Measuring and tailoring Casimir forces: from patch effects to metamaterials

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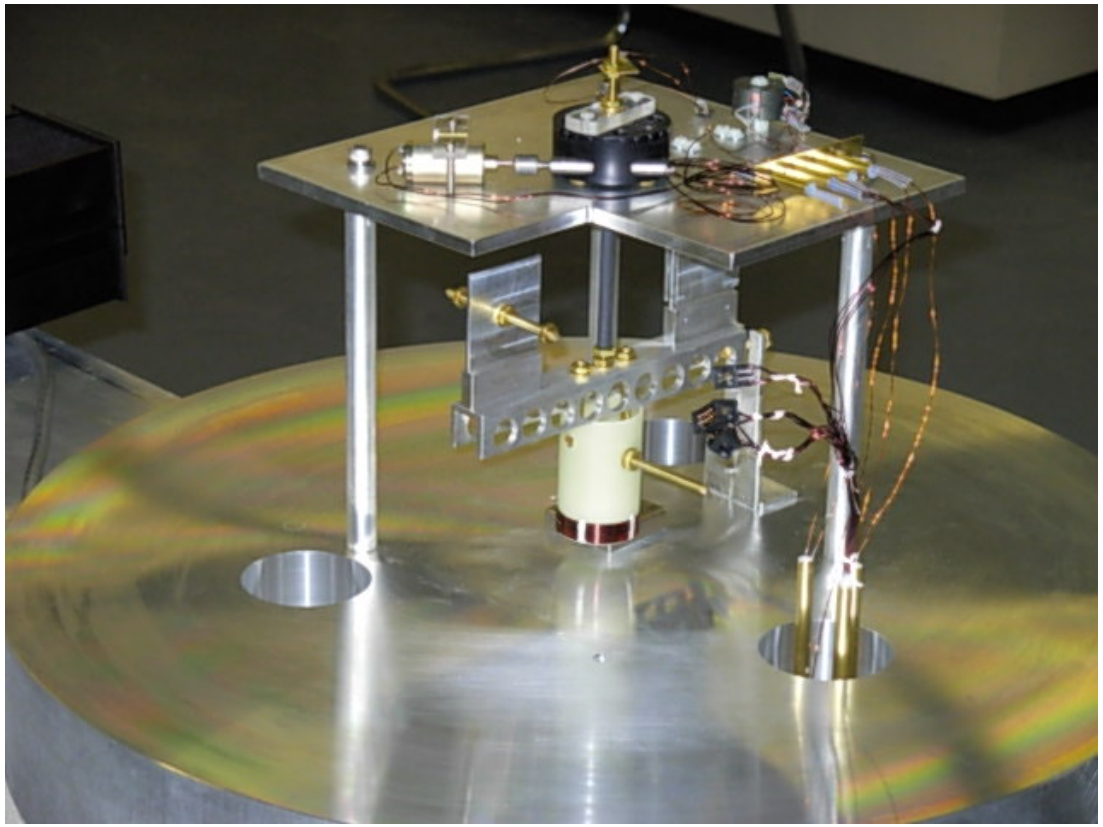
Outline of this Talk

- Electrostatic calibrations for Casimir measurements
  - Experimental set-up at Yale
  - Distance-dependent contact potential + patch effects
  - Casimir force between plates with small charge density
- Casimir forces and metamaterials
  - What is a metamaterial?
  - Proposals for Casimir force manipulation with metamaterials
  - Is Casimir repulsion possible?

Part 1: Patch effects in Casimir



Torsional Pendulum Set-up

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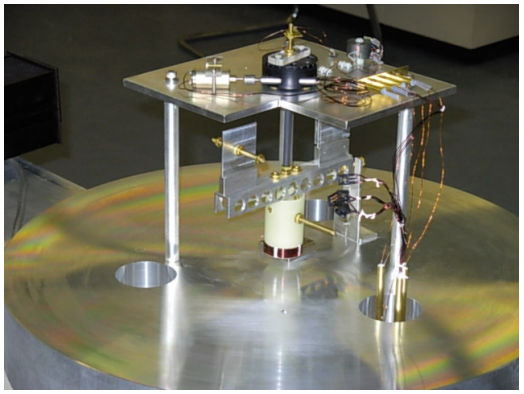
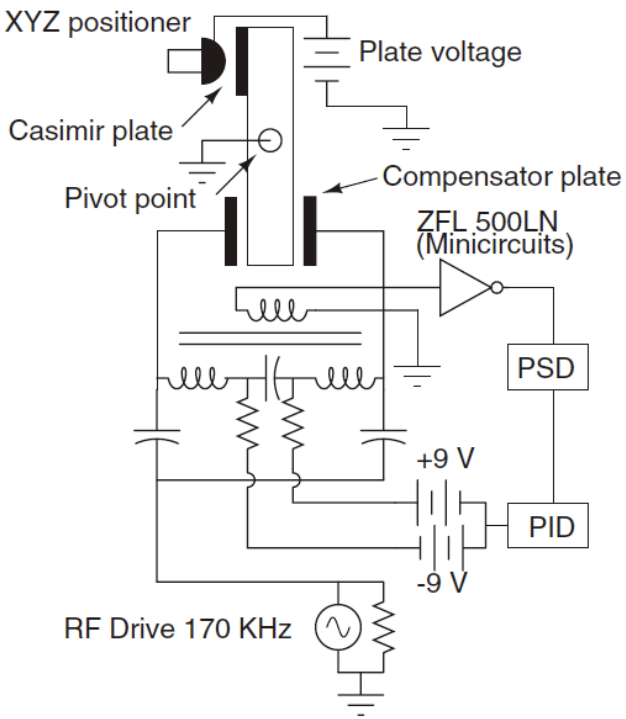
* Upgrade of Lamoreaux's 1997 experiment





An imbalance in capacitance is amplified and sent to a phase sensitive detector (PSD), which generates error signals.

A proportional integro-differential (PID) controller provides a feedback correction voltage $S_{\rm PID}(d, V_a)$ to the compensator plates, restoring equilibrium.

$$F \propto (S_{\rm PID} + 9V)^2 \approx (9V)^2 + 2S_{\rm PID} \times 9V$$

The correction voltage is the physical observable, and it is proportional to the force between the Casimir plates

Typical Casimir Measurement

$$S_{\mathrm{PID}}(d, V_a) = S_{\mathrm{dc}}(d \to \infty) + S_a(d, V_a) + S_r(d)$$

- $S_{\mathrm{dc}}(d \to \infty)$: force-free component of signal at large separations
- $S_a(d, V_a)$: electrostatic signal in response to an applied external voltage
- $S_r(d)$: residual signal due to distance-dependent forces, e.g. Casimir

The electrostatic signal between the spherical lens and the plate, in PFA ($d \ll R$), is

$$S_a(d, V_a) = \pi \epsilon_0 R (V_a - V_m)^2 / \beta d$$

β force-voltage conversion factor

This signal is minimized ($S_a = 0$) when $V_a = V_m$, and the electrostatic minimizing potential V_m is then defined to be the contact potential between the plates.

Naive picture (often used in the past):

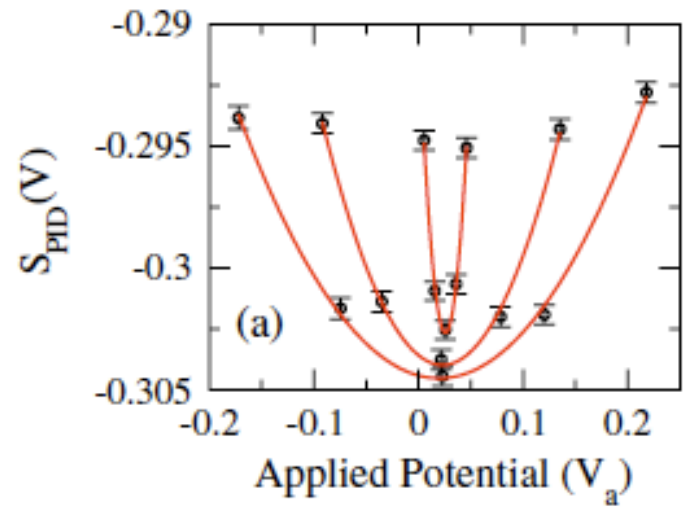
Counterbias $V_a = V_m$ fixed at large separations, and *assumed to be distance-independent* → electrostatic force is supposedly *nullified*

"Parabola" measurements

Calibration routine (Iannuzzi et al, PNAS 04)

A range of plate voltages V_a is applied, and at a given nominal absolute distance the response is fitted to a parabola

$$S_{\mathrm{PID}}(d, V_a) = S_0 + k(V_a - V_m)^2$$



Fitting parameters

$k = k(d)$ ⟶ voltage-force calibration factor + absolute distance

$V_m = V_m(d)$ ⟶ distance-dependent minimizing potential

$S_0 = S_0(d)$ ⟶ force residuals: electrostatic + Casimir + non-Newtonian gravity + ...

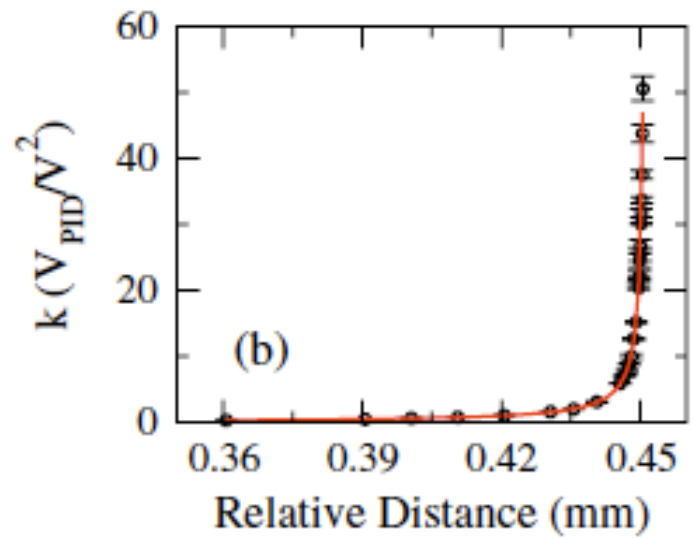
This procedure is repeated at decremental distances, from150 um down to 500 nm, completing a single experimental run.

Note: The maximum force gradient for feedback system stability is 5 nm/um, limiting the minimum distance to 500 nm.

Curvature Parameter $k(d)$

From the curvature of the different parabolas one obtains $k(d)$

$$k(d) = \frac{\pi \epsilon_0 R/\beta}{d}$$



☑ Force-voltage calibration factor

$$\beta = (1.35 \pm 0.04) \times 10^{-7}\ \mathrm{N/V}$$

☑ Sphere-plane absolute distance

$$d = d_0 - d_{\mathrm{rel}}$$

Typical uncertainty in position is about 10% at a given distance

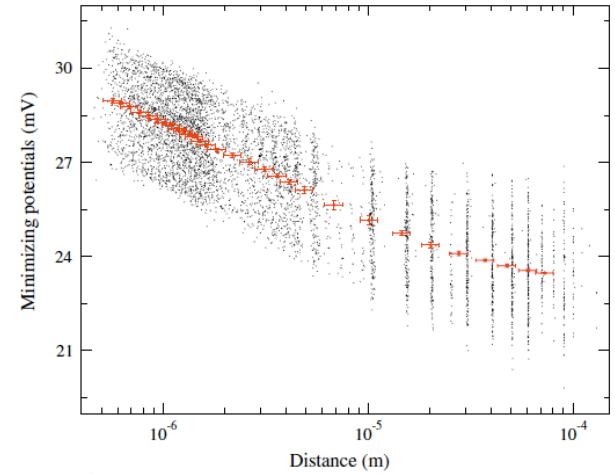
Some further details:

- Average $\chi^2 = 1.2$
- Drift of about 5 um in 3 weeks
- Single sweep suffers about 20nm drift (less than 5% at the closest approach of 500nm)

Varying Minimizing Potential

The Ge data shows a *distance-dependent* minimizing potential, of the order of 6 mV over 100 um.

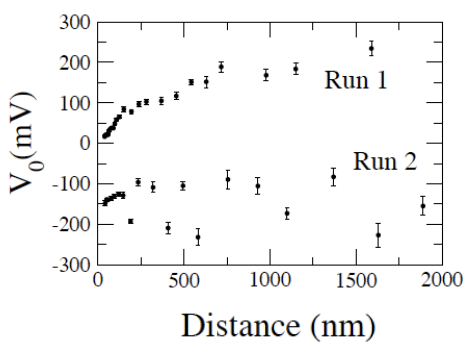
$$V_m = V_m(d)$$



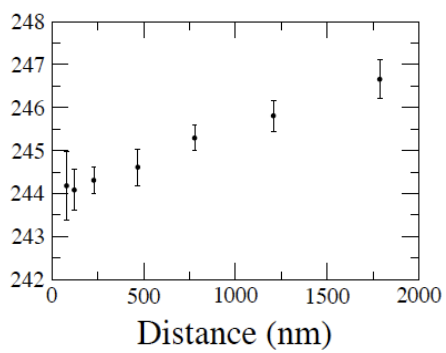
Therefore, a fixed counterbias would be incorrect!

Similar behavior has been observed in a number of experiments with Au plates (both with macro and micro spheres), including

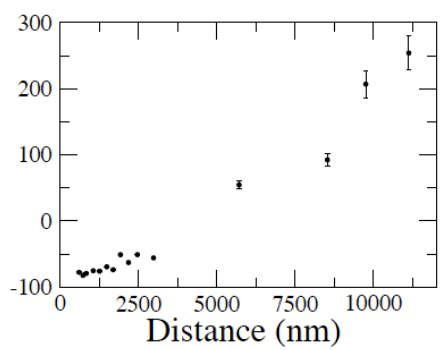
Onofrio's group



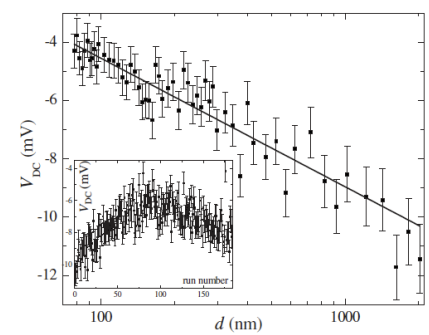
Capasso's group



Chevrier's group



Iannuzzi's group

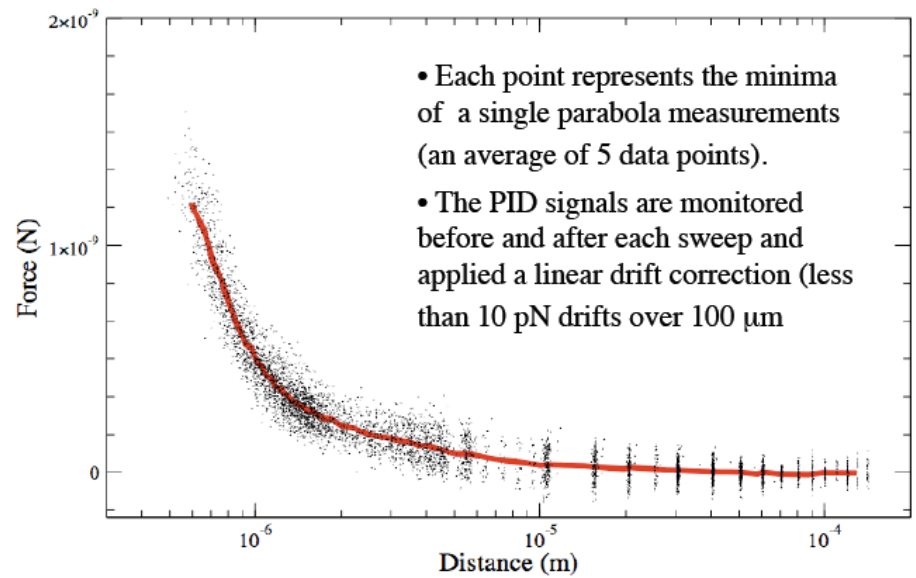


(See Kim, Brown-Hayes, DD, Brownell, Onofrio, PRA 08, 09)

Force Residuals

Residuals from Coulomb force obtained from the value of the PID signal at the minima of each parabola,

$$S_0(d) \rightarrow F_r(d)$$



In the experiment, these force residuals are *too large* to be explained just by the Casimir-Lifshitz force between the Ge plates.

In fact, the experimental data shows a $1/d$ force residual at distances $d > 5\mu\mathrm{m}$, where the Casimir force should be negligible.

What is the origin of the varying minimizing potential?

What is the origin of the additional force residual?

Surface Potentials & $V_m(d)$

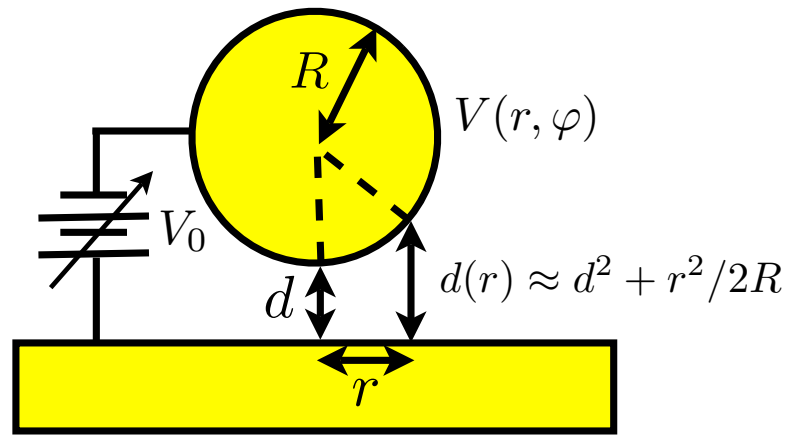
The surface of a conductor is an equipotential only for a perfectly clean surface of a homogeneous system cut along one of its crystalline planes.

It is NOT the case for any real material.

- oxide layers in "dirt" films
- local variations in the crystalline structure
- different work functions

Electrostatic force (in PFA, $R \gg d$):

$$F(d, V_0) = \frac{\epsilon_0}{2} \int_0^{2\pi} d\varphi \int_0^R r dr \frac{(V(r,\varphi) + V_0)^2}{(d + r^2/2R)^2}$$



Minimized force at a fixed distance determines the minimizing potential $V_m(d)$

$$0 = \left.\frac{\partial F(d, V_0)}{\partial V_0}\right|_{V_0 = V_m} = \epsilon_0 \int_0^{2\pi} d\varphi \int_0^R r dr \frac{V(r,\varphi) + V_m}{(d + r^2/2R)^2}$$

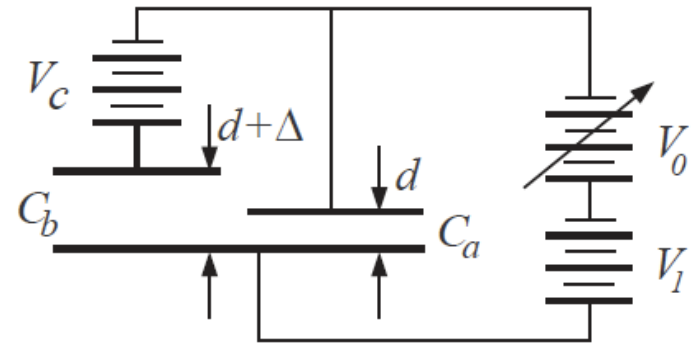
and

$$\frac{\partial^2 F(d, V_0)}{\partial V_0^2} = 2\pi\epsilon_0 \int_0^R dr \frac{r}{(d + r^2/2R)^2} \approx \frac{2\pi R \epsilon_0}{d}$$

☑ Surface patches DO NOT interfere with distance calibration!

$V_m(d)$ & Residual Elec. Force. I

A toy model illustrating the mechanism for the generation of $V_m(d)$ and $F^{\rm el}_{\rm res}(d)$



Force on lower plate:

$$F(d,V_0) = -\frac{1}{2}C'_a V_0^2 - \frac{1}{2}C'_b(V_0+V_c)^2$$

(V_0 is varied, V_c a fixed property of the plates)

$$C'_a = -\epsilon_0 A/d^2$$
$$C'_b = -\epsilon_0 A/(d+\Delta)^2$$

When force is minimized, one gets a varying minimizing potential and a varying electrostatic residual force.

$$\left.\frac{\partial F(d,V_0)}{\partial V_0}\right|_{V_0=V_m} = 0 \;\Rightarrow\; V_m(d) = -\frac{C'_b V_c}{C'_a + C'_b} = -V_c\frac{d^2}{d^2+(d+\Delta)^2}$$

$$F^{\rm el}_{\rm res}(d) \;=\; F(d,V_0=V_m(d)) \;=\; \frac{\epsilon_0 A}{2}\,\frac{V_m^2(d)[d^2+(d+\Delta)^2]}{d^4} \propto \frac{1}{d^4} \text{ for } \Delta \gg d$$

In reality, measurements can determine $V_m(d)$ up to an overall constant: $V_m(d) \to V_m(d) + V_1$

$V_m(d)$ & Residual Elec. Force. II

Sphere-plane case: $C'_a(d) = -2\pi\epsilon_0 R/d$

Dividing the sphere into infinitesimal areas, each with a random potential, and integrating over the surface to get the net residual force (as in PFA), we get

$$\boxed{F^{\rm el}_{\rm res}(d) = \pi\epsilon_0 R\,\frac{[V_m(d)+V_1]^2}{d}}$$

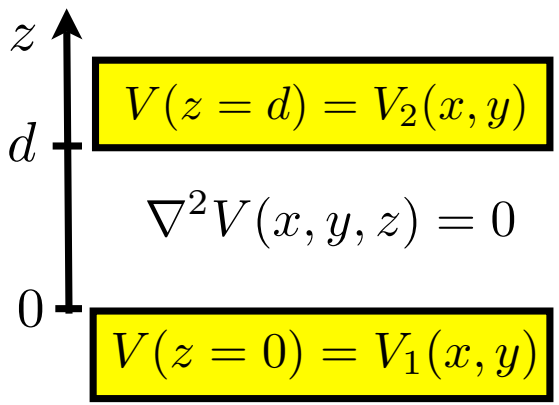
Important message from this analysis:

- ☑ Minima of parabolas DO NOT nullify all possible electrostatic forces between plates!
- ☑ Origin of $V_m(d)$ is the interplay between non-planar geometry and local variations of surface potential

Electrostatic Patch Effects. I

The patch effect is a possible systematic limitation to Casimir force measurements (Speake and Trenkel, PRL 03).

Plane-plane geometry:



$$V(x,y,z) = X(x)Y(y)Z(z)$$

$$\frac{1}{X}\frac{d^2X}{dx^2} = -\alpha^2; \quad \frac{1}{Y}\frac{d^2Y}{dy^2} = -\beta^2; \quad \frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma^2 = \alpha^2 + \beta^2$$

$$V_1(x,y) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} V_{1,\mathbf{k}} \cos(k_x x)\cos(k_y y)$$ [idem for $V_2(x,y)$]

$$V(x,y,z) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\cos(k_x x)\cos(k_y y)}{2\sinh(\gamma d)} \times \left[e^{\gamma z}\left(V_{2,\mathbf{k}} - V_{1,\mathbf{k}} e^{-\gamma d}\right) + e^{-\gamma z}\left(V_{1,\mathbf{k}} e^{\gamma d} - V_{2,\mathbf{k}}\right)\right]$$

Electrostatic energy:

$$U_{pp}(d) = \frac{\epsilon_0}{2}\int d^3\mathbf{r}\, |\nabla V|^2$$

Electrostatic Patch Effects. II

Statistical properties for patch potentials:

$$\langle V_{1,\mathbf{k}} \rangle = \langle V_{2,\mathbf{k}} \rangle = \langle V_{2,\mathbf{k}} V_{1,\mathbf{k}'} \rangle = 0;$$
$$\langle V_{1,\mathbf{k}} V_{1,\mathbf{k}'} \rangle = C_{1,\mathbf{k}}\, \delta^2(\mathbf{k} - \mathbf{k}');$$
$$\langle V_{2,\mathbf{k}} V_{2,\mathbf{k}'} \rangle = C_{2,\mathbf{k}}\, \delta^2(\mathbf{k} - \mathbf{k}'),$$

Averaging the interaction energy over different realizations of the stochastic patches, we get

$$\langle U_{pp} \rangle = \frac{\epsilon}{16} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\gamma \sinh(2\gamma d)}{\sinh^2(\gamma d)} \left[C_{1,\mathbf{k}} + C_{2,k}\right]$$

In the limit of large distances $(kd \gg 1)$, this expression has an asymptotic behavior independent of distance (self-energy of each plate). We remove the potential energy at infinite separation, to get the electrostatic interaction energy due to patch effects

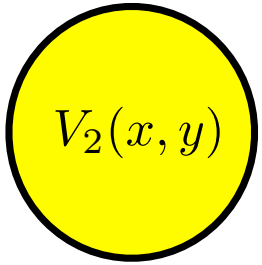
$$\boxed{\langle U_{pp} \rangle = \frac{\epsilon_0}{32\pi} \int_0^\infty dk\, \frac{k^2 e^{-kd}}{\sinh(kd)} \left[C_{1,k} + C_{2,k}\right]}$$

For large patches $(kd \ll 1)$: $\langle U_{pp} \rangle = \dfrac{\epsilon_0 V_{\mathrm{rms}}^2}{2d}$

For small patches $(kd \gg 1)$: $\langle U_{pp} \rangle \propto e^{-kd}$

Electrostatic Patch Effects. III

Sphere-plane geometry:



To compute the patch effect in the sphere-plane configuration we use PFA for the curvature effect $(d \ll R)$ but leave kd arbitrary

$$\nabla^2 V(x,y,z) = 0$$

$$F_{sp}(d) = 2\pi R \langle U_{pp}(d) \rangle = \frac{\epsilon_0 R}{16} \int_0^\infty dk \, \frac{k^2 e^{-kd}}{\sinh(kd)} \left[C_{1,k} + C_{2,k} \right]$$

$$V(z=0) = V_1(x,y)$$

Different models to describe surface potential fluctuations:

- $C_{1,k} = C_{2,k} = V_0^2$ for $k_{\min} < k < k_{\max}$

$$F_{sp} = \frac{4\pi\epsilon_0 V_{\rm rms}^2 R}{k_{\max}^2 - k_{\min}^2} \int_{k_{\min}}^{k_{\max}} dk \frac{k^2 e^{-kd}}{\sinh(kd)}$$

- $\mathcal{R}(r) = \begin{cases} V_0^2 & \text{for } r \le \lambda, \\ 0 & \text{for } r > \lambda. \end{cases}$

$$F_{sp} = 2\pi\epsilon_0 R \int_0^\infty du \; u \frac{J_1(u)}{e^{2ud/\lambda} - 1}$$

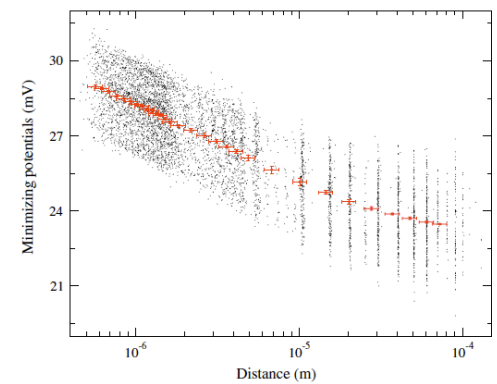
In the limit of large patches $(kd \ll 1)$:

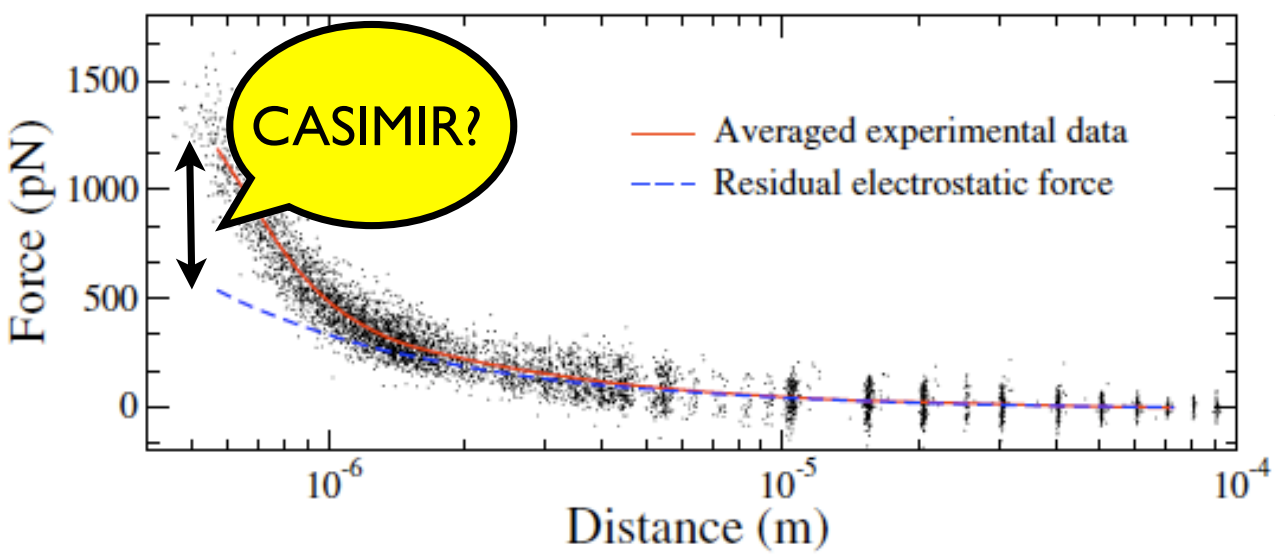
$$F_{sp}(d) = \pi\epsilon_0 R \, \frac{V_{\rm rms}^2}{d}$$

Total Elec. Residual Force

We fit the data for the *residual force at the minimizing potential* with a force of electric origin, for distances $d > 5\mu\mathrm{m}$ (negligible Casimir)

$$F_r^{\mathrm{el}}(d) = F_0 + \pi\epsilon_0 R\,\frac{[V_m(d) + V_1]^2 + V_{\mathrm{rms}}^2}{d}$$





$$F_0 = (-11 \pm 2) \times 10^{-12}\ \mathrm{N}$$

$$V_1 = (-34 \pm 3)\ \mathrm{mV}$$

$$V_{\mathrm{rms}} = (6 \pm 2)\ \mathrm{mV}$$

$$\chi_0^2 = 1.5$$

Casimir with Ge: Theory

- Casimir-Lifshitz free energy in the plane-plane geometry:

$$\frac{E}{A} = k_{\mathrm{B}}T \sum_{p} \sum_{n=0}^{\infty}{}' \int \frac{d^2\mathbf{k}}{(2\pi)^2} \ln[1 - r_1 r_2 e^{-2d\sqrt{k^2+\xi_n^2/c^2}}]$$

$r_j = r^p_{\mathbf{k},j}(i\xi_n)$ reflection amplitudes $\qquad$ $\xi_n = 2\pi n k_{\mathrm{B}} T/\hbar$ Matsubara frequencies

- Sphere-plane Casimir force is computed via PFA (well satisfied in the experiment)

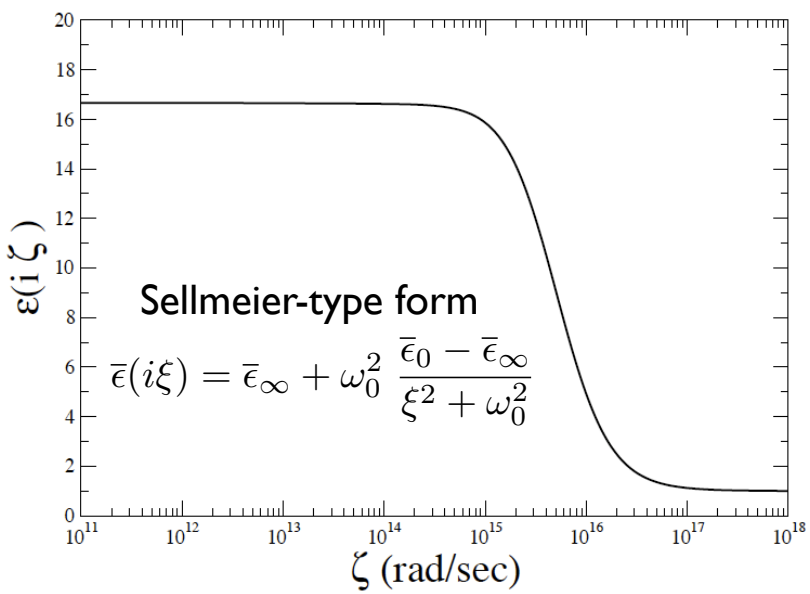
$$F_{sp}(d) = 2\pi R U_{pp}(d) = 2\pi R \, \frac{E}{A}$$

One needs to compute the reflection amplitudes $r^p_{\mathbf{k},j}(\omega)$ for a vacuum-Ge interphase. Depending on the model used to describe the optical and conductivity properties of Ge we get different reflection amplitudes.

Material properties of Ge

- intrinsic semiconductor, among the purest materials available
- small density of free carriers (electrons and holes)
- conductivity, thermal, and optical properties are well tabulated

Bare permittivity of intrinsic Ge
(not including contributions from free carriers)



Conductivity properties of intrinsic Ge

Intrinsic carrier density: $n_0(T) = \sqrt{n_c n_v}\, e^{-\frac{E_g}{2k_B T}}$

$n_c(T)$ effect. density of states in conduction band

$n_v(T)$ effect. density of states in valence band

$E_g(T)$ energy band gap

At T=300K:
$$n_c = 1.04 \times 10^{-19}\ \mathrm{cm}^{-3}$$
$$n_v = 6.0 \times 10^{-18}\ \mathrm{cm}^{-3}$$
$$E_g = 0.66\ \mathrm{eV}$$

Carrier relaxation time: $\tau \approx 3.9\ \mathrm{ps}$

Effective mass of conduction electrons: $m_e = 0.12m$

Effective mass of conduction holes: $m_h = 0.21m$

Charge-drift model. I

☑ Maxwell's eqns.: $\nabla \times \mathbf{E} = i\mu_0\omega\mathbf{H}, \quad \nabla \times \mathbf{H} = -i\bar{\epsilon}(\omega)\omega\mathbf{E} + \mathbf{J}, \quad \nabla \cdot \mathbf{E} = -\frac{en}{\bar{\epsilon}(\omega)}$

$\mathbf{J} = -en\mathbf{v}$ carrier current

☑ Classical Boltzmann transport eqn.: $\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\mathbf{v} = -\frac{e}{m}\mathbf{E} - \frac{v_T^2}{n}\nabla n - \frac{\mathbf{v}}{\tau}$

$v_T = \sqrt{k_\mathrm{B}T/m}$ mean thermal velocity

Mixing both, we get the fundamental equation for the EM field inside the semiconductor:

$$\boxed{\left[\nabla^2 + \mu_0\bar{\epsilon}(\omega)\omega^2\left(1 + i\frac{\omega_c}{\omega(1 - i\omega\tau)}\right)\right]\mathbf{E} = \left[1 + i\mu_0\bar{\epsilon}(\omega)\frac{\omega D}{1 - i\omega\tau}\right]\nabla \cdot (\nabla \cdot \mathbf{E}).}$$

$\omega_c = 4\pi e n_0 \mu/\bar{\epsilon}(\omega)$

$\mu = e\tau/m$ mobility of carriers

$D = v_T^2\tau$ diffusion constant

Charge-drift model. II

Solving the EM boundary conditions on the vacuum-Ge interphase, we obtain the following TE and TM reflection amplitudes

$$r_{\mathbf{k}}^{\mathrm{TM}}(i\xi) = \frac{\bar{\epsilon}(i\xi)\sqrt{k^2+\xi^2/c^2}-\chi}{\bar{\epsilon}(i\xi)\sqrt{k^2+\xi^2/c^2}+\chi},$$

$$r_{\mathbf{k}}^{\mathrm{TE}}(i\xi) = \frac{\sqrt{k^2+\xi^2/c^2}-\eta_T}{\sqrt{k^2+\xi^2/c^2}+\eta_T}.$$

$$\chi = \frac{1}{\eta_L}\left[k^2+\bar{\epsilon}(i\xi)\frac{\xi^2}{c^2}\frac{\eta_L\eta_T-k^2}{\eta_T^2-k^2}\right]$$

$$\eta_L(i\xi) = \sqrt{k^2+\frac{4\pi e^2 n_0}{\bar{\epsilon}(i\xi)k_{\mathrm{B}}T}+\frac{\xi(1+\xi\tau)}{v_T^2\tau}},$$

$$\eta_T(i\xi) = \sqrt{k^2+[\bar{\epsilon}(i\xi)+4\pi\sigma(i\xi)/\xi]\xi^2/c^2},$$

→ same as in the ideal dielectric+Drude conductivity model.

Some limiting behavior:

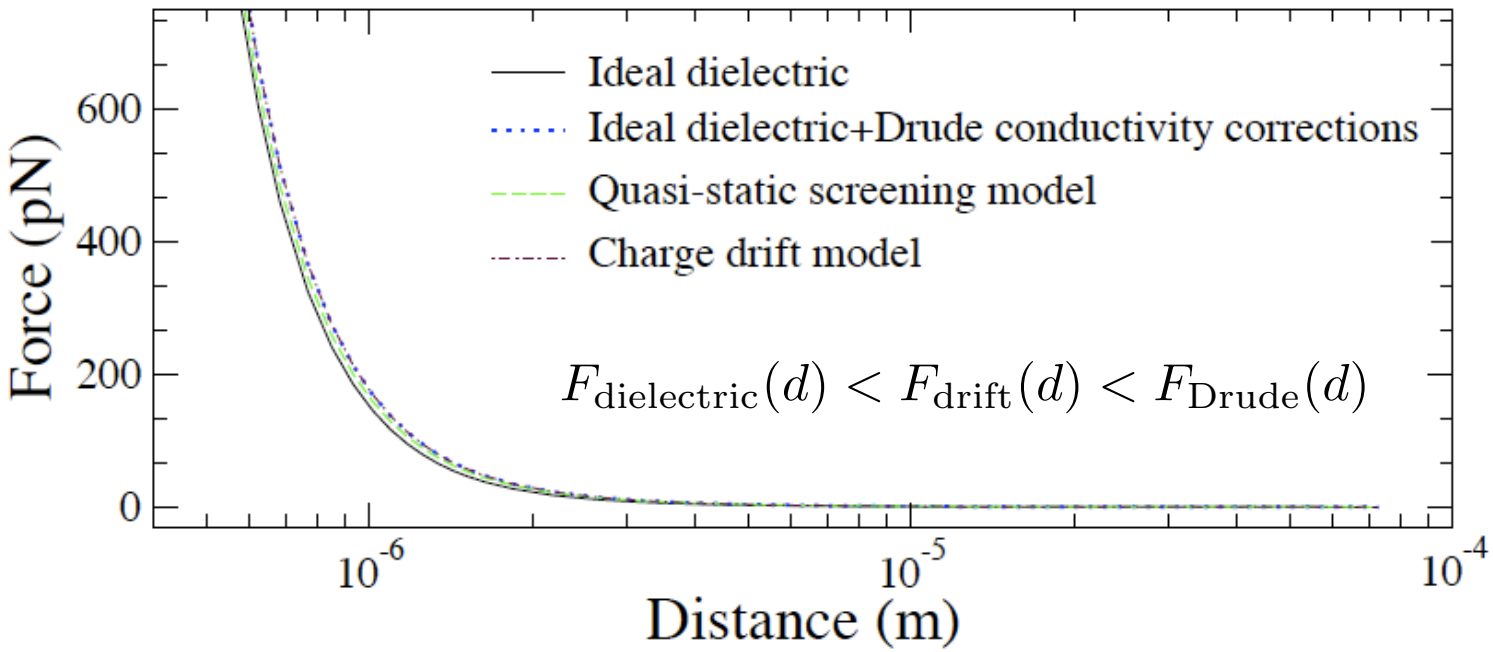
- In the quasi-static limit $\xi \to 0$ we recover Pitaevskii's results (Pitaevskii PRL 2008), namely

  $r_{\mathbf{k}}^{\mathrm{TE}}(0) = 0$ (a static TE field is a purely magnetic field, fully penetrates a non-magnetic medium)

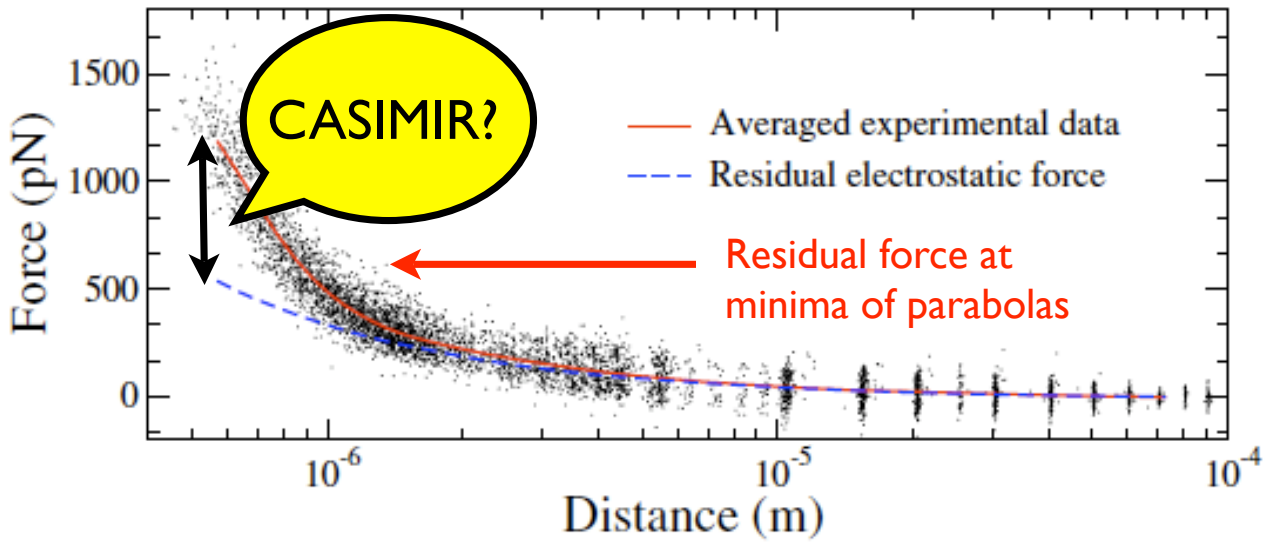
  $r_{\mathbf{k}}^{\mathrm{TM}}(\xi=0) = \frac{\bar{\epsilon}_0 q - k}{\bar{\epsilon}_0 q + k}$ (interpolates between a good conductor and an ideal dielectric)

- For $\xi \neq 0$ in the ideal dielectric limit (small free charge density and small thermal velocity), we recover Fresnel coefficients in terms of the bare permittivity $\bar{\epsilon}(\omega)$

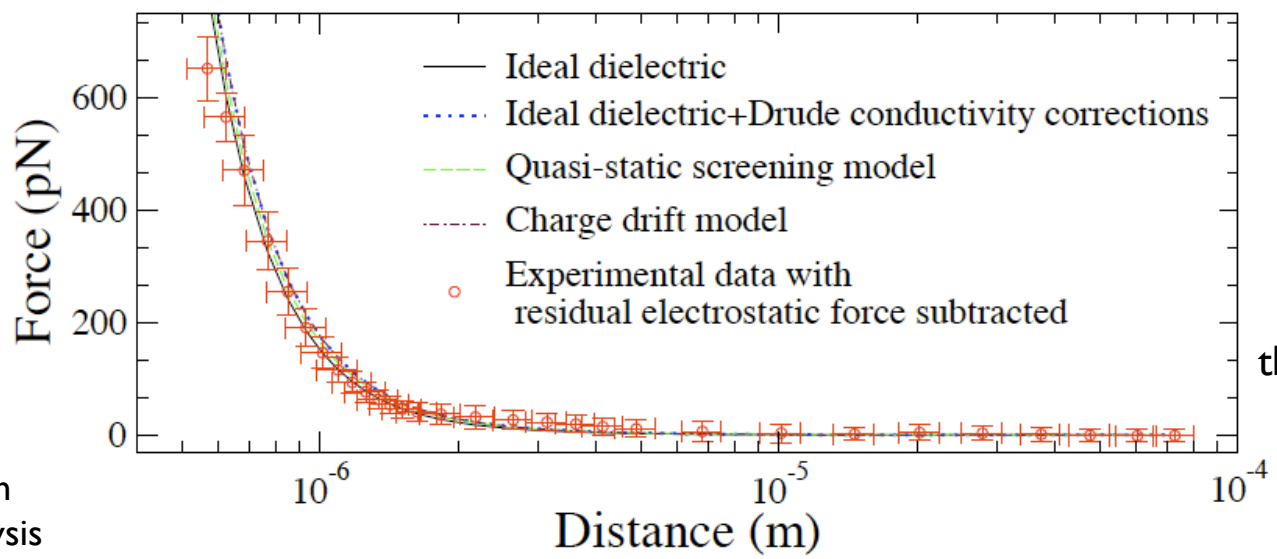
Comparison of models



Casimir force residuals



After subtraction of the electrostatic force residual $F_r^{\rm el}(d) = F_0 + \pi\epsilon_0 R\,\frac{[V_m(d)+V_1]^2 + V_{\rm rms}^2}{d}$



Error bars:

3% statistical uncertainties

10% fitting uncertainties from electrostatic analysis

For $d < 5\mu\mathrm{m}$

$\chi_0^2 \approx 1$

for all the theoretical models

Remarks: patches and Casimir

- Found a distance-dependent minimizing potential, due to large-scale variations in the contact potential along the surface of the plates. It results in a relatively large residual force of electrostatic origin $\propto [V_m(d) + V_1]^2/d$

- Found another residual force of electrostatic origin, probably due to potential patches on the surfaces that, for $d \ll \lambda \ll R$, is $\propto V_{\rm rms}^2/d$

- After subtraction of these two electrostatic residuals, we got very good agreement with a Casimir force residual. However, we do not have enough accuracy to distinguish between the different theoretical models.

- Further measurements deemed necessary to better understand the origins of the observed residual electrostatic force. For example, Kelvin probe measurements of the potential patches on the samples.

References of Part I

- Theory of patch effects in Casimir physics

  Speake and Trenkel, PRL **90**, 160403 (2003)

  Kim, Sushkov, DD, Lamoreaux, PRA **81**, 022505 (2010)

- Theory of Casimir force for materials with small carrier density

  Pitaevskii, PRL **101**, 163202 (2008)

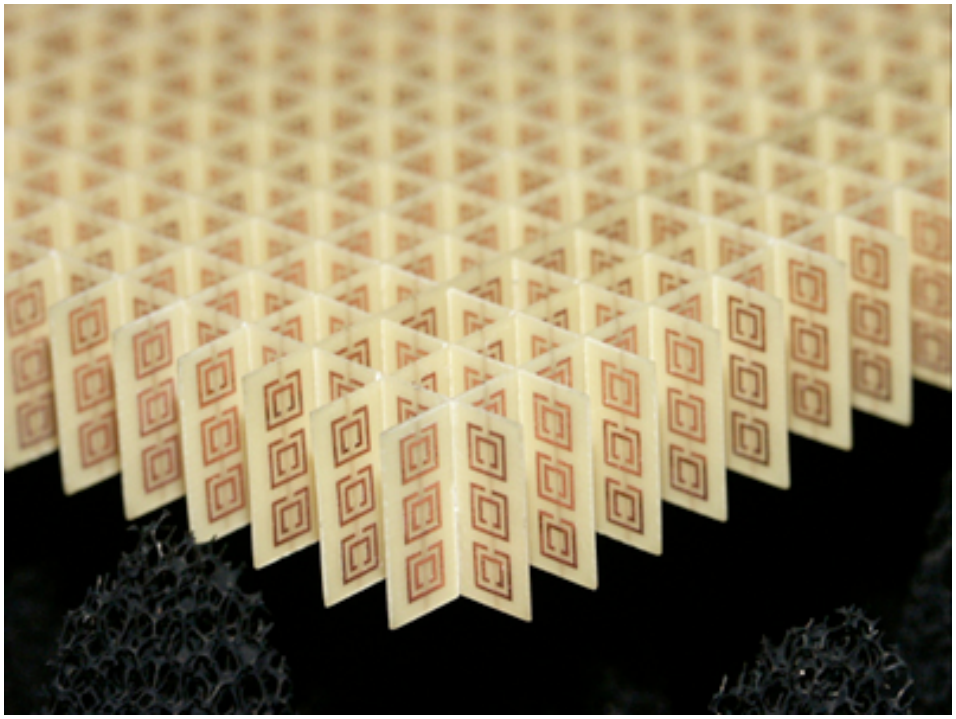
  DD and Lamoreaux, PRL **101**, 163203 (2008)

  DD and Lamoreaux, J. Phys. **161**, 012009 (2009)

- Experiment on patch effects and Casimir forces with Ge plates

  Kim, Sushkov, DD, Lamoreaux, PRL **103**, 060401 (2009)

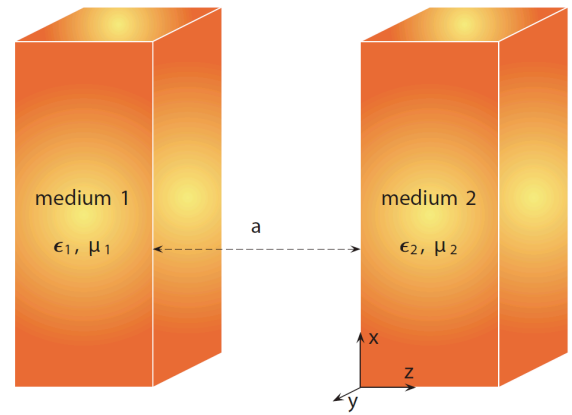
Part II: Metamaterials and Casimir



Effects of materials

The Lifshitz formula: Lifshitz (1956)

$$\frac{F}{A} = 2\hbar \,\mathrm{Im} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{2\mathrm{i}K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{2\mathrm{i}K_3 d}}$$



$$K_3 = \sqrt{\omega^2/c^2 - k_\parallel^2}$$

Reflection matrices (Fresnel formulas for isotropic media):

$$r^{\mathrm{TM,TM}}(\omega, \mathbf{k}_\parallel) = \frac{\epsilon(\omega) K_3 - \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}{\epsilon(\omega) K_3 + \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}$$

$r^{\mathrm{TE,TE}} = r^{\mathrm{TM,TM}}$ with $\epsilon \leftrightarrow \mu$

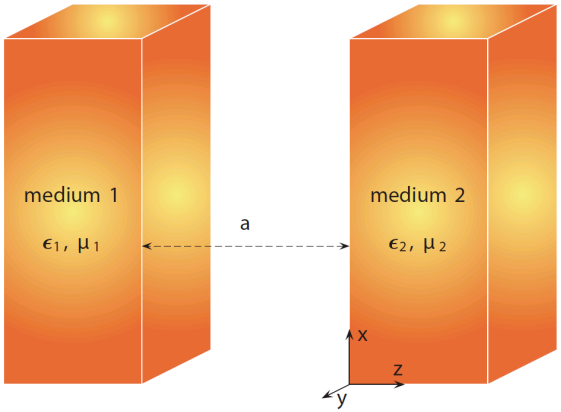
Relevant frequencies:

$$\left.\begin{array}{lll} \epsilon(\omega) \xrightarrow{\omega \gg \Omega_p} 1 & \Rightarrow & r^{p,p} \approx 0 \text{ (Transparent plates)} \\ \omega \gg c/d & \Rightarrow & e^{2\mathrm{i}K_3 d} \approx 0 \text{ (Fast oscillations)} \end{array}\right\} \Rightarrow F \approx 0$$

$$0 \leq \omega \leq \min\{\Omega_p, c/d\}$$

Going to imaginary frequencies

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$



Kramers-Kronig (causality) relations:

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega \qquad \mu(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \mu''(\omega)}{\omega^2 + \xi^2} d\omega$$

Dominant frequencies below the near-infrared/optical region of the EM spectrum (gaps d= 200-1000 nm)

The important message is that Casimir is a broad-band frequency phenomenon

The sign of the Casimir force

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$

The sign of the force is directly connected to the sign of the product of the reflection coefficients on the two plates, evaluated at imaginary frequencies. As a rule of thumb, we have (p=TE, TM)

$$R_1^p(i\xi) \cdot R_2^p(i\xi) > 0 \ (\forall\ \xi \leq c/d) \Rightarrow \text{Attraction}$$

$$R_1^p(i\xi) \cdot R_2^p(i\xi) < 0 \ (\forall\ \xi \leq c/d) \Rightarrow \text{Repulsion}$$

In terms of permittivities and permeabilities:

$$\epsilon_a(i\xi) \gg \epsilon_b(i\xi)$$

$$\mu_b(i\xi) \gg \mu_a(i\xi)$$

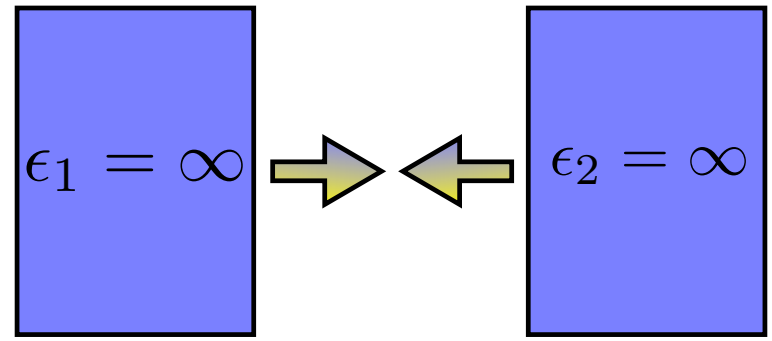
$\longrightarrow$ Repulsion

Ideal attraction-repulsion

- **Ideal attractive limit**

  Casimir (1948)

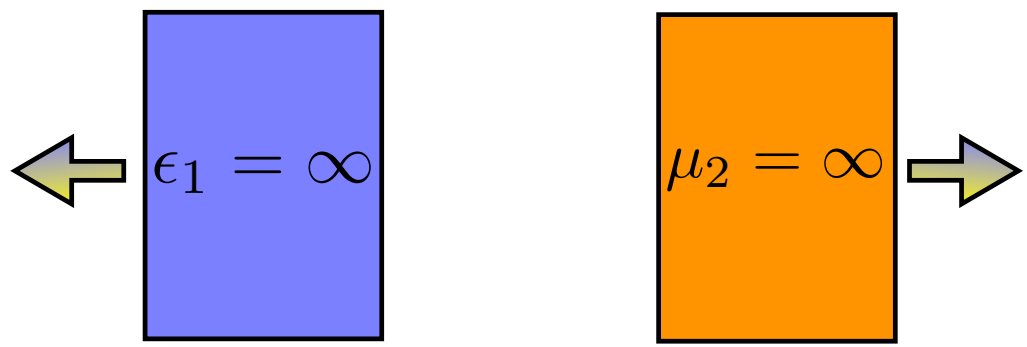
$$\frac{F}{A} = +\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$



- **Ideal repulsive limit**

  Boyer (1974)

$$\frac{F}{A} = -\frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$



- **Real repulsive limit**

  Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$ → Metamaterials

Quantum levitation with MMs?

Physicists have 'solved' mystery of levitation - Telegraph http://www.telegraph.co.uk/news/main.jhtml?xml=/news/2007/08/0...

Telegraph.co.uk

Tuesday 4 September 2007

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Physicists have 'solved' mystery of levitation

By Roger Highfield, Science Editor
Last Updated: 1:41am BST 08/08/2007

Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

In earlier work the same team of theoretical physicists showed that invisibility cloaks are feasible.

Now, in another report that sounds like it comes out of the pages of a Harry Potter book, the University of St Andrews team has created an 'incredible levitation effects' by engineering the force of nature which normally causes objects to stick together.

Professor Ulf Leonhardt and Dr Thomas Philbin, from the University of St Andrews in Scotland, have worked out a way of reversing this phenomenon, known as the Casimir force, so that it repels instead of attracts.

Their discovery could ultimately lead to frictionless micro-machines with moving parts that levitate But they say that, in principle at least, the same effect could be used to levitate bigger objects too, even a person.

In theory the discovery could be used to levitate a person

The Casimir force is a consequence of quantum mechanics, the theory that describes the world of atoms and subatomic particles that is not only the most successful theory of physics but also the most baffling.

The force is due to neither electrical charge or gravity, for example, but the fluctuations in all-pervasive energy fields in the intervening empty space between the objects and is one reason atoms stick together, also explaining a "dry glue" effect that enables a gecko to walk across a ceiling.

Now, using a special lens of a kind that has already been built, Prof Ulf Leonhardt and Dr Thomas Philbin report in the New Journal of Physics they can engineer the Casimir force to repel, rather than attact.

Because the Casimir force causes problems for nanotechnologists, who are trying to build electrical circuits and tiny mechanical devices on silicon chips, among other things, the team believes the feat could initially be used to stop tiny objects from sticking to each other.

Prof Leonhardt explained, "The Casimir force is the ultimate cause of friction in the nano-world, in particular in some microelectromechanical systems.

Such systems already play an important role - for example tiny mechanical devices which triggers a car airbag to inflate or those which power tiny 'lab on chip' devices used for drugs testing or chemical analysis.

Micro or nano machines could run smoother and with less or no friction at all if one can manipulate the force." Though it is possible to levitate objects as big as humans, scientists are a long way off developing the technology for such feats, said Dr Philbin.

The practicalities of designing the lens to do this are daunting but not impossible and levitation "could happen over quite a distance".

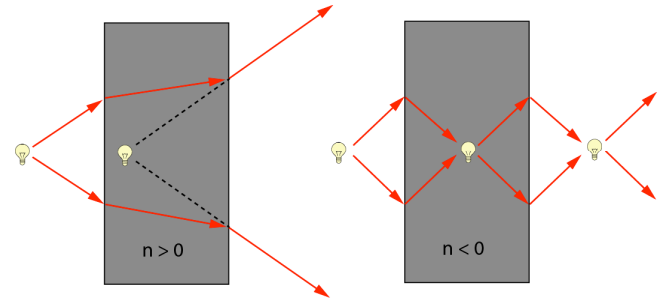
Prof Leonhardt leads one of four teams - three of them in Britain - to have put forward a theory in a peer-reviewed journal to achieve invisibility by making light waves flow around an object - just as a river flows undisturbed around a smooth rock.

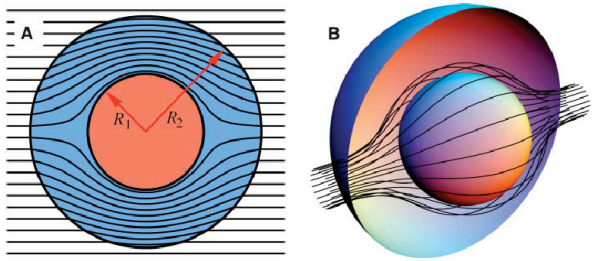
"In theory the discovery could be used to levitate a person"

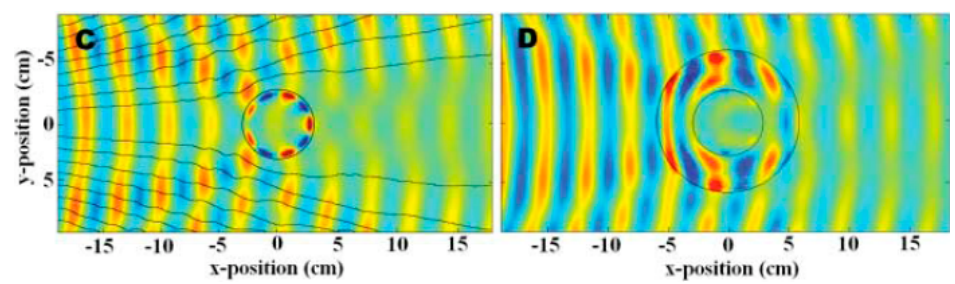
Metamaterials

- Artificial structured composites with designer electromagnetic properties
- MMs are strongly anisotropic, dispersive, magneto-dielectric media.

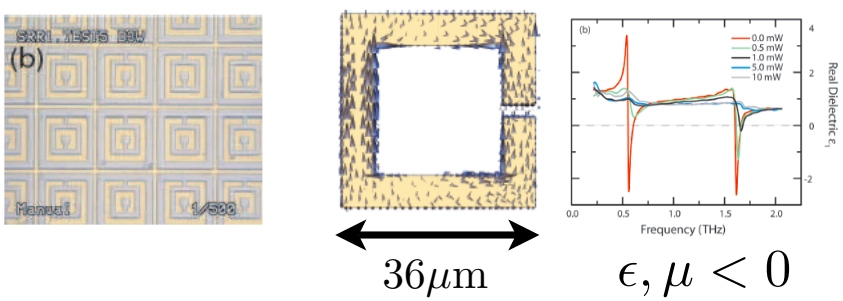
- Negative refraction — Veselago (1968), Smith et al (2000)
- Perfect lens — Pendry (2000)
- Cloaking — Smith et al (2007)



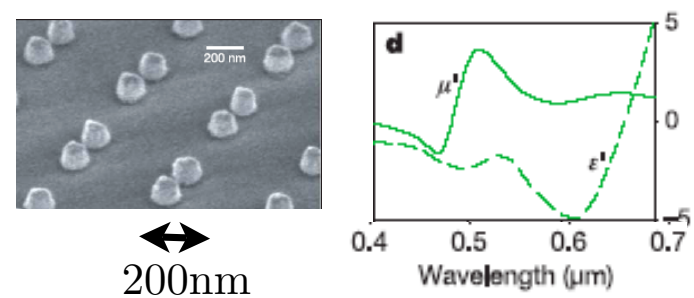




THz MMs: eg split ring resonators



Optical MMs: eg nano-pillars



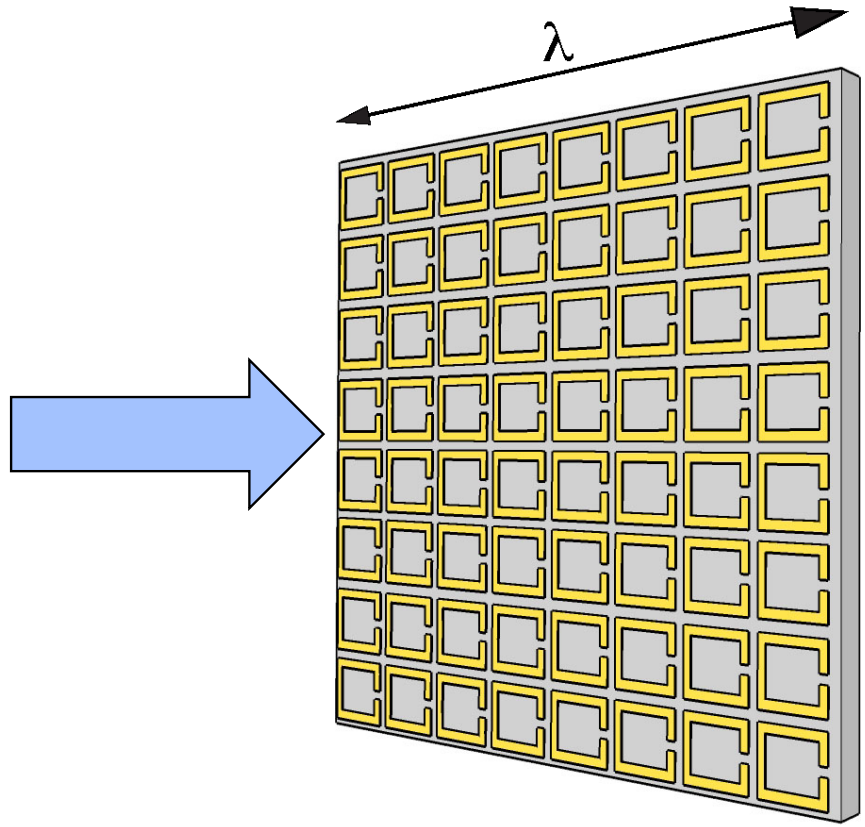
Effective medium approximation

We want to compute the Casimir force between a metallic plate and a MM. Let us assume a metallic plate in is reasonably well described by a Drude response

$$\epsilon_1(\omega) = 1 - \frac{\Omega_e^2}{\omega^2 + i\gamma_e\omega} \qquad \mu_1 = 1$$

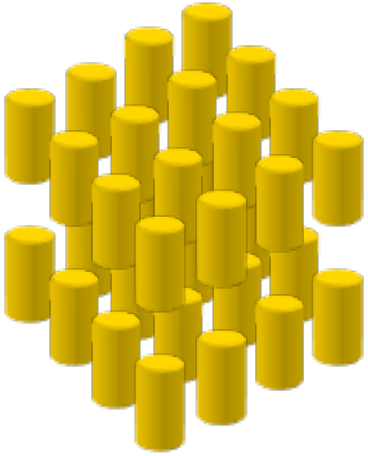
For the MM the optical response is not so simple.....

In the effective medium approximation (EMA) one describes the MM with an effective electric permittivity and an effective magnetic permeability. This is an approximation valid when the MM is probed at wavelengths much larger that the average distance between the constituent "particles" of the MM.

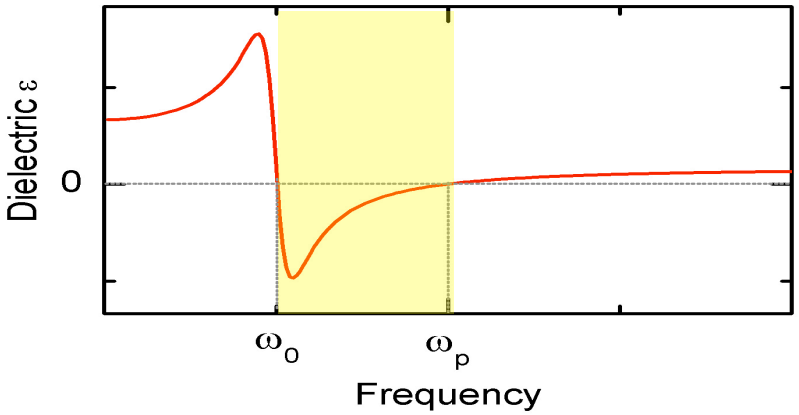


EMA: Electric response

Close to resonance, the optical response can be modeled by a Drude-Lorentz permittivity



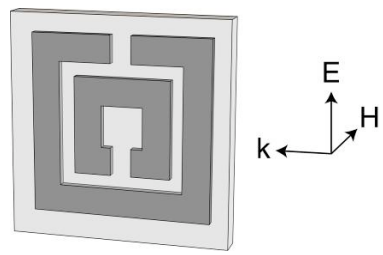
$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

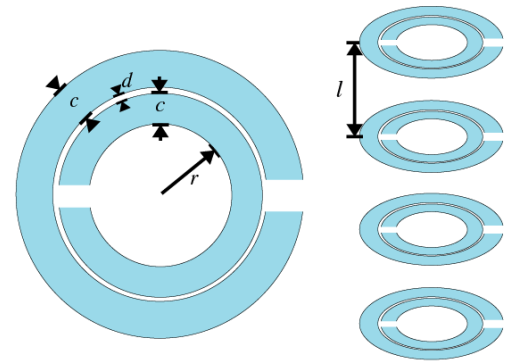


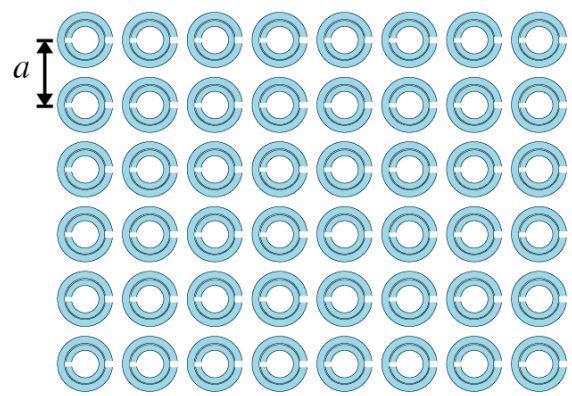
J.B. Pendry *et al.*, ***Phys. Rev. Lett.*** **76**, 4773 (1996).

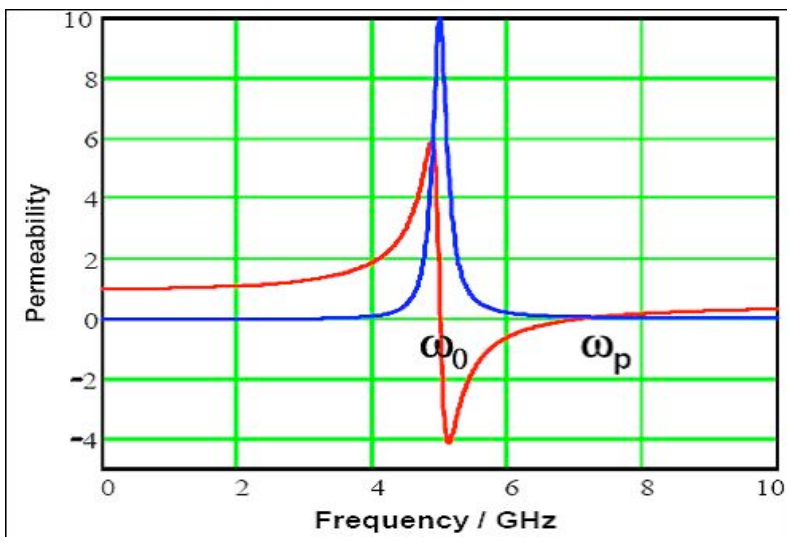
EMA: Magnetic response

$$\mu_{eff} = 1 - \frac{\frac{\pi r^2}{a^2}}{1 + \frac{2\sigma i}{\omega r \mu_0} - \frac{3}{\pi^2 \mu_0 \omega^2 C r^3}}$$









J.B. Pendry *et al.*, *IEEE Trans. Microwave Tech.* **47,** 2075 (1999).

EMA: Drude-Lorentz responses

Close to the resonance, both $\epsilon(\omega)$ and $\mu(\omega)$ can be modeled by Drude-Lorentz formulas

$$\epsilon_\alpha(\omega) = 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega}$$

$$\mu_\alpha(\omega) = 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}$$

Typical separations

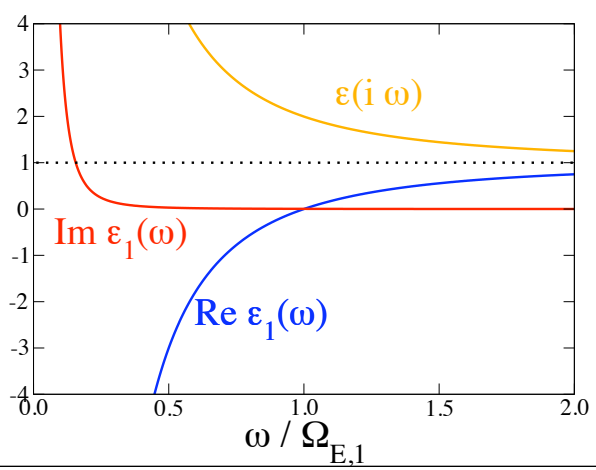
$d = 200 - 1000$ nm

Infrared-optical frequencies

$\Omega/2\pi = 5 \times 10^{14}$Hz

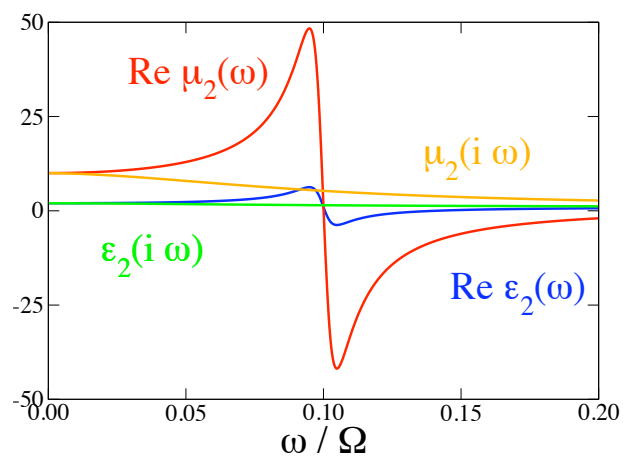
Drude metal (Au)

$\Omega_E = 9.0$ eV $\quad \Gamma_E = 35$ meV



Metamaterial

$\mathrm{Re}\ \epsilon_2(\omega) < 0 \quad \mathrm{Re}\ \mu_2(\omega) < 0$



$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$

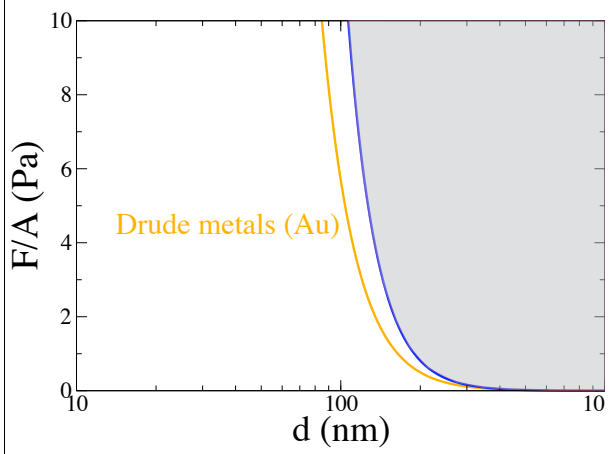
$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$

$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$

Attraction-repulsion crossover

Drude metal (Au)

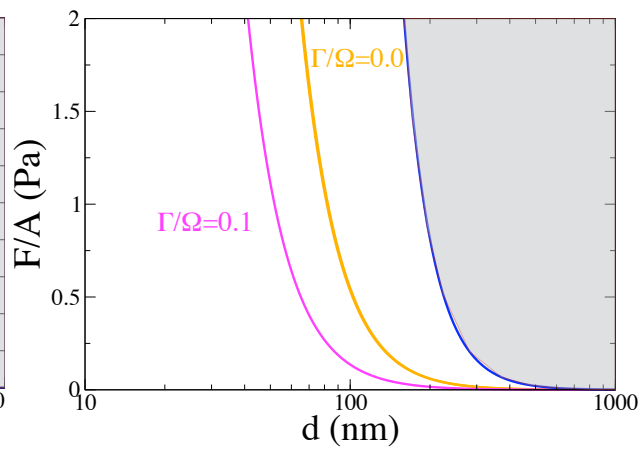
Drude metal (Au)



Only attraction

Metamaterial

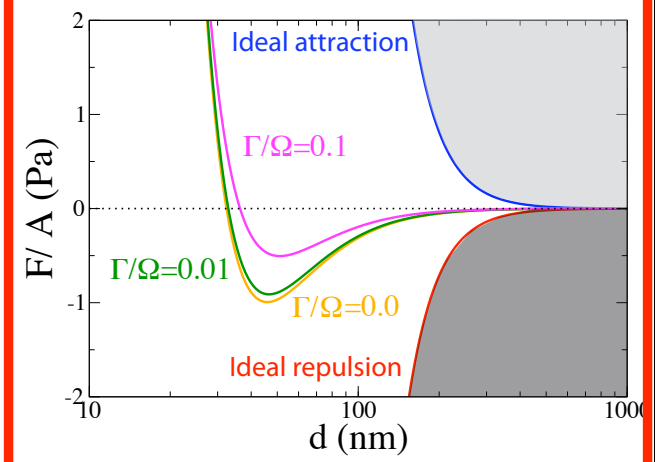
Metamaterial



Only attraction

Drude metal (Au)

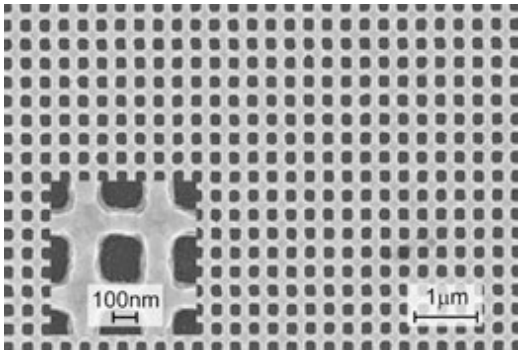
Metamaterial



Repulsion-attraction

Drude background

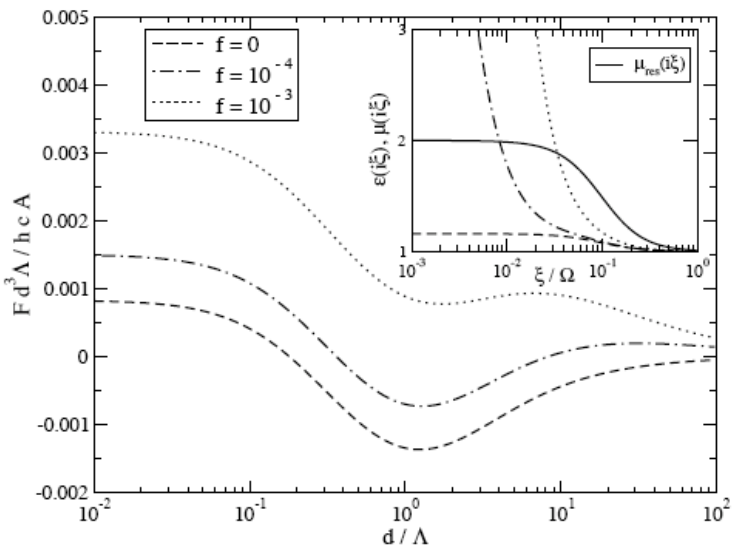
- In some metallic-based MMs, there is a net conductivity due to the metallic structure, like the fishnet design on the right.



$$\epsilon(\omega) = 1 - f\frac{\Omega_D^2}{\omega^2 - i\omega\gamma_D} - (1-f)\frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega}$$

f : filling factor

$$\mu(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}$$



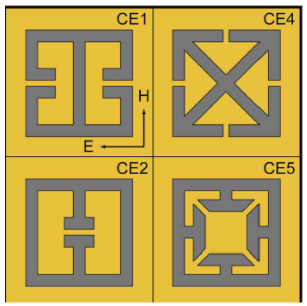
Rosa, DD, Milonni, PRL 2008

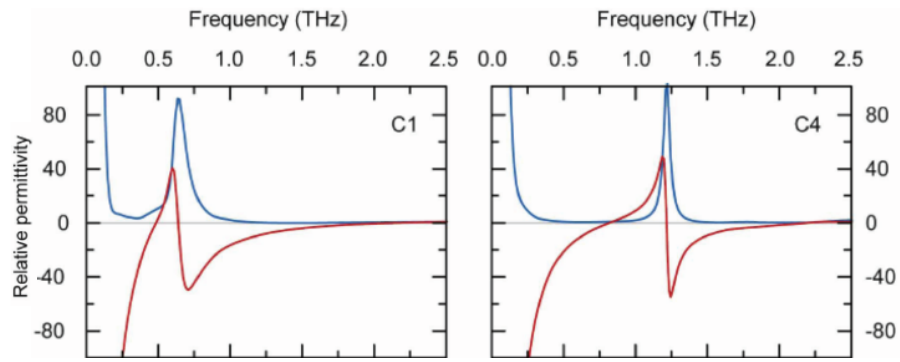
A Drude background is detrimental for Casimir force reduction or repulsion, since it results in an electric response much stronger than the magnetic one

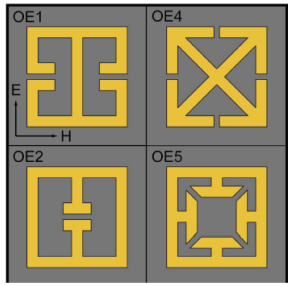
$$\epsilon_2(i\xi) \gg \mu_2(i\xi)$$

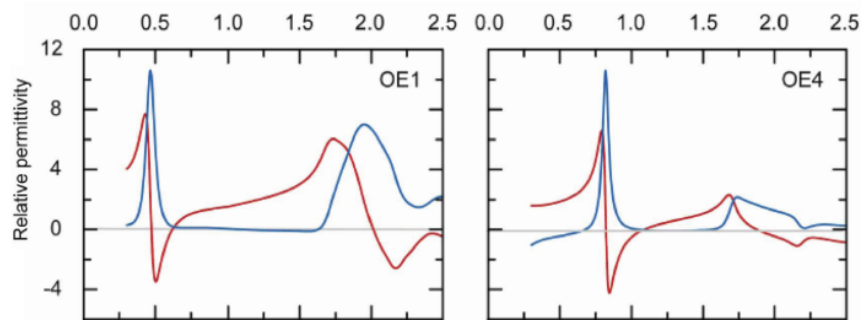
Complementary SRRs + Drude

SRRs structures provide an opportunity to avoid the large Drude background, since they can be built in two natural complementary ways









Optical anisotropy

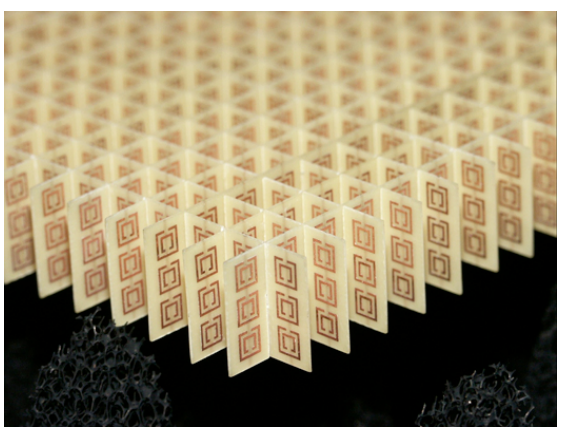
In an anisotropic medium, the constitutive relations between E, D, B, and H are more involved:

$$\mathrm{D} = \epsilon \cdot \mathrm{E} \qquad \mathrm{H} = \mu^{-1} \cdot \mathrm{B}$$

due to the tensorial nature of the permittivity and permeability

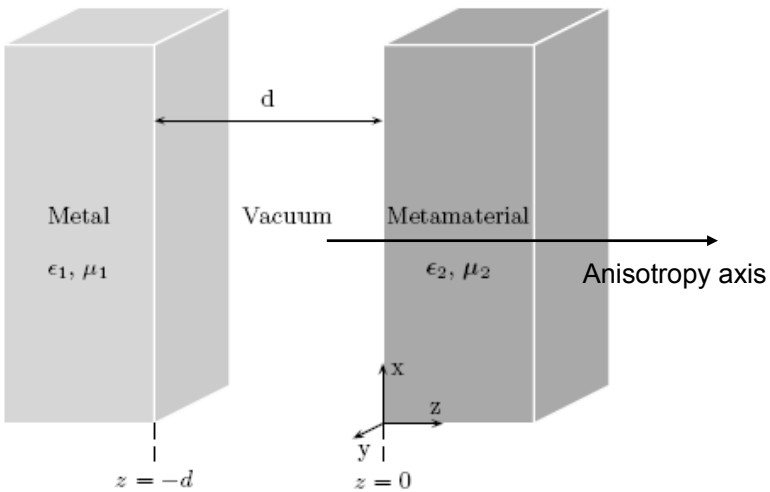
$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}$$

due to the tensorial nature of the permittivity and permeability



Examples of uniaxial anisotropy in stacked MMs

Anisotropy: Uniaxial MMs



$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \; ; \; \mu_{ij} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$

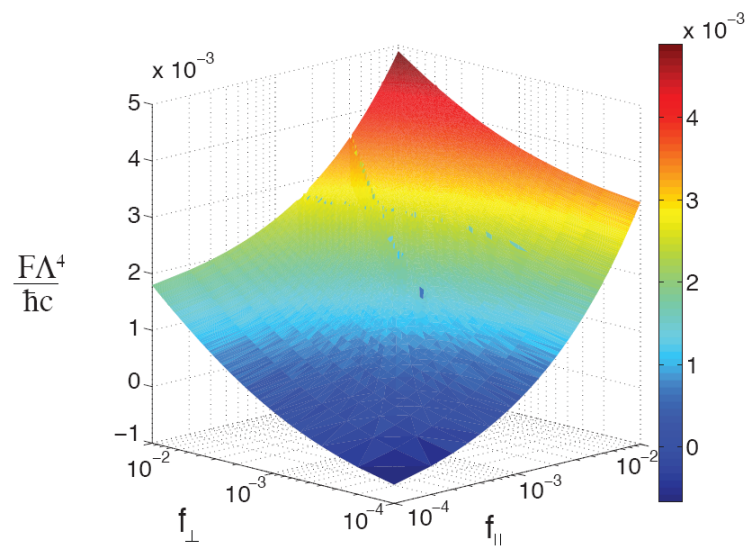
Anisotropy produces polarization mixing (non-diagonal reflection matrices)

$$\epsilon_{xx}(\omega) = \epsilon_{yy}(\omega) = 1 - (1 - f_x)\frac{\Omega_{e,x}^2}{\omega^2 - \omega_{e,x}^2 + i\gamma_{e,x}\omega} - f_x\frac{\Omega_{D,x}^2}{\omega^2 + i\gamma_{D,x}\omega},$$

$$\epsilon_{zz}(\omega) = 1 - (1 - f_z)\frac{\Omega_{e,z}^2}{\omega^2 - \omega_{e,z}^2 + i\gamma_{e,z}\omega} - f_z\frac{\Omega_{D,z}^2}{\omega^2 + i\gamma_{D,z}\omega},$$

$$\mu_{xx}(\omega) = \mu_{yy}(\omega) = 1 - \frac{\Omega_{m,x}^2}{\omega^2 - \omega_{m,x}^2 + i\gamma_{m,x}\omega},$$

$$\mu_{zz}(\omega) = 1 - \frac{\Omega_{m,z}^2}{\omega^2 - \omega_{m,z}^2 + i\gamma_{m,z}\omega}$$



Anisotropy does not help repulsion

(Rosa, DD, Milonni, PRA 2008)

EMA: correct model for μ

Drude-Lorentz for permeability is wrong. The correct expression that results in EMA from Maxwell's equations is

$$\mu_{\text{eff}}(\omega) = 1 - f\frac{\omega^2}{\omega^2 - \omega_m^2 + 2i\gamma_m\omega}$$

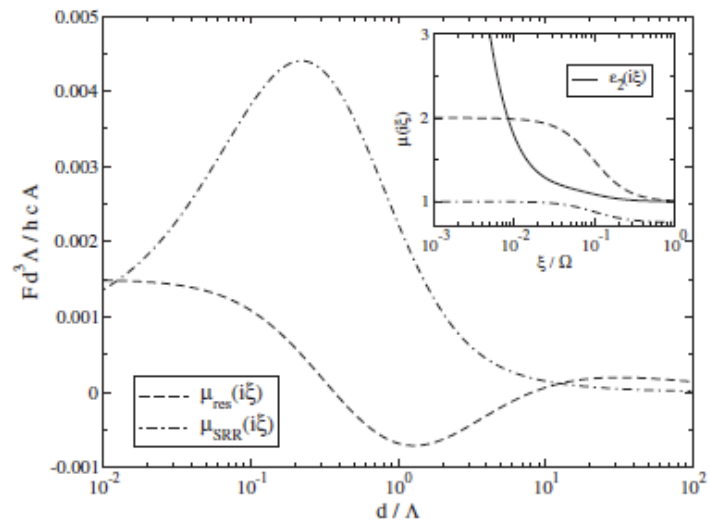
(Pendry 1999)

The appearance of the ω^2 factor in the numerator is very important:

Although close to the resonance this behaves in the same way as the Drude-Lorentz EMA permeability, it has a completely different low-frequency behavior

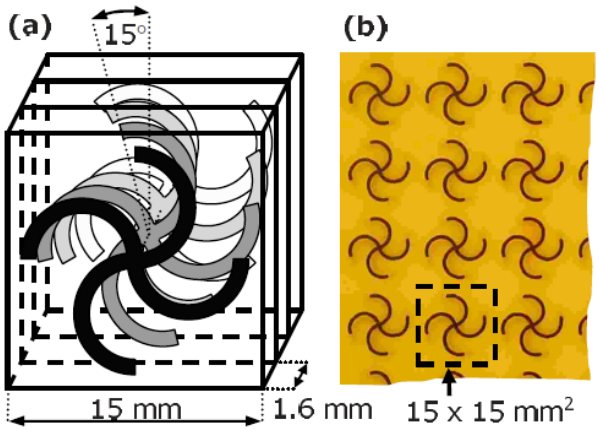
$$\mu_{\text{eff}}(i\xi) < 1 < \epsilon_{\text{eff}}(i\xi)$$

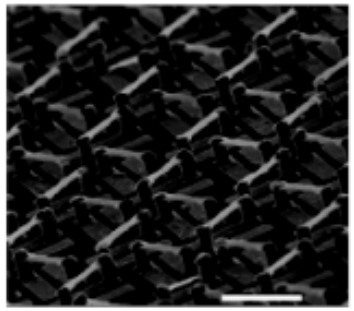
No Casimir repulsion!

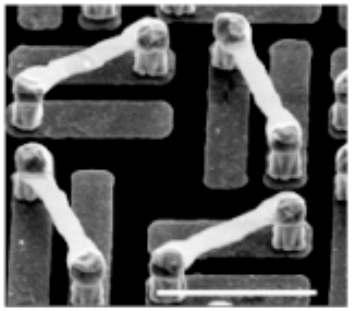


Other Casimir MMs: chirality

The chirality of a MM is defined by the chirality of its unit cell







In a chiral medium, the constitutive relations mix electric and magnetic fields

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r},\omega) - i\kappa(\omega)\mathbf{H}(\mathbf{r},\omega)$$
$$\mathbf{B}(\mathbf{r},\omega) = i\kappa(\omega)\mathbf{E}(\mathbf{r},\omega) + \mu(\omega)\mathbf{H}(\mathbf{r},\omega)$$

dispersive chirality: $\kappa(\omega) = \dfrac{\omega_k \omega}{\omega^2 - \omega_{\kappa R}^2 + i\gamma_k \omega}$

Repulsion and chiral MMs

In chiral MMs the reflection matrix is non-diagonal (mixing of E and H fields).

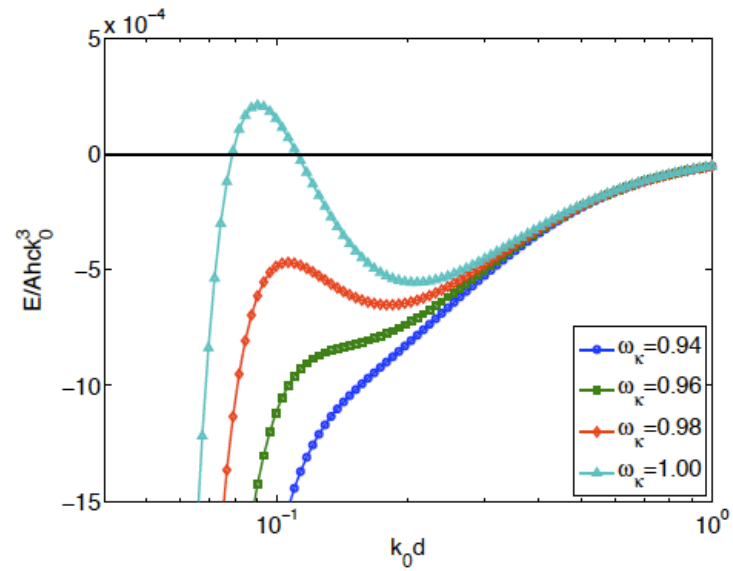
The integrand of the Casimir-Lifshitz force between two identical chiral MMs has the form:

$$F = \frac{(r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} - 2(r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}{1 - (r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} + (r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}$$

One might achieve repulsive Casimir forces with strong chirality (i.e., large values of r_{sp})

Same-chirality (SC) materials: repulsion

Opposite-chirality (OC) materials: repulsion



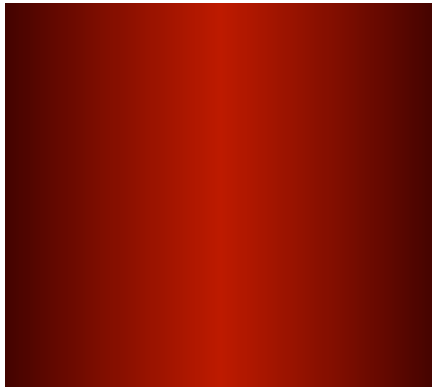
Soukoulis et al, PRL 2009

Beyond the EMA

Everything discussed so far is based on the assumption that the effective medium approximation (EMA) holds. We recall that this amounts to treating the MM in the "long-wavelength approximation", i.e., field wavelengths much larger than the typical size of the unit cell of the MM.

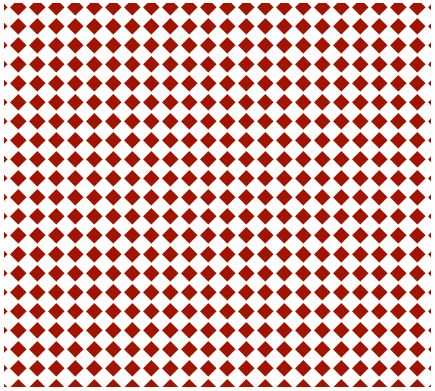
How to calculate Casimir forces when EMA does not hold?
Can one trust predictions of Casimir repulsion with MMs based on EMA?

Homogeneous medium



EMA

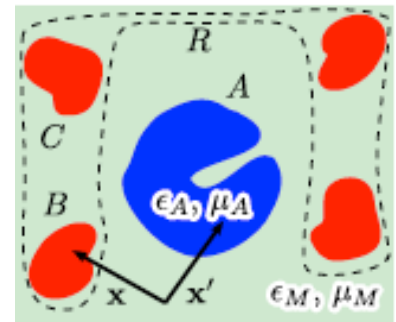
Non-homogeneous medium



beyond EMA

Constraints on stable equilibria

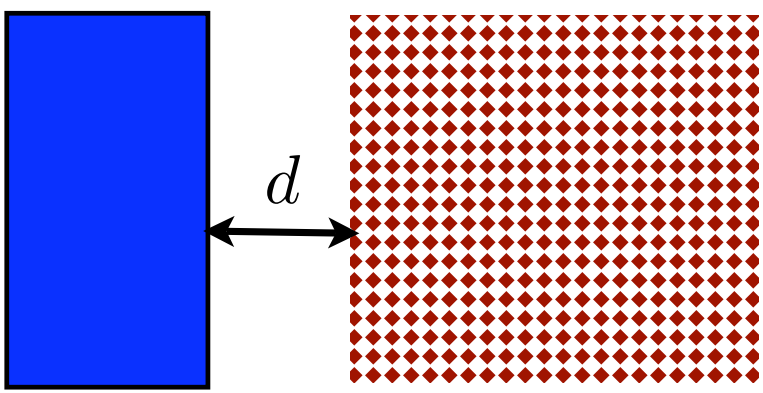
Theorem: *there are no stable equilibria with fluctuation-induced forces when all interacting objects have microscopic* $\epsilon(\mathbf{r}, i\xi) > 1$ *and* $\mu(\mathbf{r}, i\xi) \approx 1$

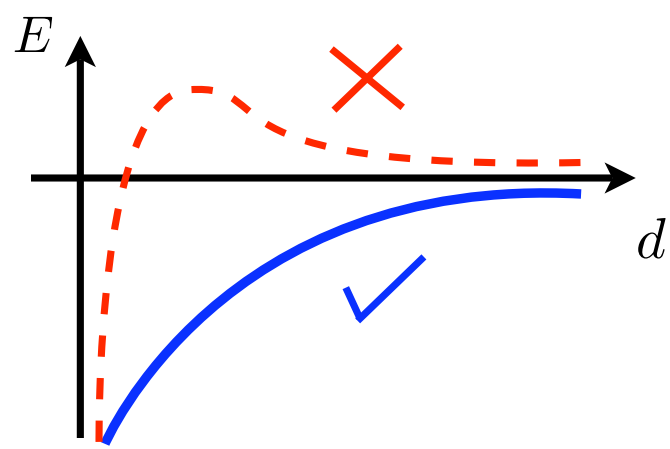


$$\nabla^2 E < 0$$

Rahi, Kardar, Emig (arXiv:0911.5364)

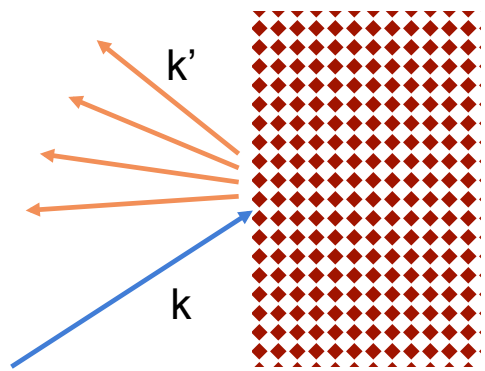
Corollary: *Casimir repulsion is impossible for any metallic/dielectric based MM in front a translationally invariant non-magnetic plate.*





Scattering theory

The Casimir force still may be described in terms of reflections (scattering theory)



Symbolically, we may write the Casimir energy as

$$\boxed{\frac{E(d)}{A} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\left[1 - \mathcal{R}_1 e^{-\mathcal{K}d} \mathcal{R}_2 e^{-\mathcal{K}d}\right]}$$

where $\mathcal{R}_i \equiv \mathcal{R}_i(\mathbf{k}_{||}, \mathbf{k}'_{||}, p, p', i\xi)$

Finding the reflection matrix

The reflection matrix can be obtained with standard methods of numerical electromagnetism. One solves Maxwell equations for the transverse fields

$$-ik\frac{\partial \mathbf{E}_t}{\partial z} = \nabla_t \left[\chi \hat{e}_3 \cdot \nabla \times \mathbf{H}_t\right] - k^2 \mu \hat{e}_3 \times \mathbf{H}_t$$
$$-ik\frac{\partial \mathbf{H}_t}{\partial z} = -\nabla_t \left[\zeta \hat{e}_3 \cdot \nabla \times \mathbf{E}_t\right] + k^2 \epsilon \hat{e}_3 \times \mathbf{E}_t$$

Assuming a two-dimensional periodic structure, we have

$$\mathbf{E}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{E}_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mathbf{H}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{H}_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

where

$$\epsilon(x,y) = \sum_{m,n} \epsilon_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mu(x,y) = \sum_{m,n} \mu_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

Exact reflection matrix

One can then write the equations for the transverse fields as

$$\boxed{-ik\frac{\partial \Psi_{m'n'}}{\partial z} = \sum_{mn} H_{m'n',mn}\Psi_{mn}} \qquad \Psi_{mn} = \begin{bmatrix} \mathcal{E}^x_{mn} \\ \mathcal{E}^y_{mn} \\ \mathcal{H}^x_{mn} \\ \mathcal{E}^y_{mn} \end{bmatrix} = \begin{bmatrix} \Psi^1_{mn} \\ \Psi^2_{mn} \\ \Psi^3_{mn} \\ \Psi^4_{mn} \end{bmatrix}$$

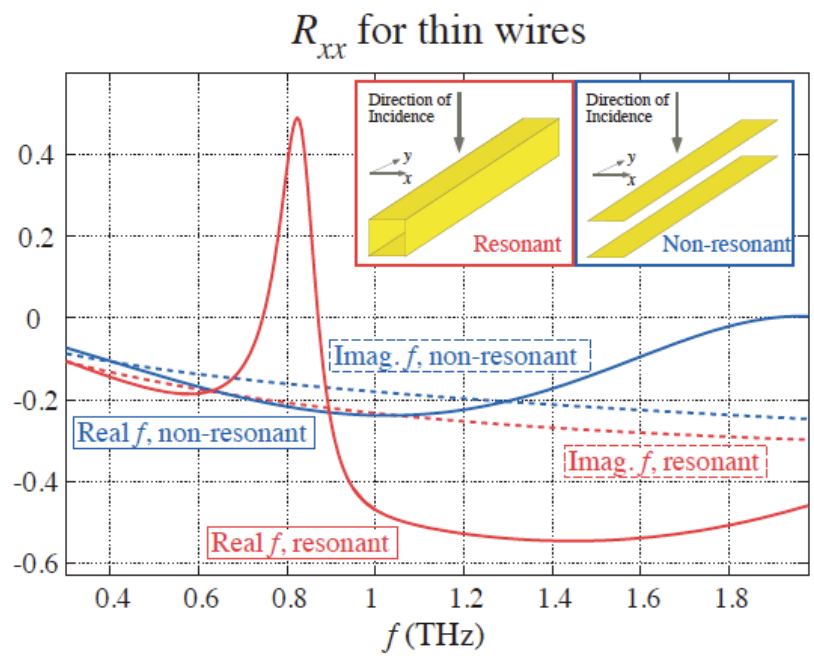
Here H is a complicated matrix, that encapsulated the coupling of modes in the periodic structure.

By numerically solving this equation and imposing the proper boundary conditions of the field on the vacuum-metamaterial interphase (RCWA or S-matrix techniques), one can find the reflection matrix of the MM.

Exact MIT numerics for MMs. I

- Magnetic metamaterials

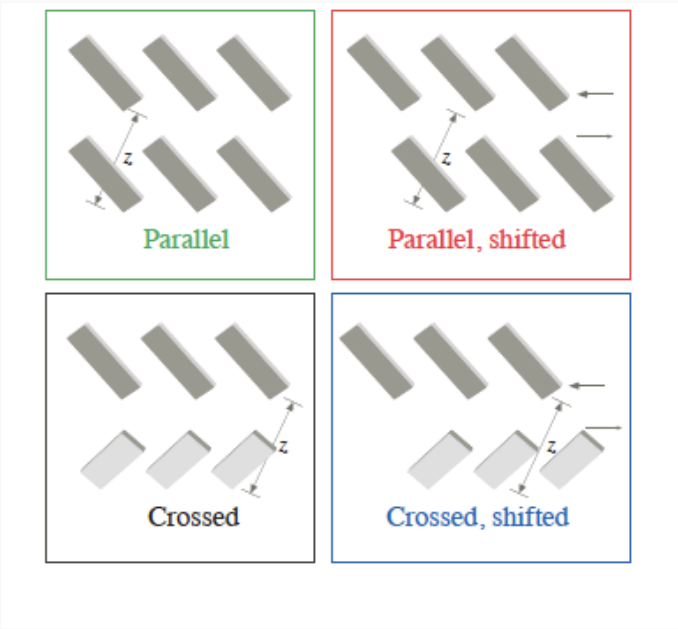
Real and imaginary-frequency reflection coefficient (at normal incidence)

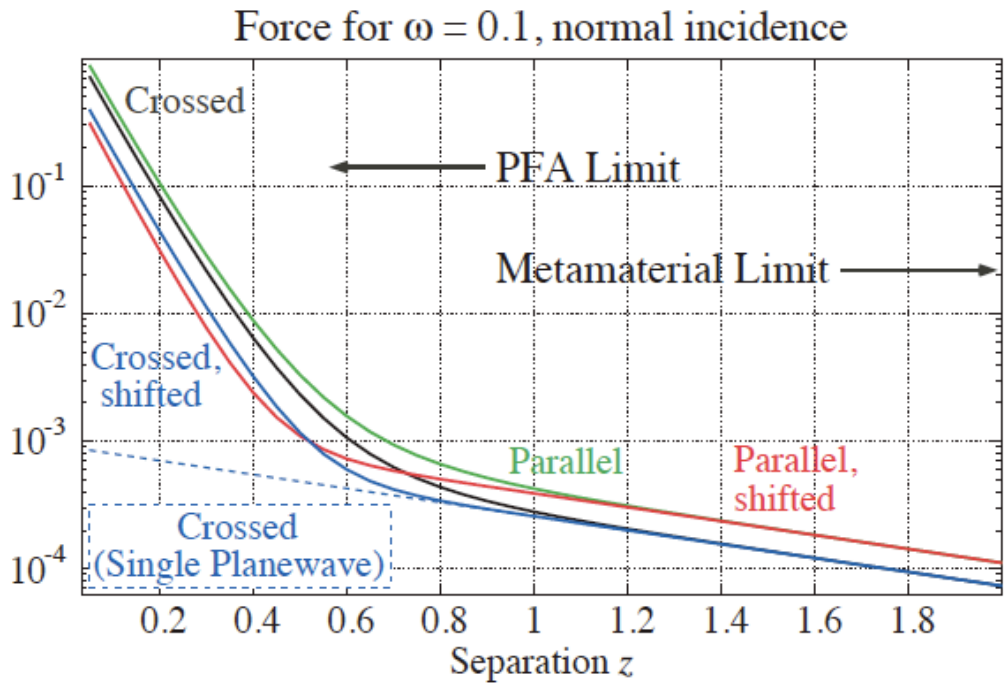


While the resonance strongly affects the real-frequency response, the imaginary-frequency response is not greatly changed.

S. Johnson's group (unpublished)

Exact MIT numerics for MMs. II

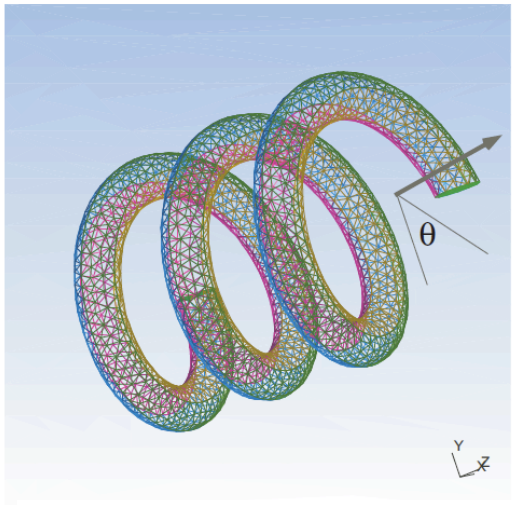


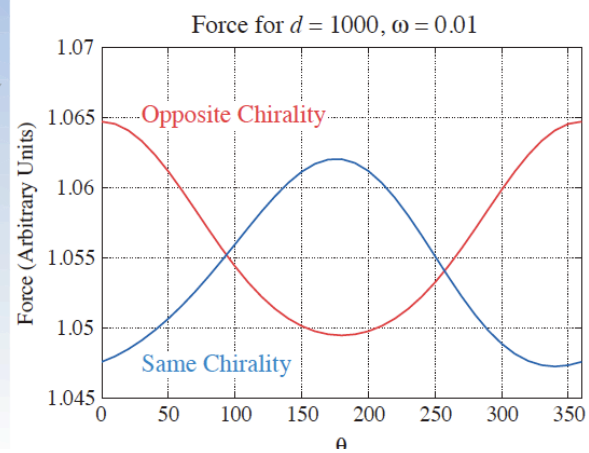


S. Johnson's group (unpublished)

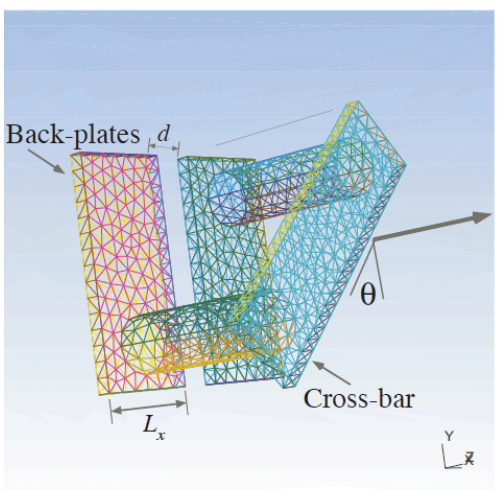
Exact MIT numerics for MMs. III

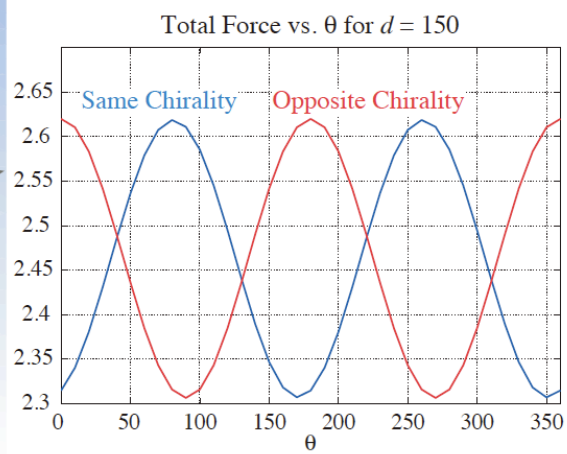
- Chiral atoms





Casimir force is not repulsive





Effect of anisotropy overwhelms any possible chiral effect

S. Johnson's group (unpublished)

Exact MIT numerics for MMs. IV

Los Alamos NATIONAL LABORATORY

Configurations

No difference in metamaterial approx.

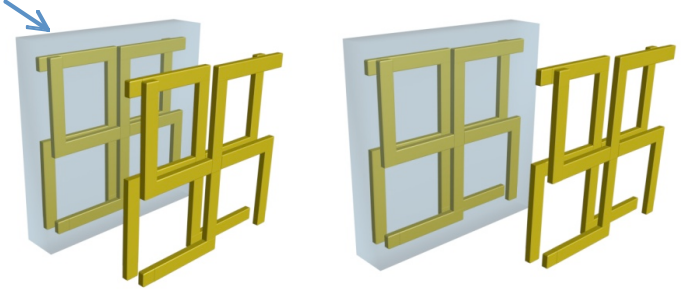
Aligned

Shifted

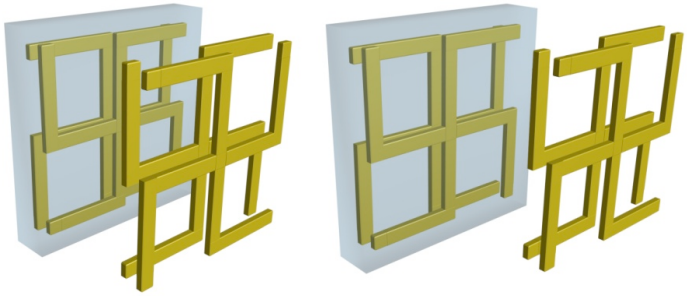
Unit Cell

Chirality

(SC)

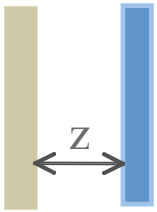


(OC)



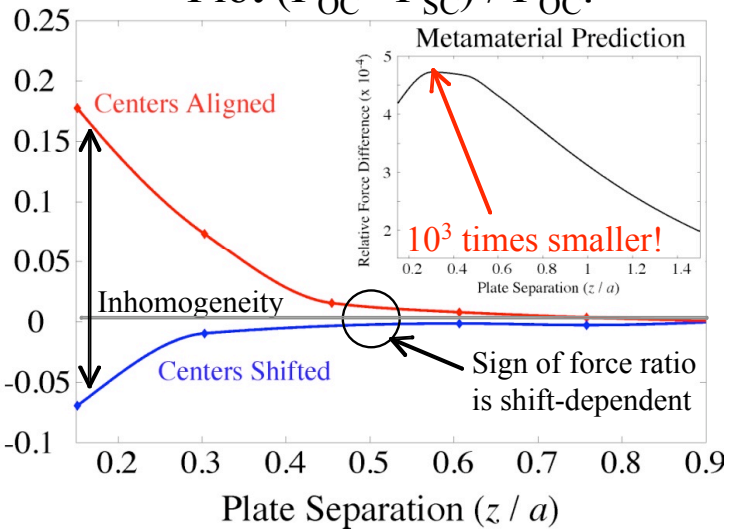
Attractive, $F_{OC} > F_{SC}$ in metamaterial limit

Results



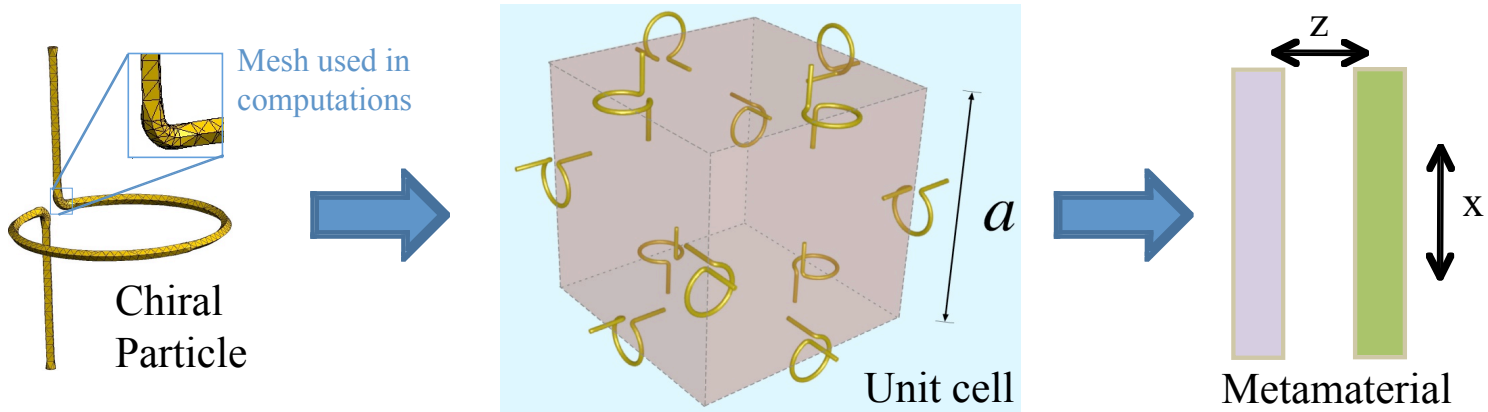
Force F in SC and OC configs.

Plot $(F_{OC} - F_{SC}) / F_{OC}$:

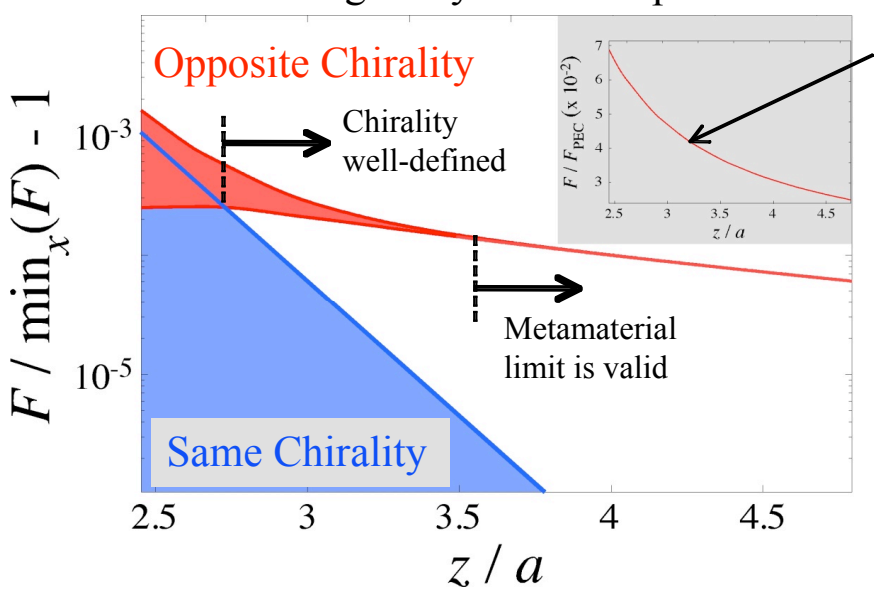


S. Johnson's group (unpublished)

Exact MIT numerics for MMs.V



Effect of inhomogeneity across displacements x



Total force relative to parallel metal plates

"repulsive" effect (force reduction) of chirality is one ten-thousandth of this!

Conclusion

In the regime where the chiral metamaterial limit is valid, the effect is **too small** to be observable.

S. Johnson's group (unpublished)

Remarks: MMs and Casimir

- Metamaterials offer an interesting possibility for Casimir force manipulation: engineered optical response, (maybe) broadband, dynamic control.

- Several proposals for MM-based Casimir force use effective medium approximation. Their predictions have to be carefully checked since EMA breaks down for electromagnetic fluctuations with wavelengths comparable to metamaterial feature sizes.

- Casimir repulsion in vacuum-separated metallic/dielectric metamaterial structures seems hard to achieve. It is certainly impossible in geometries that are effectively one-dimensional (Casimir stability considerations).

References of Part II

- Casimir-Lifshitz theory and magneto-dielectric metamaterials

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- Casimir-Lifshitz theory and chiral metamaterials

Zhao, Zhou, Economou, Soukoulis, PRL **103**, 103602 (2009)

Zhao, Koschny, Economou, Soukoulis, arXiv:0911.2019