Casimir interactions between nanostructured materials

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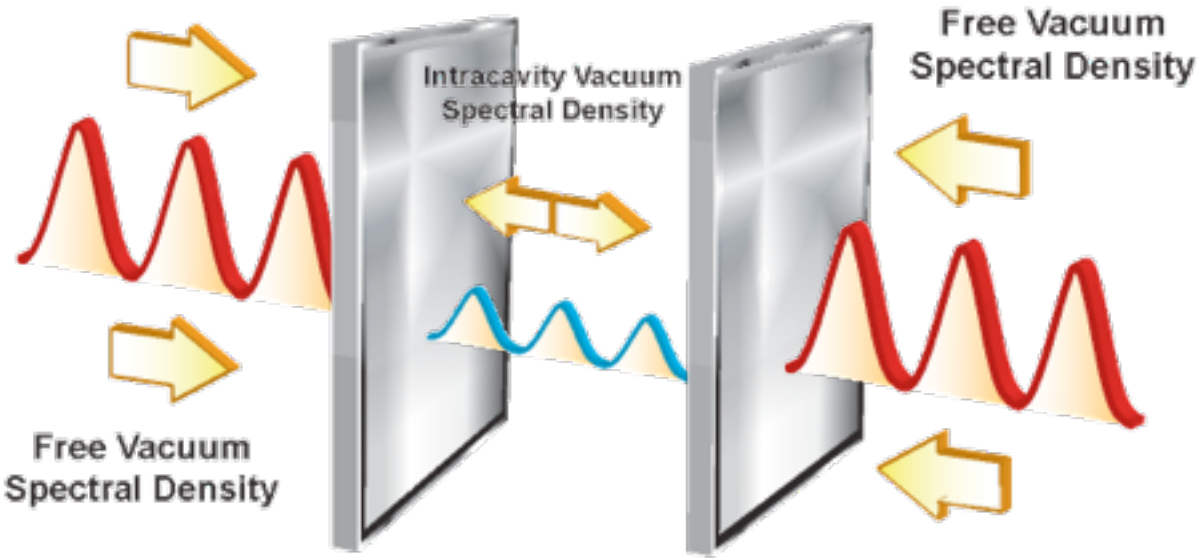
Outline of this Talk

- **Brief intro to Casimir physics**
  - Basics, modern theory and experiments
- **Tailoring Casimir forces with metamaterials**
  - Effective medium/homogenization in Casimir physics
- **Tailoring Casimir forces with nanostructures**
  - Metallic gratings for Casimir force manipulation

Brief intro to Casimir phys.



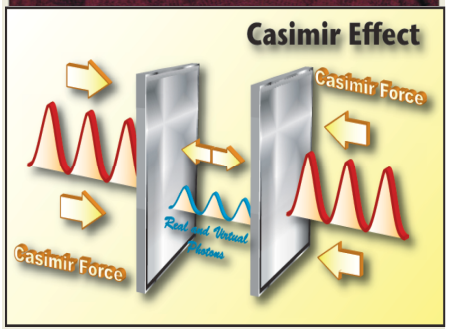
The Casimir force

- **Universal effect from confinement of vacuum fluctuations**

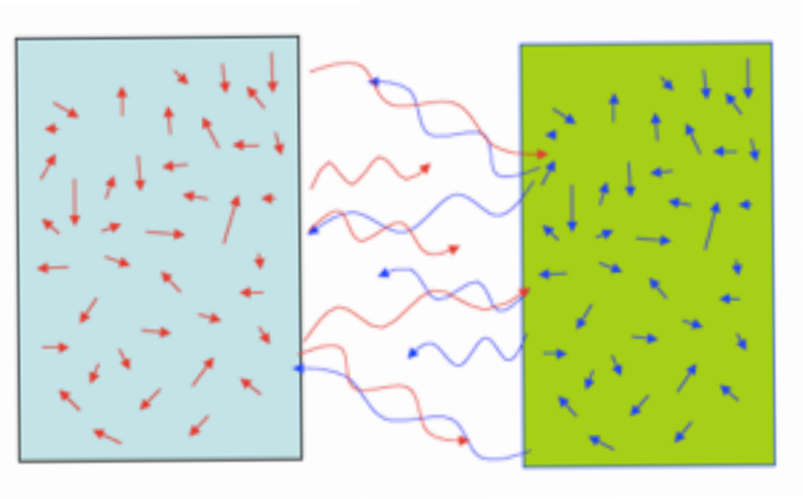
- Depends only on $\hbar, c$, and geometry

$$E = \frac{1}{2}\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \;\Rightarrow\; \boxed{\frac{F}{A} = \frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$

$$(130\mathrm{nN/cm}^2 \;@\; d = 1\mu\mathrm{m})$$



- Alternative interpretation: **fluctuating charges and currents**



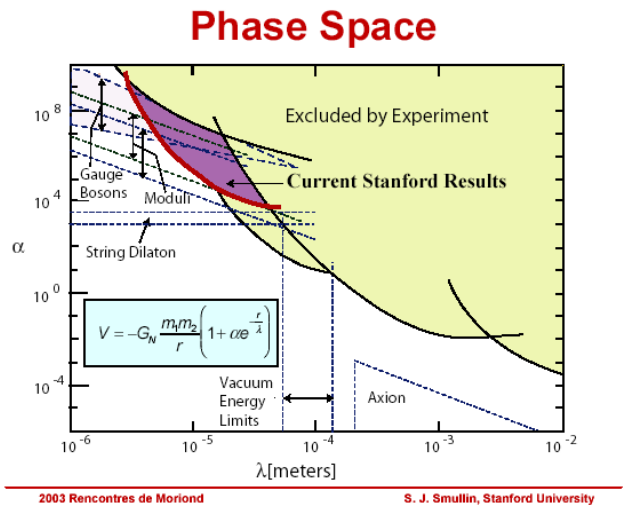
- The magnitude and sign of the force depends on geometry, materials, and temperature

Some relevant applications

Gravitation / Particle theory

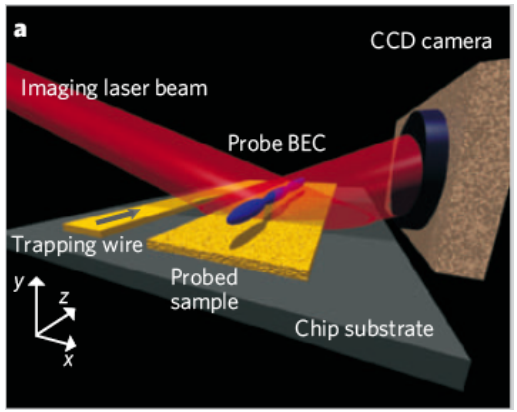
The Casimir force is the main background force to measure non-Newtonian corrections to gravity predicted by high energy physics

$$V(r) = -G\frac{m_1 m_2}{r}\left(1 + \alpha\, e^{-r/\lambda}\right)$$



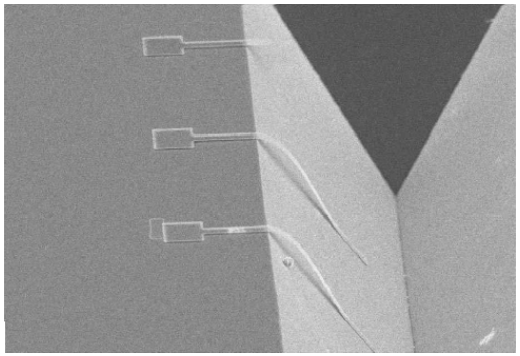
Quantum Science and Technology

Atom-surface interactions (e.g., ion traps, atom chips, BECs) and precision measurements



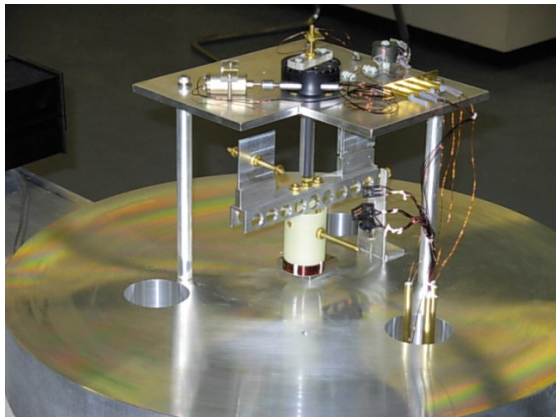
Nanotechnology

Casimir force is a challenge (stiction), but also an opportunity (contactless force transmission)



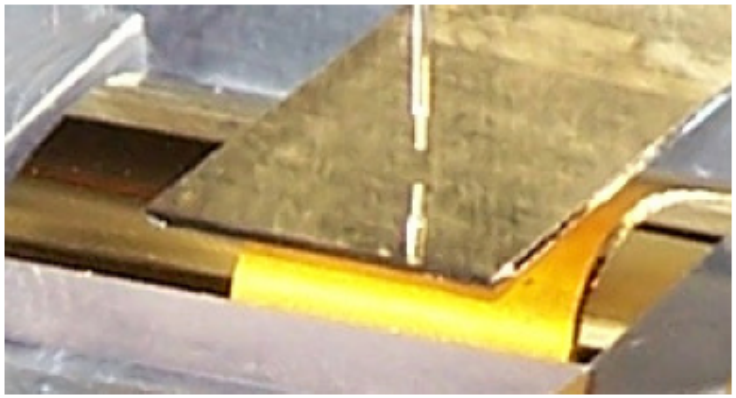
Modern experiments

- Torsion pendulum



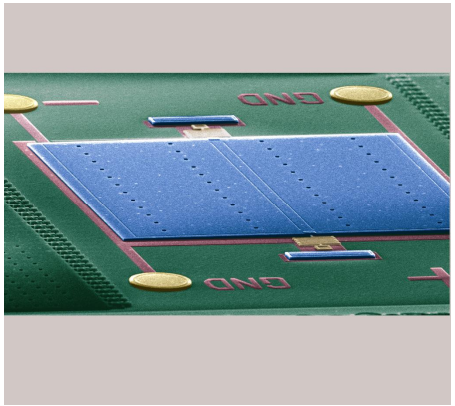
Lamoreaux (1997), 0.7-6.0 um

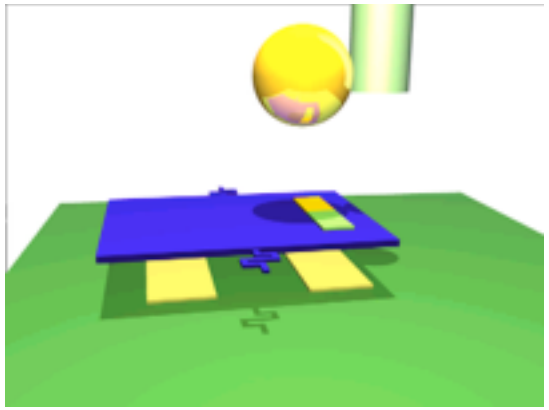
- Atomic force microscope



Mohideen (1998), 0.1-0.9 um

- MEMS and NEMS





Capasso (2001), Decca (2003), 0.2-1.0 um

The Lifshitz formula

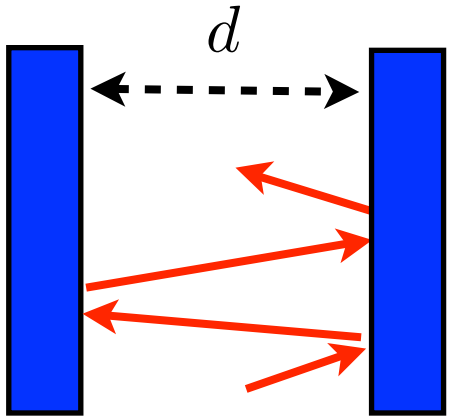
Casimir interaction energy between materials slabs (Lifshitz 1956)

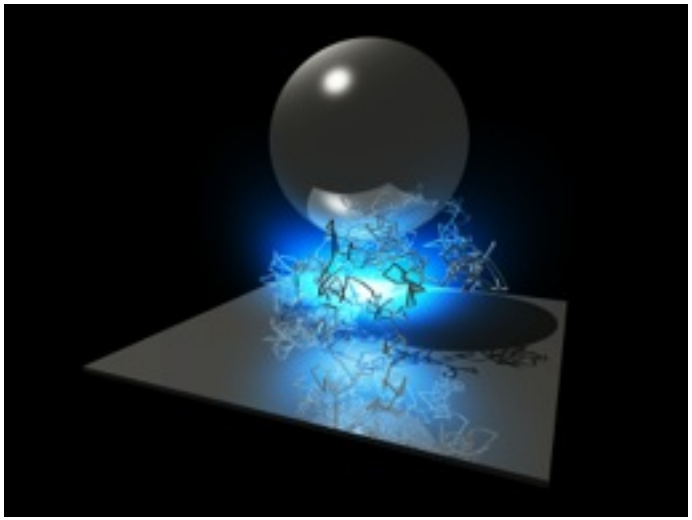
$$\frac{E(d)}{A} = \hbar \sum_p \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \coth\left(\frac{\hbar\omega}{2k_B T}\right) \mathrm{Im}\,\log[1 - R_{1,p}(\omega,k)\; R_{2,p}(\omega,k)\; e^{2id\sqrt{\omega^2/c^2-k^2}}]$$

Fresnel reflection coefficients $R_{\mathrm{TE}} = \dfrac{k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}$ $\quad R_{\mathrm{TM}} = \dfrac{\epsilon(\omega)k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{\epsilon(\omega)k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}$

The log factor can be re-written as

$$\propto \sum_{n=1}^{\infty} \frac{1}{n}[R_{1,p}\; e^{idk_z}\; R_{2,p}\; e^{idk_z}]^n$$



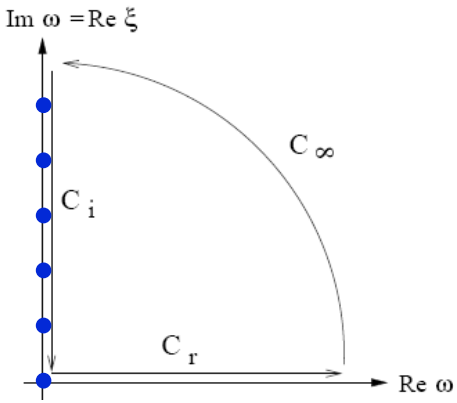


Scattering theory

Going to imaginary freq.

The function $\coth(\hbar\omega/2k_BT)$ has poles on the imaginary frequency axis at

$$\omega_m = i\xi_m \quad , \quad \xi_m = m\frac{2\pi k_B T}{\hbar}$$



After Wick rotation:

$$\frac{F}{A} = -2k_BT\sum_p \sum_{m=0}^{\infty}{}' \int \frac{d^2\mathbf{k}}{(2\pi)^2}\sqrt{\xi_m^2/c^2+k^2}\,\frac{R_{1,p}(i\xi_m,k)R_{2,p}(i\xi_m,k)e^{-2d\sqrt{\xi_m/c^2+k^2}}}{1-R_{1,p}(i\xi_m,k)R_{2,p}(i\xi_m,k)e^{-2d\sqrt{\xi_m/c^2+k^2}}}$$

$$\epsilon(i\xi) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega\epsilon''(\omega)}{\omega^2+\xi^2}d\omega$$

Kramers-Kronig (causality)

Some limiting cases:

$F \propto d^{-3}$ (non-retarded limit, small distances)

$F \propto d^{-4}$ (retarded limit, larger distances)

$F \propto Td^{-3}$ (classical limit, very large distances)

Casimir physics is a <u>broad-band</u> frequency phenomenon

The sign of the Casimir force

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$

The sign of the force is directly connected to the sign of the product of the reflection coefficients on the two plates, evaluated at imaginary frequencies. As a rule of thumb, we have (p=TE, TM)

$$R_1^p(i\xi) \cdot R_2^p(i\xi) > 0 \; (\forall\, \xi \leq c/d) \Rightarrow \text{Attraction}$$

$$R_1^p(i\xi) \cdot R_2^p(i\xi) < 0 \; (\forall\, \xi \leq c/d) \Rightarrow \text{Repulsion}$$

In terms of permittivities and permeabilities:

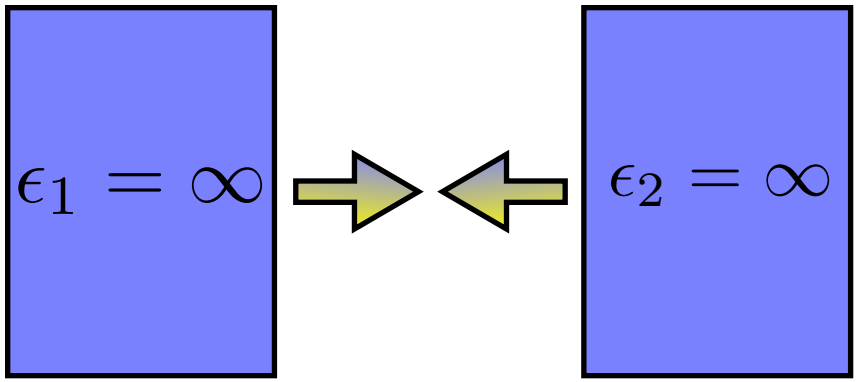
$$\left.\begin{array}{l} \epsilon_a(i\xi) \gg \epsilon_b(i\xi) \\ \mu_b(i\xi) \gg \mu_a(i\xi) \end{array}\right. \longrightarrow \text{Repulsion}$$

Ideal attraction-repulsion

- **Ideal attractive limit**

  (Casimir 1948)

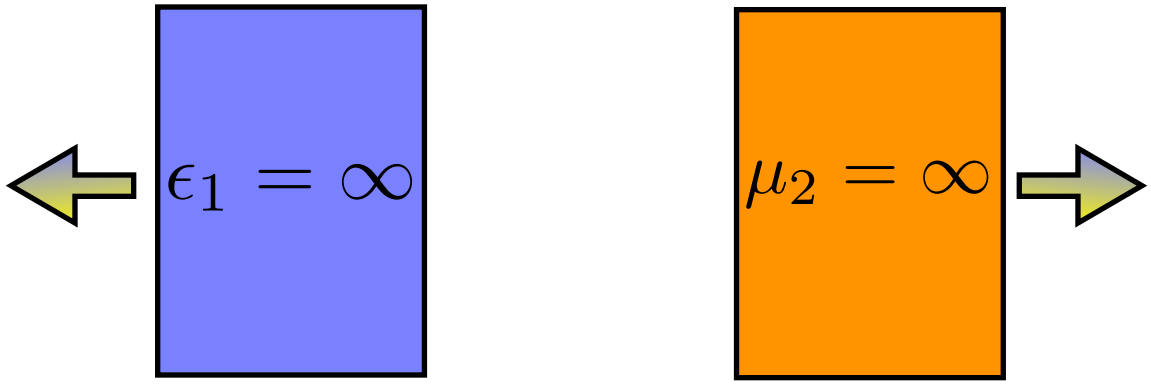
$$\frac{F}{A} = +\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$



- **Ideal repulsive limit**

  (Boyer 1974)

$$\frac{F}{A} = -\frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$



- **Real repulsion**

Natural occurring materials do **NOT** have strong magnetic response in the optical region, i.e. $\mu = 1$ $\longrightarrow$ **Metamaterials**

Quantum levitation with MMs?

Physicists have 'solved' mystery of levitation - Telegraph http://www.telegraph.co.uk/news/main.jhtml?xml=/news/2007/08/0...

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Physicists have 'solved' mystery of levitation

By Roger Highfield, Science Editor

Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

In earlier work the same team of theoretical physicists showed that invisibility cloaks are feasible.

Now, in another report that sounds like it comes out of the pages of a Harry Potter book, the University of St Andrews team has created an 'incredible levitation effects' by engineering the force of nature which normally causes objects to stick together.

Professor Ulf Leonhardt and Dr Thomas Philbin, from the University of St Andrews in Scotland, have worked out a way of reversing this phenomenon, known as the Casimir force, so that it repels instead of attracts.

Their discovery could ultimately lead to frictionless micro-machines with moving parts that levitate But they say that, in principle at least, the same effect could be used to levitate bigger objects too, even a person.

In theory the discovery could be used to levitate a person

The Casimir force is a consequence of quantum mechanics, the theory that describes the world of atoms and subatomic particles that is not only the most successful theory of physics but also the most baffling.

The force is due to neither electrical charge or gravity, for example, but the fluctuations in all-pervasive energy fields in the intervening empty space between the objects and is one reason atoms stick together, also explaining a "dry glue" effect that enables a gecko to walk across a ceiling.

Now, using a special lens of a kind that has already been built, Prof Ulf Leonhardt and Dr Thomas Philbin report in the New Journal of Physics they can engineer the Casimir force to repel, rather than attact.

Because the Casimir force causes problems for nanotechnologists, who are trying to build electrical circuits and tiny mechanical devices on silicon chips, among other things, the team believes the feat could initially be used to stop tiny objects from sticking to each other.

Prof Leonhardt explained, "The Casimir force is the ultimate cause of friction in the nano-world, in particular in some microelectromechanical systems.

Such systems already play an important role - for example tiny mechanical devices which triggers a car airbag to inflate or those which power tiny 'lab on chip' devices used for drugs testing or chemical analysis.

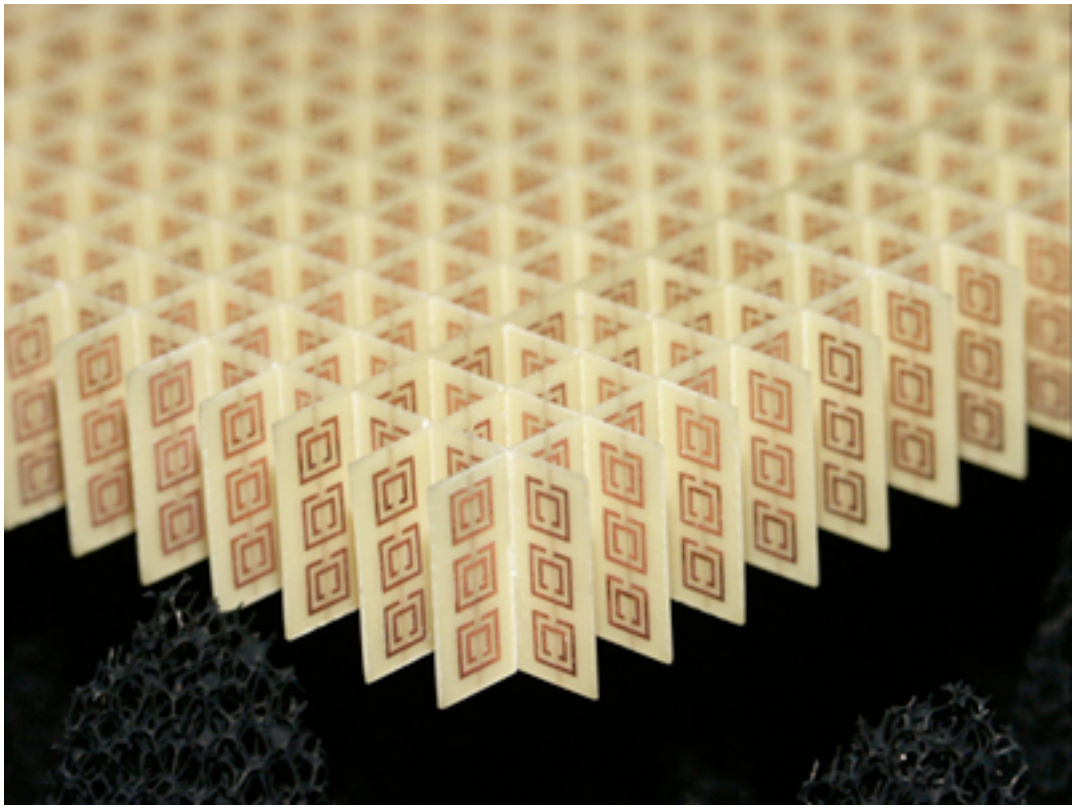
Micro or nano machines could run smoother and with less or no friction at all if one can manipulate the force." Though it is possible to levitate objects as big as humans, scientists are a long way off developing the technology for such feats, said Dr Philbin.

The practicalities of designing the lens to do this are daunting but not impossible and levitation "could happen over quite a distance".

Prof Leonhardt leads one of four teams - three of them in Britain - to have put forward a theory in a peer-reviewed journal to achieve invisibility by making light waves flow around an object - just as a river flows undisturbed around a smooth rock.

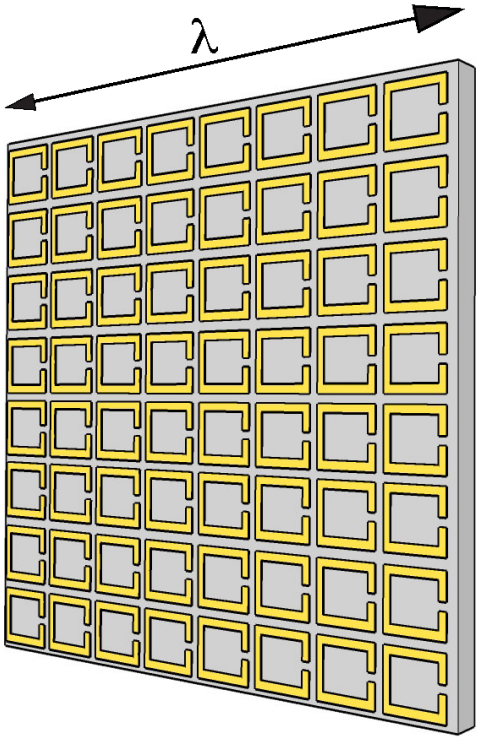
"In theory the discovery could be used to levitate a person"

Metamaterials and Casimir



Effective medium approx.

Imagine that the metamaterial is probed at wavelengths much larger that the average distance between the constituent "meta-atoms"



In this situation the MM is effectively a **continuous medium**, whose optical response can be characterized by an effective electric permittivity and an effective magnetic permeability.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

$$\mu_{eff} = 1 - \frac{\frac{\pi r^2}{a^2}}{1 + \frac{2\sigma i}{\omega r \mu_0} - \frac{3}{\pi^2 \mu_0 \omega^2 C r^3}}$$

Optical response

Close to the resonance, both $\epsilon(\omega)$ and $\mu(\omega)$ can be modeled by Drude-Lorentz formulas

$$\epsilon_\alpha(\omega) = 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega}$$

$$\mu_\alpha(\omega) = 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}$$

Typical separations

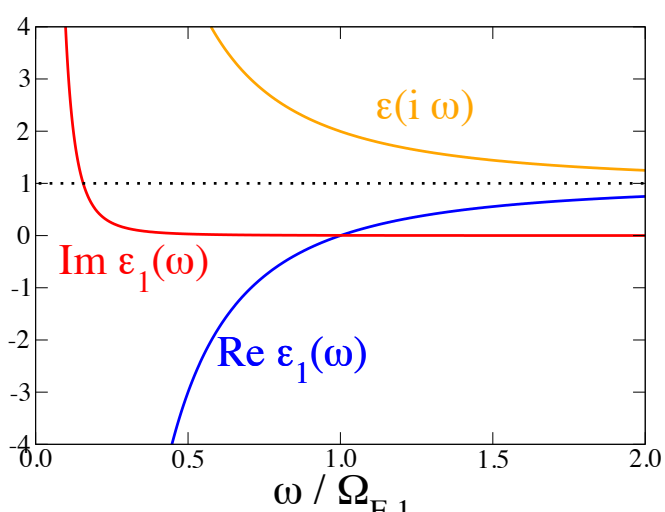
$d = 200 - 1000$ nm

Infrared-optical frequencies

$\Omega/2\pi = 5 \times 10^{14}$Hz

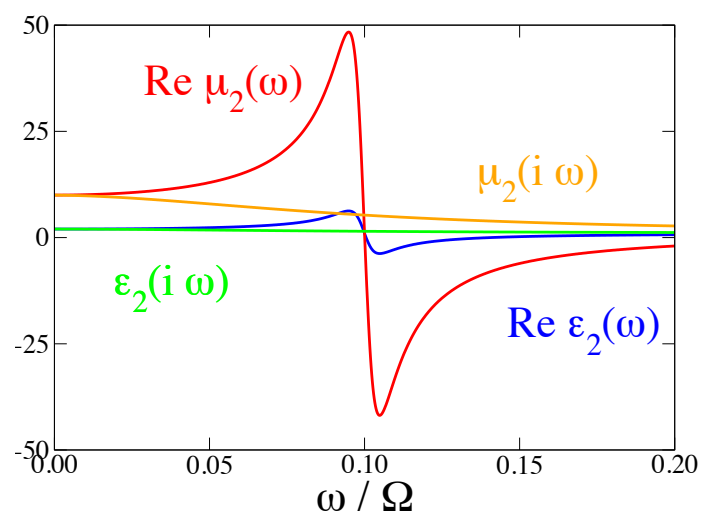
Drude metal (Au)

$\Omega_E = 9.0$ eV $\quad \Gamma_E = 35$ meV



Metamaterial

$\mathrm{Re}\ \epsilon_2(\omega) < 0 \quad \mathrm{Re}\ \mu_2(\omega) < 0$



$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$

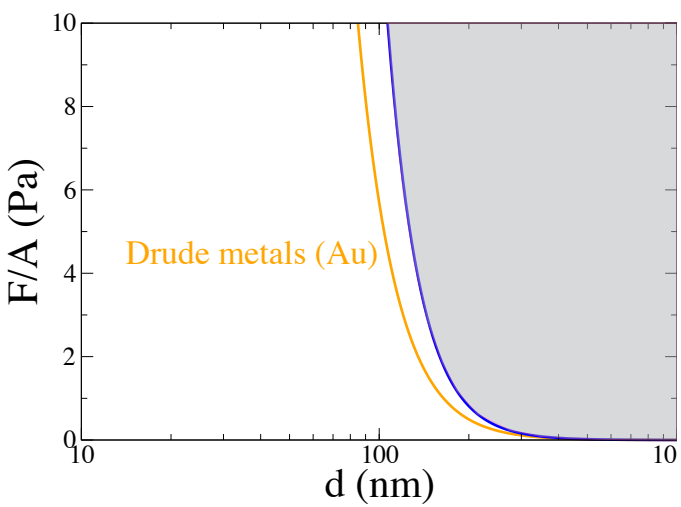
$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$

$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$

Attraction-repulsion crossover

Drude metal (Au)

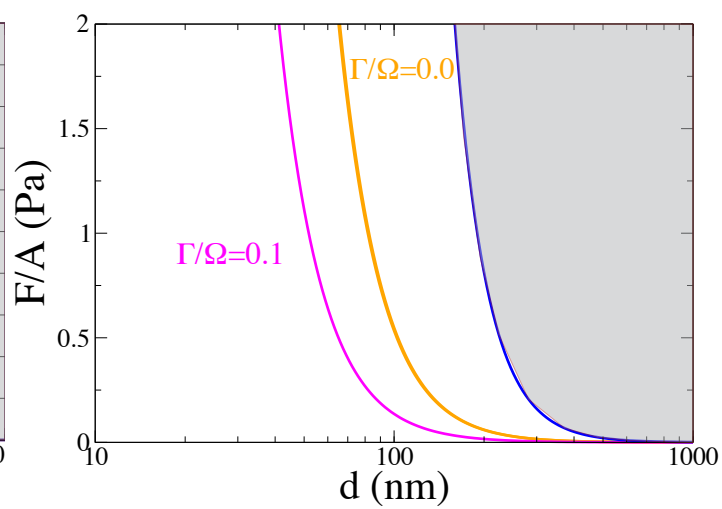
Drude metal (Au)



Only attraction

Metamaterial

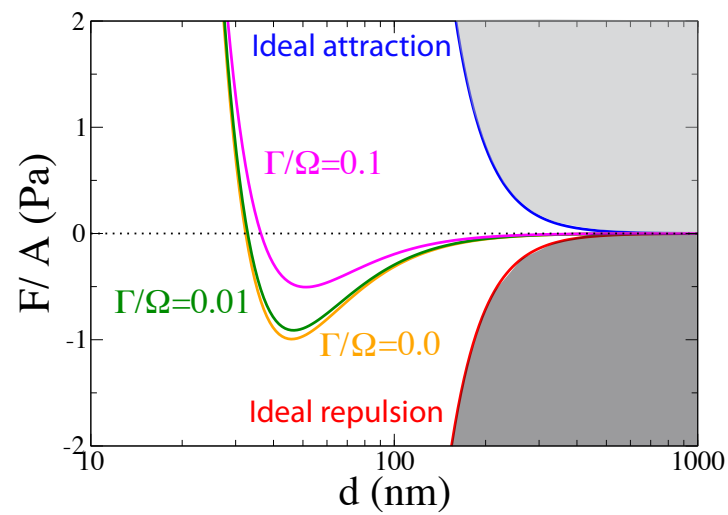
Metamaterial



Only attraction

Drude metal (Au)

Metamaterial



Repulsion-attraction

EMA: correct model for μ

- **Drude-Lorentz model for permeability is wrong!**

- **The correct expression for $\mu_{\text{eff}}(\omega)$ from Maxwell's equations**

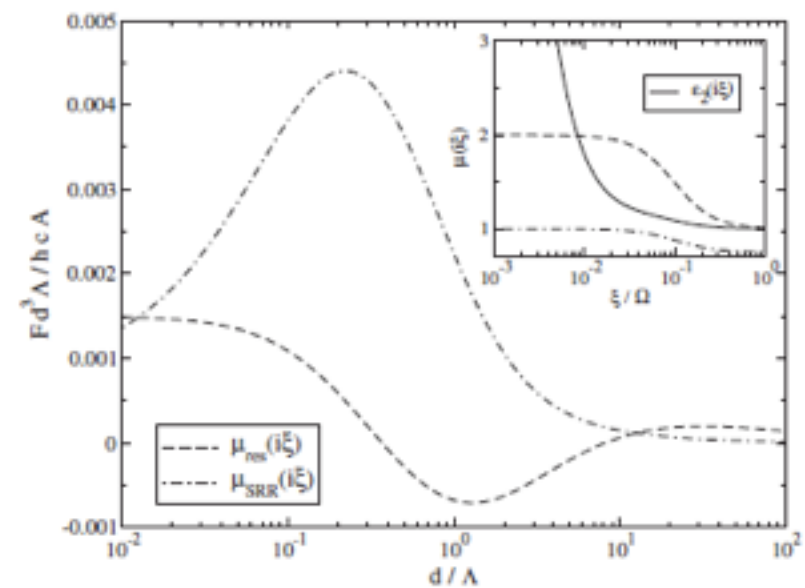
$$\mu_{\text{eff}}(\omega) = 1 - f\frac{\omega^2}{\omega^2 - \omega_m^2 + 2i\gamma_m\omega}$$

(Pendry 1999)

- Correct low frequency behavior very different from Drude-Lorentz model

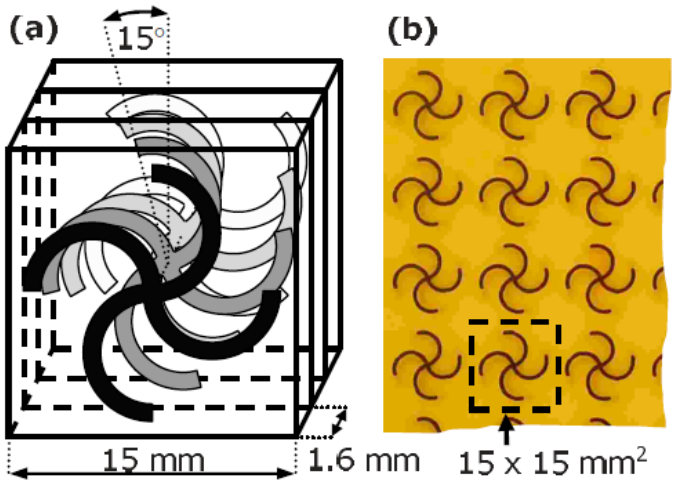
$$\mu_{\text{eff}}(i\xi) < 1 < \epsilon_{\text{eff}}(i\xi)$$

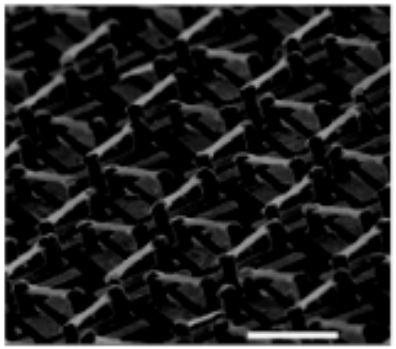
No Casimir repulsion!

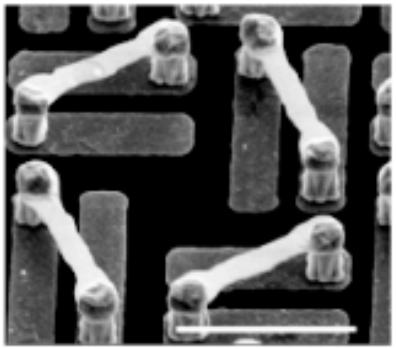


(Rosa, DD, Milonni, PRL 2008)

Other Casimir MMs: chirality







- Constitutive relations mix electric and magnetic fields

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r},\omega) - i\kappa(\omega)\mathbf{H}(\mathbf{r},\omega)$$
$$\mathbf{B}(\mathbf{r},\omega) = i\kappa(\omega)\mathbf{E}(\mathbf{r},\omega) + \mu(\omega)\mathbf{H}(\mathbf{r},\omega)$$

dispersive chirality: $\kappa(\omega) = \dfrac{\omega_k \omega}{\omega^2 - \omega_{\kappa R}^2 + i\gamma_k \omega}$

Reflection matrices become non-diagonal

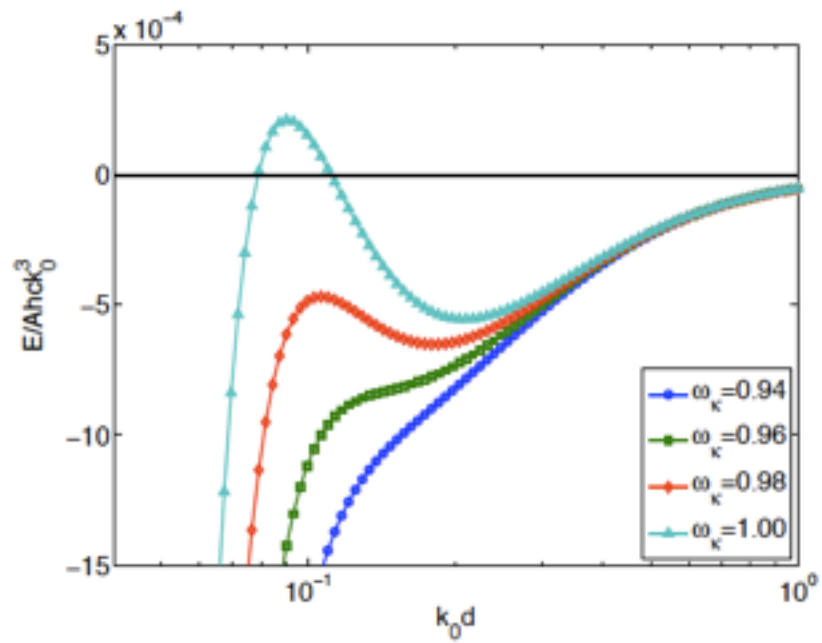
Repulsion and chiral MMs

- Casimir force between two chiral materials

$$F = \frac{(r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} - 2(r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}{1 - (r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} + (r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}$$

Repulsion can be achieved with strong chirality, which results in large values of r_{sp}

(Soukoulis *et al.*, PRL 2009)

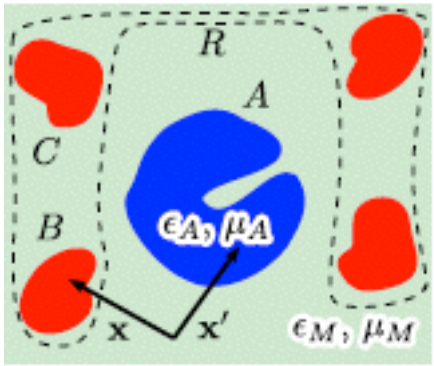


- However, predictions are based on EMA in a region of parameters where EMA is expected to fail!

- **Exact numerics shows that there is no repulsion**

Constraints on stable equilib.

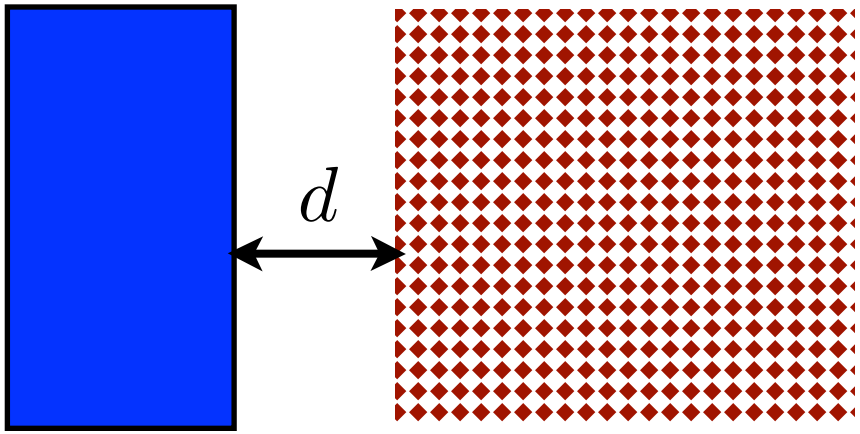
Theorem: *there are no stable equilibria with fluctuation-induced forces when all interacting objects have microscopic* $\epsilon(\mathbf{r}, i\xi) > 1$ *and* $\mu(\mathbf{r}, i\xi) \approx 1$

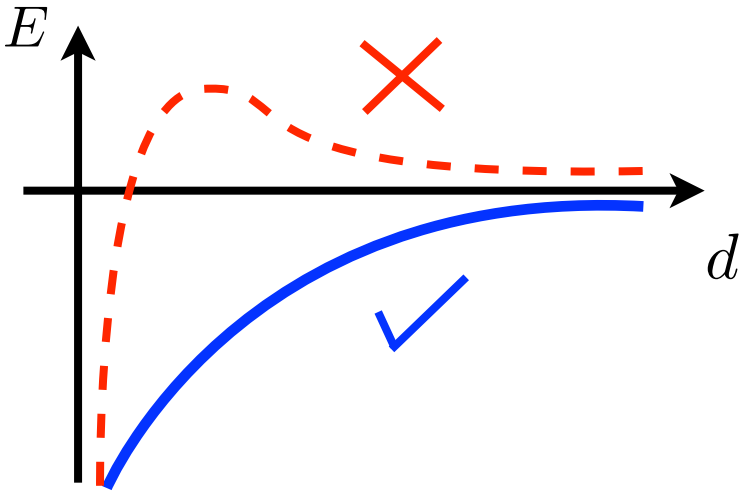


$$\nabla^2 E < 0$$

(Rahi, Kardar, Emig, PRL 2010)

Corollary: *Casimir repulsion is impossible for any metallic/dielectric based MM in front a translationally invariant non-magnetic plate.*





Going beyond EMA

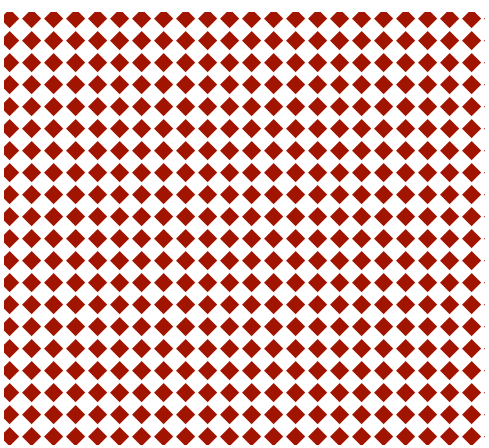
So far, we have treated the MM in the "long-wavelength approximation", i.e., field wavelengths much larger than the typical size of the unit cell of the MM.

- **How to calculate Casimir forces when EMA does not hold?**

Homogeneous medium

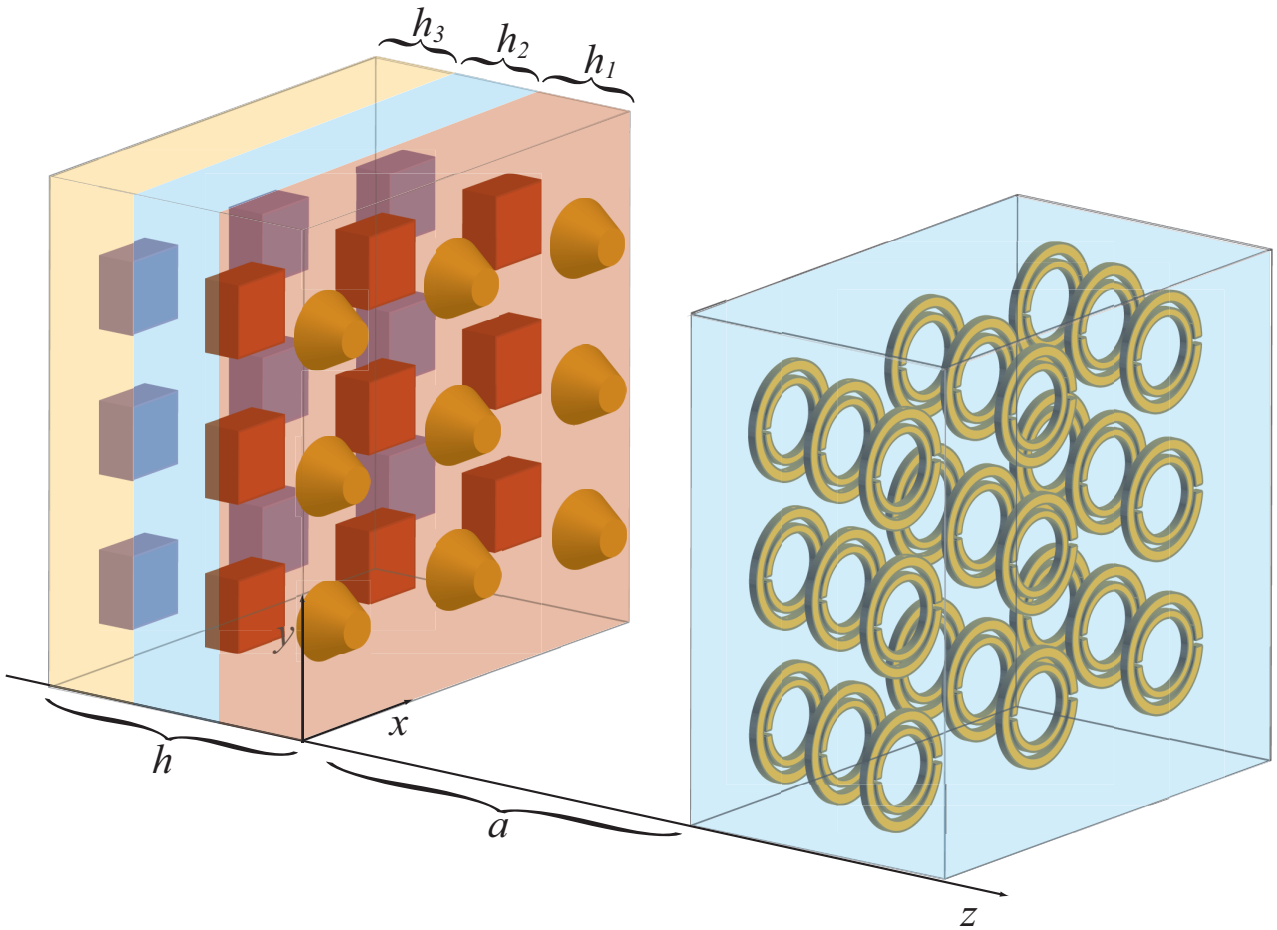
EMA

Non-homogeneous medium

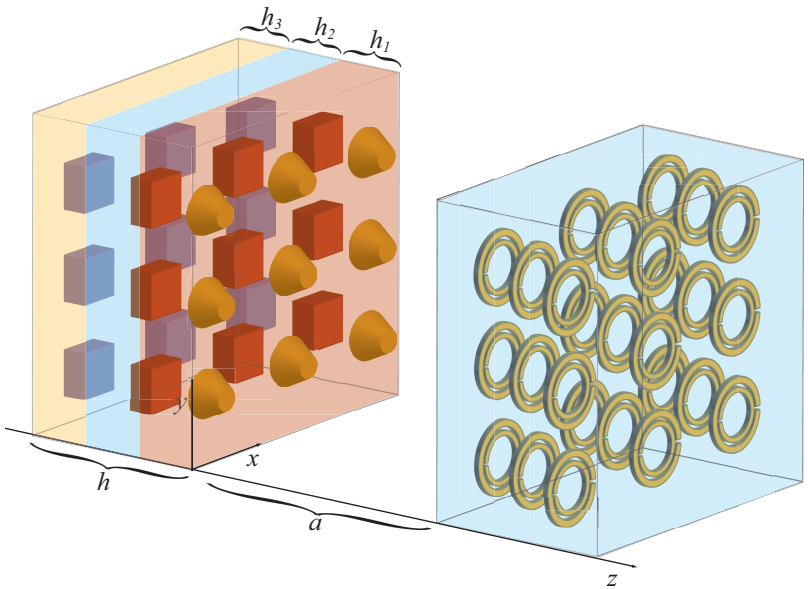


beyond EMA

Casimir nanostructures

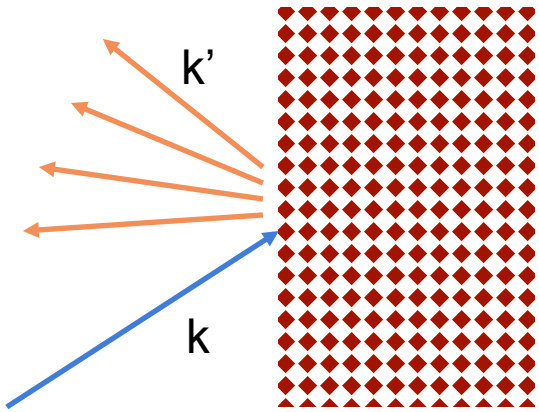


Scattering theory



The Casimir force still may be described in terms of reflections **(scattering theory)**

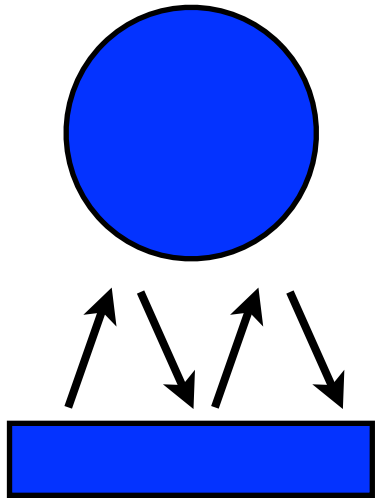
$$\mathcal{R}_i(\omega, \mathbf{k}, \mathbf{k}', p, p')$$



Symbolically, we may write the Casimir energy as

$$\boxed{\frac{E(d)}{A} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\left[1 - \mathcal{R}_1 e^{-\mathcal{K}d} \mathcal{R}_2 e^{-\mathcal{K}d}\right]}$$

$$\propto \sum_{n=1}^{\infty} \frac{1}{n} \left[\mathcal{R}_1(i\xi) e^{-d\mathcal{K}(i\xi)} \mathcal{R}_2(i\xi) e^{-d\mathcal{K}(i\xi)}\right]^n$$



Finding the reflection matrix

The reflection matrix can be obtained with standard methods of numerical electromagnetism. One way is to solve Maxwell equations for the transverse fields

$$-ik\frac{\partial \mathbf{E}_t}{\partial z} = \nabla_t \left[\chi \hat{e}_3 \cdot \nabla \times \mathbf{H}_t\right] - k^2 \mu \hat{e}_3 \times \mathbf{H}_t$$
$$-ik\frac{\partial \mathbf{H}_t}{\partial z} = -\nabla_t \left[\zeta \hat{e}_3 \cdot \nabla \times \mathbf{E}_t\right] + k^2 \epsilon \hat{e}_3 \times \mathbf{E}_t$$

Assuming a two-dimensional periodic structure, we have

$$\mathbf{E}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{E}_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mathbf{H}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{H}_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

where

$$\epsilon(x,y) = \sum_{m,n} \epsilon_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mu(x,y) = \sum_{m,n} \mu_{m,n} \ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

Exact reflection matrix

One can then write the equations for the transverse fields as

$$\boxed{-ik\frac{\partial \Psi_{m'n'}}{\partial z} = \sum_{mn} H_{m'n',mn}\Psi_{mn}}$$

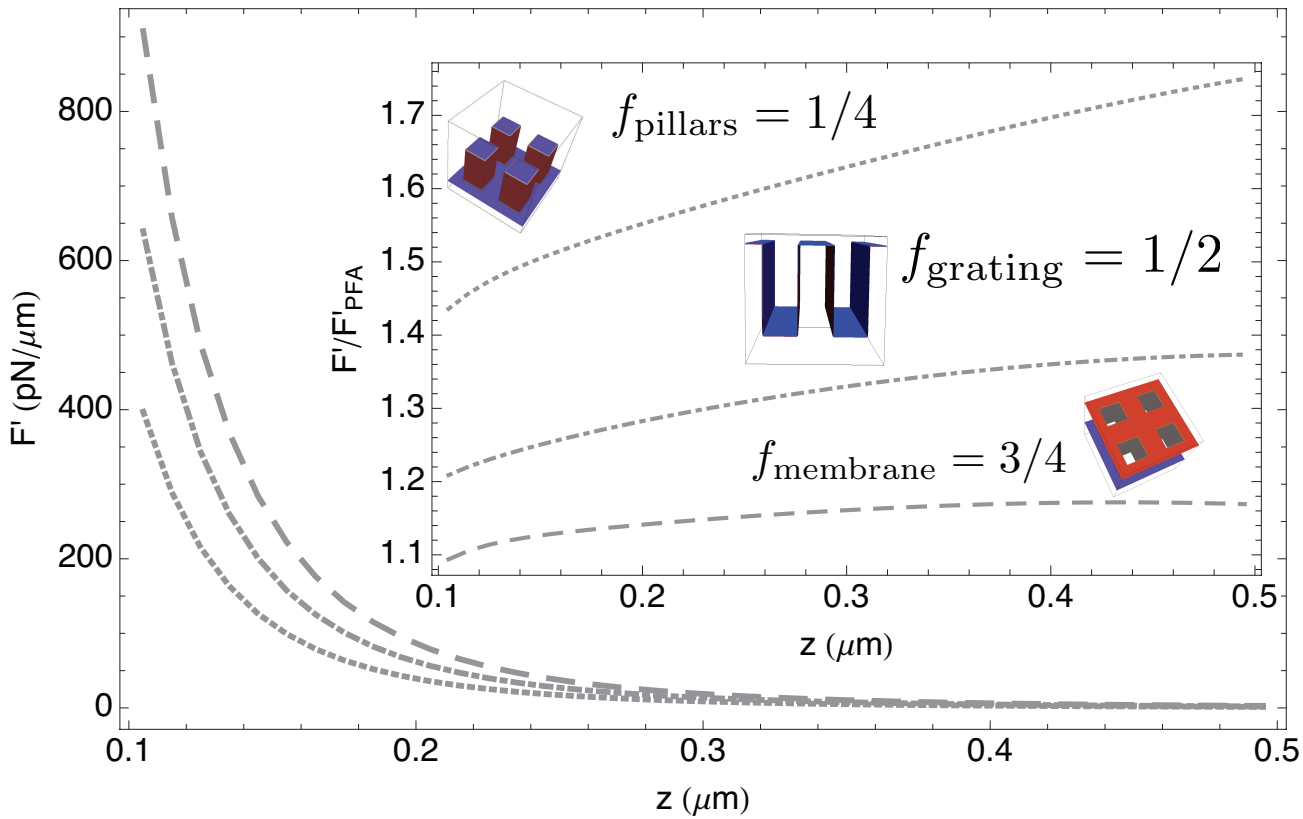
$$\Psi_{mn} = \begin{bmatrix} \mathcal{E}^x_{mn} \\ \mathcal{E}^y_{mn} \\ \mathcal{H}^x_{mn} \\ \mathcal{E}^y_{mn} \end{bmatrix} = \begin{bmatrix} \Psi^1_{mn} \\ \Psi^2_{mn} \\ \Psi^3_{mn} \\ \Psi^4_{mn} \end{bmatrix}$$

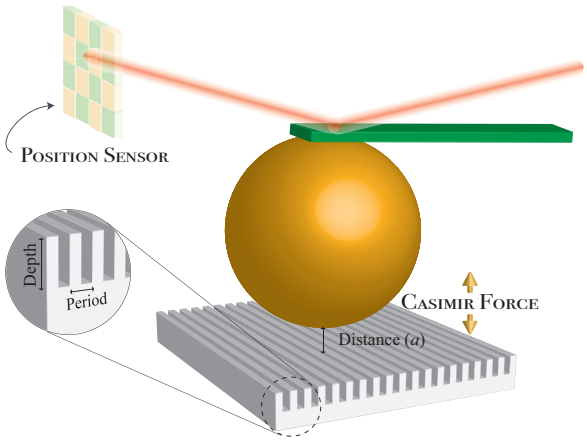
Here H is a complicated matrix, that encapsulates the coupling of modes in the periodic structure.

By numerically solving this equation and imposing the proper boundary conditions of the field on the vacuum-metamaterial interphase (**RCWA or S-matrix techniques**), one can find the reflection matrix of the MM.

2D periodic structures

Example: Casimir force between a Au plane and Si pillars/grating/membrane @ T=300 K





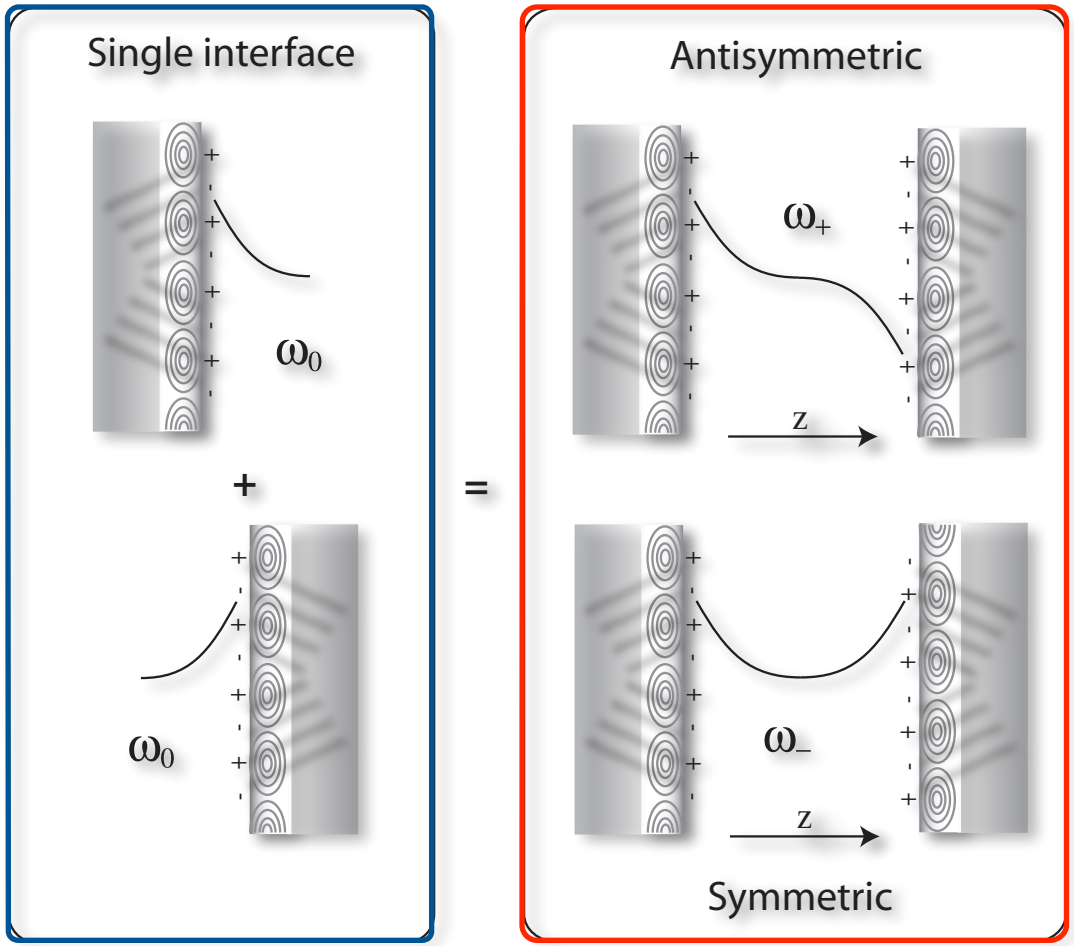
$R = 50\mu\text{m}$

period $= 400$ nm

depth $= 1070$ nm

(Davids, Intravaia, Rosa, DD, PRA 2010)

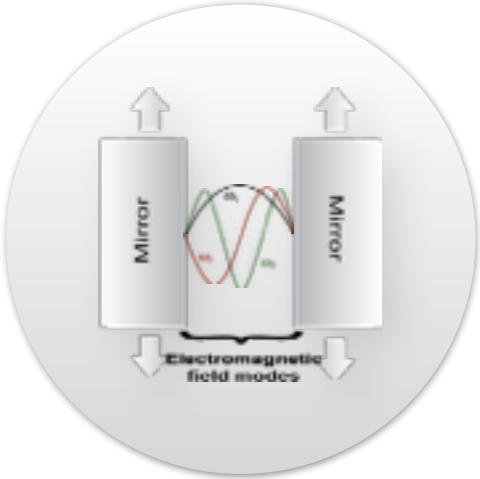
Casimir plasmonics



Mode summation approach

- **Alternative approach: compute Casimir energy as a sum over zero-point energies**

$$E = \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p \right]_L}_{\text{Infinite zero point energy}} - \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p \right]_{L\to\infty}}_{\text{Setting the zero}}$$

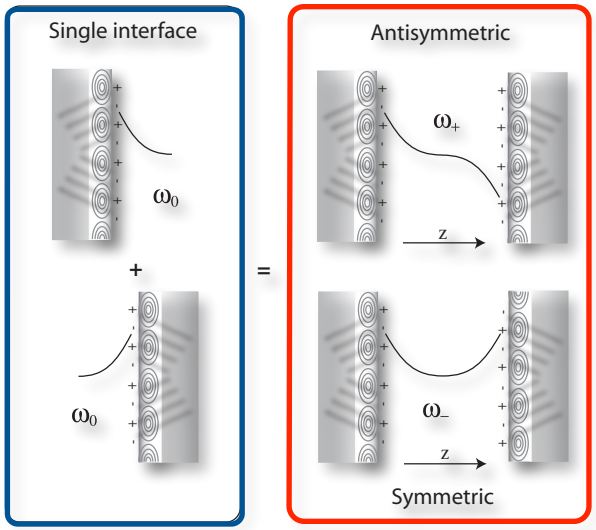


In the case of metallic plates described by the plasma model

$$\left.\begin{aligned} \mu[\omega] &= 1 \\ \epsilon[\omega] &= 1 - \frac{\omega_p^2}{\omega^2} \end{aligned}\right\} \rightarrow \quad E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2} \left[\omega_+ + \omega_-\right]_{L\to\infty}^{L}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m^p \right]_{L\to\infty}^{L}}_{\text{Photonic contribution } (E_{ph})}$$

Surface plasmons interaction

- **Surface plasmons:** evanescent modes of the EM field associated with electronic density oscillations at the metal-vacuum interface.



- When the tails of the evanescent fields overlap, the two surface plasmons hybridize

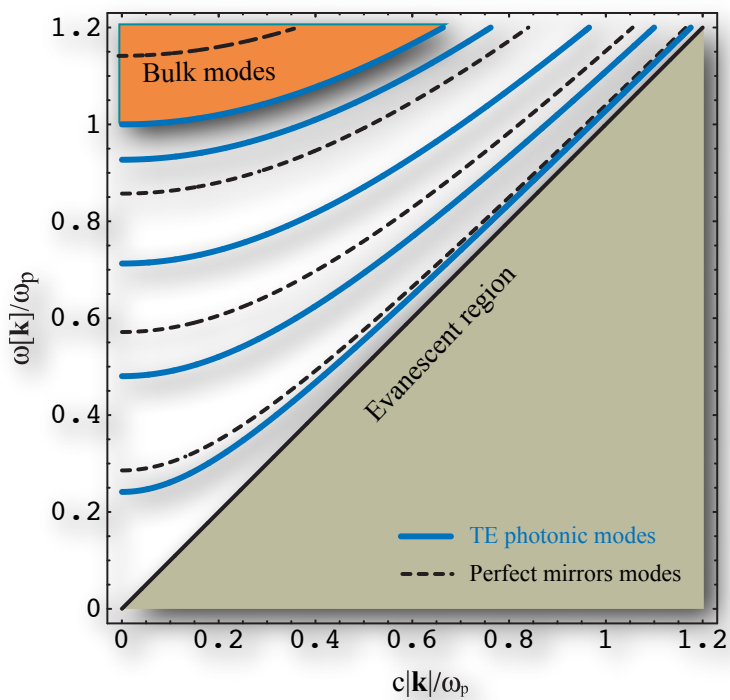
$$2\times\omega_{sp}[\mathbf{k}] \rightarrow \omega_+[\mathbf{k}],\ \omega_-[\mathbf{k}]$$

- At short distances the Casimir energy is given by the shift in the zero-point energy of the surface plasmons due to their Coulomb (electrostatic) interaction)

$$E_{sp} = A\int \frac{d^2\mathbf{k}}{(2\pi)^2}\left(\frac{\hbar\omega_+}{2}+\frac{\hbar\omega_-}{2}-2\frac{\hbar\omega_{sp}}{2}\right) = -\frac{\hbar c\alpha\pi^2 A}{580\lambda_p L^2}$$

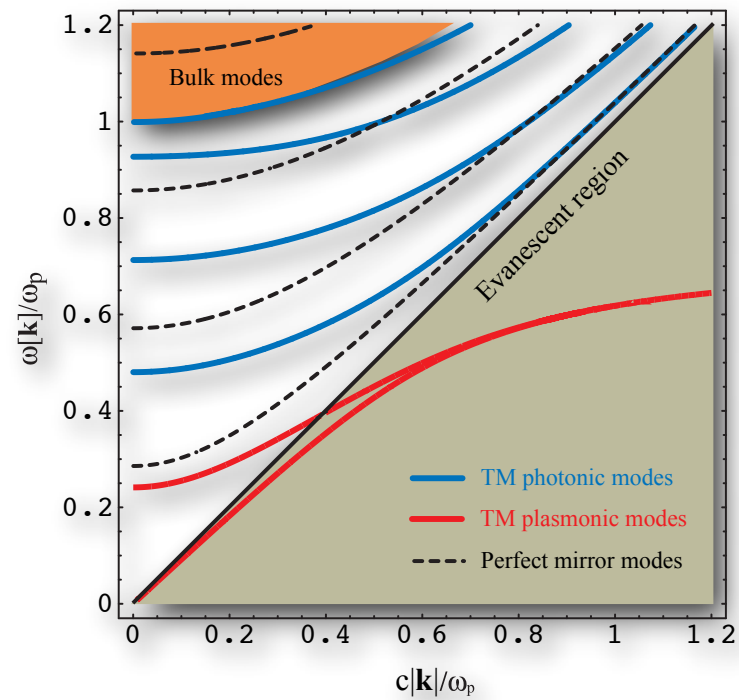
Mode spectrum in a cavity

$$E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2} \left[\omega_+ + \omega_-\right]_{L\to\infty}^{L}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m^p\right]_{L\to\infty}^{L}}_{\text{Photonic contribution } (E_{ph})}$$



All the TE-modes belong to the propagative sector

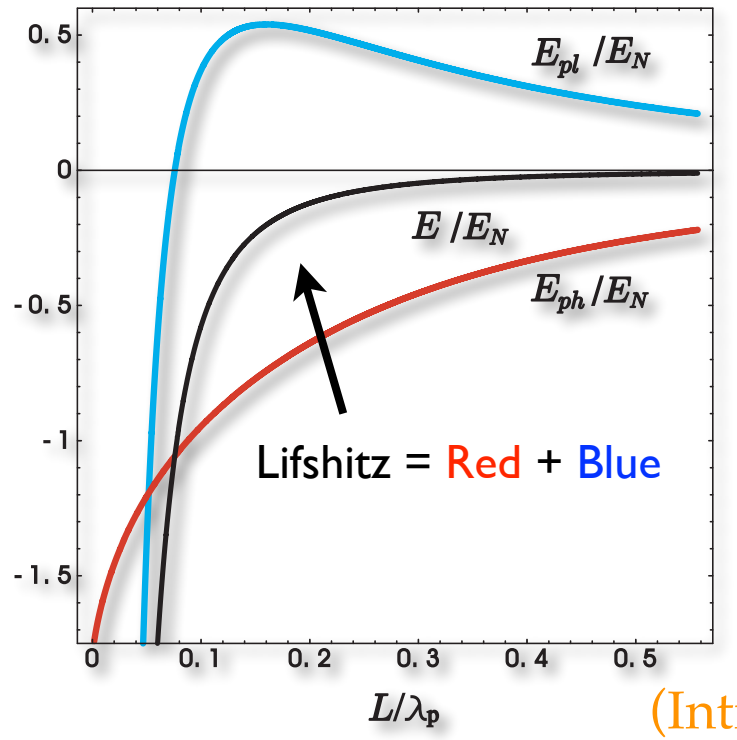
They differ from the perfect mirrors modes because of the dephasing due to the non perfect reflection coefficient.



TM-modes propagative modes look qualitatively like TE modes.

There are only two evanescent modes. They are the generalization to all distances of the coupled plasmon modes.

Plasmonic & photonic parts

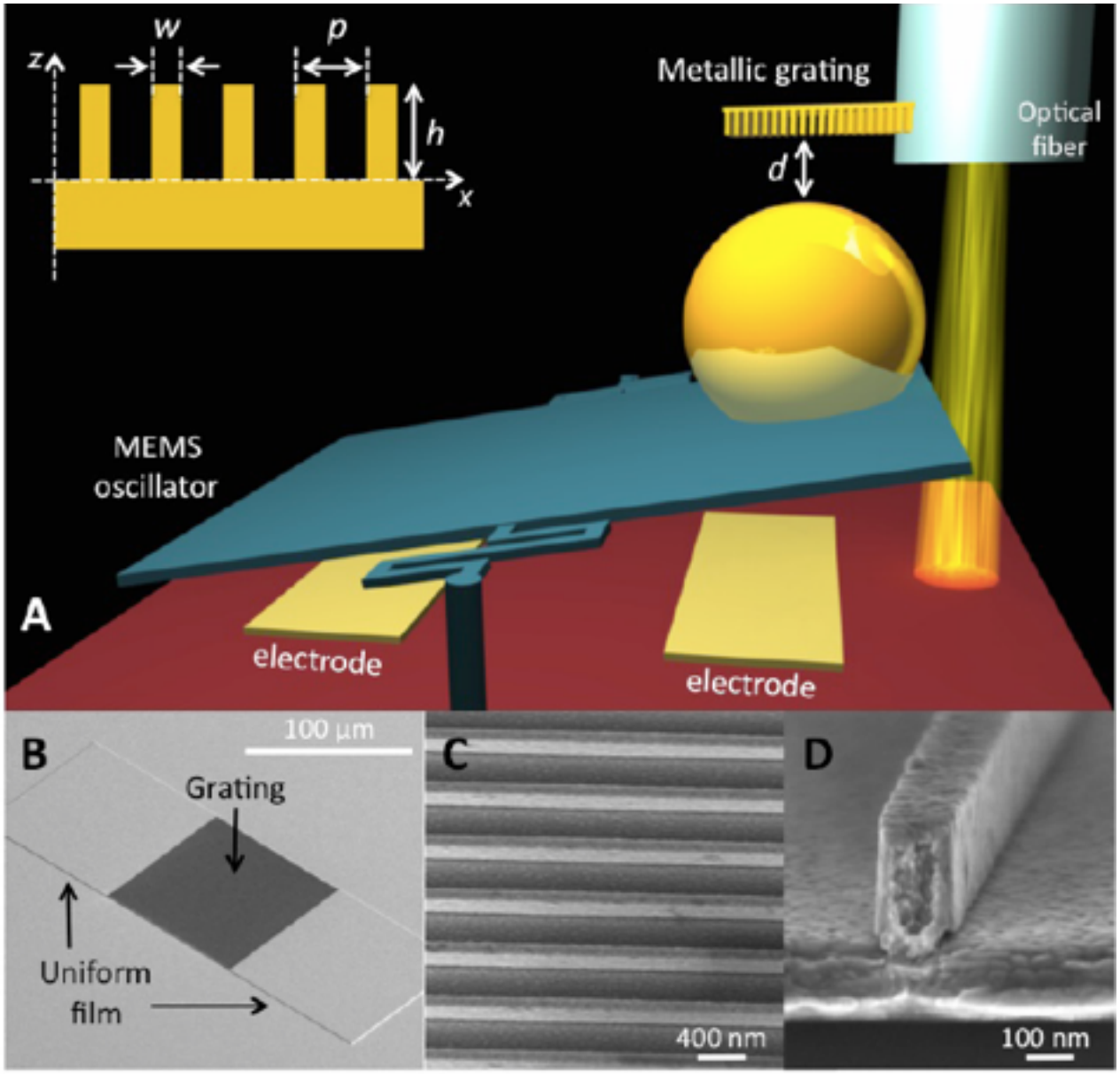


(Intravaia, Lambrecht , PRL 2005)

- The photonic contribution is always attractive
- The plasmonic contribution is repulsive at large distances, and attractive at short distances
- Their sum is always attractive due to a delicate cancellation

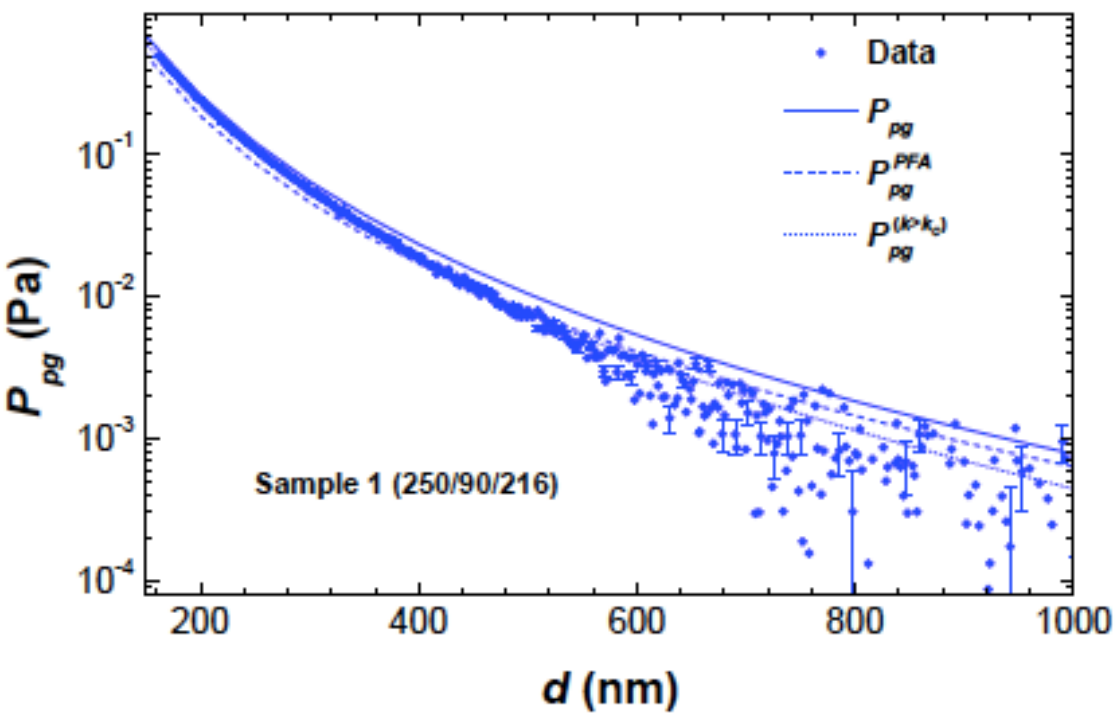
Can one control the Casimir force by changing the balance of the two contributions?

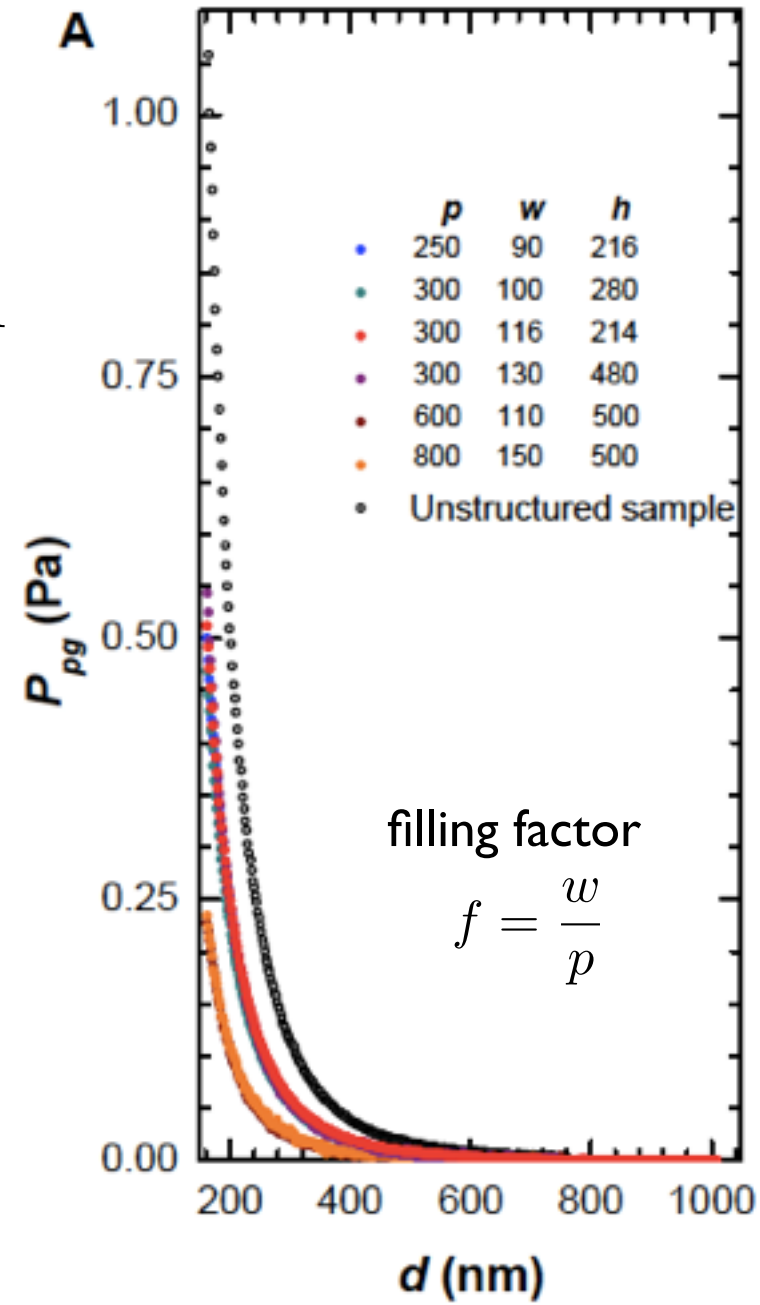
Metallic nano-gratings



Strong force reduction

- Torsional balance set-up
- Metallic sphere $(R = 150\ \mu\mathrm{m})$
- Metallic nanostructures $w, p, h \approx 100$ nm
- Sputtering and electroplating

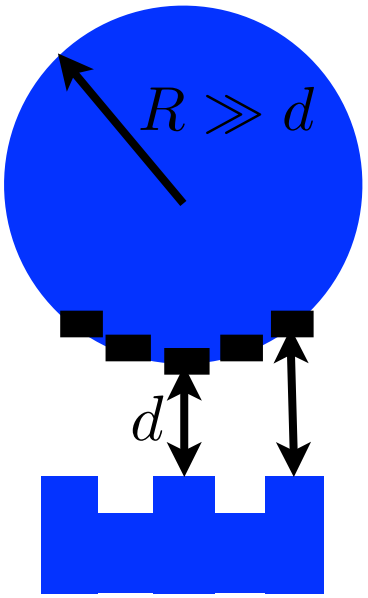




Modeling and simulation

- Use of standard **PFA to treat the sphere's curvature**

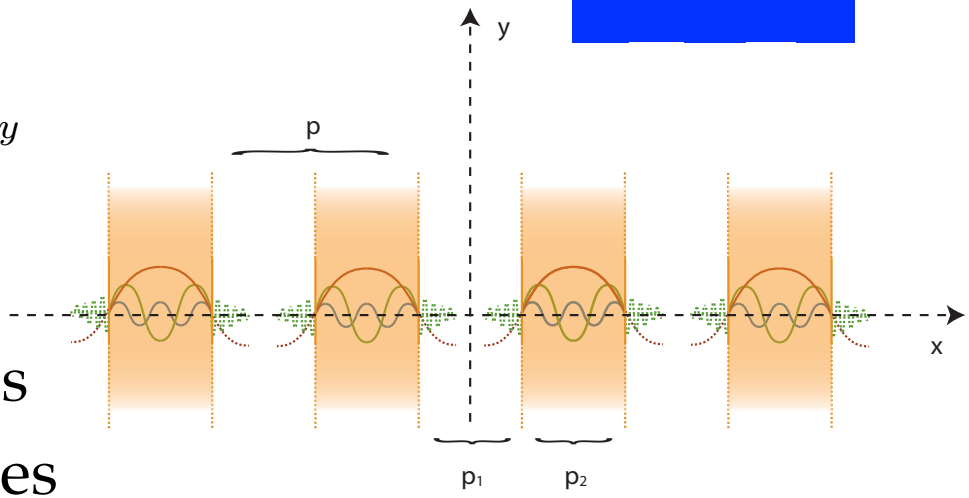
$$F'_{sg} \approx 2\pi R P_{pg} \qquad d/R < 6 \times 10^{-3}$$



- **Exact plane-grating pressure** P_{pg}

Scattering approach + modal expansions (Li 1993)

$$\begin{pmatrix} E_z(x,y) \\ E_x(x,y) \\ H_z(x,y) \\ H_x(x,y) \end{pmatrix}_i = \sum_{\nu,s} A_\nu^{(s,i)} \mathbf{Y}^{(s,i)}[x, \eta_\nu^{(s,i)}] e^{\imath\lambda[\eta_\nu^{(s,i)}]y}$$



Analytical expressions for eigenvectors

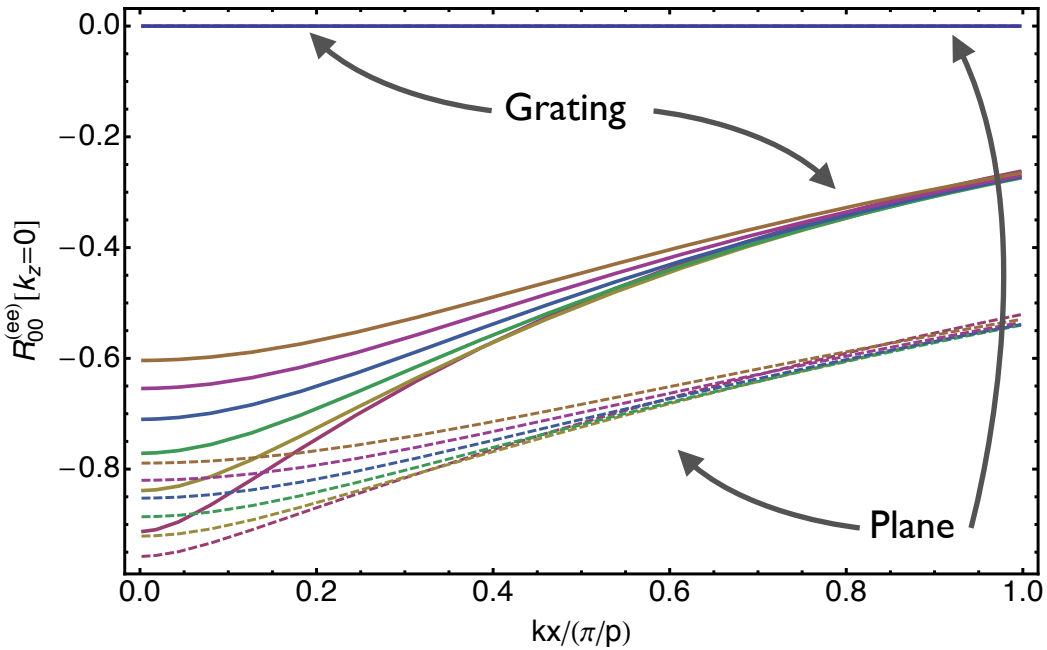
Transcendental equation for eigenvalues

$$0 = \tilde{D}^{(s)}(\eta) = -\cos(\alpha_0 p) + \cos(p_1\sqrt{\eta})\cos(p_2\sqrt{\eta - [\epsilon(i\xi)-1]\xi^2})$$
$$-\frac{1}{2}\left(\frac{\sqrt{\eta - [\epsilon(i\xi)-1]\xi^2}}{\sigma_2^{(s)}(i\xi)\sqrt{\eta}} + \frac{\sigma_2^{(s)}(i\xi)\sqrt{\eta}}{\sqrt{\eta - [\epsilon(i\xi)-1]\xi^2}}\right)\sin(p_1\sqrt{\eta})\sin(p_2\sqrt{\eta - [\epsilon(i\xi)-1]\xi^2}),$$

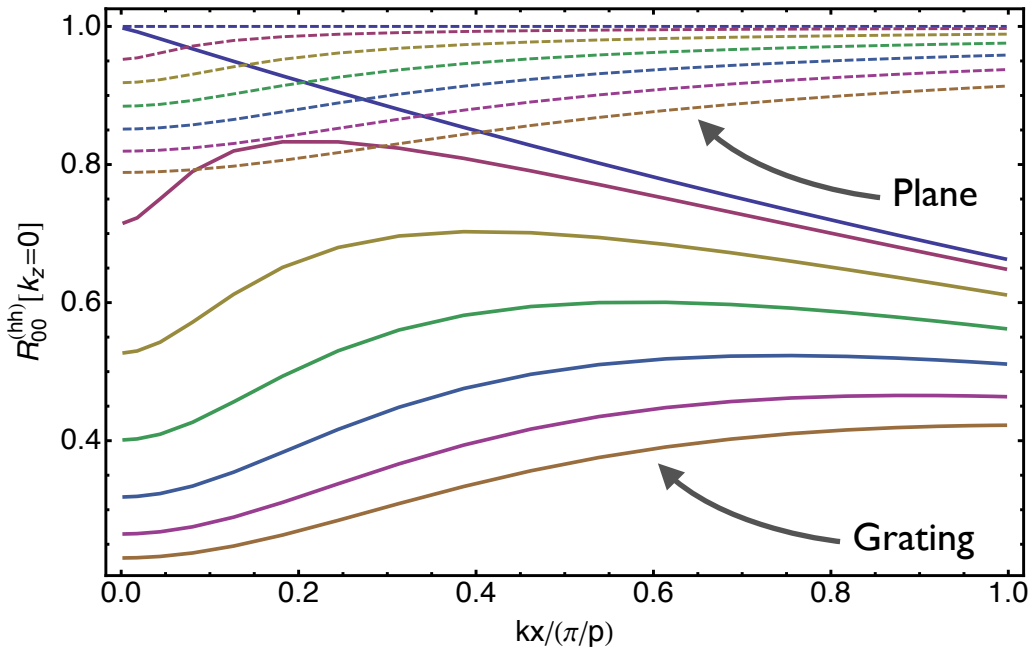
(Intravaia, DD *et al.* PRA 2012)

Reflection matrices

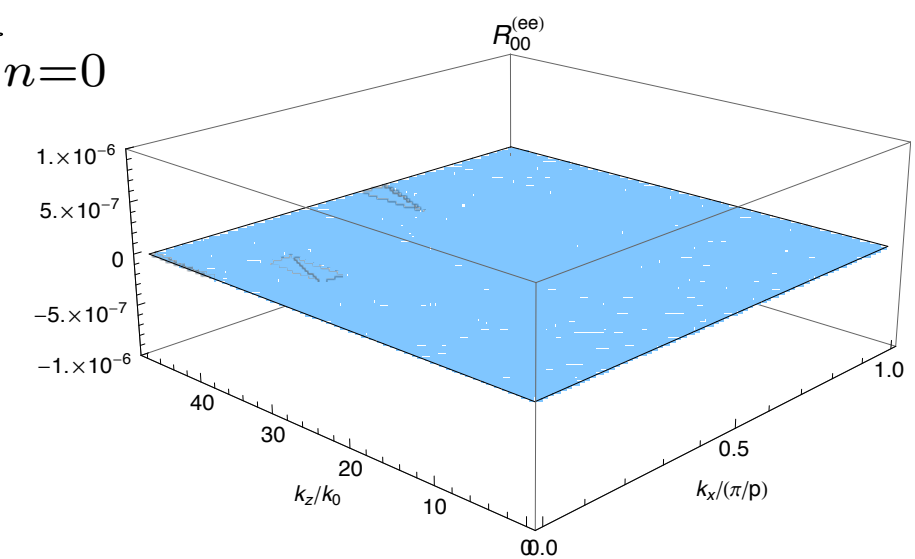
e-polarization (first 7 ξ_n's)



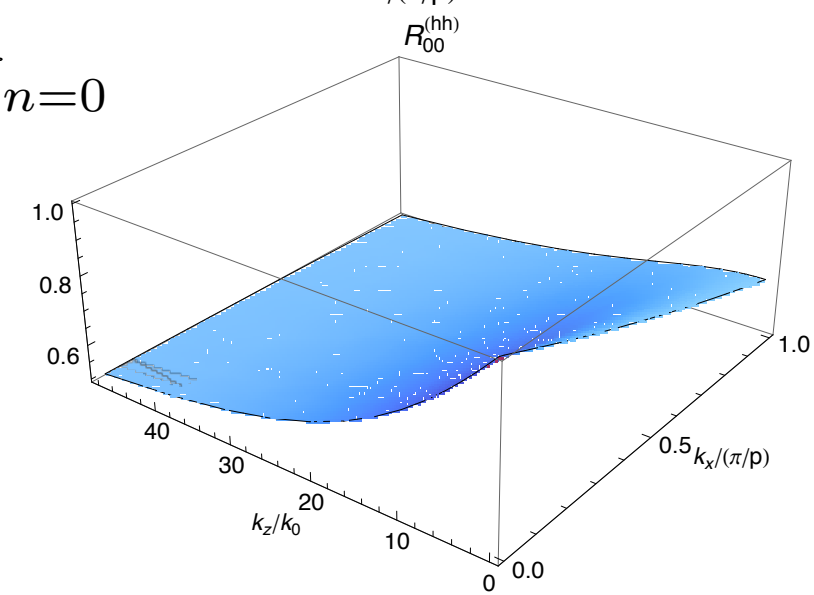
h-polarization (first 7 ξ_n's)



$\xi_{n=0}$

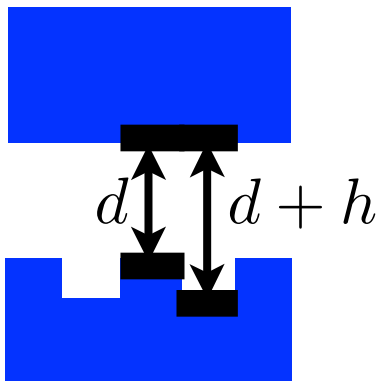


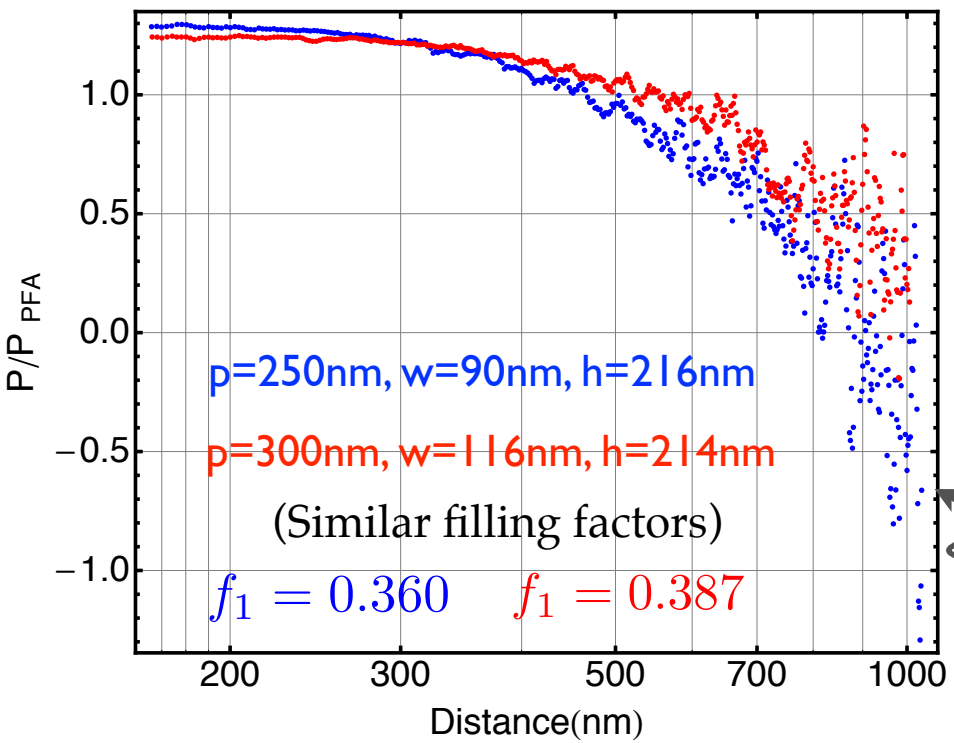
$\xi_{n=0}$



Normalizing to grating's PFA

$$P_{pg}^{\mathrm{PFA}}(d) = f P_{pp}(d) + (1-f) P_{pp}(d+h)$$





Small separations: PFA underestimates the total pressure

Large separations: PFA overestimates the exact pressure

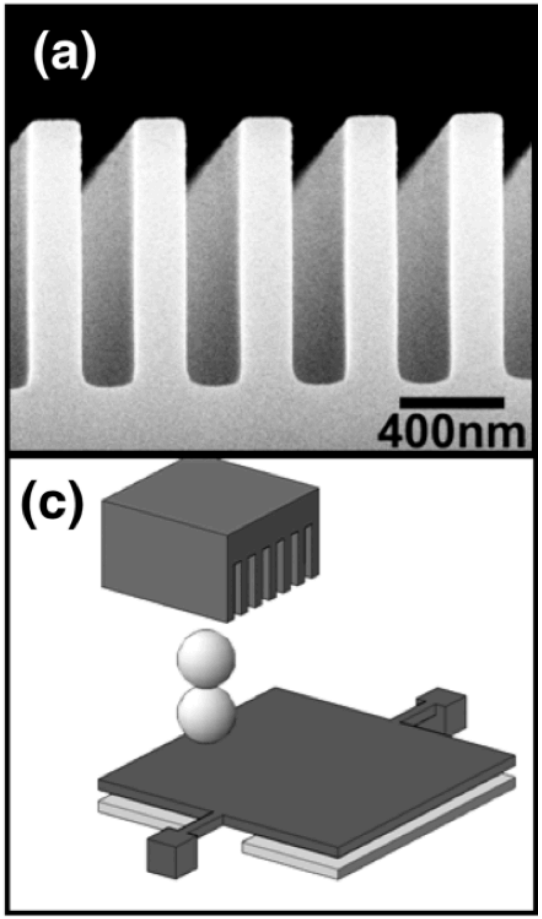
Pressure is going to zero faster than d^{-4}

Strong suppression of the Casimir force

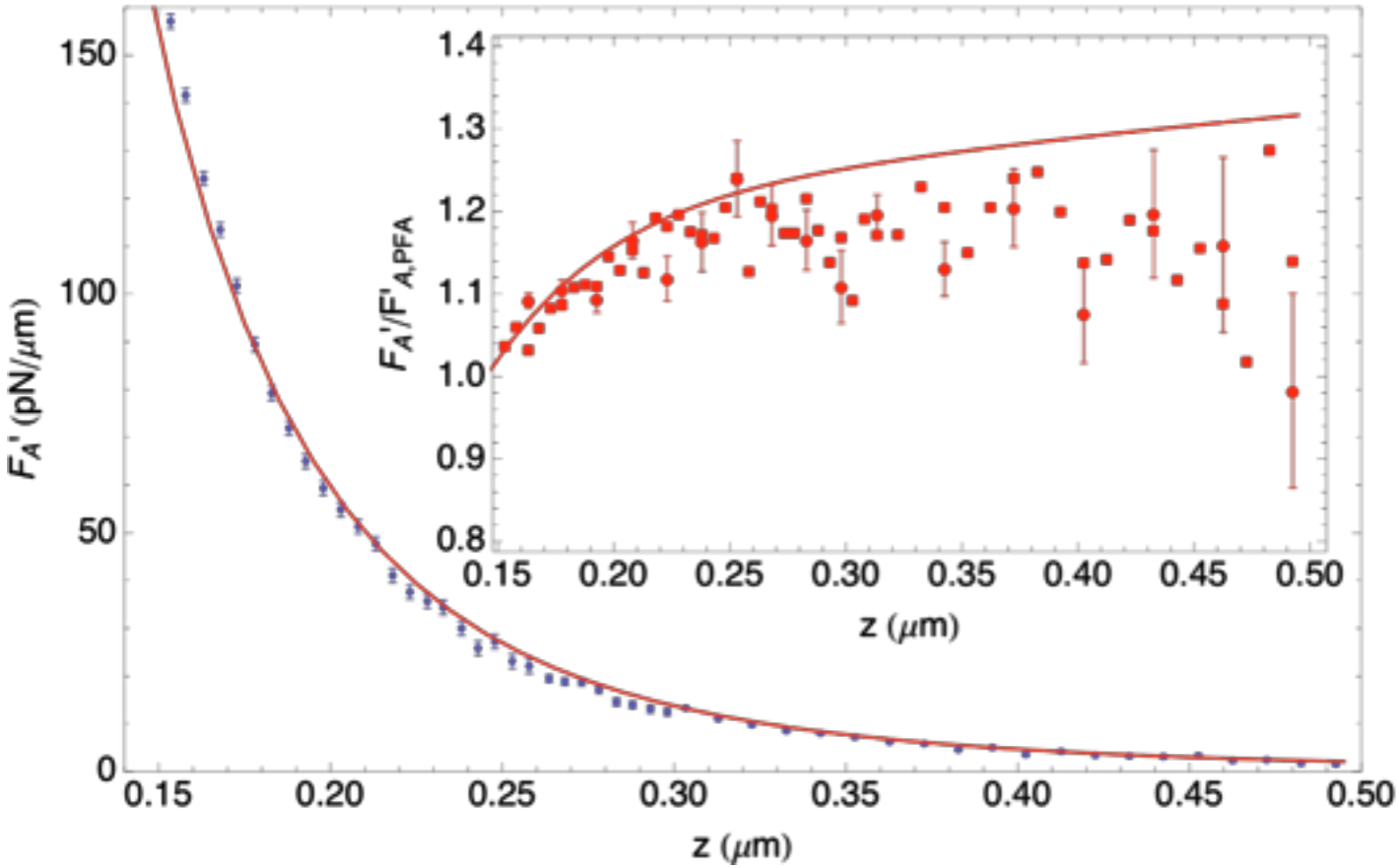
(Intravaia, DD *et al.*, Nature Communications 2013)

Previous works on Si gratings

(Chan *et al*, PRL 2008)



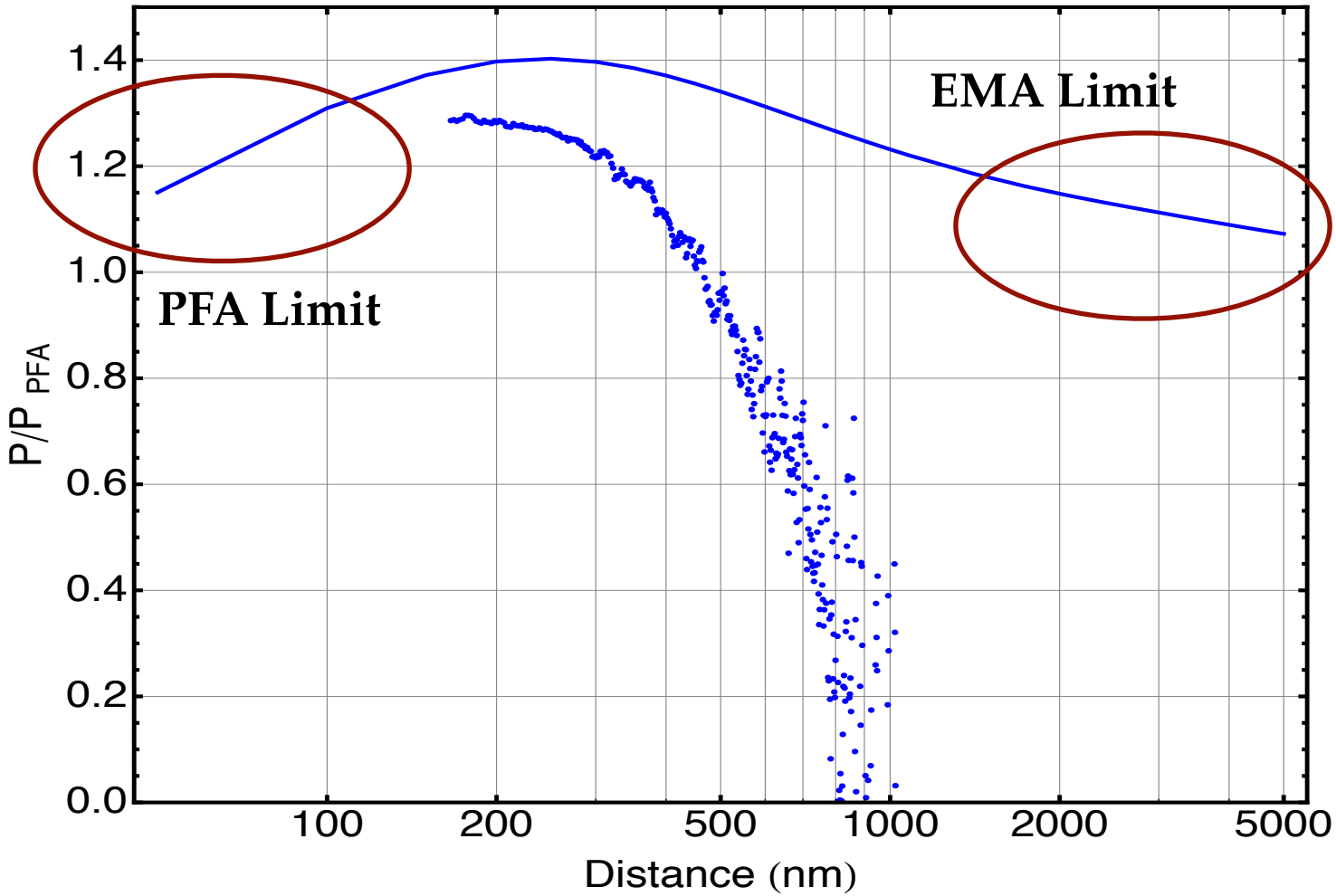
Sphere radius of 50 μm



period=1μm, depth = 1070 nm, and filling factor = 0.510

PFA underestimates the real force

Open problem



Numerical crosschecks show that the theory is accurate within few %

Double checks on the experiment show no apparent mistakes

Experiment/theory discrepancy: open problem in Casimir physics

nature COMMUNICATIONS

ARTICLE

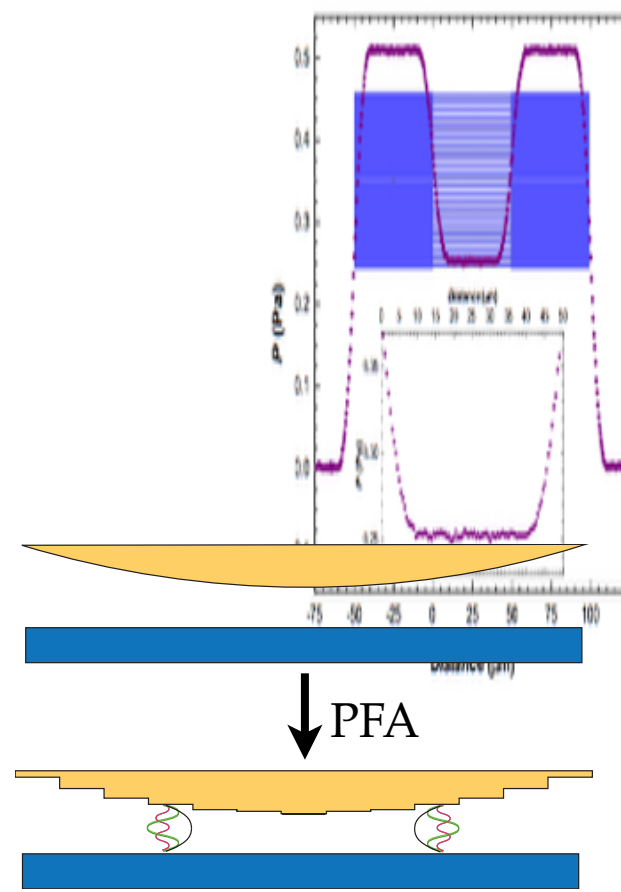
Received 5 Feb 2013 | Accepted 28 Aug 2013 | Published 27 Sep 2013 OPEN

Strong Casimir force reduction through metallic surface nanostructuring

Francesco Intravaia[1], Stephan Koev[2,3], Il Woong Jung[4], A. Alec Talin[2], Paul S. Davids[5], Ricardo S. Decca[6], Vladimir A. Aksyuk[2], Diego A.R. Dalvit[1] & Daniel López[4]

What is going on?

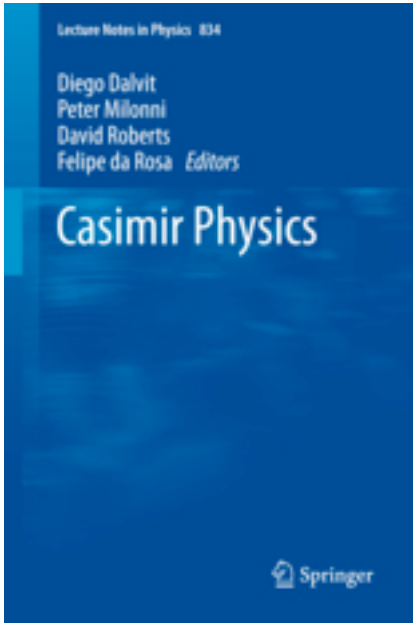
- **Are there problems with the experiment?**
  - set-up similar to previous ones
  - sphere-plane force re-obtained with new set-up
- **Are we correctly describing the experiment?**
  - finite-size grating
  - thermal equilibrium
- **Is something wrong with the theory?**
  - Reflection matrices
  - Optical properties
  - Surface roughness
  - Electrostatic patches
  - Validity of PFA for the sphere's curvature
  - etc



Final comments

- Importance of correct description of optical properties
- Narrow-band intuition (as in standard photonics) does not always work in Casimir physics
- Care must be exercised when using effective medium approximations in Casimir physics
- There are still open problems

Thank you!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