Recent progress in Casimir physics

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Collaborators

Los Alamos NATIONAL LABORATORY

❑ Theory:

- Peter Milonni (LANL) → Metamaterials
- Diego Mazzitelli (Buenos Aires) → Non-planar geometries
- Paulo Maia Neto (Rio de Janeiro) → Casimir-Polder forces
- Serge Reynaud (ENS, Paris) → Lateral Casimir forces
- Dima Mozyrsky (LANL) → Damping-Decoherence

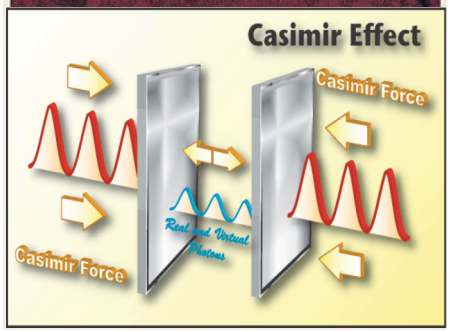
❑ Experiment:

- Roberto Onofrio (Dartmouth) → Cantilevers
- Ricardo Decca (Indiana) → NEMS and MEMS
- Steve Lamoreaux (Yale) → Torsional pendulums
- Toni Taylor (LANL, CINT) → Metamaterials, plasmonics
- Malcolm Boshier (LANL) → Bose-Einstein condensates

Outline

- ✓ The Casimir force: theory and experiments
- ✓ Tailoring the Casimir force:
  - ❑ by engineering different geometries
  - ❑ by engineering different materials
- ✓ The Casimir-Polder force
  - ❑ theory and experiments
  - ❑ non-trivial effects of geometry: the lateral CP force

The Casimir force



Casimir forces originate from changes in quantum vacuum fluctuations imposed by surface boundaries. Predicted by the Dutch physicist H. Casimir in 1948.

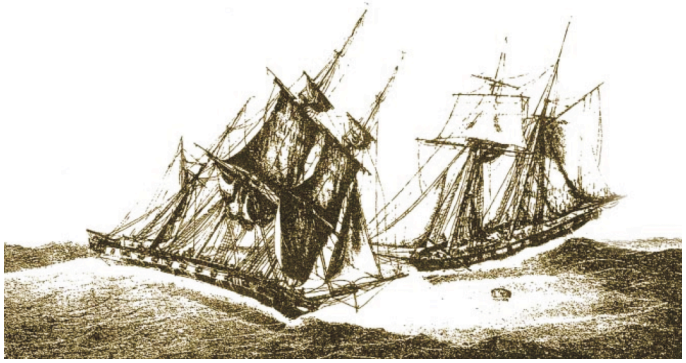
Dominant interaction in the micron and submicron lengthscales

$$\frac{F}{A} = \frac{\pi^2}{240}\frac{\hbar c}{d^4}$$

$$(130\mathrm{nN/cm^2} \ @ \ d = 1\mu\mathrm{m})$$

Classical analog

L'Album du Marin (1836)



Lifshitz theory

In reality, one needs to include corrections due to finite conductivity, finite temperature, roughness, etc.

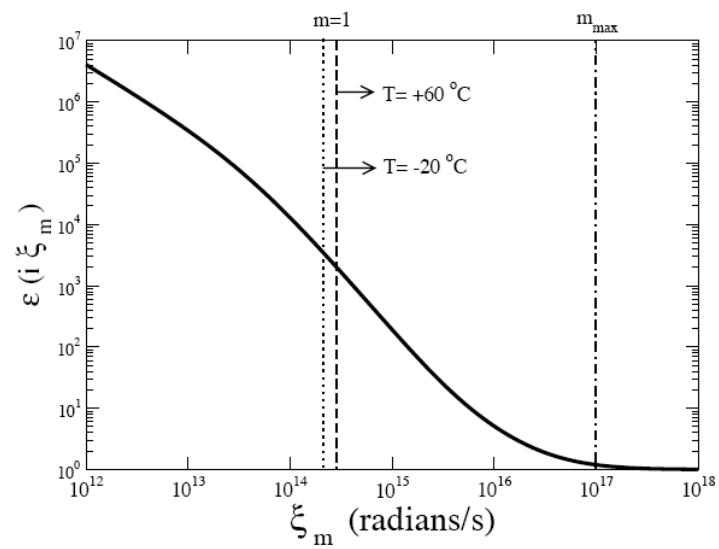
$$\frac{F}{A} = 2k_B T \sum_{n=0}^{\infty} \int_{\xi_n/c}^{\infty} \frac{d\kappa}{2\pi} \kappa^2 \sum_{\lambda=\mathrm{TE,TM}} \left(\frac{e^{2\kappa d}}{r_{\lambda_1} r_{\lambda_2}} - 1 \right)^{-1}$$

Matsubara frequencies: $\omega_n = i\xi_n = 2\pi i n k_B T/\hbar$

Reflection coefficients:

$$r_{\mathrm{TM}} = \frac{\epsilon(i\xi_n)c\kappa - \sqrt{\xi_n^2[\epsilon(i\xi_n)\mu(i\xi_n) - 1] + \kappa^2 c^2}}{\epsilon(i\xi_n)c\kappa + \sqrt{\xi_n^2[\epsilon(i\xi_n)\mu(i\xi_n) - 1] + \kappa^2 c^2}}$$

$$r_{\mathrm{TE}} = r_{\mathrm{TM}} \text{ with } \epsilon \leftrightarrow \mu$$

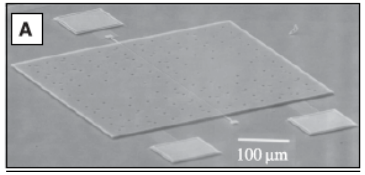


Optical data for gold

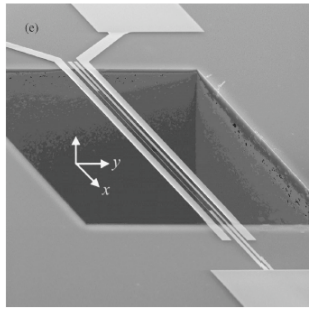
Why does it matter?

Nanotechnology:

- Actuation in NEMS and MEMS driven by Casimir forces
- Stiction of movable parts in MEMS and NEMS
- Tailoring the Casimir force with designer materials



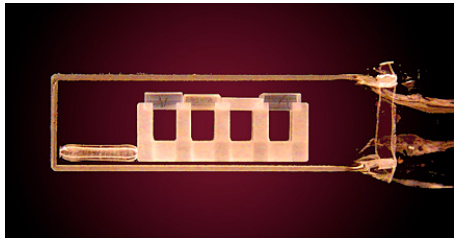
Capasso group



Roukes group

Quantum science and technology:

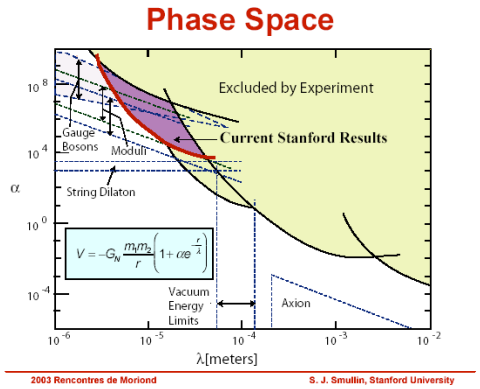
- Atom-surface interactions
- Atom chips: trapping and transport
- Precision measurements



Cornell group

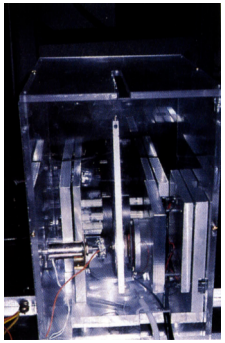
Quantum field theory - Gravitation:

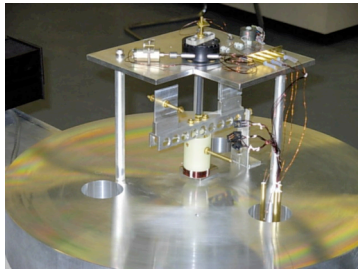
- Thermal dependence
- Background to measure non-Newtonian forces



Modern experiments

❑ Torsion pendulum (Lamoreaux)

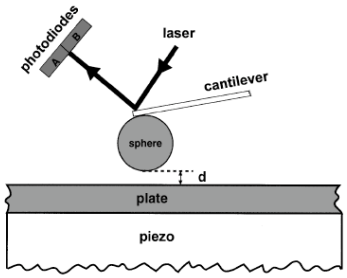




2007 prototype

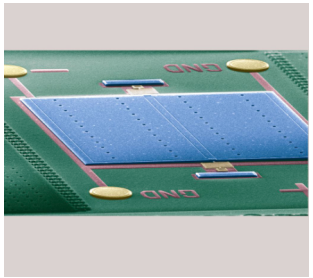
sphere-plane, d=1-10 um, precision 5%

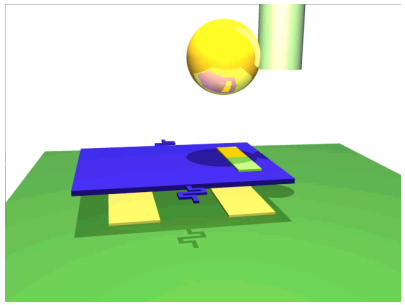
❑ AFM (Mohideen)



sphere-plane, d=200-1000 nm, prec. 2%

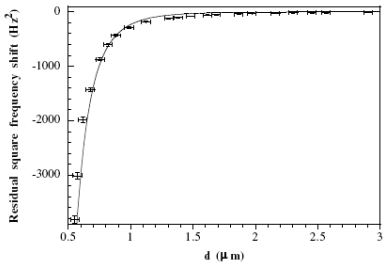
❑ MEMS and NEMS (Capasso, Decca)





sphere-plane, d=200-1000 nm, prec. 1%

❑ Micro-cantilevers (Onofrio)



plane-plane, d=0.5-3 um, prec. 15%
cylinder-plane, ongoing experiment

Tailoring the Casimir force

The magnitude and the sign of the Casimir force depend on the geometry and composition of surfaces

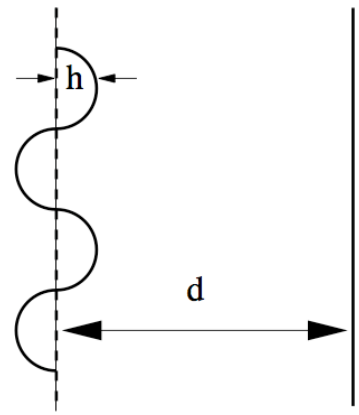
- **Vacuum drag via lateral Casimir forces**
- **Strongly modified and repulsive Casimir forces**
- **Temperature corrections to the Casimir force**

- **Analytical methods for non-planar geometries and real materials**
- **Numerical methods for computing the force for arbitrary geometries and materials**

Effects of geometry

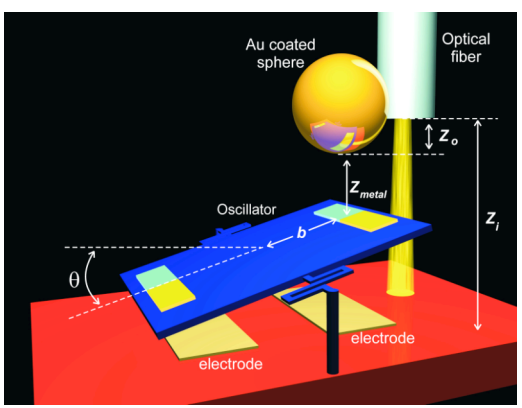
A common method for dealing with non-flat geometries is the **Proximity Force Approximation**, which effectively is an averaging over local plane-plane geometries

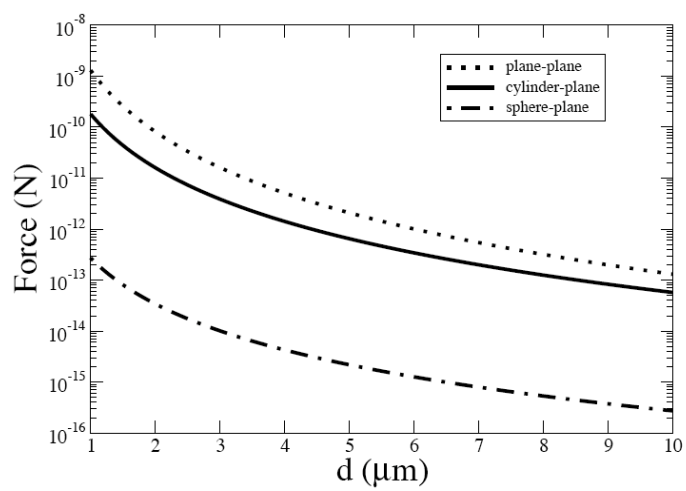
$$E_{\text{non-planar}} \approx \int \frac{d^2\mathbf{r}}{A} E_{\text{flat}}[d - h(\mathbf{r})] \qquad [\text{PFA}]$$



❑ sphere-plane

$$F_{sp} \approx \frac{\pi^3 \hbar c R}{360 d^3} \qquad (d \ll R)$$



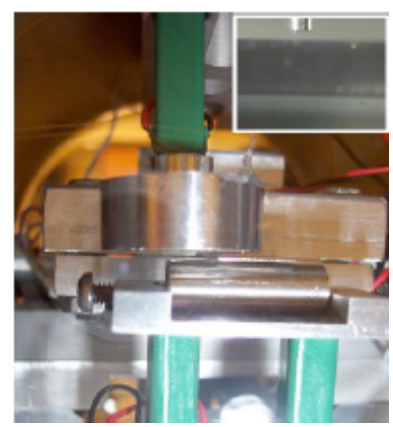


$A = 1\text{mm}^2 \quad L = 1\text{cm}$

$a = 100\mu\text{m} \quad R = 100\mu\text{m}$

❑ cylinder-plane

$$F_{cp} \approx \frac{\pi^3 \hbar c L a^{1/2}}{384\sqrt{2} d^{7/2}} \qquad (d \ll a)$$



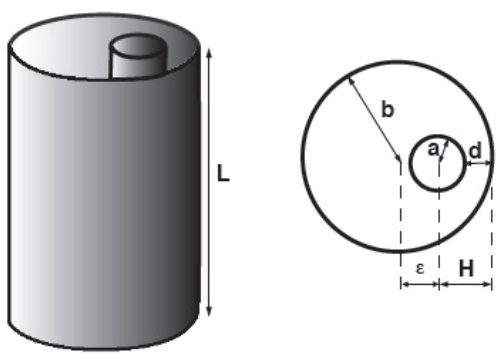
Beyond PFA: exact calculations

For highly symmetric problems, it is possible to calculate exactly the Casimir energy, solving an **eigenvalue problem**

$$E = \frac{\hbar}{2}\sum_p (\omega_p - \tilde{\omega}_p)$$

Argument theorem: $$\frac{1}{2\pi i}\int_C d\lambda\, \lambda\, e^{-\sigma\lambda}\frac{d}{d\lambda}\ln f(\lambda) = \sum_i \lambda_i\, e^{-\sigma\lambda_i} \qquad f(\lambda_i) = 0,\ \text{analytic}$$

❑ Eccentric cylinders



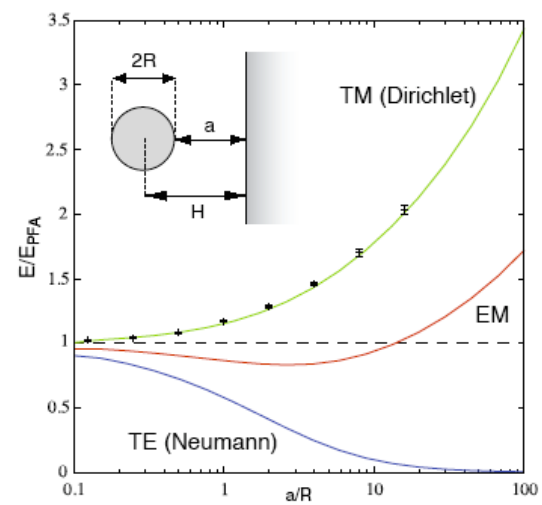
$$\lambda_i: \quad \det[Q_{mn}^{\mathrm{TM,TE}}] = 0$$

$$Q_{mn}^{\mathrm{TM}} = [J_n(\lambda a)N_m(\lambda b) - J_m(\lambda b)N_n(\lambda a)]J_{n-m}(\lambda\epsilon)$$

$$Q_{mn}^{\mathrm{TE}} = [J_n'(\lambda a)N_m'(\lambda b) - J_m'(\lambda b)N_n'(\lambda a)]J_{n-m}(\lambda\epsilon)$$

$$\epsilon, b \to \infty$$
$$a, d \text{ fixed}$$

❑ Cylinder-plane



- *"Exact Casimir interaction between eccentric cylinders"*, Dalvit *et al*, PRA **74**, 020101 (R) (2006)
- *"Casimir interaction between a cylinder and a plane"*, Emig *et al*, PRL **96**, 080403 (2006)

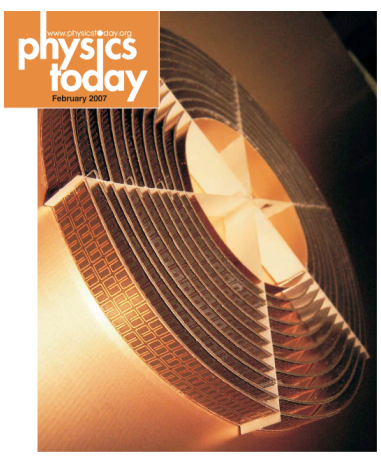
Effects of material structure

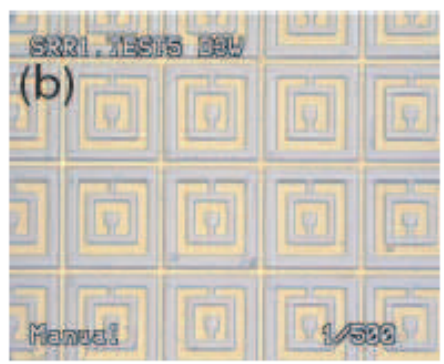
The Casimir force can be tailored via the **frequency-dependent reflection amplitudes** of the materials. The main contributing frequencies are in the **near-infrared/optical region** (gaps: 200 nm- 10 um)

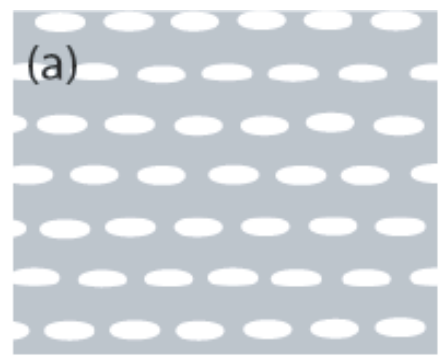
$$\frac{F}{A} = 2k_B T \sum_{n=0}^{\infty} \int_{\xi_n/c}^{\infty} \frac{d\kappa}{2\pi} \kappa^2 \sum_{\lambda=\mathrm{TE,TM}} \left(\frac{e^{2\kappa d}}{\boxed{r_{\lambda_1} r_{\lambda_2}}} - 1 \right)^{-1} \rightarrow \epsilon(\omega), \mu(\omega)$$

Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials <u>do not have</u> strong magnetic response close to the optical region, i.e. $\mu = 1$

Possible solution: artificial materials (MMs and plasmonic structures)



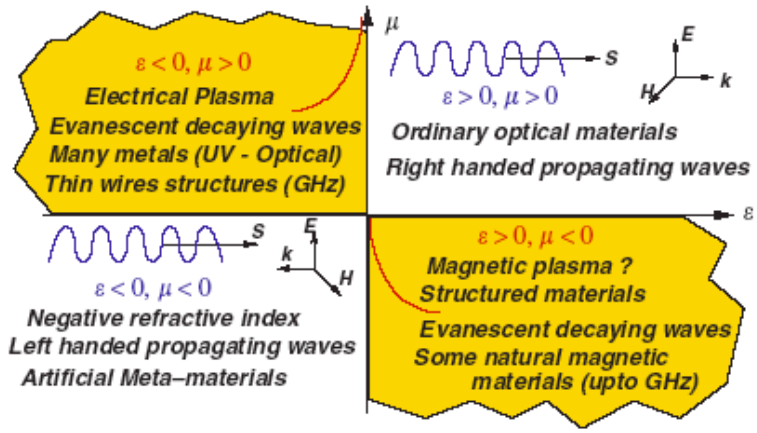




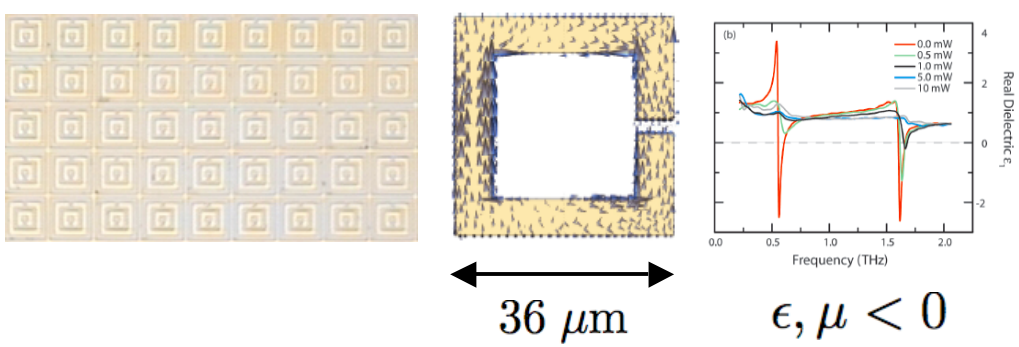
Metamaterials

- Artificially structures composites with designer electromagnetic properties
- Maxwell eqns for **dispersive magneto-dielectric media**

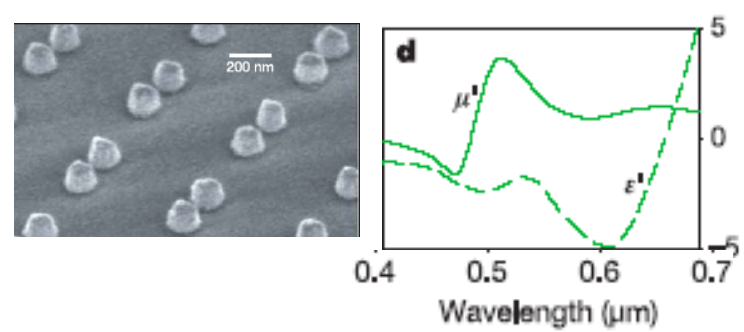
- **Negative-refraction** th: Veselago (1968)
  exp: Smith group (2000)
- **Perfect lens** th: Sir Pendry (2000)
- **Cloaking** exp: Smith group (2007)



THz MMs:
Split Ring Resonators



Optical MMs:
Nanopillars

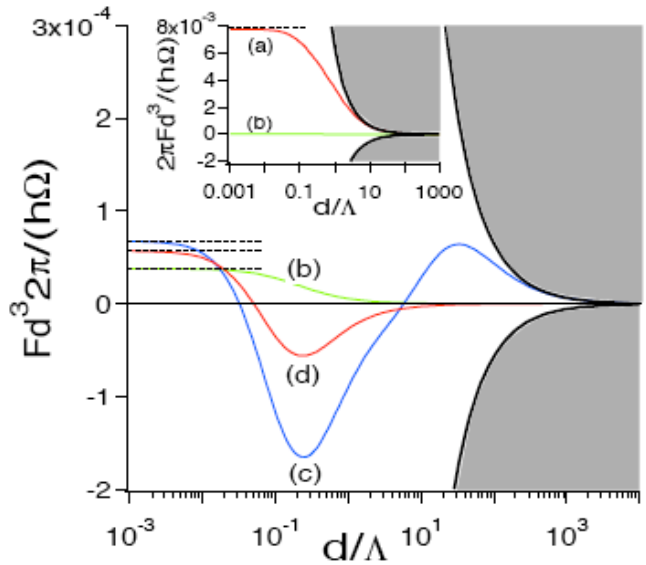


Casimir repulsion

Drude-Lorentz model:

$$\epsilon_\alpha(i\xi_n) = 1 + \frac{\Omega_\alpha^2}{\omega_\alpha^2 + \xi_n^2}$$

$$\mu_\alpha(i\xi_n) = 1 + \frac{\Theta_\alpha^2}{\nu_\alpha^2 + \xi_n^2}$$



$$\Lambda = 2\pi c/\Omega \simeq 200\text{nm} - 10\mu\text{m}$$

Attraction

(a) Two identical non-magnetic Drude metals

(b) Two identical MMs

Repulsion

(c) Drude metal + MM

(d) Purely dielectric MM + mainly magnetic MM

- *"Casimir force between designed materials: what is possible and what not"*, Henkel and Joulain, EPL **72**, 929 (2005)
- *"Quantum levitation by left-handed metamaterials"*, Leonhard and Philbin, quant-ph/0608115v4
- *"van der Waals interactions in a magnetodielectric medium"*, Dalvit et al, PRA **75**, 052117 (2007)
- *"Surface plasmon modes and the Casimir energy"*, Intravaia and Lambrecht, PRL **94**, 110404 (2005)

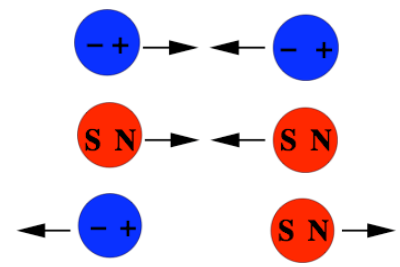
Main experimental difficulty: fabrication of **nano-structured** MMs with strong magnetic response at optical frequencies

The Casimir-Polder force

Casimir-Polder forces are dispersive forces between polarizable objects. They are Casimir force "cousins".

atom ⟷ atom atom ⟷ body

1. Two electrically polarizable objects ($\alpha_e \longleftrightarrow \alpha_e$) attract
2. Two magnetically polarizable objects ($\alpha_m \longleftrightarrow \alpha_m$) attract
3. Two objects of opposed polarizability ($\alpha_e \longleftrightarrow \alpha_m$) repel



Revelant applications:

- ❑ **Atom-surface interactions (atom optics, atom chips)**
- ❑ **Precision measurements**

van der Waals/ Casimir-Polder

Atom+body interaction potential:

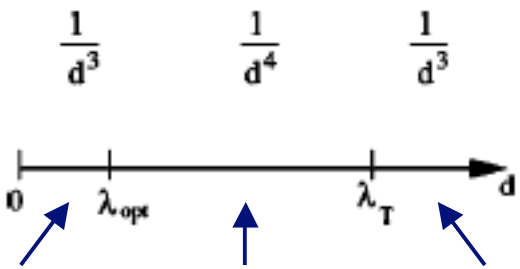
$$U_A(\mathbf{r}_A) = \frac{\hbar}{2\pi}\int_0^\infty du u^2 \alpha_e^A(iu)\mathrm{Tr}[G(\mathbf{r}_A,\mathbf{r}_A,iu)]$$

Scattering Green tensor:

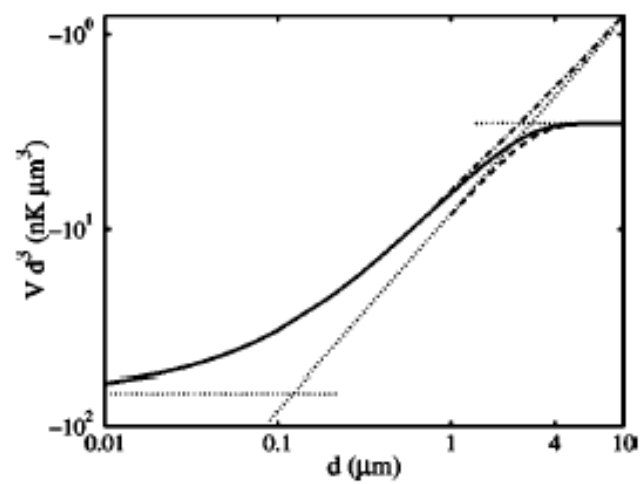
$$\left[\nabla\times\kappa(\mathbf{r},\omega)\nabla\times - \frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\right]G(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r},\mathbf{r}',\omega)$$

$$G(\mathbf{r},\mathbf{r}',\omega)\to 0 \;\text{ for }\; |\mathbf{r}-\mathbf{r}'|\to\infty$$

$$\kappa(\mathbf{r},\omega) = \mu^{-1}(\mathbf{r},\omega)$$



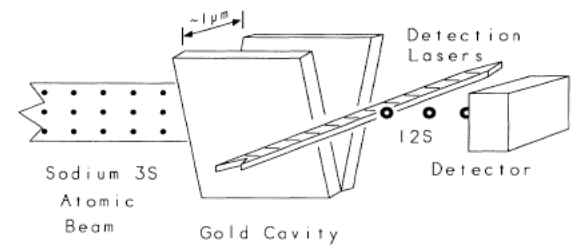
van der Waals (non-retarded limit) **Casimir-Polder** (retarded limit) **Thermal regime**



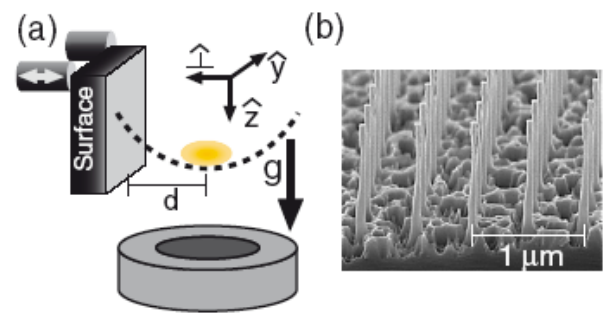
$$\lambda_{\mathrm{opt}} \approx 500\ \mathrm{nm} \qquad \lambda_T = \frac{\hbar c}{k_B T} = 7.6\mu\mathrm{m}\ @\ T = 300\mathrm{K}$$

Experimental detection

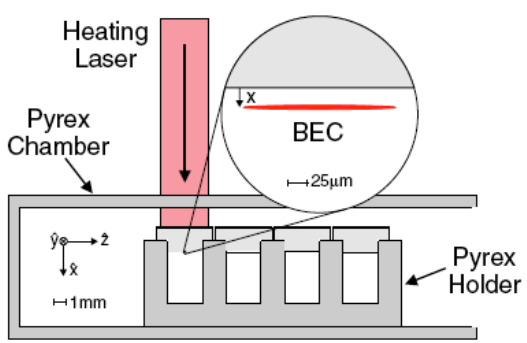
- **Deflection of Na atoms** — Hinds group



- **Quantum reflection** — Aspect group, Shimizu Vuletic group Ketterle group

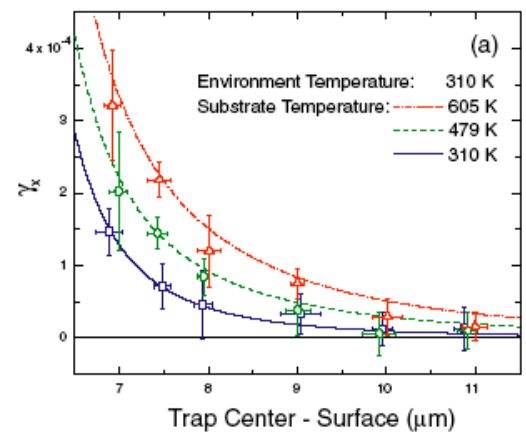


- **BEC oscillations** — Cornell group



$$\gamma_x \equiv \frac{\omega_o - \omega_x}{\omega_o} \simeq \frac{1}{2m\omega_o^2}\langle \partial_x F_{CP} \rangle,$$

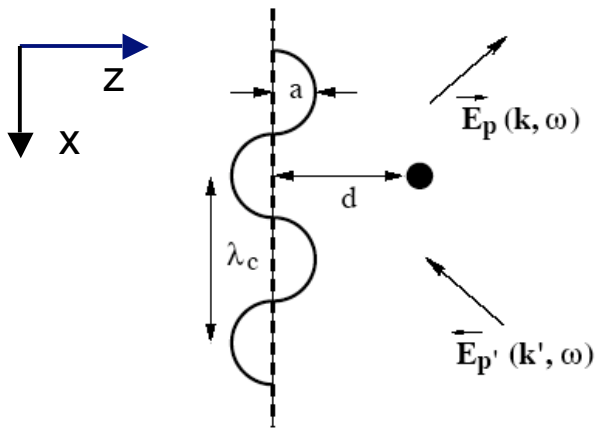
$$U_{NEQ} \sim (T_S^2 - T_E^2)/x^2.$$



Temperature corrections

Lateral Casimir-Polder force

Non-trivial effects of geometry: corrugated surfaces



$$\vec{E}_p = (\mathbf{k},\omega) = \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \sum_{p'} \langle \mathbf{k}, p | \mathcal{R}(\omega) | \mathbf{k}', p' \rangle \; \overleftarrow{E}_{p'}(\mathbf{k}',\omega)$$

Non specular reflection matrices

$$h(\mathbf{r}) = a\cos(k_c x)$$

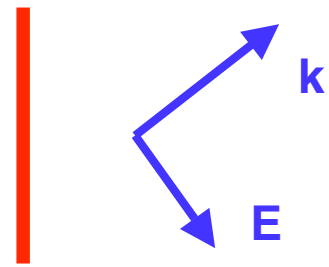
Interaction energy

$$\boxed{U^{(1)}(\mathbf{R}_A) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}_A}\, g(\mathbf{k}, z_A)\, H(\mathbf{k})}$$

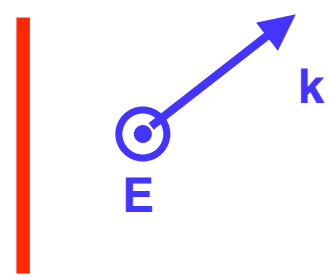
$$g(\mathbf{k}, z_A) = \frac{\hbar}{c^2\epsilon_0} \int_0^\infty \frac{d\xi}{2\pi}\, \xi^2\, \alpha(i\xi) \int \frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\mathbf{k}',\mathbf{k}'-\mathbf{k}}(z_A, \xi)$$

$$a_{\mathbf{k},\mathbf{k}'}(z_A, \xi) = \frac{1}{2\kappa'} \sum_{p,p'} \hat{\epsilon}_p^+ \cdot \hat{\epsilon}_{p'}^-\, e^{-(\kappa+\kappa')z_A} R_{pp'}(\mathbf{k}, \mathbf{k}')$$

TE modes



TM modes



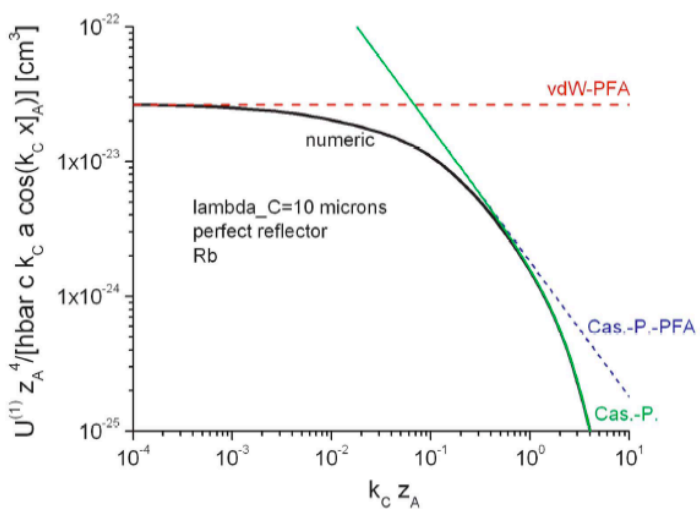
Beyond PFA geometry effects

- **PFA limit: smooth surface** $k_c z_A \to 0 \qquad (k_c = 2\pi/\lambda_c)$

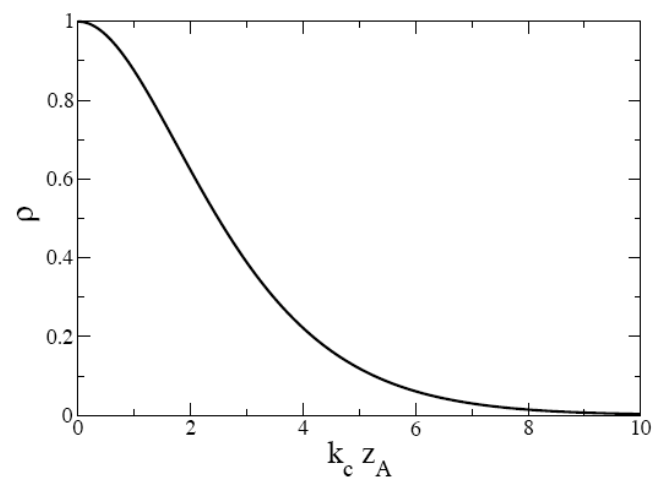
$$U(\mathbf{R}_A) \approx U^{(0)}(z_A) - h(\mathbf{r}_A)\,\frac{dU^{(0)}(z_A)}{dz_A}$$

- **Deviations from PFA** $\rho = \dfrac{g(k_c, z_A)}{g(0, z_A)}$

Eg: Rb atom in front of a perfectly conducting sinusoidal corrugated surface



Exact interaction potential



Deviations from PFA

Lateral force BEC measurement

Measure dipole oscillations of an elongated BEC perpendicular to the corrugation lines (along the x direction)

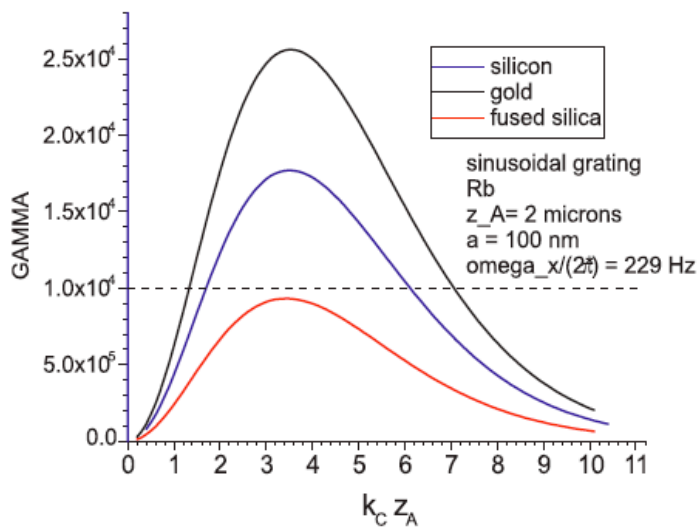
BEC density (Thomas-Fermi limit): $n(x,z) = \frac{15}{6\pi}\frac{1}{R^5}\left[R^2 - (x^2+z^2)\right]^{3/2}$

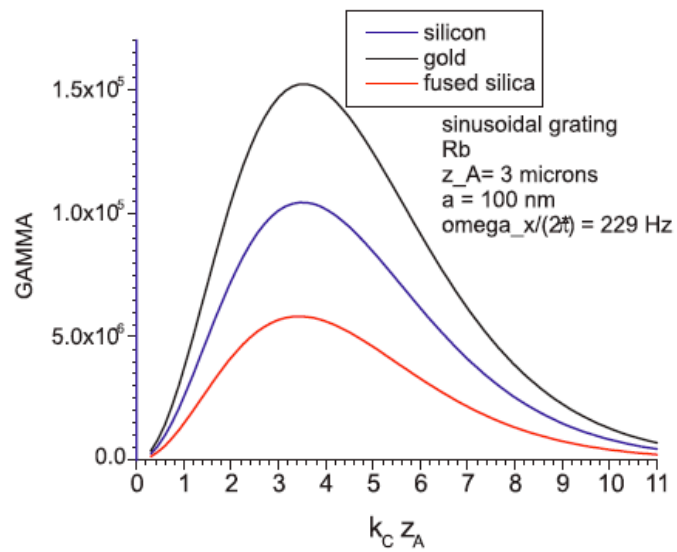
Frequency shift: $\omega_{x,\mathrm{CM}}^2 = \omega_x^2 + \frac{1}{m}\int dx dz\, n(x,z)\,\frac{\partial^2}{\partial x^2}V_{\mathrm{CP}}(x,z)$

$$\gamma = \frac{\omega_{x,\mathrm{CM}} - \omega_x}{\omega_x}$$

Calculations for Rb atoms and real materials (conductors, semiconductors, insulators)

$z_A = 2\mu\mathrm{m}$





$z_A = 3\mu\mathrm{m}$

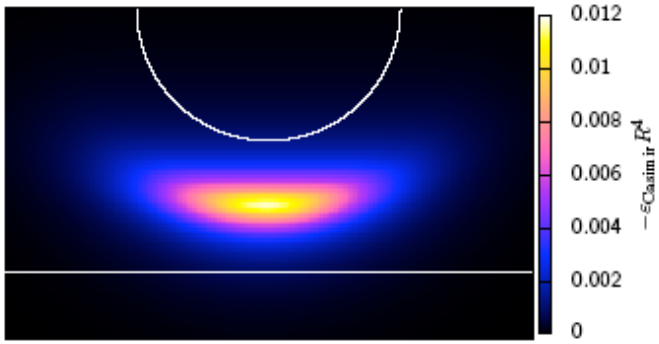
Numerical algorithms for Casimir

❑ Wordline path-integral approach

- *"Wordline algorithms for Casimir configurations"*, Gies *et al*, PRD **74**, 045002 (2006)

Only for scalar fields,

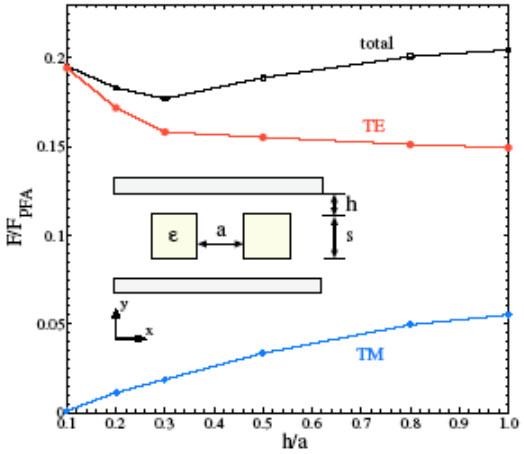
ideal (Dirichlet) boudary conditions



❑ Finite-element method for solving EM Green function

- *"Computation and visualization of Casimir forces in arbitrary geometries"*, Joannopoulus *et al*, arXiv: 0704.1890v2

Electromagnetic field, real boundary conditions, real materials



Summary

After 60 years the Casimir force is still surprising and challenging

- ✓ **Engineering the Casimir force via geometry and designer materials**
- ✓ **Precision measurements with MEMS, NEMS, BECs, optical lattices,…**
- ✓ **Applications in nanotechnology and quantum science**