

# Recent progress in Casimir physics

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# Collaborators



## ■ Theory:

Engineering the Casimir force with geometry: Paulo Maia Neto (Rio de Janeiro)  
Diego Mazzitelli (Buenos Aires)  
Astrid Lambrecht (LKB, Paris)  
Serge Reynaud (LKB, Paris)

Engineering the Casimir force with metamaterials: Peter Milonni (LANL)  
Felipe da Rosa (LANL)

## ■ Experiments:

BECs for Casimir-Polder force: Malcolm Boshier (LANL)

Metamaterials for Casimir force: Antoniette Taylor (CINT, LANL)

Casimir force measurements: Roberto Onofrio (Dartmouth)  
Steve Lamoreaux (Yale)  
Ricardo Decca (Indiana)

# Outline of this talk

## ○ Part I

- Review of theory and experiments on the Casimir force
- Review of theory and experiments on the Casimir-Polder force

## ○ Part II

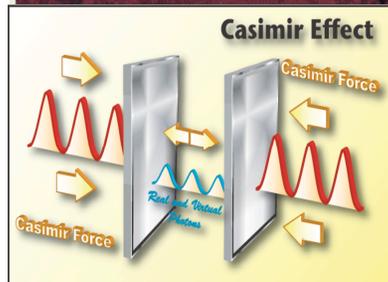
- Casimir forces within scattering theory
- Cold atoms for probing CP forces

## ○ Part III

- Metamaterials for quantum levitation?

# Part I: Casimir force

# The Casimir force



Casimir forces originate from changes in quantum vacuum fluctuations imposed by surface boundaries

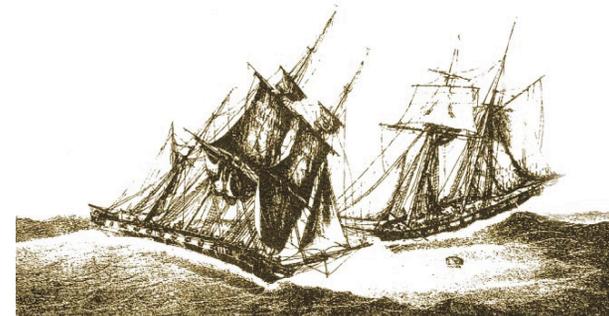
They were predicted by the Dutch physicist Hendrik Casimir in 1948

Dominant interaction in the micron and sub-micron lengthscales

$$\frac{F}{A} = \frac{\pi^2}{240} \frac{\hbar c}{d^4}$$

(130nN/cm<sup>2</sup> @  $d = 1\mu\text{m}$ )

**Classical Analog:** L'Album du Marin (1836)



# Relevant applications

## ■ Gravitation / Particle theory:

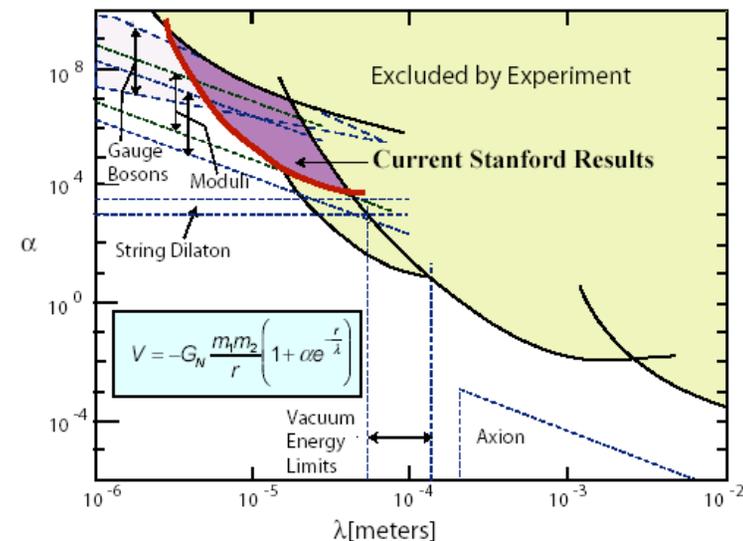
Some theories of particle physics predict deviations from the Newtonian gravitational potentials in the micron and submicron range

The Casimir force is the main background force to measure these non-Newtonian corrections to gravity

Yukawa-like potential:

$$V(r) = -G \frac{m_1 m_2}{r} \left( 1 + \alpha e^{-r/\lambda} \right)$$

## Phase Space



# Relevant applications

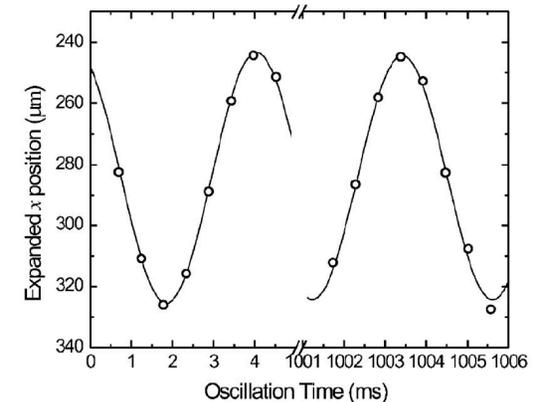
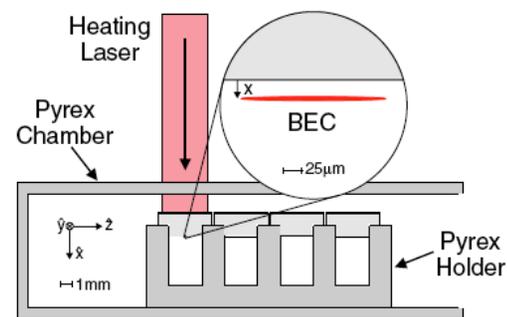
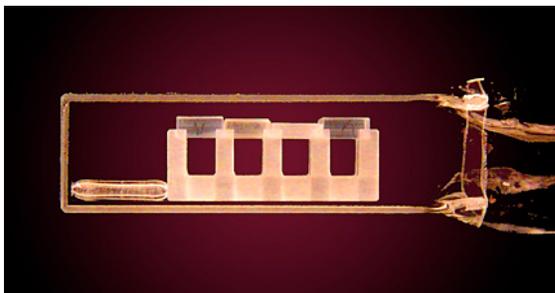
## Quantum Science and Technology:

Atom-surface interactions

Precision measurements

Example: Casimir-Polder interaction between a BEC and a surface

Cornell et al (2007)



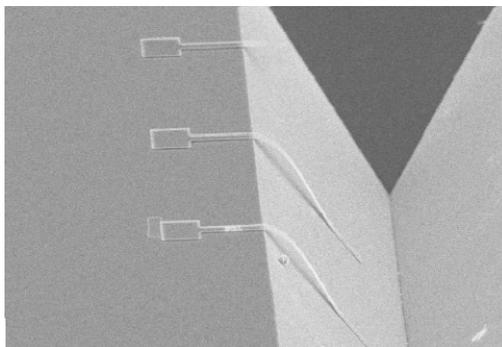
$$\gamma_x \equiv \frac{\omega_x - \omega'_x}{\omega_x} \approx -\frac{1}{2\omega_x^2 m} \partial_x^2 U^*$$

# Relevant applications

## ■ Nanotechnology:

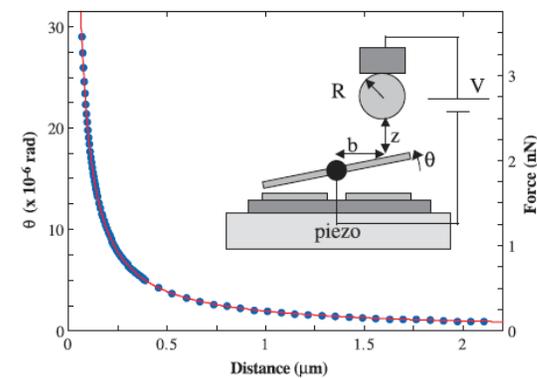
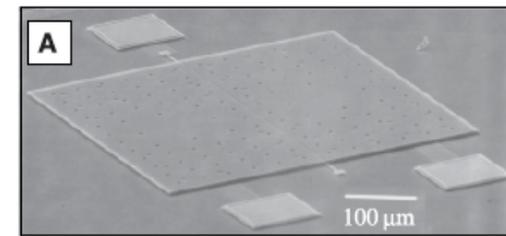
Problems with stiction of  
movable parts in MEMS

“pull-in” effect



Zhao et al (2003)

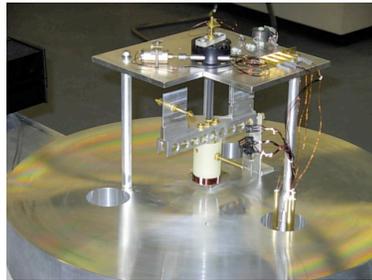
Actuation in NEMS and MEMS  
driven by Casimir forces



Capasso et al (2001)

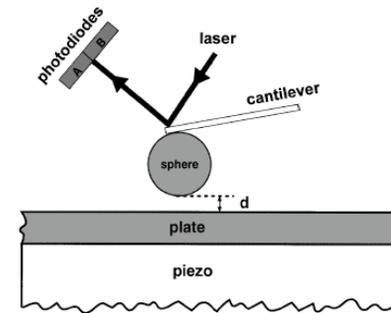
# Modern Casimir experiments

## Torsion pendulum



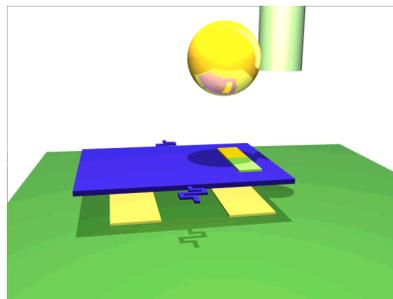
sphere-plane,  $d=1-10 \text{ um}$   
Lamoreaux

## Atomic force microscope



sphere-plane,  $d=200-1000 \text{ nm}$   
Mohideen et al

## MEMS and NEMS



sphere-plane,  $d=200-1000 \text{ nm}$   
Capasso et al, Decca et al

## Micro-cantilever



plane-plane, cylinder-plane,  $d=1-3 \text{ um}$   
Onofrio et al

# Tailoring the Casimir force

■ The magnitude and sign of the Casimir force depend on the geometry and composition of surfaces

Engineer geometries and designer materials for various applications:

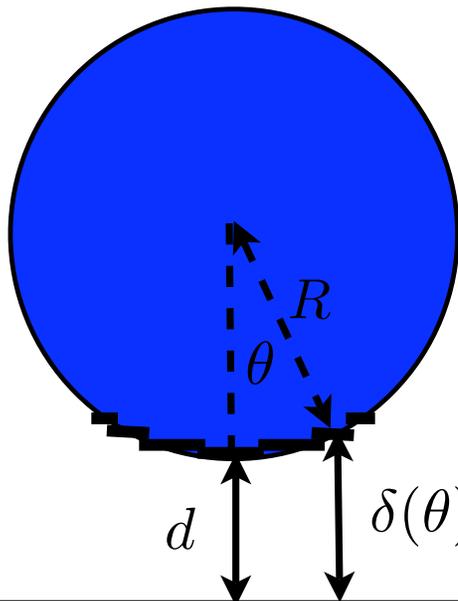
- Demonstration of strongly modified/repulsive Casimir forces
- Demonstration of vacuum drag via lateral Casimir forces

🌐 Effects of geometry: **proximity force approx and beyond**

🌐 Effects of materials: **Lifshitz formula and beyond**

# Geometry effects: PFA

- The Proximity Force Approximation (PFA) corresponds to approximating the Casimir energy by its expression for the planar case, **averaging over local planes**



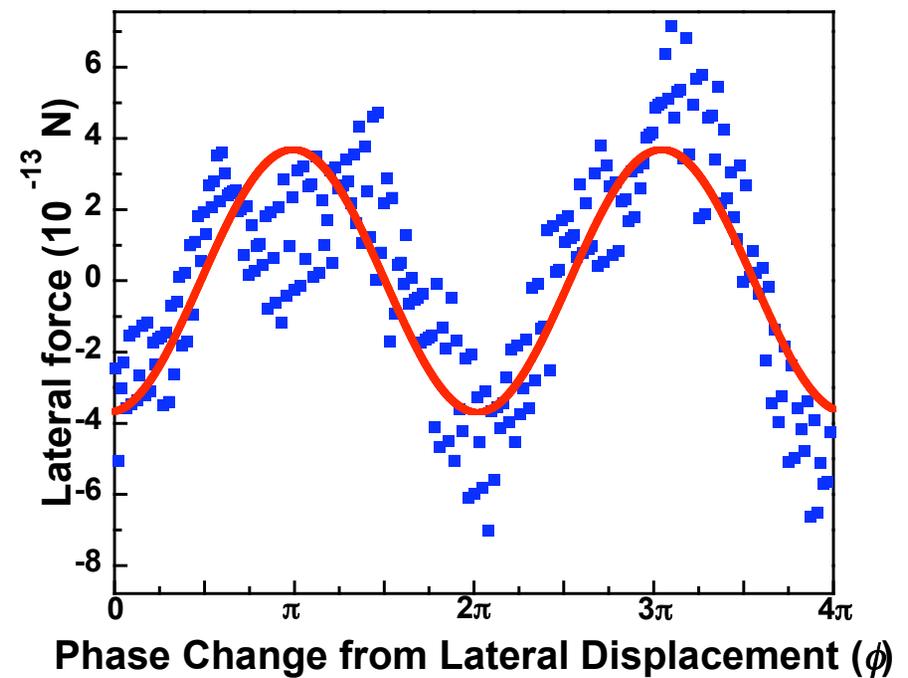
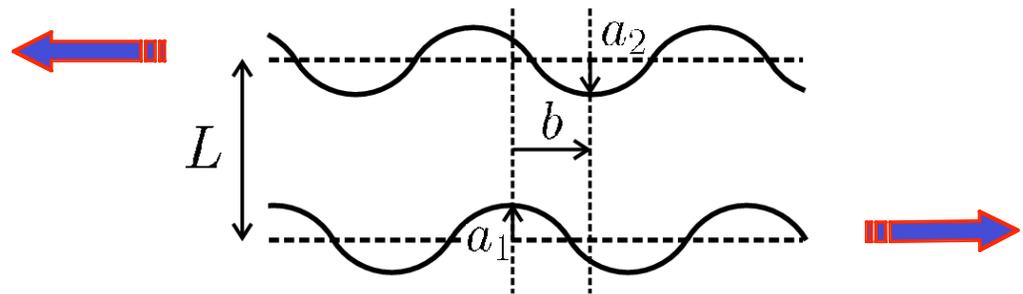
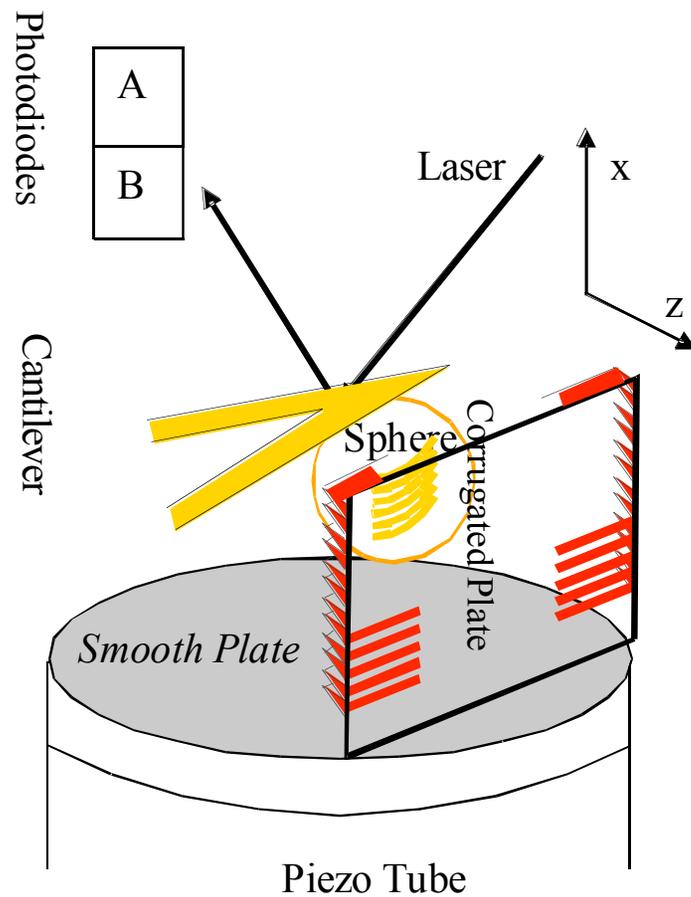
$$E_{\text{SP}}^{\text{PFA}}(d) \approx 2\pi R^2 \int_0^{\theta_m} d\theta \sin \theta \frac{E_{\text{PP}}(\delta(\theta))}{A}$$

It is a good approximation when  $R \gg d$

- There are a few **perturbative methods to go beyond PFA**, and also **exact results for a few geometries** with perfectly conducting surfaces (cylinder-plane, eccentric cylinders, etc).

# Geometry effects: lateral force

Mohideen et al (2002)

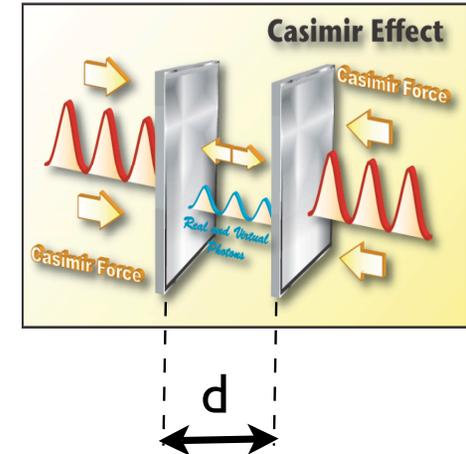


# Materials effects: Lifshitz eqn.

The Lifshitz formula: Lifshitz (1956)

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \text{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$

$$K_3 = \sqrt{k_\parallel^2 + \xi^2/c^2}$$



Reflection matrices (Fresnel formulas for isotropic media):

$$r^{\text{TM, TM}}(i\xi, \mathbf{k}_\parallel) = \frac{\epsilon(i\xi)K_3 - \sqrt{k_\parallel^2 + \epsilon(i\xi)\xi^2/c^2}}{\epsilon(i\xi)K_3 + \sqrt{k_\parallel^2 + \epsilon(i\xi)\xi^2/c^2}}$$

$$r^{\text{TE, TE}} = r^{\text{TM, TM}} \quad \text{with} \quad \epsilon \leftrightarrow \mu$$

Dominant frequencies below the near-infrared/optical region of the EM spectrum (gaps  $d = 200\text{-}1000$  nm)

Kramers-Kronig (causality) relations:

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{x\epsilon''(x)}{x^2 + \xi^2} dx$$

$$\mu(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{x\mu''(x)}{x^2 + \xi^2} dx$$

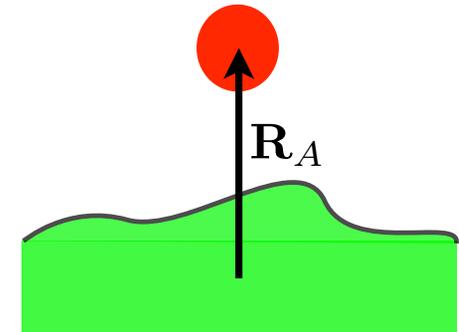
# Part I: Casimir-Polder force

# The Casimir-Polder force

## ■ vdW - CP interaction Casimir and Polder (1948)

The interaction energy between a ground-state atom and a surface is given by

$$U_{\text{CP}}(\mathbf{R}_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \text{Tr} \mathbf{G}(\mathbf{R}_A, \mathbf{R}_A, i\xi)$$



Atomic polarizability:  $\alpha(\omega) = \lim_{\epsilon \rightarrow 0} \frac{2}{3\hbar} \sum_k \frac{\omega_{k0} |\mathbf{d}_{0k}|^2}{\omega_{k0}^2 - \omega^2 - i\omega\epsilon}$

Scattering Green tensor:  $\left( \nabla \times \nabla \times - \frac{\omega^2}{c^2} \epsilon(\mathbf{r}, \omega) \right) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}')$

## ■ Eg: Ground-state atom near planar surface @ T=0

Non-retarded (vdW) limit  $z_A \ll \lambda_A$

$$U_{\text{vdW}}(z_A) = -\frac{\hbar}{8\pi\epsilon_0} \frac{1}{z_A^3} \int_0^\infty \frac{d\xi}{2\pi} \alpha(i\xi) \frac{\epsilon(i\xi) - 1}{\epsilon(i\xi) + 1}$$

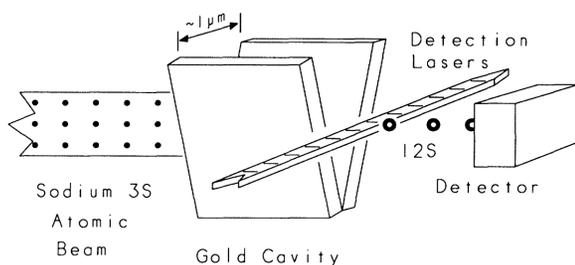
Retarded (CP) limit  $z_A \gg \lambda_A$

$$U_{\text{CP}}(z_A) = -\frac{3\hbar c \alpha(0)}{8\pi} \frac{1}{z_A^4} \frac{\epsilon_0 - 1}{\epsilon_0 + 1} \phi(\epsilon_0)$$

# Modern CP experiments

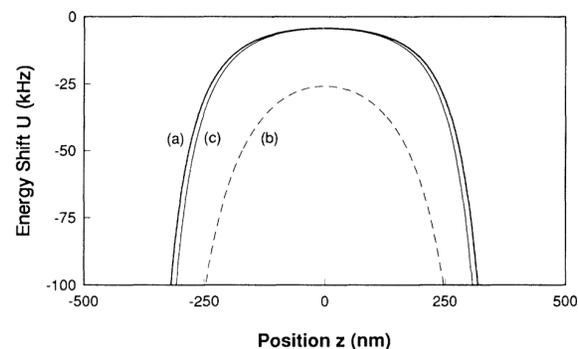
## Deflection of atoms

Hinds et al (1993)



$L = 0.7 - 1.2 \text{ } \mu\text{m}$

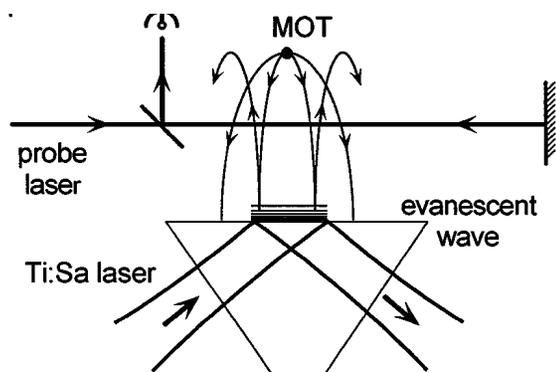
Exp-Th agreement @ 10%



$$U_{CP} = -\frac{1}{4\pi\epsilon_0} \frac{\pi^3 \hbar c \alpha(0)}{L^4} \left[ \frac{3 - 2 \cos^2(\pi z/L)}{8 \cos^4(\pi z/L)} \right]$$

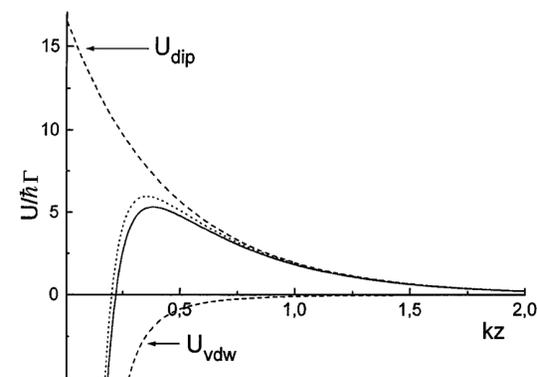
## Classical reflection on atomic mirror

Aspect et al (1996)



$$U_{\text{dip}} = \frac{\hbar}{4} \frac{\Omega^2}{\Delta} e^{-2kz}$$

$$U_{\text{vdW}} = -\frac{\epsilon - 1}{\epsilon + 1} \frac{1}{48\pi\epsilon_0} \frac{D^2}{z^3}$$

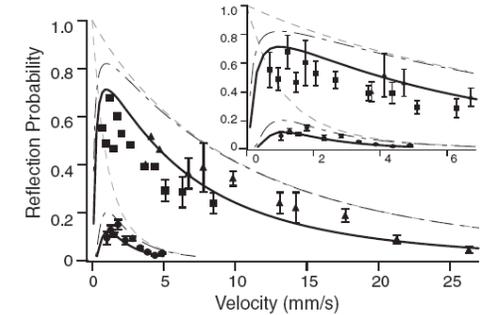
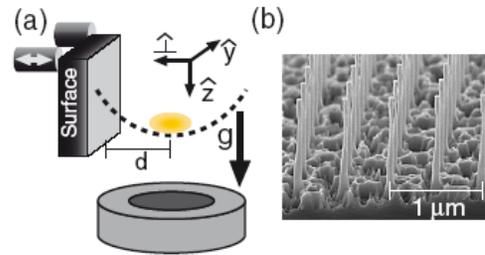


Exp-Th agreement @ 30%

# Modern experiments (cont'd)

## Quantum reflection

Wave-nature of atoms implies that slow atoms can reflect from purely attractive potentials

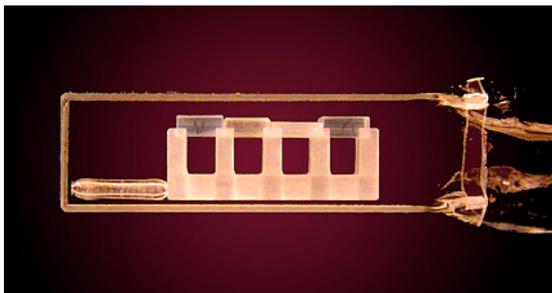


$$k = \sqrt{k_0^2 - 2mU/\hbar^2} \quad \phi = \frac{1}{k^2} \frac{dk}{dr} > 1$$

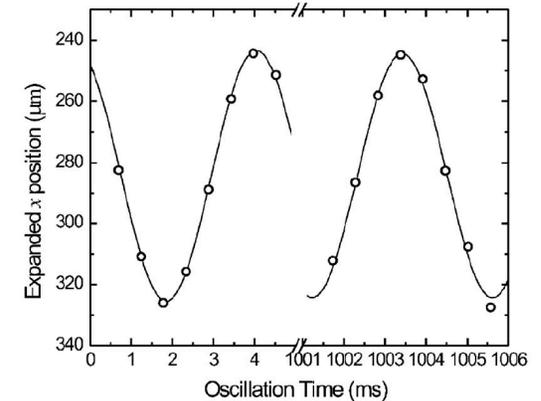
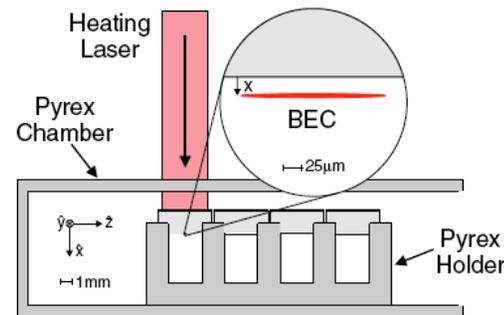
$$U = -C_n/r^n \quad (n > 2)$$

Shimizu (2001)    Ketterle et al (2006)  
DeKieviet et al (2003)

## BEC oscillator



Cornell et al (2007)

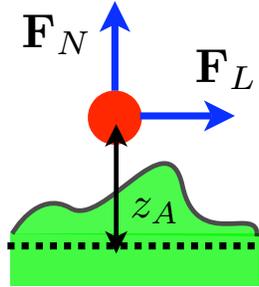


$$\gamma_x \equiv \frac{\omega_x - \omega'_x}{\omega_x} \simeq -\frac{1}{2\omega_x^2 m} \partial_x^2 U^*$$

# Part II: Cold atoms and Casimir



# CP within scattering theory



$$U_{\text{CP}} = U_{\text{CP}}^{(0)}(z_A) + U_{\text{CP}}^{(1)}(z_A, x_A)$$

■ Normal CP force:

$$U_{\text{CP}}^{(0)}(z_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{1}{2\kappa} \sum_p \hat{\epsilon}_p^+ \cdot \hat{\epsilon}_p^- r_p(\mathbf{k}, \xi) e^{-2\kappa z_A}$$

■ Lateral CP force:

$$U_{\text{CP}}^{(1)}(z_A, x_A) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} e^{i\mathbf{k} \cdot \mathbf{r}_A} g(\mathbf{k}, z_A) H(\mathbf{k})$$

Response function  $g$ :

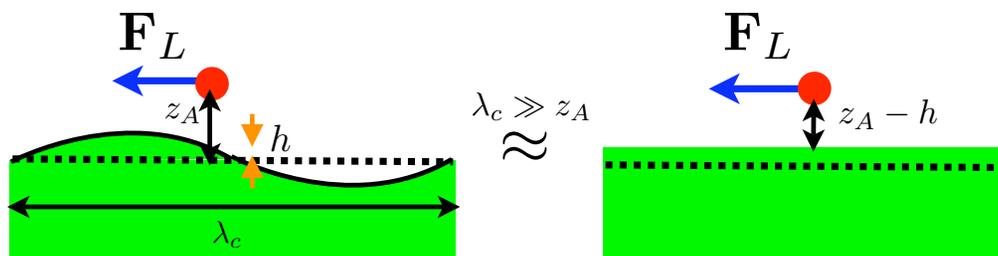
$$g(\mathbf{k}, z_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \int \frac{d^2 \mathbf{k}'}{(2\pi)^2} a_{\mathbf{k}', \mathbf{k}' - \mathbf{k}}(z_A, \xi)$$

$$a_{\mathbf{k}', \mathbf{k}''} = \sum_{p', p''} \hat{\epsilon}_{p'}^+ \cdot \hat{\epsilon}_{p''}^- \frac{e^{-(\kappa' + \kappa'')z_A}}{2\kappa''} R_{p', p''}(\mathbf{k}', \mathbf{k}'')$$

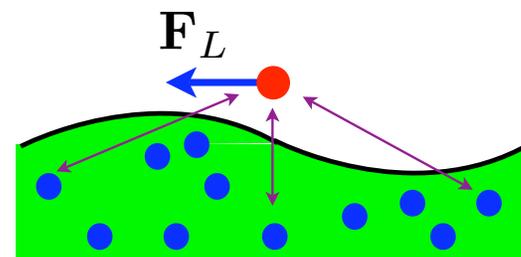
Our approach is perturbative in  $h(\mathbf{x}, y)$ , which should be the smallest length scale in the problem  $\hbar \ll z_A, \lambda_c, \lambda_A, \lambda_0$

# Geometry effects: Lateral CP

## Proximity Force Approximation (PFA)

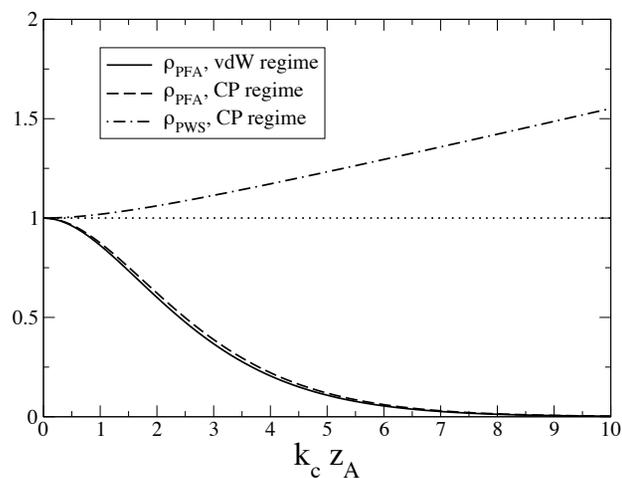


## Pair-wise Summation (PWS)



## Deviations from PFA and PWS

$$\rho_{\text{PFA}} = \frac{g(k_c, z_A)}{g(0, z_A)} \quad \rho_{\text{PWS}} \equiv \frac{g(k_c, z_A)}{g_{\text{PWS}}(k_c, z_A)}$$



### Example:

atom-surface distance  $z_A = 2\mu\text{m} \gg \lambda_A$

corrugation wavelength  $\lambda_c = 3.5\mu\text{m}$

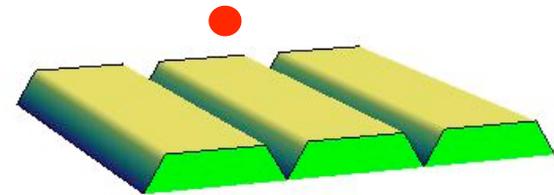
➔  $\rho_{\text{PFA}} \approx 30\% \quad \rho_{\text{PWS}} \approx 115\%$

PFA largely overestimates the lateral CP force  
PWS underestimates the lateral CP force

# Atoms as local probes

In contrast to the case of the lateral Casimir force between corrugated surfaces, an atom is a **local probe** of the lateral Casimir-Polder force. **Deviations from the PFA can be much larger than for the force between two surfaces!**

✓ Even larger deviations from PFA can be obtained for a **periodically grooved surface**.



📍 If the atom is located above one plateau, **the PFA predicts that the lateral Casimir-Polder force should vanish**, since the energy is thus unchanged in a small lateral displacement.

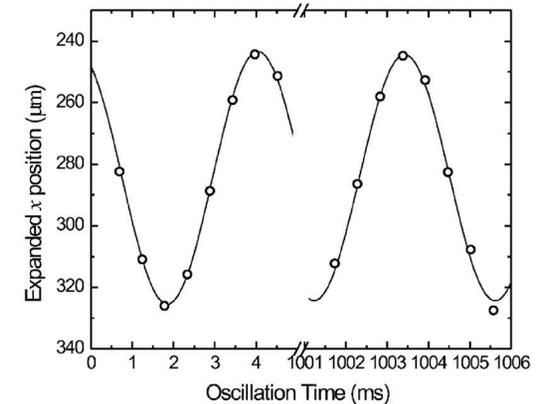
📍 **A non-vanishing force appearing when the atom is moved above the plateau thus clearly signals a deviation from PFA!**

# BEC as a field sensor

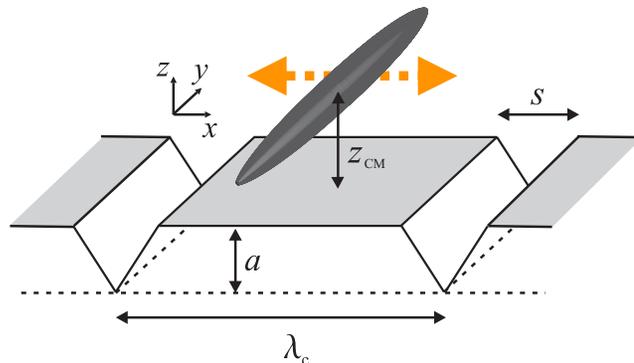
## BEC oscillator

The normal component of Casimir-Polder force  $U_{CP}^{(0)}(z)$  shifts the **normal dipolar oscillation frequency** of a BEC trapped above a surface

Antezza et al (2004) Cornell et al (2005, 2007)



In order to measure the lateral component  $U_{CP}^{(1)}(x, z)$ , a cigar-shaped BEC could be trapped parallel to the corrugation lines, and the **lateral dipolar oscillation** measured as a function of time



$$V(\mathbf{r}) = V_{ho}(\mathbf{r}) + U_{CP}(\mathbf{r})$$

$$V_{ho}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \quad \omega_y \ll \omega_x = \omega_z$$

**Lateral frequency shift:**

$$\omega_{x,CM}^2 = \omega_x^2 + \frac{1}{m} \int dx dz n_0(x, z) \frac{\partial^2}{\partial x^2} U_{CP}^{(1)}(x, z)$$

# BEC as a field sensor (cont'd)

## Density variations of a BEC above an atom chip

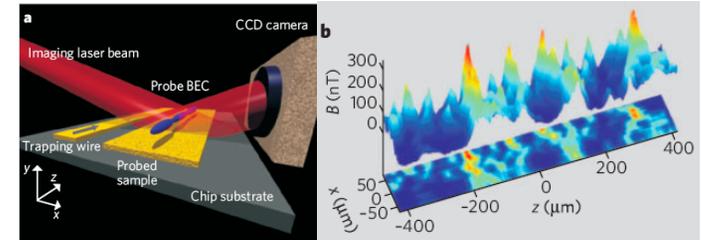
- For a quasi one-dimensional BEC, the potential is related to the 1D density profile as

$$V_{\text{ho}}(x) + U_{\text{CP}}(x) = -\hbar\omega_x \sqrt{1 + 4a_{\text{scat}}n_{1d}(x)}$$

For the lateral CP force, perfect conductor, sinusoidal corrugation ( $a = 100\text{nm}$ ), distance  $z_A = 2\mu\text{m}$ , PFA limit ( $k_c z_A \ll 1$ )

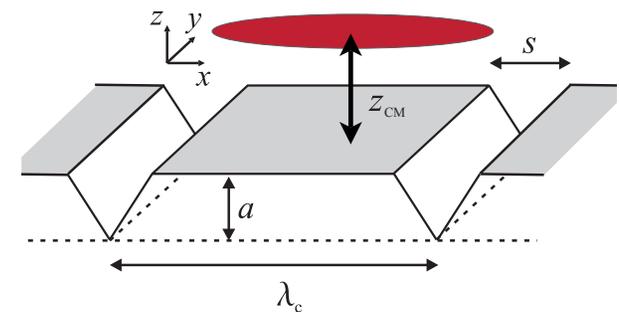
$$\Delta U_{\text{CP}}^{(1)} \simeq 10^{-14} \text{ eV}$$

- To measure the lateral CP force, the elongated BEC should be aligned along the x-direction, and a **density modulation** along this direction **above the plateau** would be a signature of a nontrivial (beyond-PFA) geometry effect.



Measurement of the magnetic field variations along a current-carrying wire

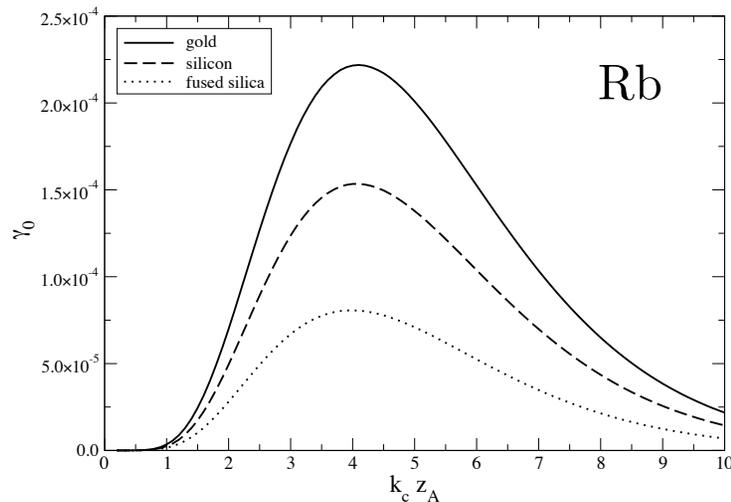
Schmiedmayer et al (2005)



# Single-atom/BEC frequency shift

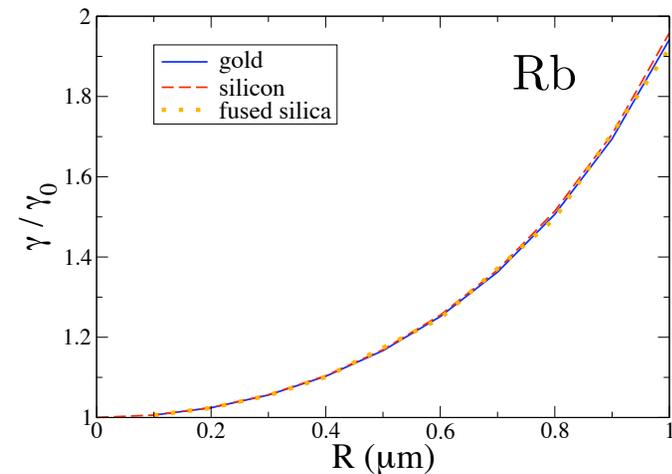
$$\gamma_0 \equiv \frac{\omega_{x,CM} - \omega_x}{\omega_x}$$

## Single-atom lateral freq. shift



$$\begin{aligned}\omega_x/2\pi &= 229 \text{ Hz} \\ z_{CM} &= 2 \mu\text{m} \\ \lambda_c &= 4 \mu\text{m} \\ a &= 250 \text{ nm} \\ s &= \lambda_c/2\end{aligned}$$

## Single-atom / BEC comparison



Given the **reported sensitivity**  $\gamma = 10^{-5} - 10^{-4}$  for relative frequency shifts from E. Cornell's experiment, we expect that **beyond-PFA lateral CP forces on a BEC above a plateau of a periodically grooved silicon surface should be detectable for distances  $z_{CM} < 3 \mu\text{m}$ , groove period  $\lambda_c = 4 \mu\text{m}$ , groove amplitude  $a = 250 \text{ nm}$ , and a BEC radius of, say,  $R \approx 1 \mu\text{m}$**

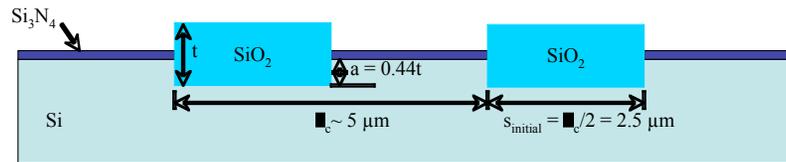
# Towards the experiment

- Surfaces are being fabricated at Sandia Labs (Matt Blain)
- CP force measurements with BEC will be done at LANL (Malcolm Boshier)

Number of periods, 100/set  
 •Length of “grooves”, 2 mm  
 •see next slide

**Process sequence**

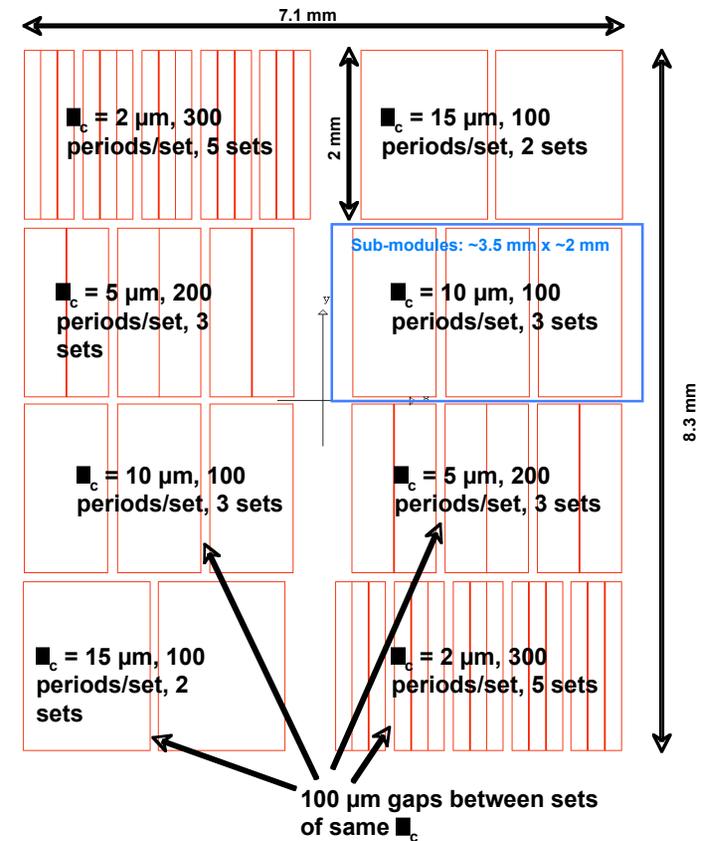
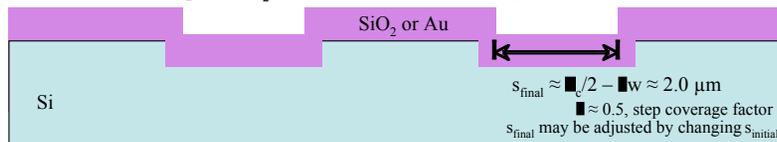
1. Deposit/pattern SiN oxidation mask
2. Grow SiO<sub>2</sub> in exposed Si to thicknesses of  $t = 100, 200, 500$  and  $2000$  nm



3. Strip SiN and SiO<sub>2</sub>



4. Deposit SiO<sub>2</sub> or Au to a thickness of  $w = 1$   $\mu\text{m}$



# Summary part II

- Novel cold atoms techniques open a promising way of investigating nontrivial geometrical effects on quantum vacuum
- Important feature of atoms: they can be used as local probes of quantum vacuum fluctuations
- Non-trivial, beyond-PFA effects should be measurable using a BEC as a vacuum field sensor with available technology

For more details see:

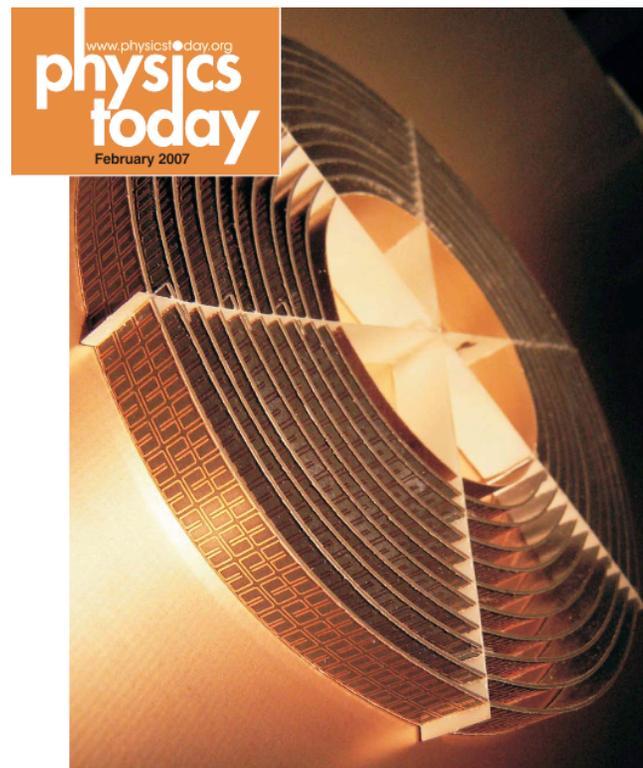
Dalvit, Maia Neto, Lambrecht, and Reynaud,  
Phys. Rev. Lett. 100, 040405 (2008)  
J. Phys.A 41, 164028 (2008)

# Part III: MMs and Casimir



# Metamaterials and Casimir

- Artificial materials for engineering the Casimir force



Invisibility by design

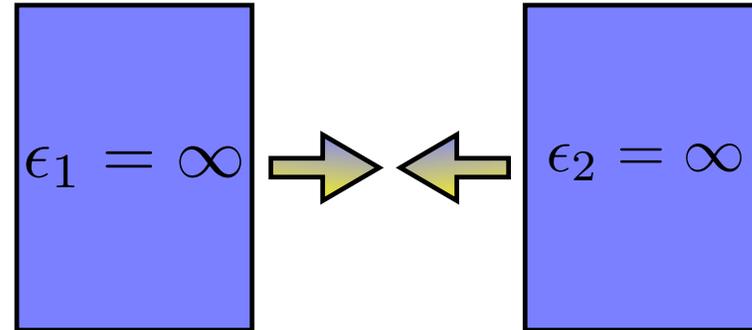
Smith et al (2007)

# Casimir attraction-repulsion

## ■ Ideal attractive limit

Casimir (1948)

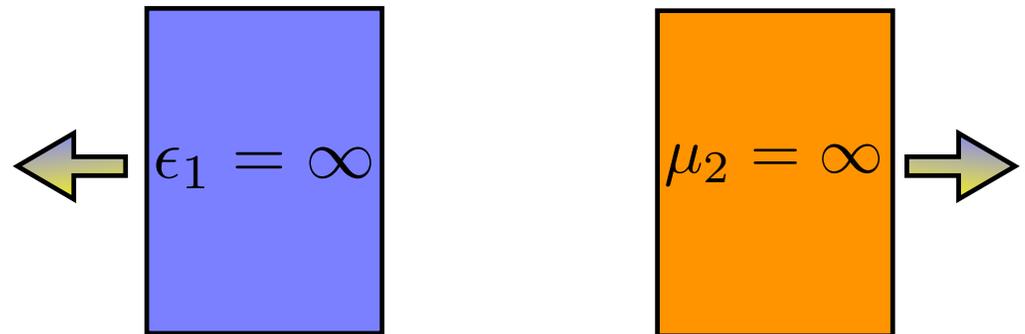
$$\frac{F}{A} = + \frac{\pi^2}{240} \frac{\hbar c}{d^4}$$



## ■ Ideal repulsive limit

Boyer (1974)

$$\frac{F}{A} = - \frac{7}{8} \frac{\pi^2}{240} \frac{\hbar c}{d^4}$$



## ■ Real repulsive limit

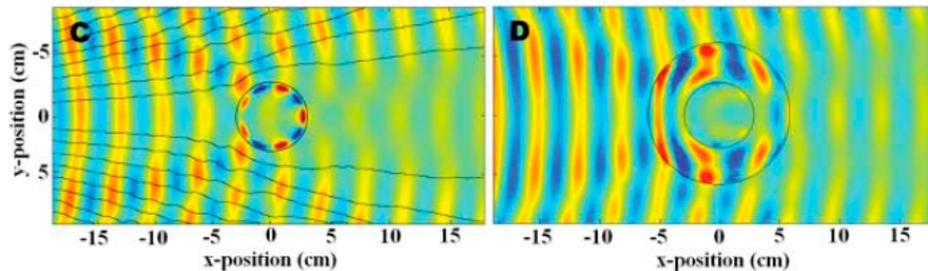
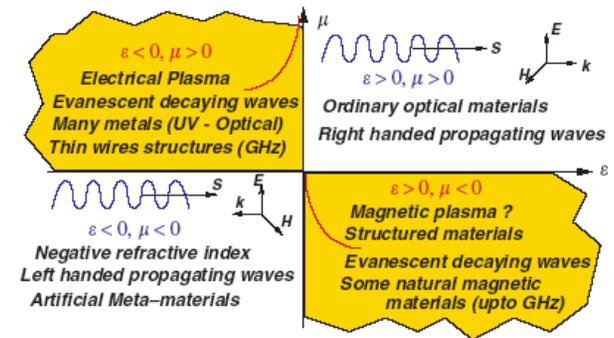
Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e.  $\mu = 1$

→ **Metamaterials**

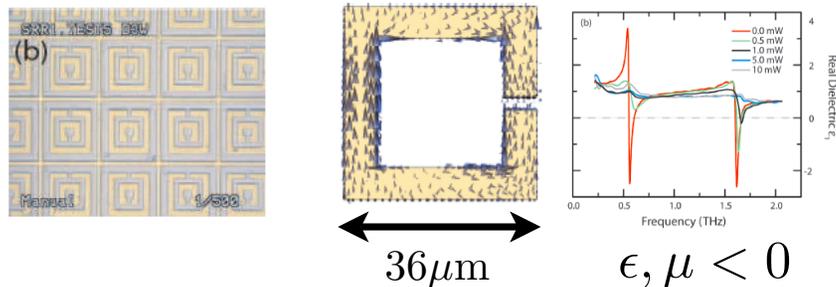
# Metamaterials

- Artificial structured composites with designer electromagnetic properties
- MMs are strongly anisotropic, dispersive, magneto-dielectric media.

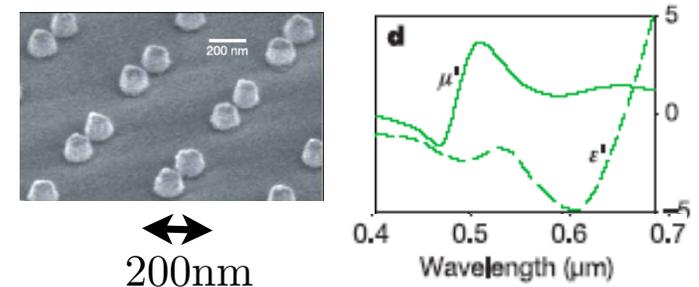
- Negative refraction** Veselago (1968), Smith et al (2000)
- Perfect lens** Pendry (2000)
- Cloaking** Smith et al (2007)



THz MMs: eg split ring resonators



Optical MMs: eg nano-pillars



# Quantum levitation with MMs?

Physicists have 'solved' mystery of levitation - Telegraph

http://www.telegraph.co.uk/news/main.jhtml?xml=/news/2007/08/0...

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### Physicists have 'solved' mystery of levitation

By Roger Highfield, Science Editor  
Last Updated: 1:45pm BST 08/08/2007

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FEATURE FOCUS 

Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

In earlier work the same team of theoretical physicists showed that invisibility cloaks are feasible.

Now, in another report that sounds like it comes out of the pages of a Harry Potter book, the University of St Andrews team has created an 'incredible levitation effect' by engineering the force of nature which normally causes objects to stick together.

Professor Ulf Leonhardt and Dr Thomas Philbin, from the University of St Andrews in Scotland, have worked out a way of reversing this phenomenon, known as the Casimir force, so that it repels instead of attracts.

Their discovery could ultimately lead to frictionless micro-machines with moving parts that levitate. But they say that, in principle at least, the same effect could be used to levitate bigger objects too, even a person.



The Casimir force is a consequence of quantum mechanics, the theory that describes the world of atoms and subatomic particles that is not only the most successful theory of physics but also the most baffling.

The force is due to neither electrical charge or gravity, for example, but the fluctuations in all-pervasive energy fields in the intervening empty space between the objects and is one reason atoms stick together, also explaining a "dry glue" effect that enables a gecko to walk across a ceiling.

Now, using a special lens of a kind that has already been built, Prof Ulf Leonhardt and Dr Thomas Philbin report in the New Journal of Physics they can engineer the Casimir force to repel, rather than attract.

Because the Casimir force causes problems for nanotechnologists, who are trying to build electrical circuits and tiny mechanical devices on silicon chips, among other things, the team believes the feat could initially be used to stop tiny objects from sticking to each other.

Prof Leonhardt explained, "The Casimir force is the ultimate cause of friction in the nano-world, in particular in some microelectromechanical systems.

Such systems already play an important role - for example tiny mechanical devices which triggers a car airbag to inflate or those which power tiny 'lab on chip' devices used for drugs testing or chemical analysis.

Micro or nano machines could run smoother and with less or no friction at all if one can manipulate the force." Though it is possible to levitate objects as big as humans, scientists are a long way off developing the technology for such feats, said Dr Philbin.

The practicalities of designing the lens to do this are daunting but not impossible and levitation "could happen over quite a distance".

Prof Leonhardt leads one of four teams - three of them in Britain - to have put forward a theory in a peer-reviewed journal to achieve invisibility by making light waves flow around an object - just as a river flows undisturbed around a smooth rock.

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In theory the discovery could be used to levitate a person

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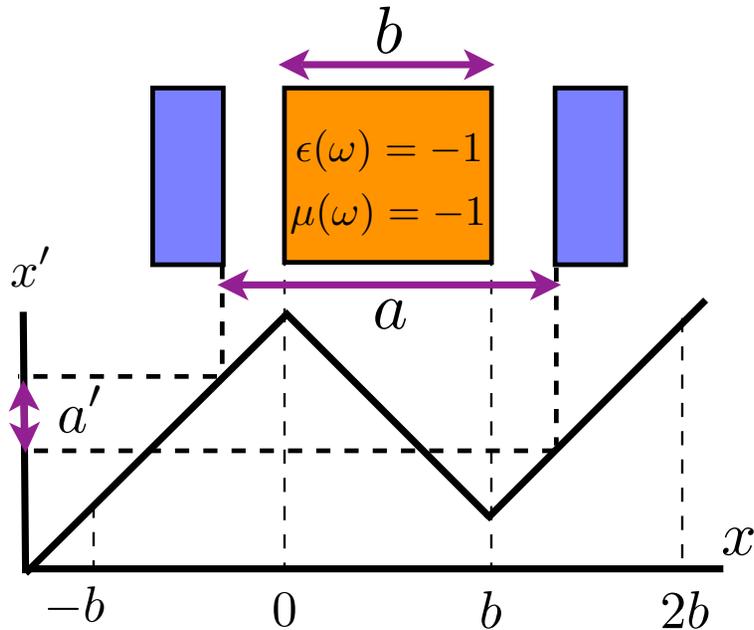
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“In theory the discovery could be used to levitate a person”

# Quantum levitation with MMs?

## Transformation media

Leonhardt et al (2007)



**Perfect lens:** EM field in  $-b < x < 0$  is mapped into  $x'$ . There are two images, one inside the device and one in  $b < x < 2b$ .

**Casimir cavity:**  $a' = |a - 2b|$

When  $a < 2b$  (plates within the imaging range of the perfect lens)

$$\Rightarrow f = -\frac{\partial U}{\partial a'} \frac{\partial a'}{\partial a} = +\frac{\hbar c \pi^2}{240 a'^4} \Rightarrow \text{Repulsion}$$

For real materials, however .....

- According to causality, no **passive** medium ( $\epsilon''(\omega) > 0$ ) can sustain  $\epsilon, \mu \simeq -1$  over a wide range of frequencies. In fact,  $\epsilon(i\xi), \mu(i\xi) > 0$
- Another proposal is to use an **active** MM ( $\epsilon''(\omega) < 0$ ) in order to get repulsion. But then the whole approach breaks down, as real photons would be emitted into the quantum vacuum.

# Modelling optical response

Close to resonance, the optical response can be modeled by Drude-Lorentz permittivities and permeabilities

$$\epsilon_{\alpha}(\omega) = 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega}$$

$$\mu_{\alpha}(\omega) = 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}$$

Typical separations

$$d = 200 - 1000 \text{ nm}$$

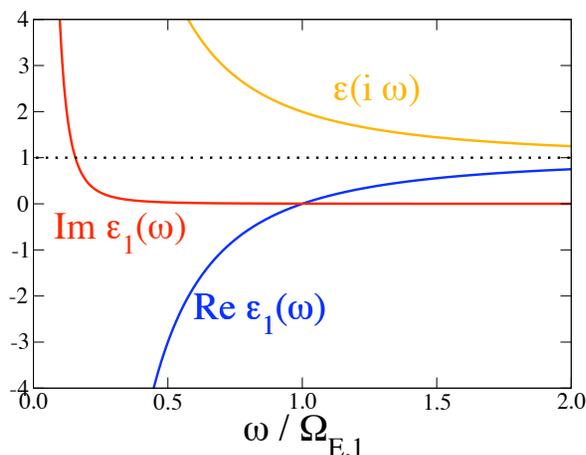


Infrared-optical frequencies

$$\Omega/2\pi = 5 \times 10^{14} \text{ Hz}$$

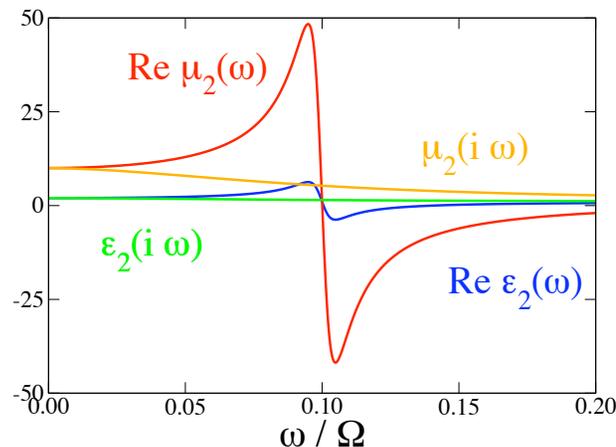
**Drude metal (Au)**

$$\Omega_E = 9.0 \text{ eV} \quad \Gamma_E = 35 \text{ meV}$$



**Metamaterial**

$$\text{Re } \epsilon_2(\omega) < 0 \quad \text{Re } \mu_2(\omega) < 0$$



$$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$$

$$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$$

$$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$$

# Attraction-repulsion crossover

Drude metal (Au)

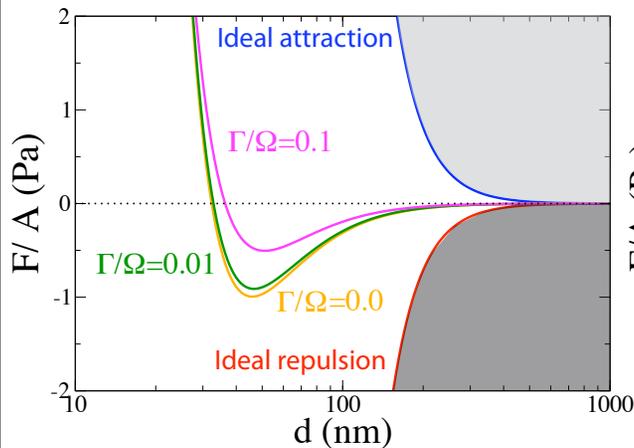
Metamaterial

Drude metal (Au)

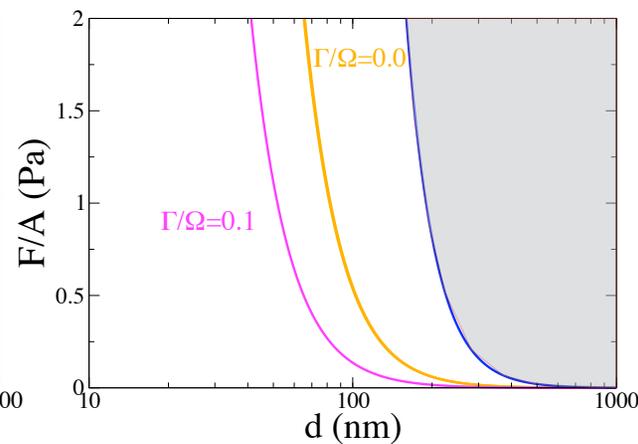
Metamaterial

Metamaterial

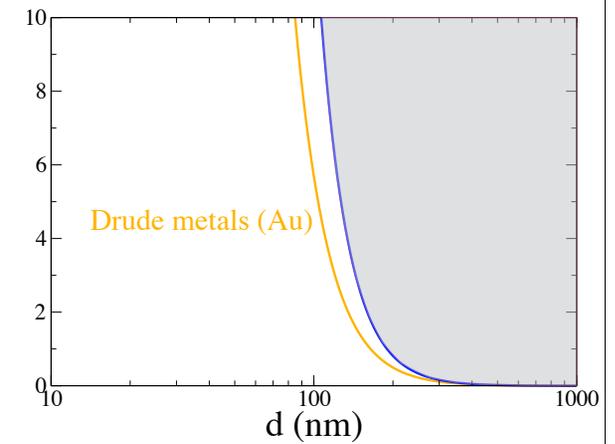
Drude metal (Au)



Repulsion-attraction



Only attraction



Only attraction

A slab made of Au ( $\rho = 19.3 \text{ gr/cm}^3$ ) of width  $\delta = 1 \mu\text{m}$  could levitate in front of one of these MMs at a distance of  $d \approx 110 \text{ nm}$  !!!

Casimir and metamaterials, Henkel et al (2005)

Casimir and surface plasmons, Intravaia et al (2005)

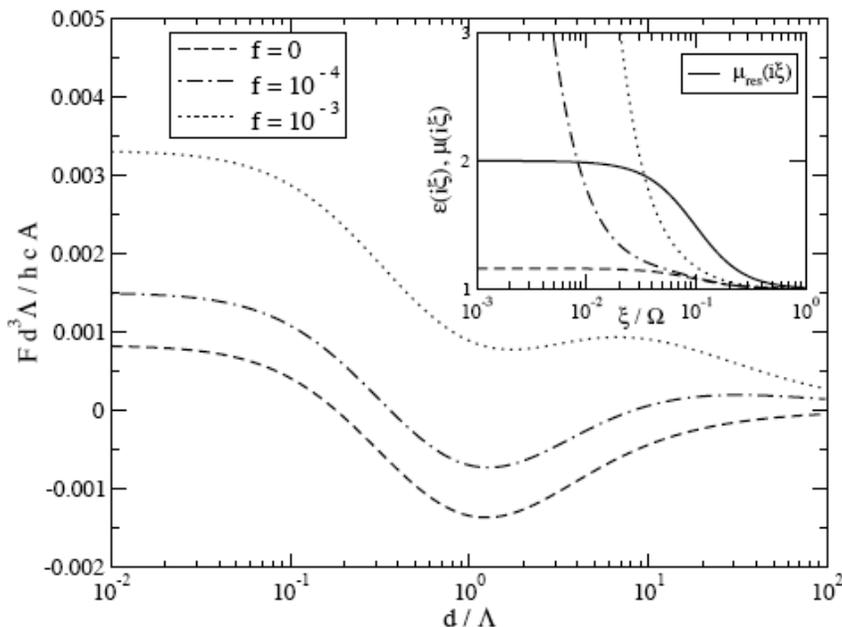
van der Waals in magneto-dielectrics, Spagnolo et al (2007)

# Drude background

Real metamaterials typically have metallic inclusions, and the corresponding Drude background makes attractive a Casimir force that would otherwise be predicted to be repulsive

$$\epsilon(\omega) = 1 - f \frac{\Omega_D^2}{\omega^2 - i\omega\gamma_D} - (1 - f) \frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega} \quad f : \text{filling factor}$$

$$\mu(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}$$



Gold plate in front of a silver-based MM with resonance in the near-infrared:

$$\Omega_D = 1.37 \times 10^{16} \text{ rad/sec} \quad \text{Ag plasma frequency}$$

$$\text{Gold plate: } \Omega_1/\Omega_D = 0.96 \quad \gamma_1/\Omega_D = 0.004$$

$$\text{Silver-based MM: } \Omega_e/\Omega_D = 0.04$$

$$\Omega_m/\Omega_D = 0.1$$

$$\gamma_D/\Omega_D = 0.006 \quad \omega_e/\Omega_D = \omega_m/\Omega_D = 0.1$$

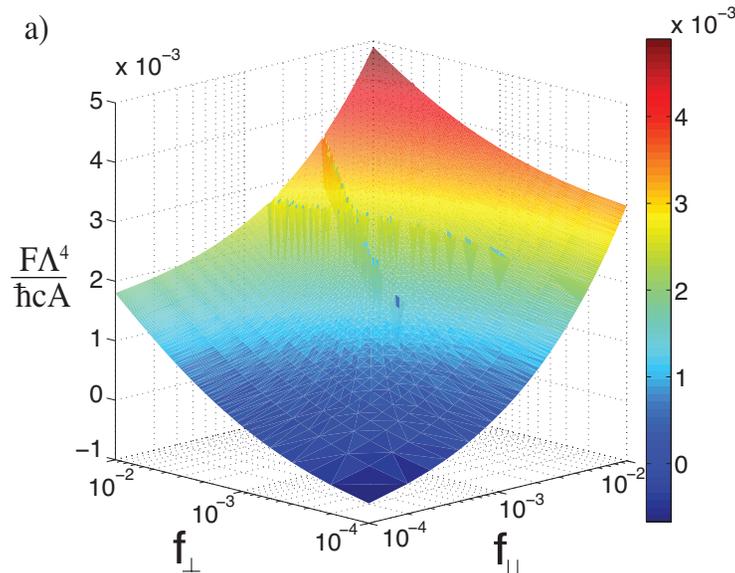
$$\gamma_e/\Omega_D = \gamma_m/\Omega_D = 0.005$$

# Optical anisotropy

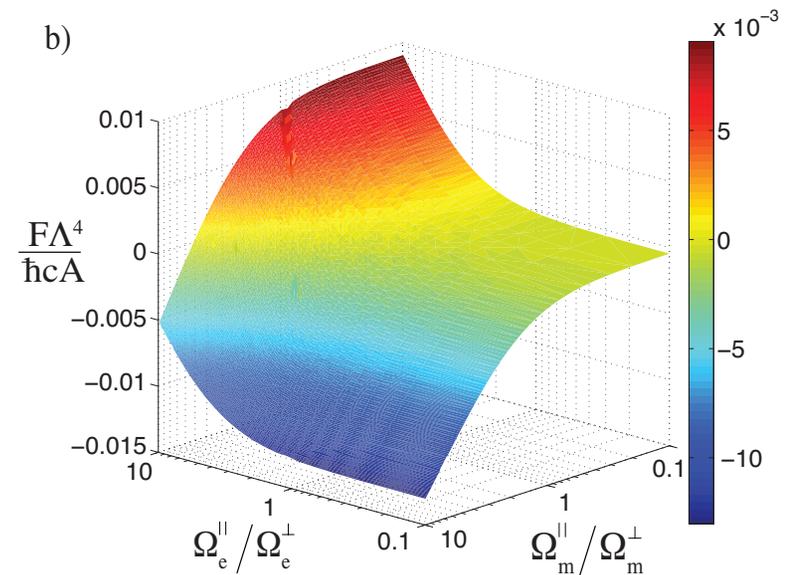
Uni-axial MMs:  $\epsilon = \text{diag}(\epsilon_{\parallel}, \epsilon_{\parallel}, \epsilon_{\perp})$        $\mu = \text{diag}(\mu_{\parallel}, \mu_{\parallel}, \mu_{\perp})$

$$r^{\text{TE,TE}}(i\xi, \mathbf{k}_{\parallel}) = \frac{\mu_{\parallel} K_3 - \sqrt{\frac{\mu_{\parallel}}{\mu_{\perp}} k_{\parallel}^2 + \mu_{\parallel} \epsilon_{\parallel} \xi^2 / c^2}}{\mu_{\parallel} K_3 + \sqrt{\frac{\mu_{\parallel}}{\mu_{\perp}} k_{\parallel}^2 + \mu_{\parallel} \epsilon_{\parallel} \xi^2 / c^2}} \quad (K_3 = \sqrt{k_{\parallel}^2 + \xi^2 / c^2})$$

$$r^{\text{TM,TM}} = r^{\text{TE,TE}} \quad \text{with} \quad \mu \leftrightarrow \epsilon \quad r^{\text{TE,TM}} = r^{\text{TM,TE}} = 0$$



Metallic-based MM with only  
electric anisotropy ( $\mu_{\parallel} = \mu_{\perp}$ )



Dielectric-based MM ( $f = 0$ )

# Summary part III

- Metamaterials might strongly influence the quantum vacuum, providing a route towards quantum levitation.
- Drude backgrounds in metallic-based MMs can spoil Casimir repulsion.
- MMs with weak Drude backgrounds (polaritonic photonic crystals and dielectric structures) might lead to quantum levitation.

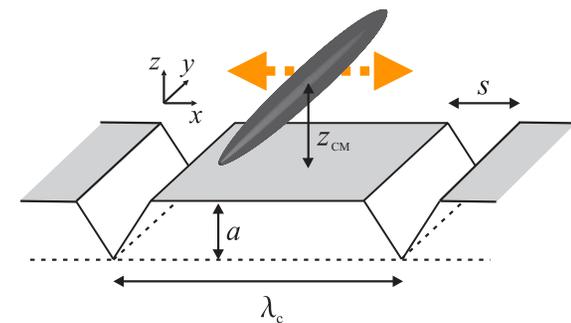
For more details see:

Rosa, Dalvit, Milonni, arXiv: 0803.2908  
to appear in Phys. Rev. Lett.

# General conclusions

Casimir forces: still surprising after 60 years

Non-trivial geometry effects



Non-trivial materials effects

