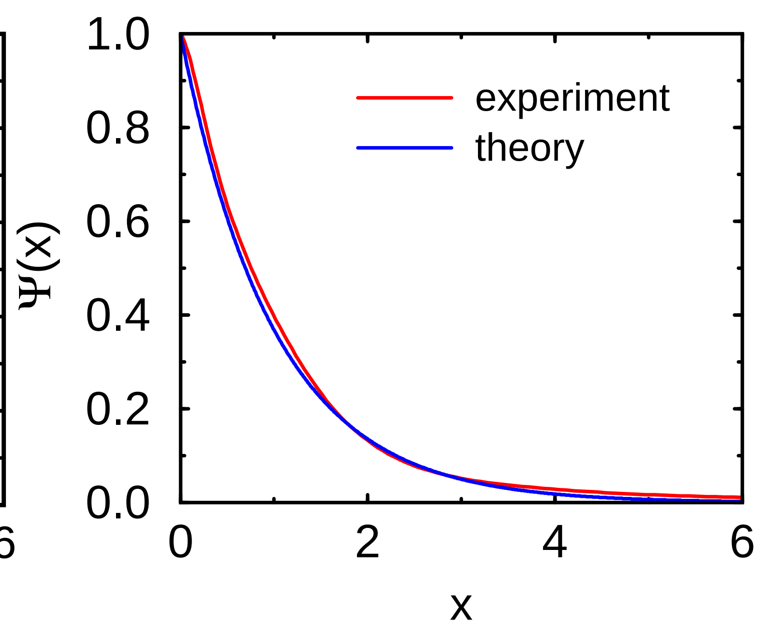
$$k_* \sim R_*^2 \sim t$$

- Compare cumulative distribution

$$\Psi(x) = \int_x \Phi(y)dy$$

$$\Psi(x) = \Phi(x) = \exp(-x)$$





Conclusions

- Phases: Growth, Gelation, Instant Gelation
- Growth, Gelation: self-similar distributions
- Marginal cases: log-normal and power-law distributions
- Marginal cases: multiscaling of moments
- Instant gelation: logarithmic gelation times

Spatial correlations?