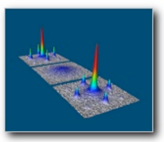
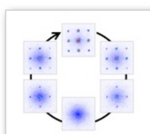
Quantum dynamics, measurement and decoherence in Bose-Einstein condensates

Diego A. R. Dalvit

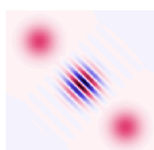
- **What is a Bose-Einstein condensate?**
- **Early experiments** ⇒ **Mean field description**
- **Recent experiments** ⇒ **Quantum correlations become important**



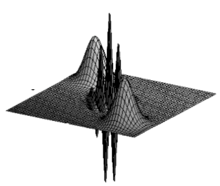
Quantum phase transitions in optical lattices



Collapses and revivals of coherent matter waves

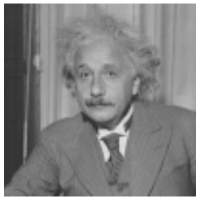


Schroedinger cat states of BECs



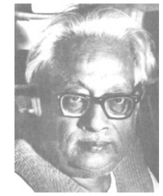
Effects of decoherence

What is a Bose-Einstein condensate?

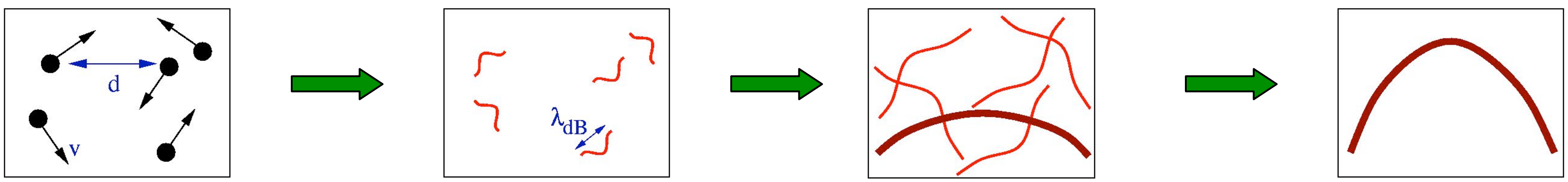


BECs were predicted in 1925 by Einstein,

based on a 1924 work by Bose.



What happens when identical bosons are cooled down to very low temperatures ? $T \approx \mathrm{nK}$



High temperature T **:**

thermal velocity v

density d^{-3}

"Billiard balls"

Low temperature T **:**

De Broglie wavelength

$\lambda_{\mathrm{dB}} = (2\pi\hbar^2/mk_{\mathrm{B}}T)^{1/2}$

"Wave packets"

$T = T_{\mathrm{crit}}$

BEC

$\lambda_{\mathrm{dB}} \approx d$

"Matter wave overlap"

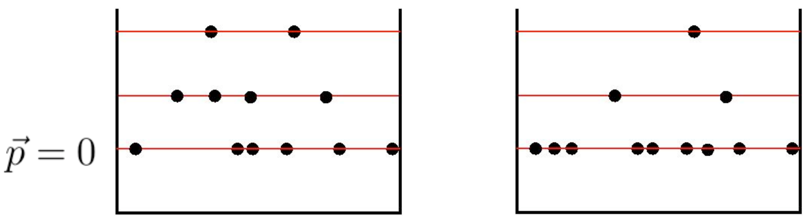
$T = 0$

Pure BEC

"Giant matter wave"

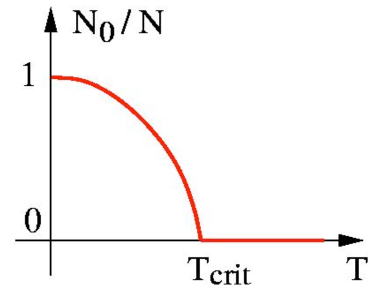
✓ **Phase transition due to quantum statistical effects** ← **Bose-Einstein statistics**

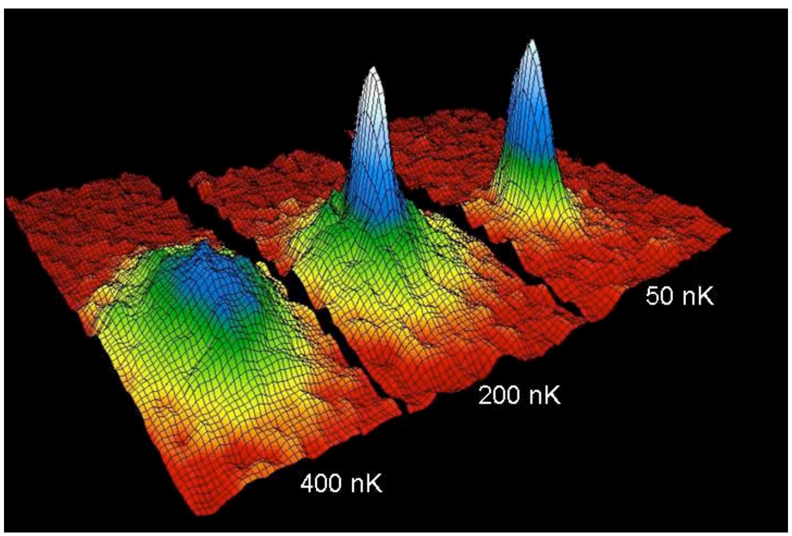
Atoms condense into the state of lowest energy (zero momentum)



$T > T_{\rm crit}$ $\qquad$ $T < T_{\rm crit}$

Bose-Einstein condensation:
macroscopic occupation of ground state





Advance in AMO trapping and cooling machanisms
(magnetic trapping-laser cooling)

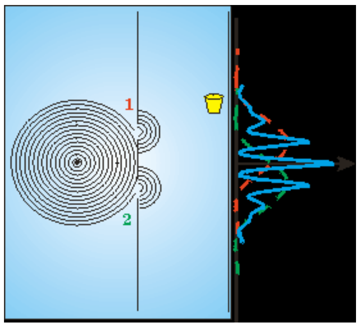
Experimentally realized in 1995
2001 Physics Nobel Prize

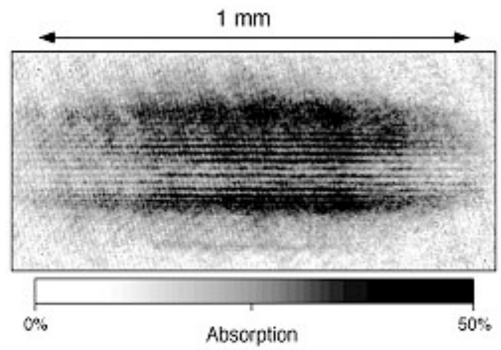
$$N/V = 2.5 \times 10^{12} \mathrm{cm}^{-3} \qquad T_{\rm crit} = 170 \mathrm{nK}$$

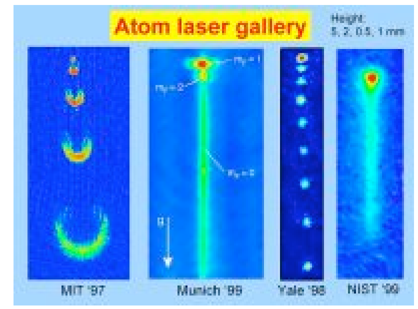
^{87}Rb (1995)	H (1998)
^{23}Na (1995)	^{4}He (2001)
^{7}Li (1995)	^{41}K (2001)

Analogy Laser-BEC:

- ❑ **Laser** ⟹ **many photons with the same phase, frequency and polarization**
- ❑ **BEC** ⟹ **many atoms in the same quantum state, perfectly coherent**



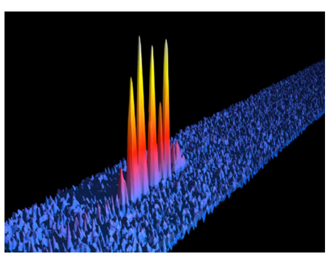




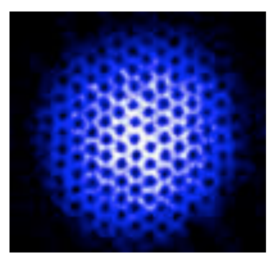
✓ **BECs are a new state of matter, in which a macroscopic number of atoms are in perfect, coherent lock-step.**

✓ **BECs represent the ultimate in control over matter at the quantum level. One can play with many knobs (trapping potentials, interactions, density, temperature, etc.)**

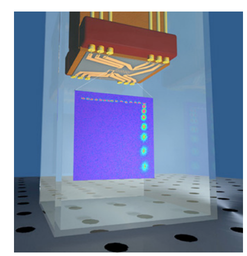
✓ **BECs have produced many interesting results in many-body quantum physics, from nonlinear quantum optics to condensed matter physics.**



solitons



vortices



atomic waveguides

Theoretical description

Mean field approach: **all atoms of the condensate are in the same mean field** Φ

$$\Psi(x_1, \ldots, x_N) = \prod_{j=1}^{N} \Phi(x_j)$$

Φ **is the condensate wavefunction, and it satisfies a nonlinear Schrödinger equation, called the Gross-Pitaevskii equation (GPE).**

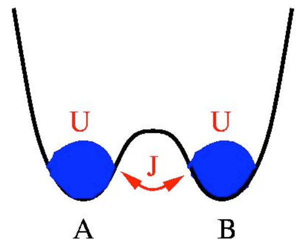
$$\boxed{i\hbar \frac{\partial \Phi}{\partial t} = \left[\underbrace{-\frac{\nabla^2}{2m}}_{\text{kinetic energy}} + \underbrace{V_{\text{trap}}(x)}_{\text{trapping potential}} + \underbrace{g|\Phi(x)|^2}_{\text{collisions between atoms}} \right] \Phi}$$

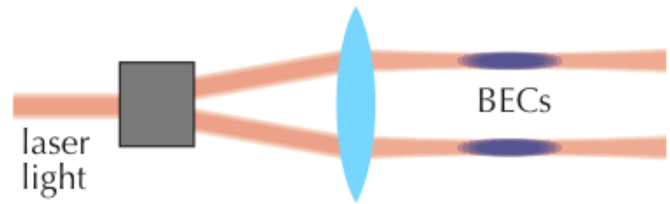
The GPE is a classical nonlinear PDE that has successfully explained many of the BEC physics, including solitons, vortices, collective excitations, Josephson junction arrays of BECs, and even molecular condensates.

BECs beyond mean field theory

Recent experiments go beyond mean field → **quantum correlations become important**

Double well potential:



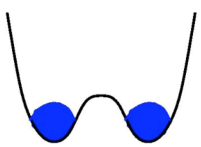


$$H = \hbar\omega(a^\dagger a + b^\dagger b) - J(a^\dagger b + b^\dagger a) + \frac{U}{2}[(a^\dagger a)^2 + (b^\dagger b)^2]$$

ω harmonic trap frequency
J tunneling rate
$U > 0$ repulsive interactions

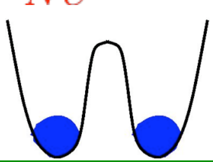
Ground states:

- when $\frac{J}{NU} \gg 1$ → $\boxed{|\Psi\rangle_{\rm GS} \propto (a^\dagger + b^\dagger)^N |0\rangle}$

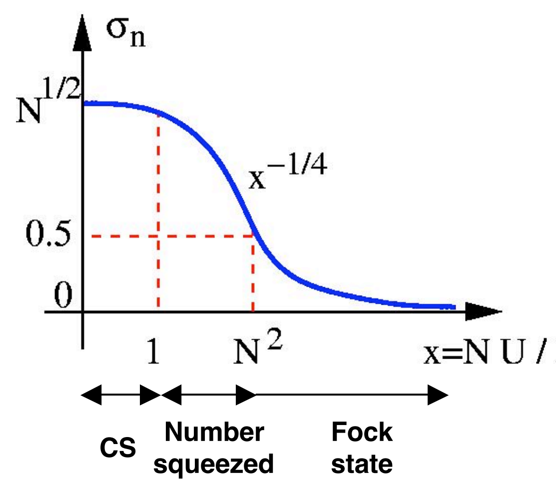


Coherent state
phase well defined

- when $\frac{J}{NU} \ll 1$ → $\boxed{|\Psi\rangle_{\rm GS} \propto (a^\dagger)^{N/2}\,(b^\dagger)^{N/2} |0\rangle}$



Fock state
number well defined

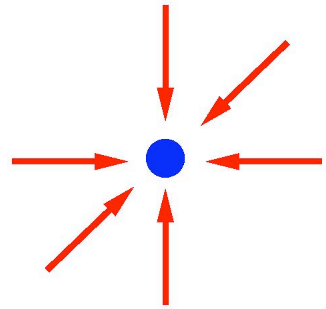


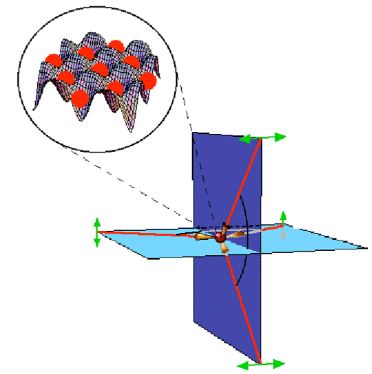
Optical lattice: **Artificial crystal of light - a periodic intensity pattern that is formed by the interference of two or more laser beams.**

Photon-atom interaction (AC Stark shift):

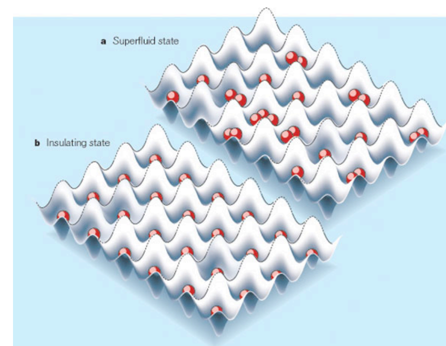
$$\mathcal{E} = -\frac{1}{2}\langle \vec{d} \rangle \cdot \vec{E}$$

$$V_{\text{trap}} = V_0[\sin^2(k_x x) + \sin^2(k_y y) + \sin^2(k_z z)]$$





Mott insulator-superfluid quantum phase transition



Bose-Hubbard Hamiltonian:

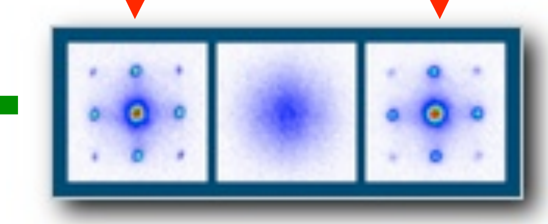
$$\boxed{H = \sum_i \epsilon_i n_i - J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1)}$$

$$n_i \equiv a_i^\dagger a_i$$

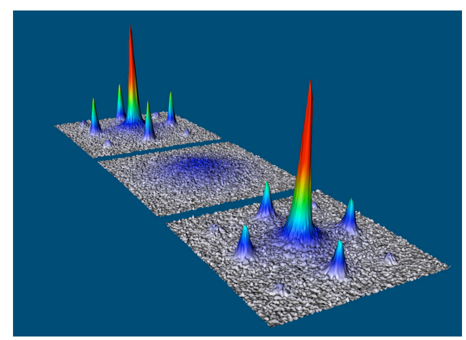
coherent state

coherent state

Initial state preparation for quantum computing with neutral atoms



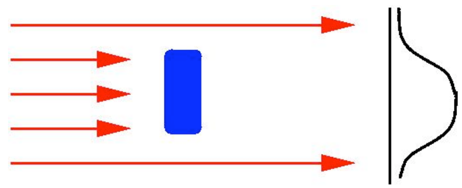
Fock state



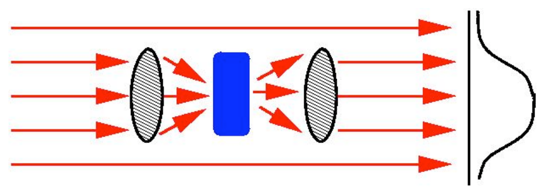
Measurement backaction in BECs

BECs are measured using two optical techniques:

✓**Absorption imaging:** **laser in resonance with an atomic transition. Destructive measurement. Absorption of photons causes heating** → **single shot**



✓**Phase contrast imaging:** **laser off-resonance. Non-destructive measurement. BEC acts as a dielectric media for the light, changing the refraction index.** → **multiple shots, and in the limit of continuous measurement, a "movie" of the evolving condensate.**



The light field is

- **A "meter" for the BEC, it gets entangled with the BEC and it is finally measured by the classical aparatus (lens, detectors)**
- **An "environment" for the BEC. Information about the state of the condensate can be obtained through measurements of the light field.**

What is the effect of continuous quantum measurement on the BEC dynamics?

$$H_{\text{int}} = \frac{\chi}{2}\int d\vec{x}\; n(\vec{x})E^2(\vec{x})$$

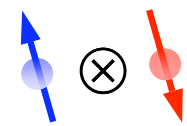
χ is the effective electric susceptibility of the BEC

During the measurement process, the BEC ("the system") gets entangled with the laser ("the bath").

What is entanglement ?

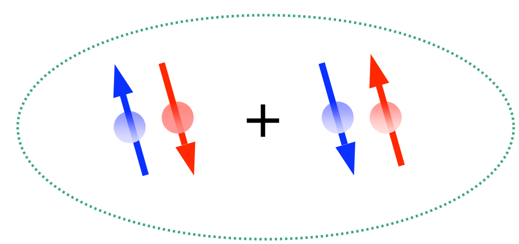
- **Separable state of two qubits:**

$$|\varphi\rangle = |0\rangle_A |1\rangle_B$$



- **Entangled state of two qubits: Bell state**

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right)$$

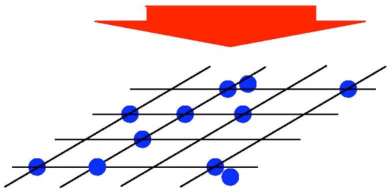


Non separable $\Longleftrightarrow$ **Entangled**

To follow the dynamics of the BEC, one traces over (ignores) the degrees of freedom of the light field. One obtains an evolution equation for the density matrix ρ_{BEC} of the BEC.

$$\rho(t=0) = \rho_{\text{BEC}} \otimes \rho_{\text{laser}}$$ **separable** $\xrightarrow{e^{-iH_{\text{int}}t/\hbar}}$ $\rho(t>0)$ **entangled**

Imagine a BEC in a 2D optical lattice, and the phase contrast imaging laser is shined onto the 2D lattice. There is measurement backaction on the BEC



$$\frac{d\rho_{\mathrm{BEC}}}{dt} = -\frac{i}{\hbar}[H_{\mathrm{BEC}}, \rho_{\mathrm{BEC}}] - S \sum_{\text{sites } l} [n_l, [n_l, \rho_{\mathrm{BEC}}]]$$

measurement strength (QND measurement)

Two competing factors:

✓ **hopping (J) ⟶ delocalization ⟶ coherent states**

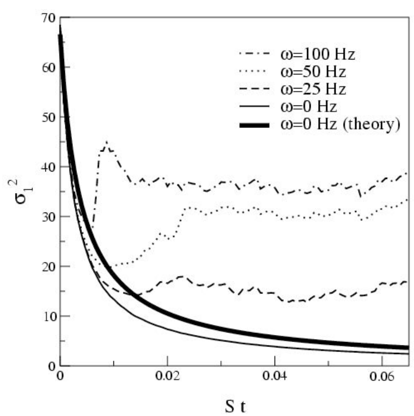
✓ **measurement (S) ⟶ localization ⟶ Fock states**

$n_l(t=0) = 100$

$\sigma_l^2(t=0) = 66.6$

$S = 2\ \mathrm{sec}^{-1}$

1D, 3 modes



Measurement drives the quantum state of the BEC towards number-squeezed states

$n_l\ S/J \gg 1$

Similar to atomic spin squeezing via QND measurements (Bigelow et al. PRL 85, 1594 (2000))

Collapses and revivals of coherent matter waves

$$H = \epsilon\, n + \frac{U}{2}\, n(n-1) \;\Rightarrow\; |n\rangle$$ **Fock states are eigenstates**

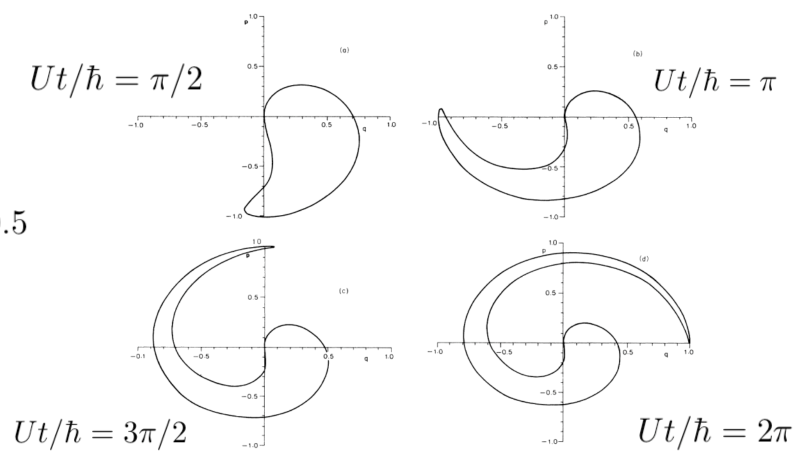
Initial state = coherent state $$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\, |n\rangle$$

Macroscopic matter wave field $$\Psi(t) = \langle \alpha(t)|a|\alpha(t)\rangle = \alpha\, e^{-\frac{i}{\hbar}(\epsilon+U/2)t}\, e^{(e^{-iUt/\hbar}-1)|\alpha|^2}$$

revival time: $$\boxed{t_{\mathrm{rev}} = \frac{2\pi\hbar}{U}}$$

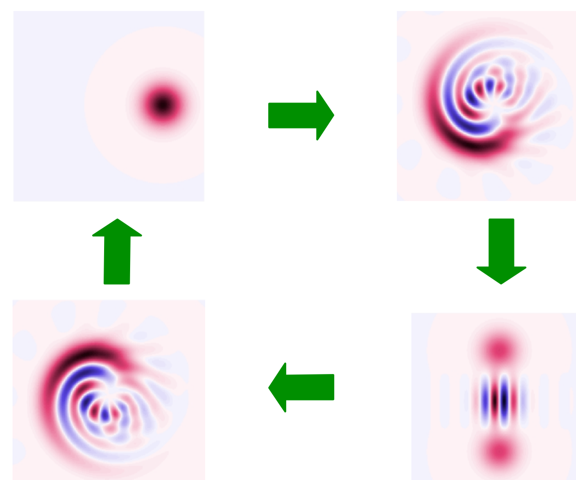
Ehrenfest time = Quantum-classical separation time-scale $$\boxed{t_{\mathrm{E}} = \frac{\hbar}{U|\alpha|}}$$

Classical dynamics



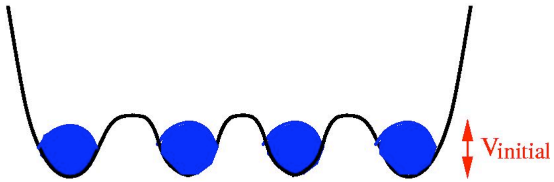
$\alpha = 0.5$
$\epsilon = 1$

Quantum dynamics



How to measure this in the BEC?

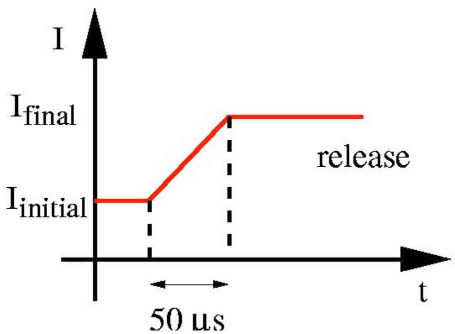
superfluid regime



$^{87}\mathrm{Rb}$ in a 3D optical lattice

150.000 lattice sites (approx n=2 atoms per site)

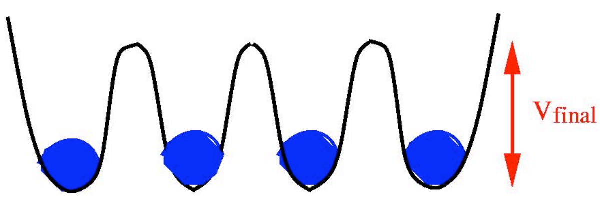
Fast ramp-up of V

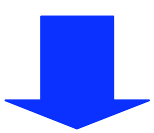


- Lattice potential depth is rapidly increased from Vinitial to Vfinal.
- Ramping time fast compared to tunnelling, but slow enough to ensure that all atoms remain in the ground state of each lattice site.

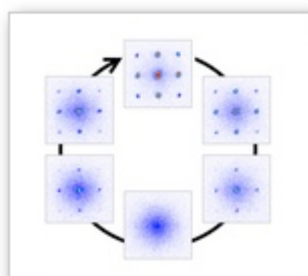
Preservation of the atom number distribution from Vinitial at the new Vfinal

Negligible tunnelling

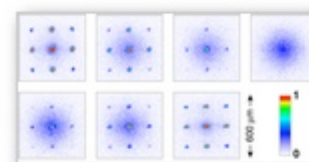




release



Collapses and revivals



$$t_{\mathrm{rev}} = \frac{2\pi\hbar}{U} \approx 500\mu\mathrm{s} \qquad t_{\mathrm{E}} = \frac{\hbar}{Un} \approx t_{\mathrm{rev}}$$

- The experiment was performed in the deep quantum regime $n \simeq O(1)$
- It would be interesting to study the dynamics of the quantum-classical separation for macroscopic objects, in the quasiclassical regime $n \gg 1$
- In this regime, the effects of the bath (e.g., thermal atoms colliding with the BEC) become important, and may affect the quantum dynamics.

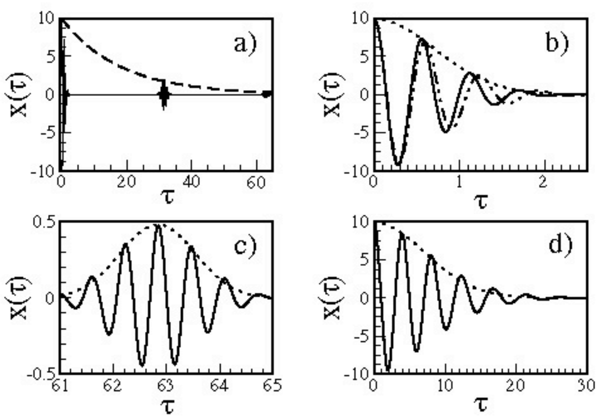
noise and dissipation

Toy model for the system- bath: $H_{\text{total}} = H_{\text{BEC}} + H_{\text{bath}} + H_{\text{int}}$

$H_{\text{BEC}} = \hbar\omega_{\text{clas}}\, a^\dagger a + \frac{U}{2}\, a^\dagger a\, (a^\dagger a - 1)$ → **BEC as a non-linear oscillator**

$H_{\text{bath}} = \sum_j \hbar\omega_j\, b_j^\dagger\, b_j$ → **set of harmonic oscillators**

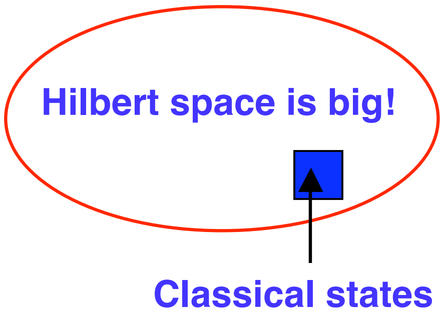
$H_{\text{int}} = \sum \lambda_j\, x\, q_j$ → **position-position coupling** $x = \frac{1}{2}(a^\dagger + a)$ $q_j = \frac{1}{2}(b_j^\dagger + b_j)$

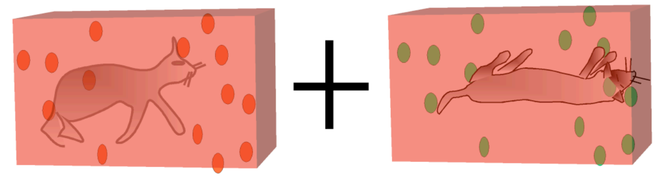


✓ **Quantum revivals are damped out due to the bath on a "decoherence" time scale τ_{D}**

✓ **Even when $\tau_{\text{D}} \ll \tau_{\text{E}}$, quantum effects (width of the revival bumps, given by τ_{E}) may survive bath-induced decoherence.**

Decoherence in Bose-Einstein condensates





A quantum system can explore states in the Hilbert space that are highly non-classical, such as Schrödinger cat states

How does the classical world appears from the underlying quantum substrate?

Interaction with the environment $\Longleftrightarrow$ decoherence

$$|\Psi\rangle_{\text{initial}} = (|\text{cat alive}\rangle + |\text{cat dead}\rangle) \otimes |\Phi_{\text{env}}\rangle$$

separable state

interaction

$$|\Psi\rangle_{\text{final}} = |\text{cat alive}\rangle \otimes |\Phi_1\rangle + |\text{cat dead}\rangle \otimes |\Phi_2\rangle$$

entangled state

(reduced) density matrix of the system

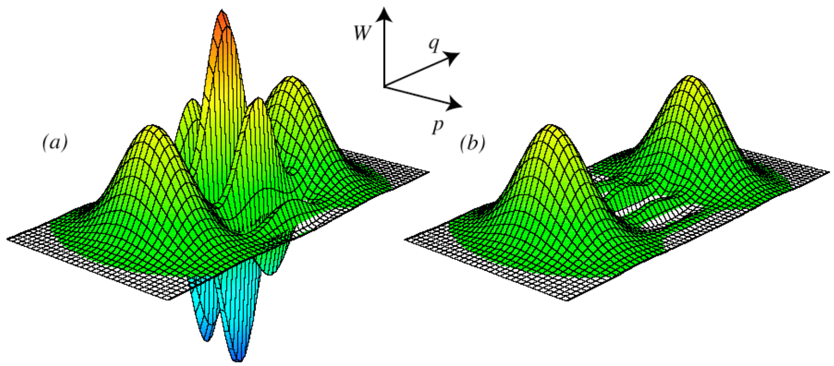
$$\rho_{\text{system}} = \text{Tr}_{\text{env}}(|\Psi_{\text{final}}\rangle\langle\Psi_{\text{final}}|) \approx |\text{cat alive}\rangle\langle\text{cat alive}| + |\text{cat dead}\rangle\langle\text{cat dead}|$$

when $\langle\Phi_1|\Phi_2\rangle \approx 0$

Statistical mixture of distinct classical possibilities

Wigner distribution $W(q,p)$

- quasi-probability distribution in phase-space (q,p)
- allows the representation of a quantum state in close analogy to a classical representation (q,p)



without decoherence

with decoherence

Decoherence is:

- **Ally:** results in the quantum-classical transition, explains the emergence of the classical world
- **Enemy:** as it kills quantum superpositions, it is the main obstacle towards quantum computation and quantum information processing

Possible cat states in BECs $(N \approx 10^3 - 10^6)$

✓**Atomic cats:** **either with hyperfine levels or with double well potential**

$$H_{\rm int} \propto a^\dagger b + b^\dagger a \;\Rightarrow\; \boxed{|\Psi\rangle = |N,0\rangle + |0,N\rangle}$$ **aka "NOON" states**

✓**Atomic-molecular cats:** **either via photoassociation or Feshbach resonances**

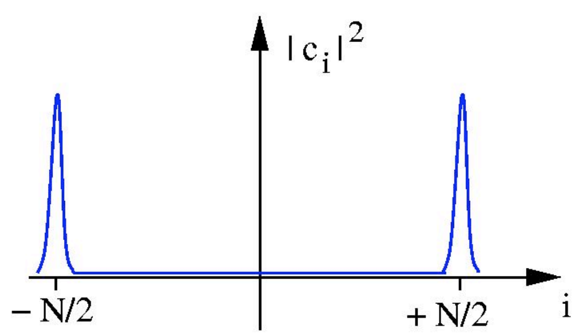
$$H_{\rm int} \propto m^\dagger\, a_1\, a_2 + a_1^\dagger\, a_2^\dagger\, m \;\Rightarrow\; |\Psi\rangle = |2N_{\rm (atoms)}\,,\; 0_{\rm (molecules)}\rangle + |0_{\rm (atoms)}\,,\; N_{\rm (molecules)}\rangle$$

BEC atomic cats (NOON states)

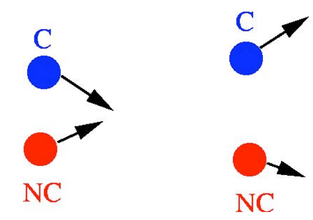
$$H_{\rm BEC} = \epsilon(a^\dagger a + b^\dagger b) + \frac{U}{2}((a^\dagger a)^2 + (b^\dagger b)^2) - J(a^\dagger b + b^\dagger a)$$

For attractive interactions $U < 0$ and $N|U| \gg J$ NOON states have the lowest energy

$$|\Psi\rangle = \sum_i c_i\, |\frac{N}{2} + i, \frac{N}{2} - i\rangle$$



Does a BEC cat live long enough?



- Collisions with the thermal cloud cause decoherence
- Decoherence rate:

$$\boxed{\tau_{\rm decoh}^{-1} \approx \left(4\pi a^2 \frac{N_{\rm E}}{V} v_T\right) N^2}$$

time to "kill the cat"

where a is the scattering length (proportional to U)

$N_{\rm E}/V$ is density of thermal (non-condensed) atoms

v_T is thermal velocity of non-condensed atoms

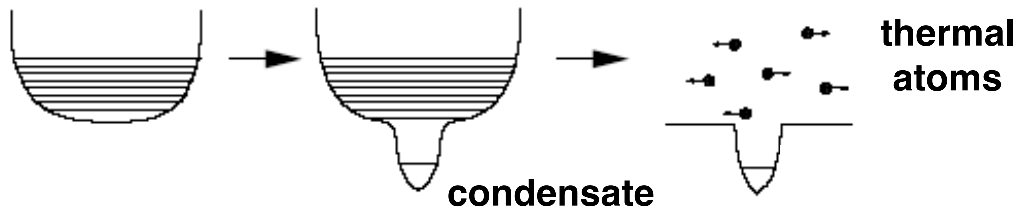
For typical parameters

$$T = 1\mu{\rm K} \qquad a = 5{\rm nm}$$
$$v_T = 10^{-2}\ {\rm m/s} \qquad V = 10^{-15}\ {\rm m}^3$$
$$N = 10^5 \qquad N_{\rm E} = 100$$

$\Rightarrow$ $\tau_{\rm decoh} \approx 10^{-7}{\rm s}$ $\Rightarrow$ **very short time!**

What can we do to minimize decoherence effects ?

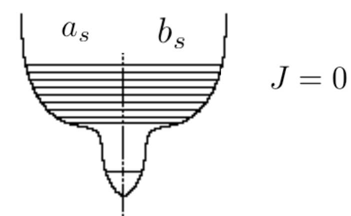
- **Trap engineering**



- **Symmetrization of the environment**

$$H_{\mathrm{E}} = \sum_s [\epsilon_s (a_s^\dagger a_s + b^\dagger b_s) - J(a_s^\dagger b_s + b_s^\dagger a_s)] \quad \Longrightarrow \quad H_{\mathrm{E}} = \sum_s [(\epsilon_s - J)\, S_s^\dagger S_s + (\epsilon_s + J)\, O_s^\dagger O_s]$$

$S_s = a_s + b_s$ (symmetric states)
$O_s = a_s - b_s$ (anti-symmetric states)



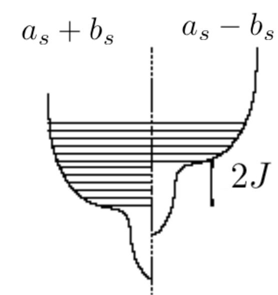
For $2J \gg k_{\mathrm{B}}\, T$, only symmetric states are occupied

⬇

Collisions affect $|N, 0\rangle$ and $|0, N\rangle$ in the same way!

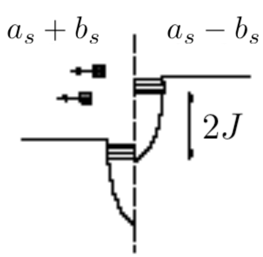
⬇

No decoherence



However, in real situations *some* antisymmetric states will be occupied

$$\tau_{\mathrm{decoh}}^{-1} \approx \left(4\pi a^2 \frac{N_{\mathrm{E}}^{\mathrm{antisym}}}{V} v_T \right) N^2$$



Ongoing reseach

- ✓ **Dynamics of quantum phase transitions in optical lattices**
- ✓ **Atom interferometry with BECs**
- ✓ **Quantum information**
- ✓ **Casimir forces**

References:

- "*Conditional quantum dynamics with several observers*", Phys. Rev. A **69,** 022109 (2004)
- "*Survival of quantum effects for observables after decoherence*", Phys. Rev. A **69,** 062110 (2004)
- "*Decoherence and the Loschmidt echo*", Phys. Rev. Lett. **91,** 210403 (2003)
- "*Measurement-induced squeezing of a Bose-Einstein condensate*", Phys. Rev. A **65,** 033610 (2002)
- "*Continuous quantum measurement a Bose-Einstein condensate: a stochastic Gross-Pitaevskii equation*", Phys. Rev. A **65,** 053604 (2002)
- "*Unconditional pointer states from conditional master equations*", Phys. Rev. Lett. **86,** 373 (2001)
- "*Decoherence in Bose-Einstein condensates: towards bigger and better Schrödinger cats*", Phys. Rev. A **62,** 013607 (2000)