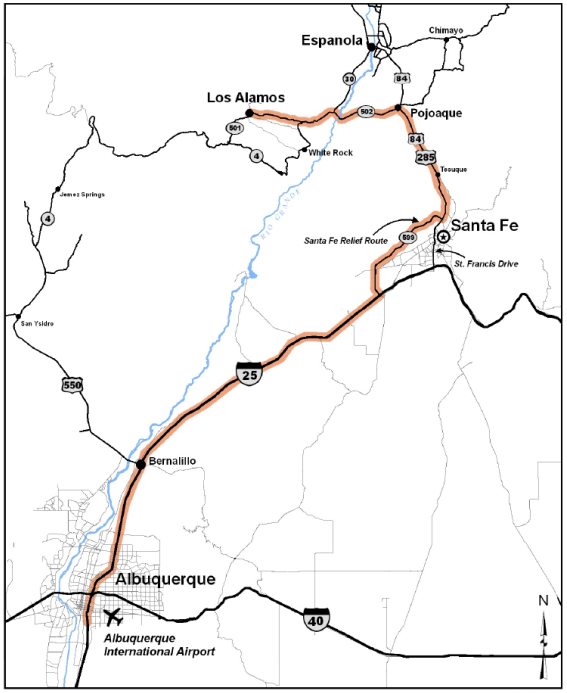
Towards Casimir force repulsion with metamaterials

Diego A. R. Dalvit
Los Alamos National Laboratory



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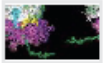
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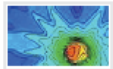
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Collaborators

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Theory:

Engineering the Casimir force with metamaterials and plasmonics

Peter Milonni (LANL)
Felipe da Rosa (LANL)
Francesco Intravaia (LANL)

Experiments:

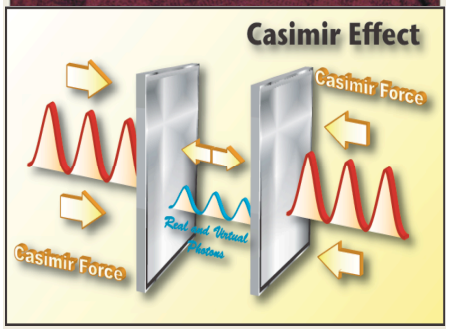
Casimir force measurements:

Roberto Onofrio (Dartmouth)
Steve Lamoreaux (Yale)
Ricardo Decca (Indiana)
Maarten de Boer, Stephen Howell (Sandia)

Metamaterials for Casimir force:

John O'Hara, Toni Taylor (LANL)
Igal Brener, Paul Davids (Sandia)

The Casimir force



Casimir forces originate from changes in quantum vacuum fluctuations imposed by surface boundaries

They were predicted by the Dutch physicist Hendrik Casimir in 1948

$$E = \frac{1}{2}\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \Rightarrow \boxed{\frac{F}{A} = \frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$

$$(130\mathrm{nN/cm}^2 \ @ \ d = 1\mu\mathrm{m})$$

Dominant interaction in the micron and sub-micron lengthscales

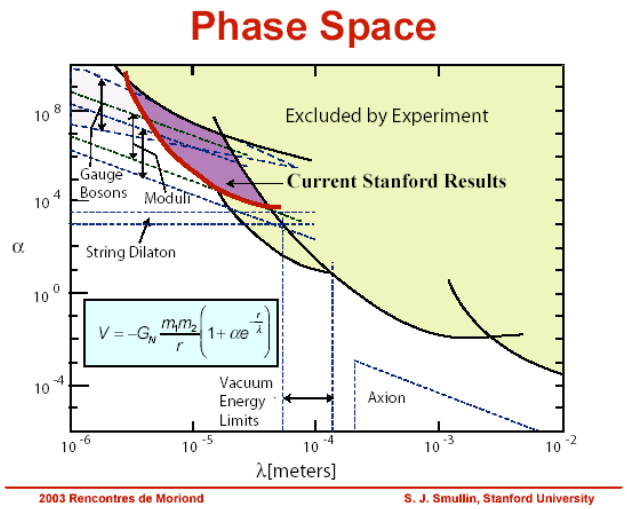
The magnitude and sign of the Casimir force depend on the geometry and composition of surfaces

Some relevant applications

Gravitation / Particle theory:

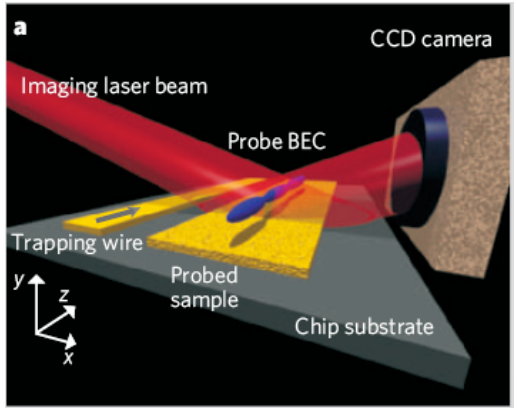
The Casimir force is the main background force to measure these non-Newtonian corrections to gravity

$$V(r) = -G\frac{m_1 m_2}{r}\left(1 + \alpha\, e^{-r/\lambda}\right)$$



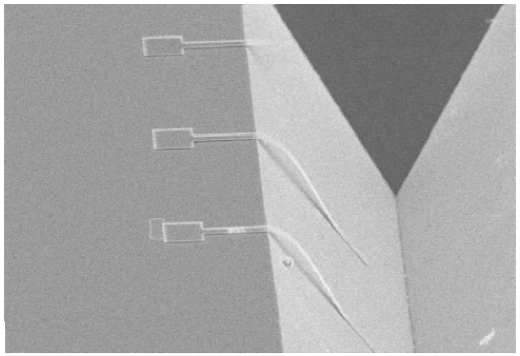
Quantum Science and Technology:

Atom-surface interactions (e.g., atom chips, BECs) and precision measurements



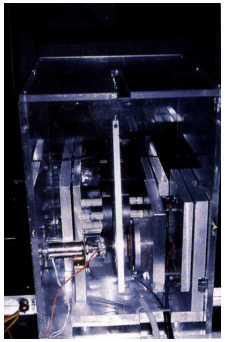
Nanotechnology:

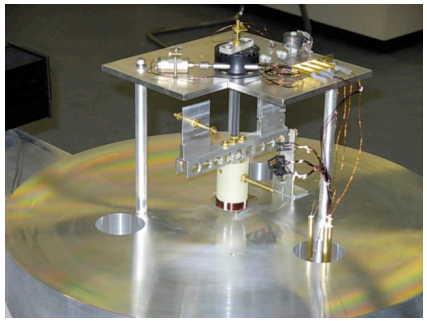
Casimir force is a challenge (stiction), but also an opportunity (contactless force transmission)



Modern Casimir experiments

- Torsion pendulum



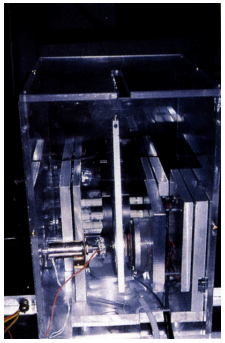


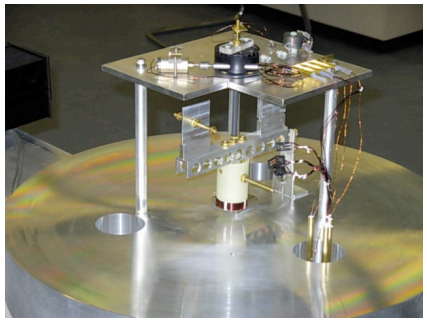
sphere-plane, d=1-10 um

Lamoreaux

Modern Casimir experiments

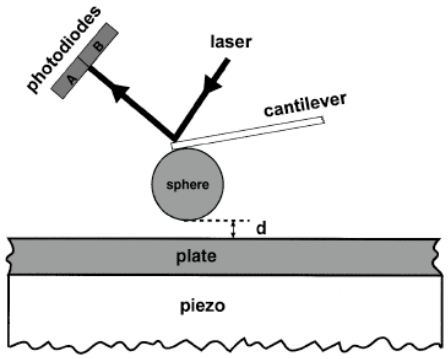
- Torsion pendulum





sphere-plane, d=1-10 um
Lamoreaux

- Atomic force microscope



sphere-plane, d=200-1000 nm
Mohideen et al

Modern Casimir experiments

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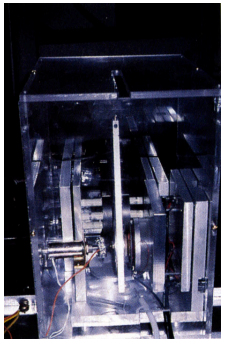
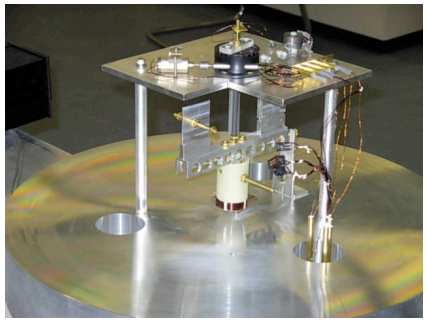
Torsion pendulum

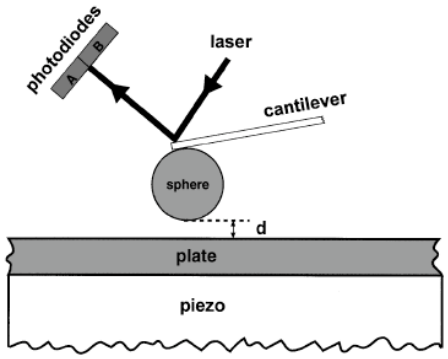




sphere-plane, d=1-10 um

Lamoreaux

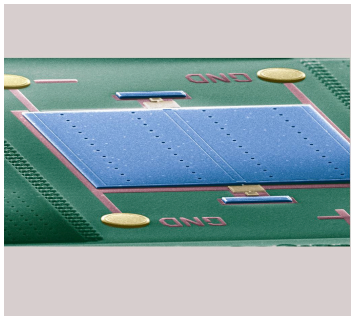
Atomic force microscope



sphere-plane, d=200-1000 nm

Mohideen et al

MEMS and NEMS



sphere-plane, d=200-1000 nm

Capasso et al, Decca et al

Modern Casimir experiments

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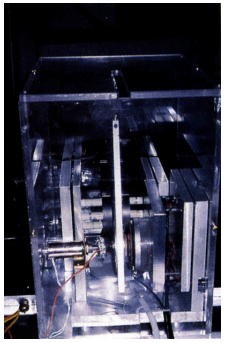
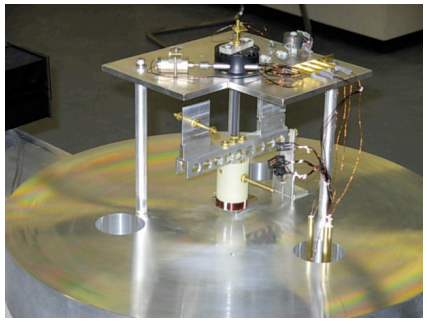
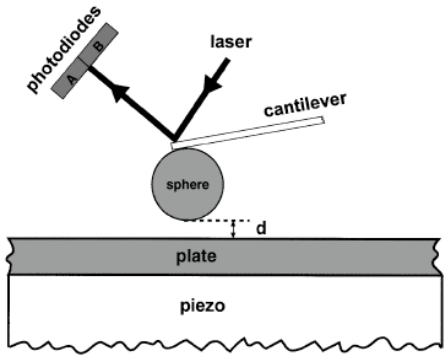




sphere-plane, d=1-10 um

Lamoreaux

Atomic force microscope



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Mohideen et al

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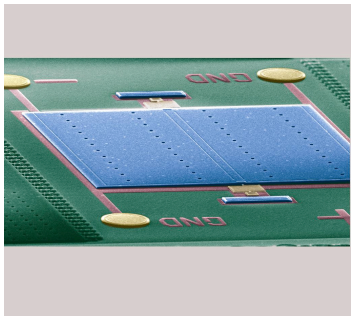
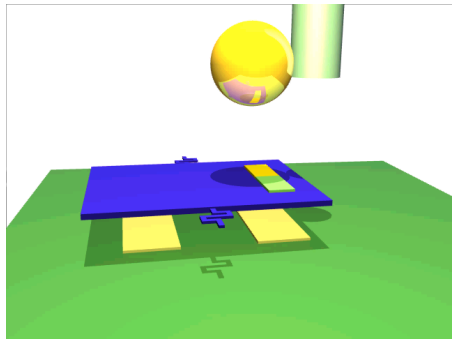


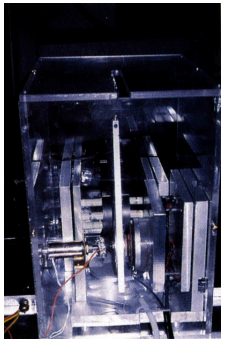


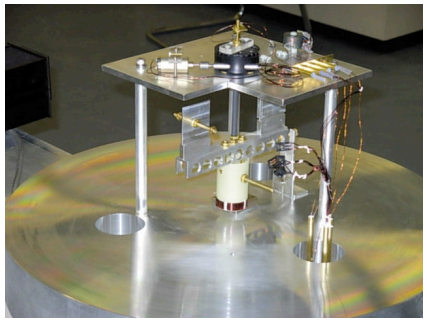
sphere-plane, d=200-1000 nm

Capasso et al, Decca et al

Modern Casimir experiments

Torsion pendulum





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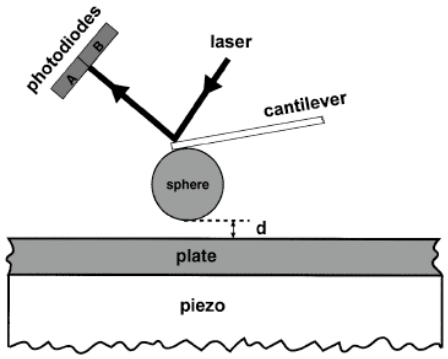

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Mohideen et al

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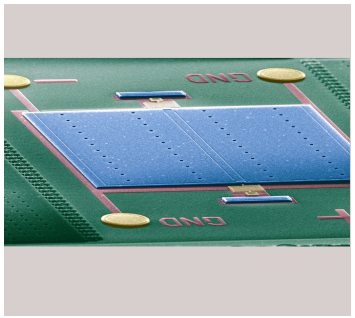
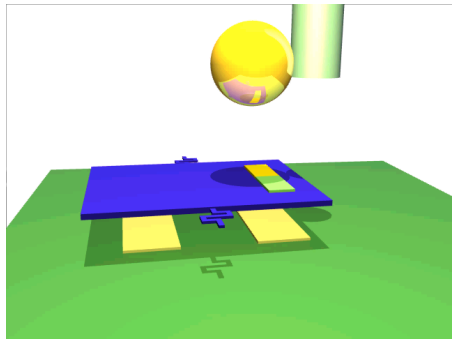
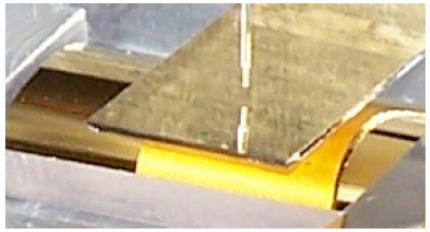




sphere-plane, d=200-1000 nm

Capasso et al, Decca et al

Micro-cantilever



plane-plane, cylinder-plane, d=1-3 um

Onofrio et al

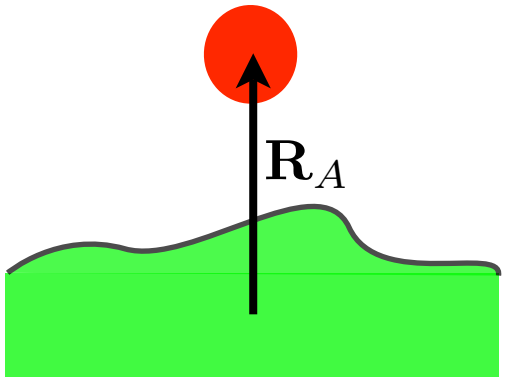
The Casimir-Polder force

vdW - CP interaction

Casimir and Polder (1948)

The interaction energy between a ground-state atom and a surface is given by

$$U_{\mathrm{CP}}(\mathbf{R}_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\,\mathrm{Tr}\,\mathbf{G}(\mathbf{R}_A,\mathbf{R}_A,i\xi)$$



Atomic polarizability: $\alpha(\omega) = \lim_{\epsilon\to 0}\frac{2}{3\hbar}\sum_k \frac{\omega_{k0}|\mathbf{d}_{0k}|^2}{\omega_{k0}^2-\omega^2-i\omega\epsilon}$

Scattering Green tensor: $\left(\nabla\times\nabla\times-\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\right)\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)=\delta(\mathbf{r}-\mathbf{r}')$

The Casimir-Polder force

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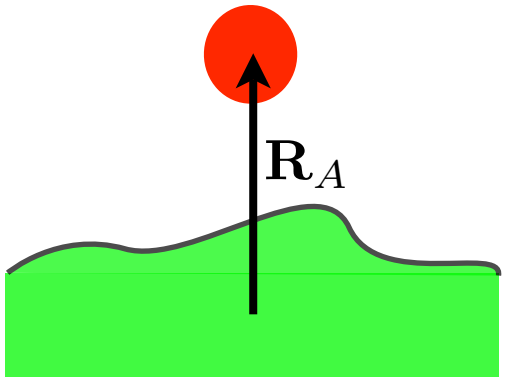

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Eg: Ground-state atom near planar surface @ T=0

Non-retarded (vdW) limit $z_A \ll \lambda_A$

$$U_{\rm vdW}(z_A) = -\frac{\hbar}{8\pi\epsilon_0}\frac{1}{z_A^3}\int_0^\infty \frac{d\xi}{2\pi}\alpha(i\xi)\frac{\epsilon(i\xi)-1}{\epsilon(i\xi)+1}$$

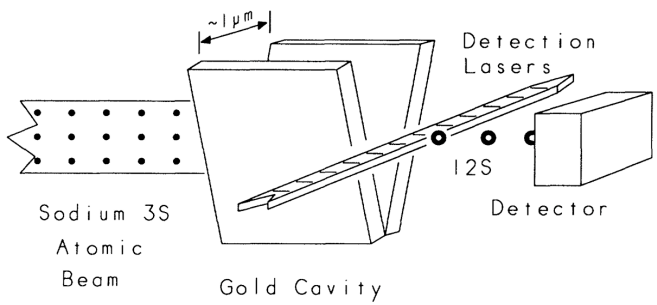
Retarded (CP) limit $z_A \gg \lambda_A$

$$U_{\rm CP}(z_A) = -\frac{3\hbar c\alpha(0)}{8\pi}\frac{1}{z_A^4}\frac{\epsilon_0-1}{\epsilon_0+1}\phi(\epsilon_0)$$

Modern CP experiments

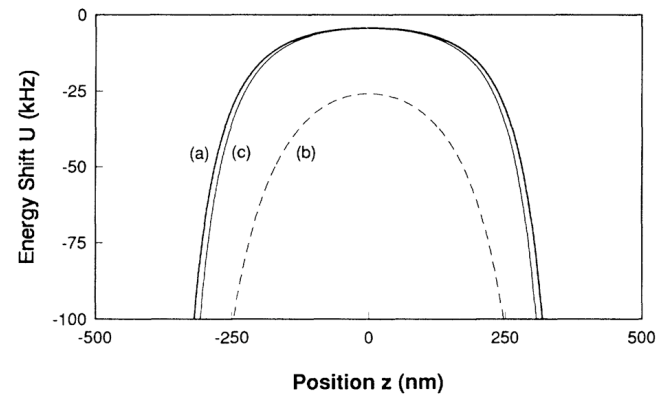
- Deflection of atoms

Hinds et al (1993)



L= 0.7-1.2 um

Exp-Th agreement @ 10%



$$U_{CP} = -\frac{1}{4\pi\epsilon_0}\frac{\pi^3\hbar c\alpha(0)}{L^4}\left[\frac{3-2\cos^2(\pi z/L)}{8\cos^4(\pi z/L)}\right]$$

Modern CP experiments

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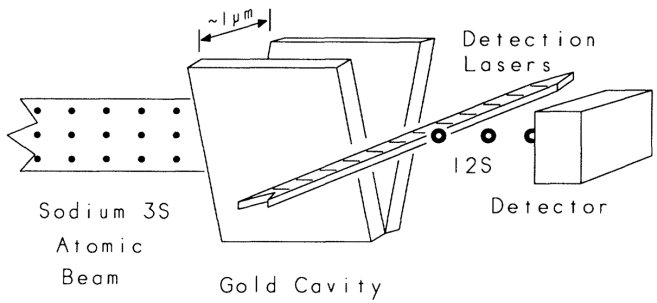

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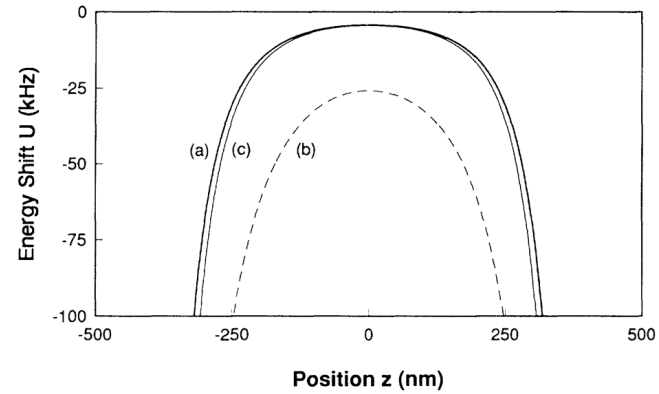

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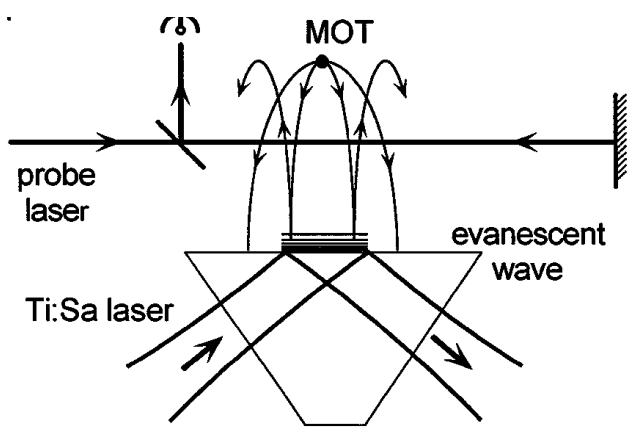
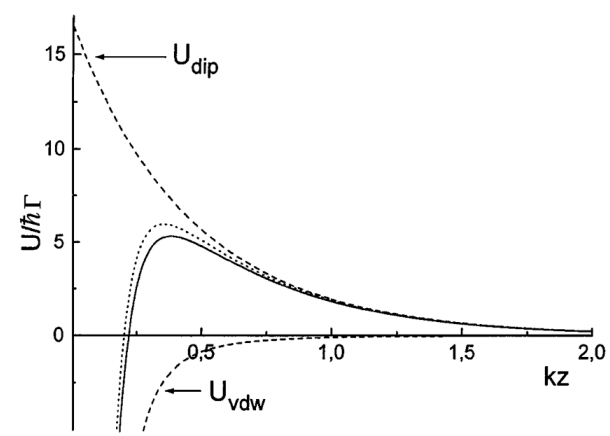
Classical reflection on atomic mirror

Aspect et al (1996)



$$U_{\rm dip} = \frac{\hbar}{4}\frac{\Omega^2}{\Delta}e^{-2kz}$$

$$U_{\rm vdW} = -\frac{\epsilon-1}{\epsilon+1}\frac{1}{48\pi\epsilon_0}\frac{D^2}{z^3}$$

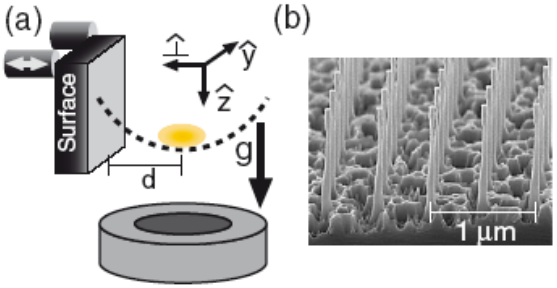


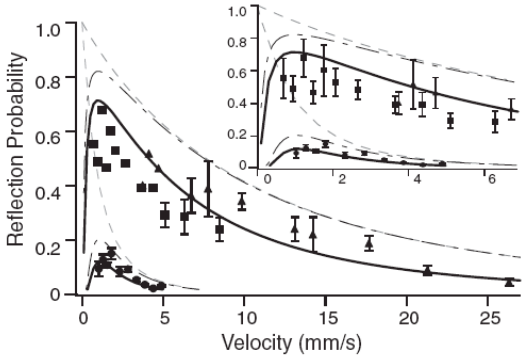
Exp-Th agreement @ 30%

Modern experiments (cont'd)

Quantum reflection

Wave-nature of atoms implies that slow atoms can reflect from purely attractive potentials





$$k = \sqrt{k_0^2 - 2mU/\hbar^2} \qquad \phi = \frac{1}{k^2}\frac{dk}{dr} > 1$$

$$U = -C_n/r^n \ (n > 2)$$

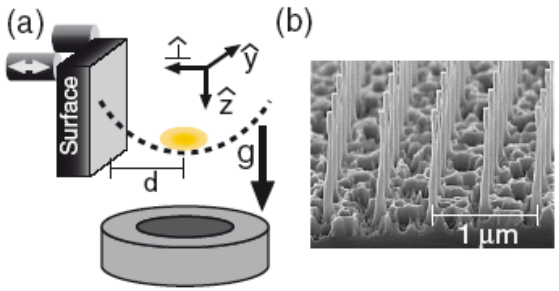
Shimizu (2001) Ketterle et al (2006)

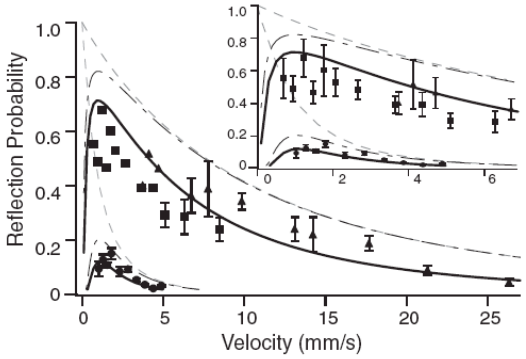
DeKievet et al (2003)

Modern experiments (cont'd)

Quantum reflection

Wave-nature of atoms implies that slow atoms can reflect from purely attractive potentials





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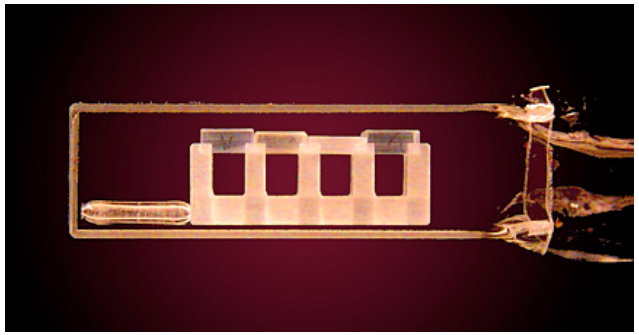
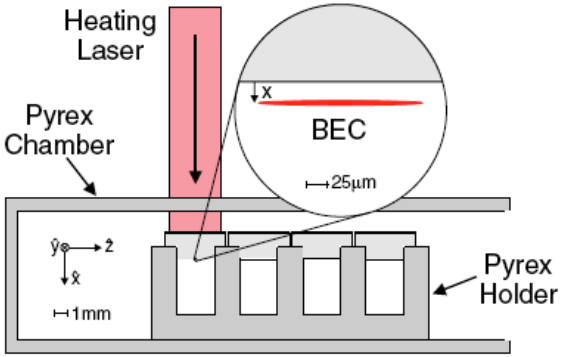
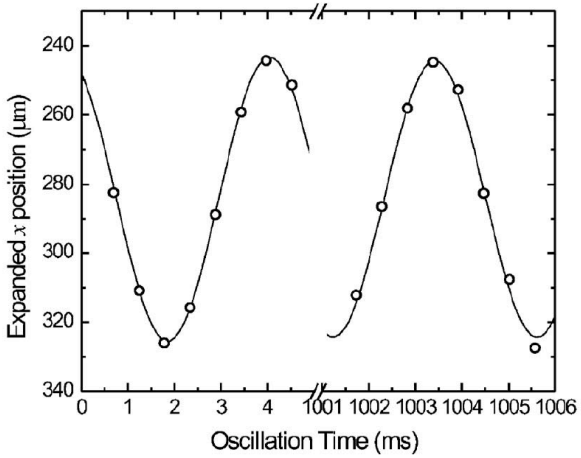
BEC oscillator

Cornell et al (2007)



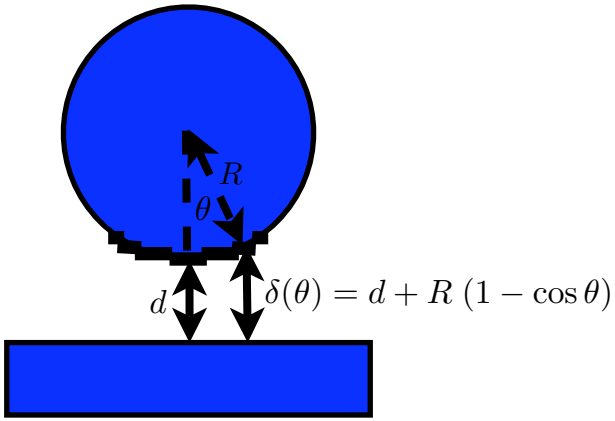




$$\gamma_x \equiv \frac{\omega_x - \omega_x'}{\omega_x} \simeq -\frac{1}{2\omega_x^2 m}\partial_x^2 U^*$$

Effects of geometry

Approximation methods (PFA, PWS)



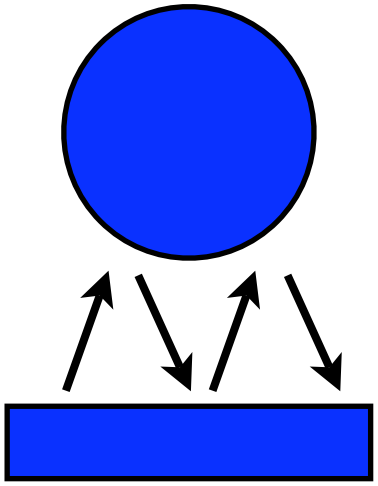
$$E_{\rm SP}^{\rm PFA}(d) \approx 2\pi R^2 \int_0^{\theta_m} d\theta \sin\theta \, \frac{E_{\rm PP}(\delta(\theta))}{A}$$

$$R \gg d$$

Exact methods: scattering theory, Green function

$$\boxed{E_{\rm SP}^{\rm exact}(d) = \hbar \int_0^\infty d\xi \, {\rm Tr} \log[1 - \mathcal{R}_{\rm S}(i\xi)\, e^{-d\,\mathcal{K}(i\xi)}\, \mathcal{R}_{\rm P}(i\xi)\, e^{-d\,\mathcal{K}(i\xi)}]}$$

$$\propto \sum_{n=1}^{\infty} \frac{1}{n} \, [\mathcal{R}_{\rm S}(i\xi) e^{-d\mathcal{K}(i\xi)} \mathcal{R}_{\rm P}(i\xi) e^{-d\mathcal{K}(i\xi)}]^n$$



Effects of materials

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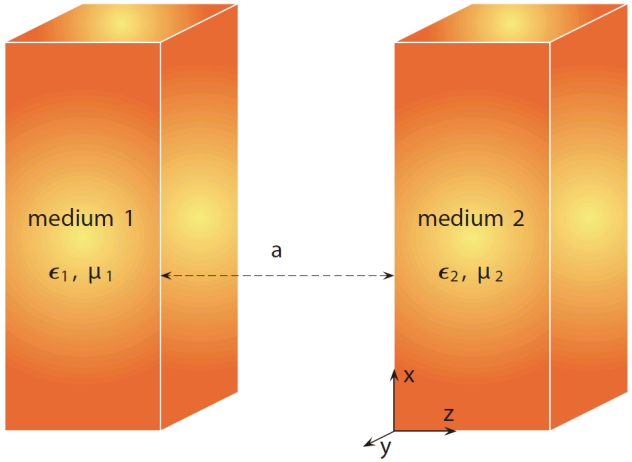
The Lifshitz formula: Lifshitz (1956)

$$\boxed{\frac{F}{A} = 2\hbar\,\mathrm{Im}\int_0^\infty \frac{d\omega}{2\pi}\int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr}\frac{\mathbf{R}_1\cdot\mathbf{R}_2 e^{2\mathrm{i}K_3 d}}{1-\mathbf{R}_1\cdot\mathbf{R}_2 e^{2\mathrm{i}K_3 d}}}$$



$$K_3 = \sqrt{\omega^2/c^2 - k_\parallel^2}$$

Reflection matrices (Fresnel formulas for isotropic media):

$$r^{\mathrm{TM,TM}}(\omega,\mathbf{k}_\parallel) = \frac{\epsilon(\omega)K_3 - \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}{\epsilon(\omega)K_3 + \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}} \qquad r^{\mathrm{TE,TE}} = r^{\mathrm{TM,TM}} \quad \text{with} \quad \epsilon \leftrightarrow \mu$$

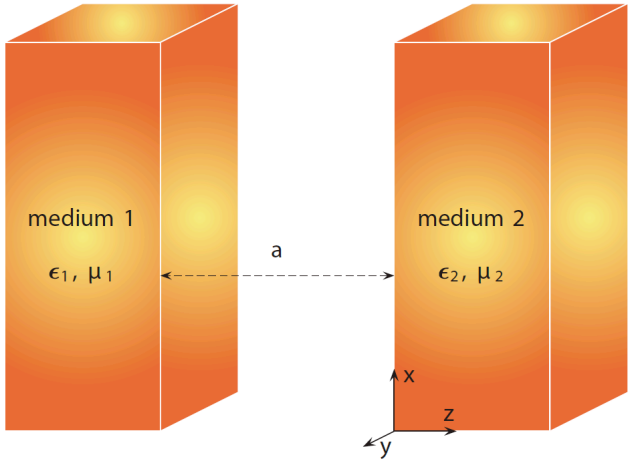
Relevant frequencies:

$$\left.\begin{array}{lll} \epsilon(\omega) \xrightarrow{\omega \gg \Omega_p} 1 & \Rightarrow & r^{p,p} \approx 0 \text{ (Transparent plates)} \\ \omega \gg c/d & \Rightarrow & e^{2\mathrm{i}K_3 d} \approx 0 \text{ (Fast oscillations)} \end{array}\right\} \Rightarrow F \approx 0$$

$$\boxed{0 \le \omega \le \min\{\Omega_p, c/d\}}$$

Going to imaginary frequencies

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$



Kramers-Kronig (causality) relations:

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega \qquad \mu(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \mu''(\omega)}{\omega^2 + \xi^2} d\omega$$

Dominant frequencies below the near-infrared/optical region of the EM spectrum (gaps d= 200-1000 nm)

The important message is that Casimir is a broad-band frequency phenomenon

The sign of the Casimir force

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$

The sign of the force is directly connected to the sign of the product of the reflection coefficients on the two plates, evaluated at imaginary frequencies. As a rule of thumb, we have (p=TE,TM)

$$R_1^p(i\xi) \cdot R_2^p(i\xi) > 0 \; (\forall \; \xi \leq c/d) \Rightarrow \text{Attraction}$$

$$R_1^p(i\xi) \cdot R_2^p(i\xi) < 0 \; (\forall \; \xi \leq c/d) \Rightarrow \text{Repulsion}$$

In terms of permittivities and permeabilities:

$$\epsilon_a(i\xi) \gg \epsilon_b(i\xi)$$

$$\mu_b(i\xi) \gg \mu_a(i\xi)$$

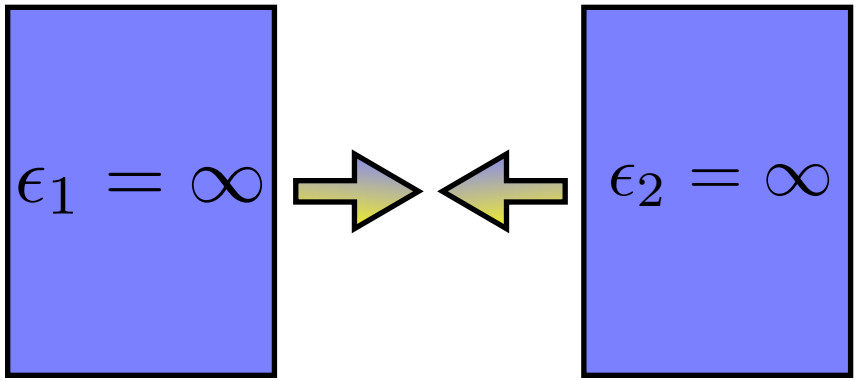
$\longrightarrow$ Repulsion

Ideal attraction-repulsion

- Ideal attractive limit

  Casimir (1948)

$$\frac{F}{A} = +\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$



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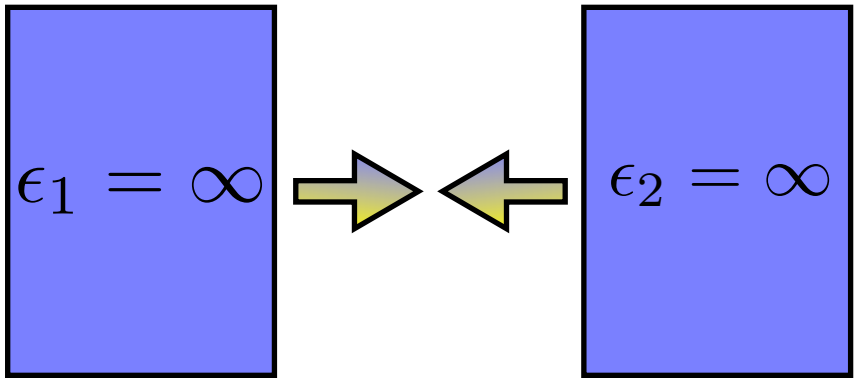
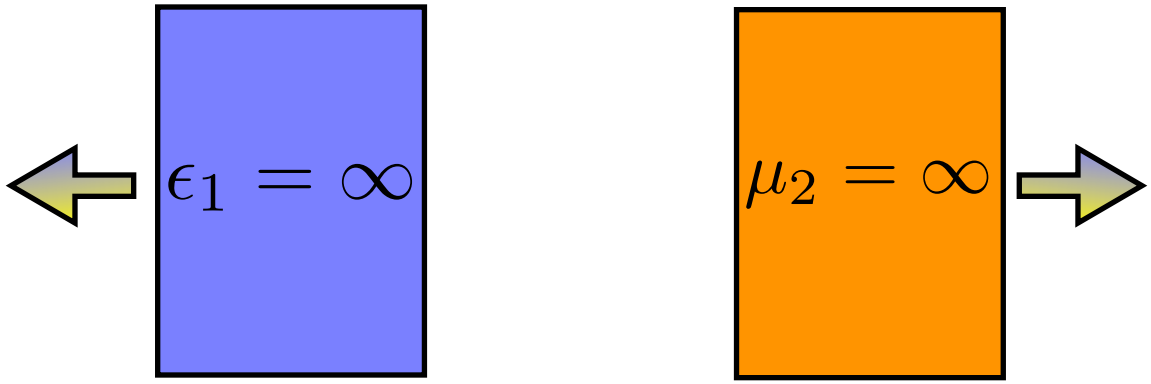


- **Ideal repulsive limit**

  Boyer (1974)

$$\frac{F}{A} = -\frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$



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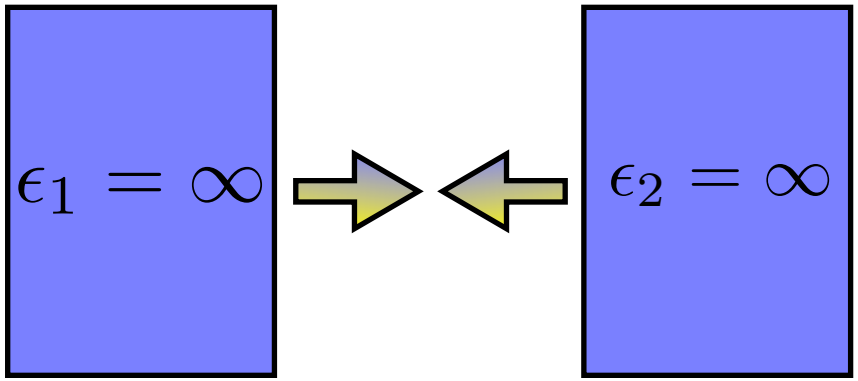

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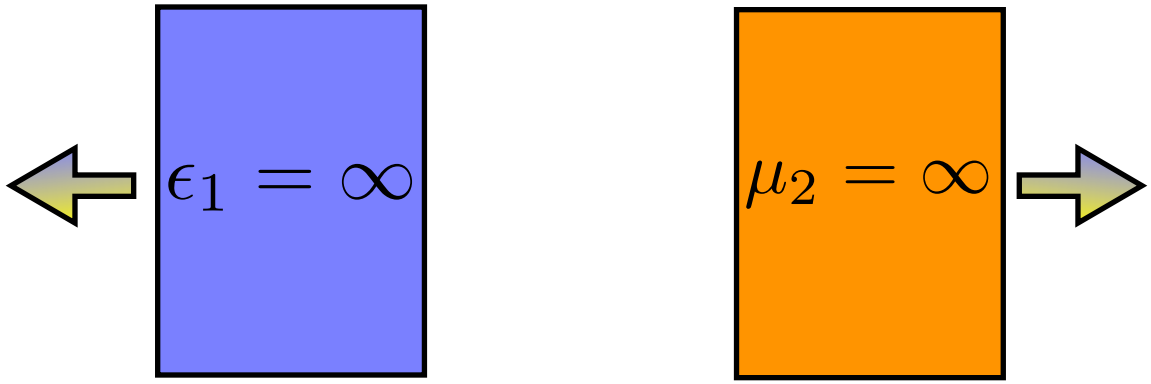

- **Real repulsive limit**

  Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$

Ideal attraction-repulsion

- **Ideal attractive limit**

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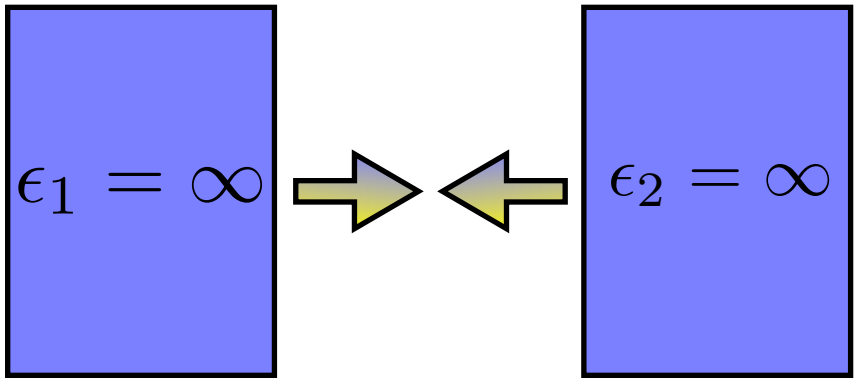

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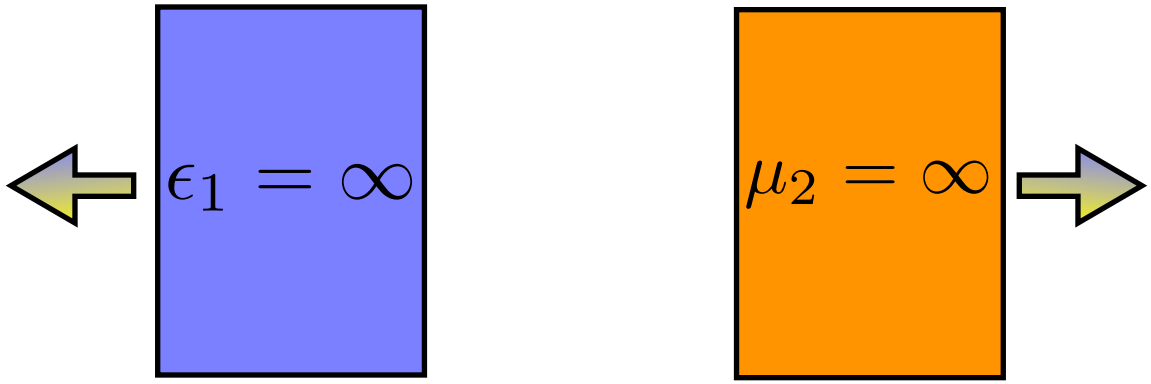

- **Real repulsive limit**

  Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$ → Metamaterials

Quantum levitation with MMs?

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Physicists have 'solved' mystery of levitation - Telegraph http://www.telegraph.co.uk/news/main.jhtml?xml=/news/2007/08/0...

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FEATURE FOCUS

Physicists have 'solved' mystery of levitation

By Roger Highfield, Science Editor
Last Updated: 1:45am BST 08/08/2007

Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

In earlier work the same team of theoretical physicists showed that invisibility cloaks are feasible.

Now, in another report that sounds like it comes out of the pages of a Harry Potter book, the University of St Andrews team has created an 'incredible levitation effects' by engineering the force of nature which normally causes objects to stick together.

Professor Ulf Leonhardt and Dr Thomas Philbin, from the University of St Andrews in Scotland, have worked out a way of reversing this phenomenon, known as the Casimir force, so that it repels instead of attracts.

Their discovery could ultimately lead to frictionless micro-machines with moving parts that levitate But they say that, in principle at least, the same effect could be used to levitate bigger objects too, even a person.

In theory the discovery could be used to levitate a person

advertisement

The Casimir force is a consequence of quantum mechanics, the theory that describes the world of atoms and subatomic particles that is not only the most successful theory of physics but also the most baffling.

The force is due to neither electrical charge or gravity, for example, but the fluctuations in all-pervasive energy fields in the intervening empty space between the objects and is one reason atoms stick together, also explaining a "dry glue" effect that enables a gecko to walk across a ceiling.

Now, using a special lens of a kind that has already been built, Prof Ulf Leonhardt and Dr Thomas Philbin report in the New Journal of Physics they can engineer the Casimir force to repel, rather than attact.

Because the Casimir force causes problems for nanotechnologists, who are trying to build electrical circuits and tiny mechanical devices on silicon chips, among other things, the team believes the feat could initially be used to stop tiny objects from sticking to each other.

Prof Leonhardt explained, "The Casimir force is the ultimate cause of friction in the nano-world, in particular in some microelectromechanical systems.

Such systems already play an important role - for example tiny mechanical devices which triggers a car airbag to inflate or those which power tiny 'lab on chip' devices used for drugs testing or chemical analysis.

Micro or nano machines could run smoother and with less or no friction at all if one can manipulate the force." Though it is possible to levitate objects as big as humans, scientists are a long way off developing the technology for such feats, said Dr Philbin.

The practicalities of designing the lens to do this are daunting but not impossible and levitation "could happen over quite a distance".

Prof Leonhardt leads one of four teams - three of them in Britain - to have put forward a theory in a peer-reviewed journal to achieve invisibility by making light waves flow around an object - just as a river flows undisturbed around a smooth rock.

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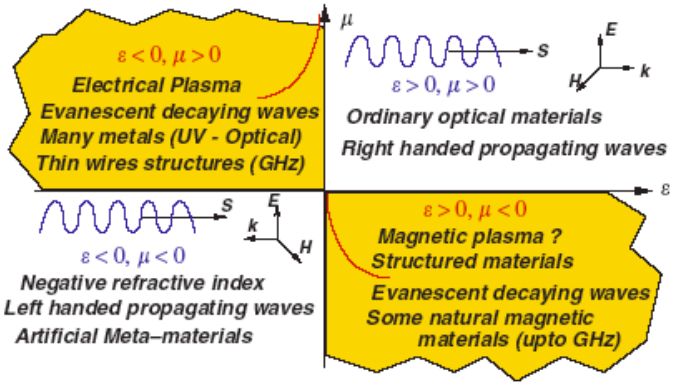
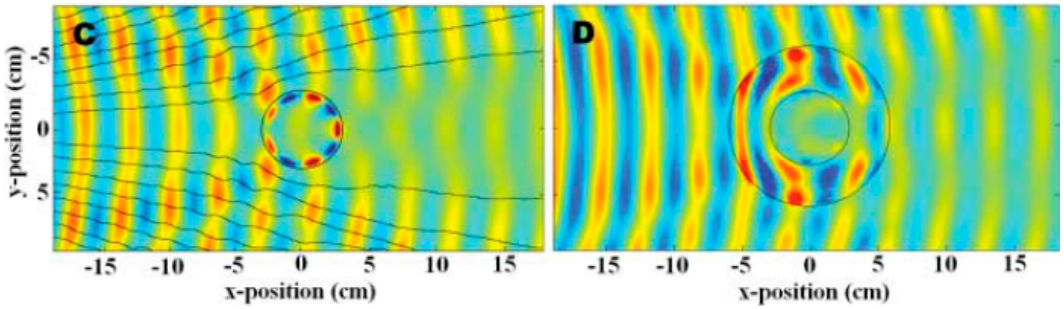
"In theory the discovery could be used to levitate a person"

Metamaterials

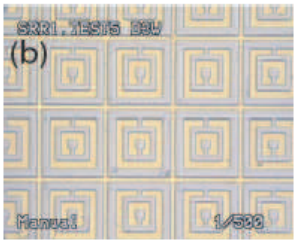
- Artificial structured composites with designer electromagnetic properties
- MMs are strongly anisotropic, dispersive, magneto-dielectric media.

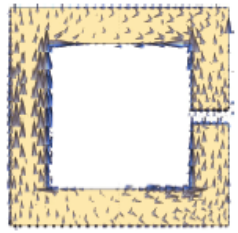
- Negative refraction — Veselago (1968), Smith et al (2000)
- Perfect lens — Pendry (2000)
- Cloaking — Smith et al (2007)



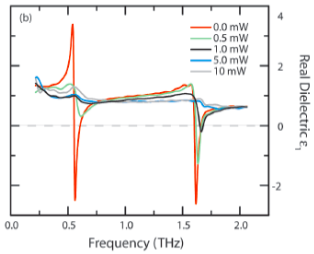


THz MMs: eg split ring resonators



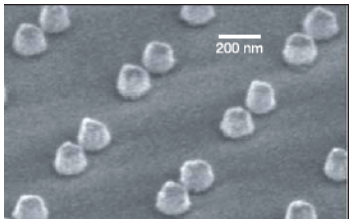


$36\mu m$

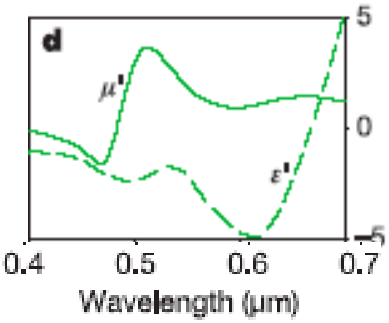


$\epsilon, \mu < 0$

Optical MMs: eg nano-pillars

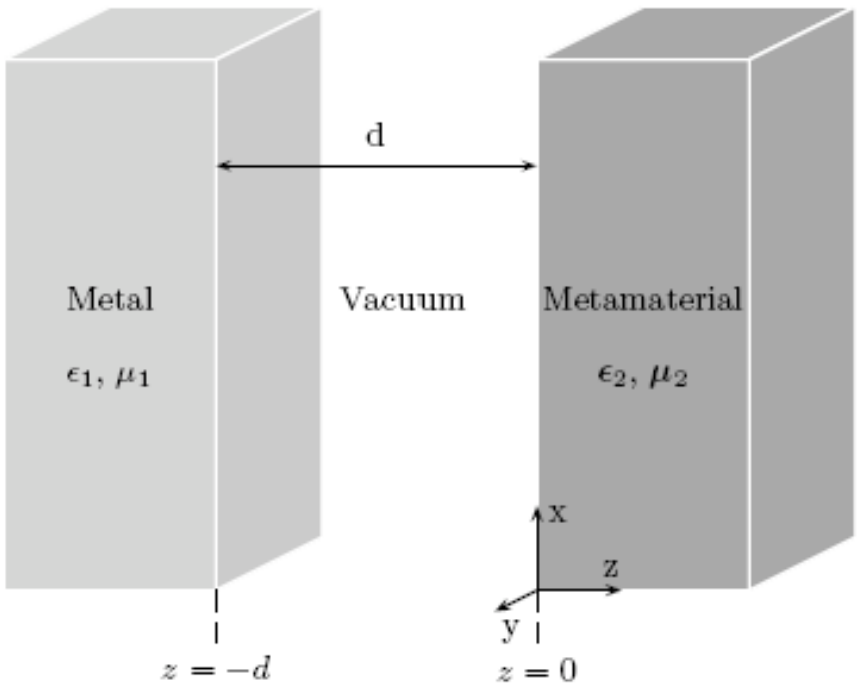


200nm



MMs + Casimir effect

In order to fix ideas, let us consider a metallic half-space facing a magneto-dielectric (MM) half-space.



The experiments in this project will actually be a metallic Au sphere glued to an AFM above a metamaterial planar structure.

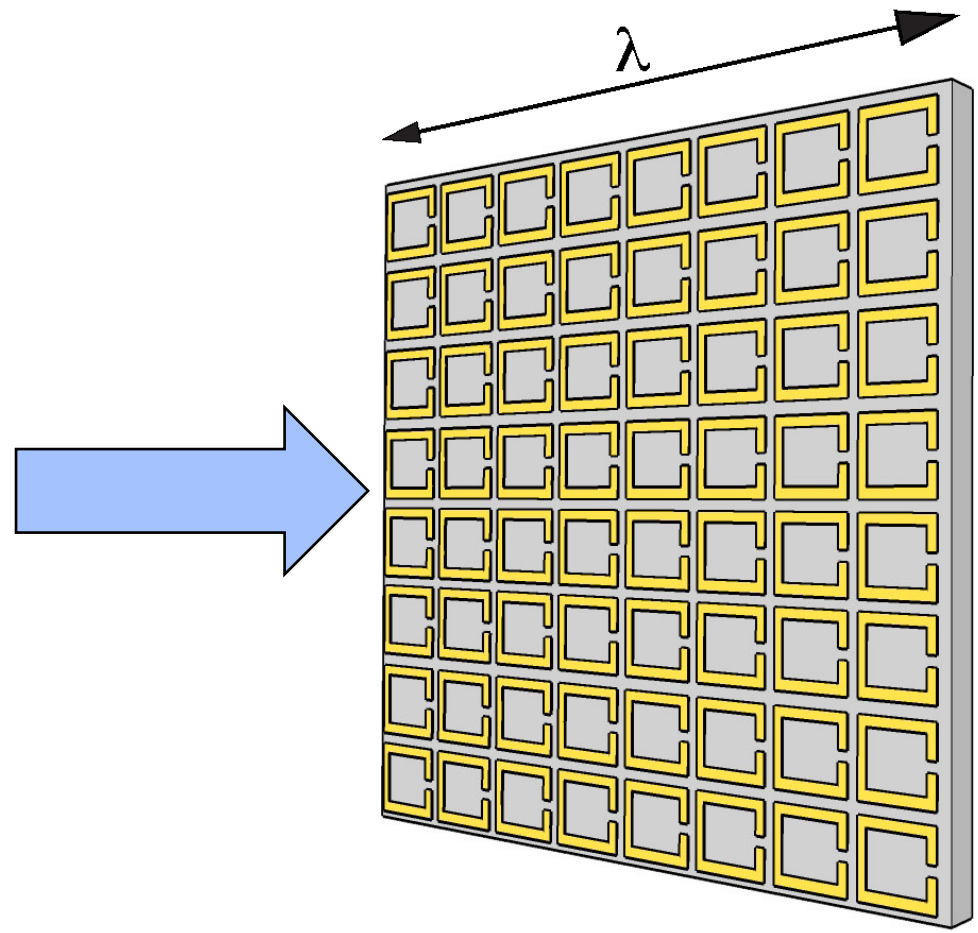
Effective medium approximation

Let us assume the metal is reasonably well described by a Drude response

$$\epsilon_1(\omega) = 1 - \frac{\Omega_e^2}{\omega^2 + i\gamma_e\omega} \qquad \mu_1 = 1$$

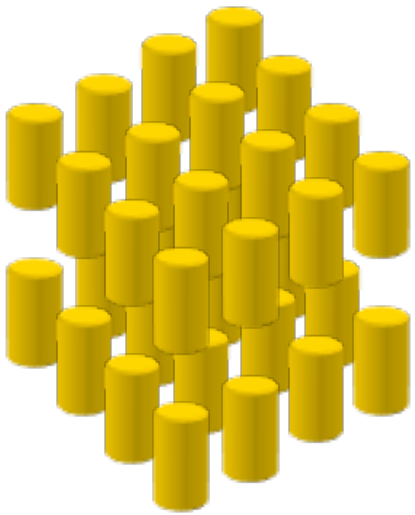
For the MM the optical response is not so simple.....

In the effective medium approximation (EMA) one describes the MM with an effective electric permittivity and an effective magnetic permeability. This is an approximation valid when the MM is probed at wavelengths much larger that the average distance between the constituent "particles" of the MM.

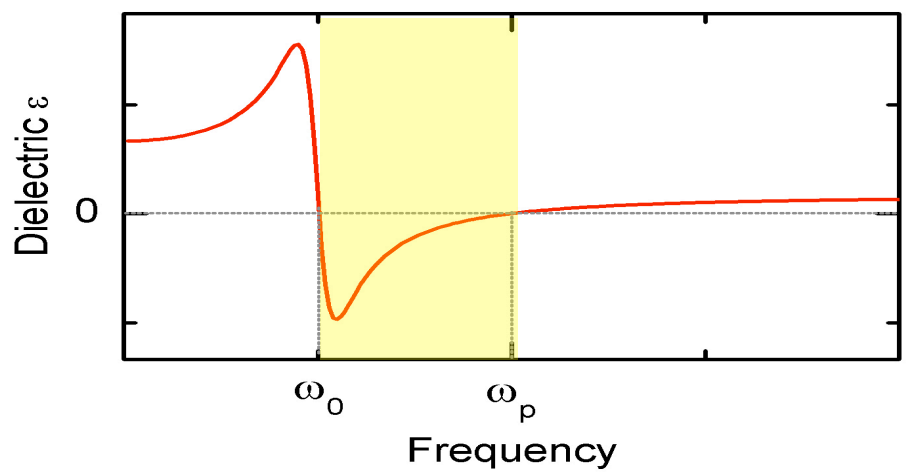


Electric response

Close to resonance, the optical response can be modeled by a Drude-Lorentz permittivity



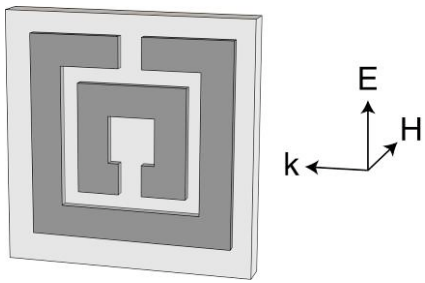
$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

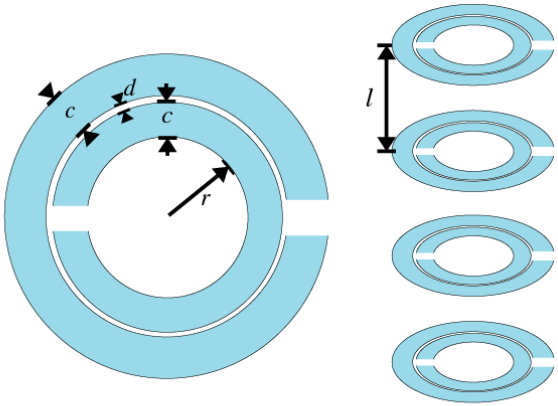


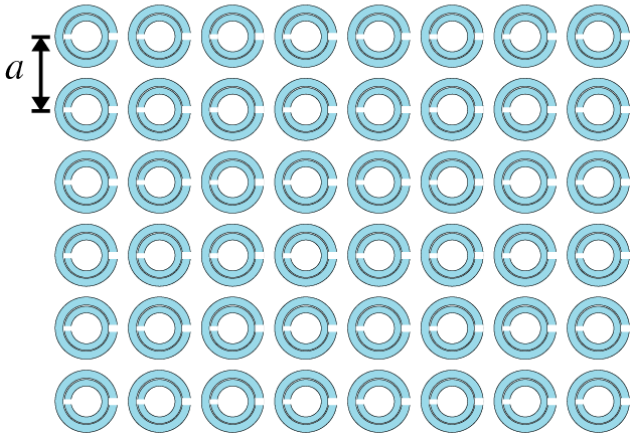
J.B. Pendry ***et al.***, ***Phys. Rev. Lett.*** **76**, 4773 (1996).

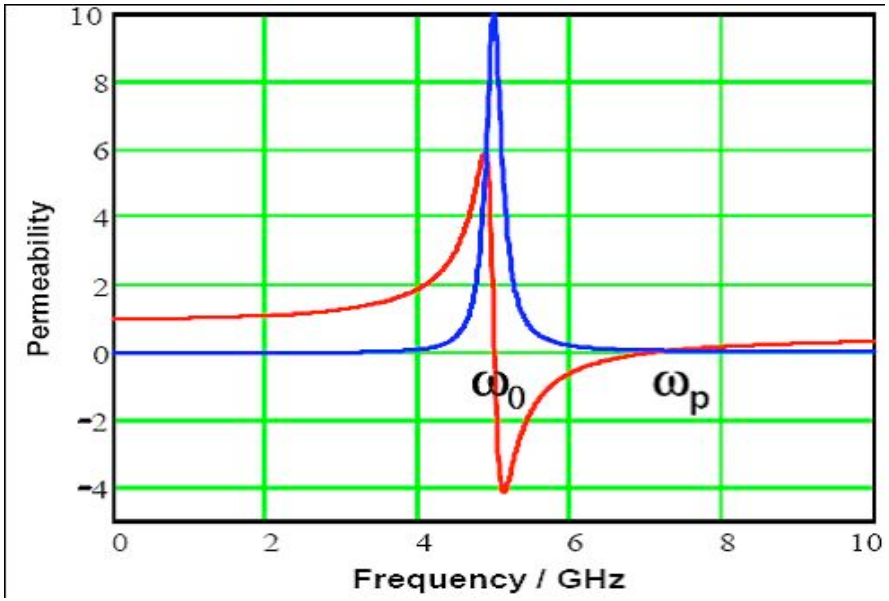
Magnetic response

$$\mu_{eff} = 1 - \frac{\dfrac{\pi r^2}{a^2}}{1 + \dfrac{2\sigma i}{\omega r \mu_0} - \dfrac{3}{\pi^2 \mu_0 \omega^2 C r^3}}$$









J.B. Pendry *et al.*, *IEEE Trans. Microwave Tech.* **47,** 2075 (1999).

Drude-Lorentz responses

Close to the resonance, both $\epsilon(\omega)$ and $\mu(\omega)$ can be modeled by Drude-Lorentz formulas

$$\epsilon_\alpha(\omega) = 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega}$$

$$\mu_\alpha(\omega) = 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}$$

Typical separations

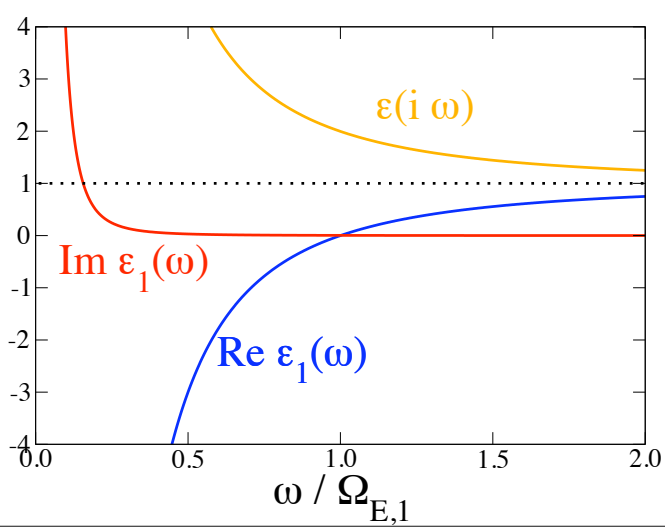
$$d = 200 - 1000 \text{ nm}$$

Infrared-optical frequencies

$$\Omega/2\pi = 5 \times 10^{14}\text{Hz}$$

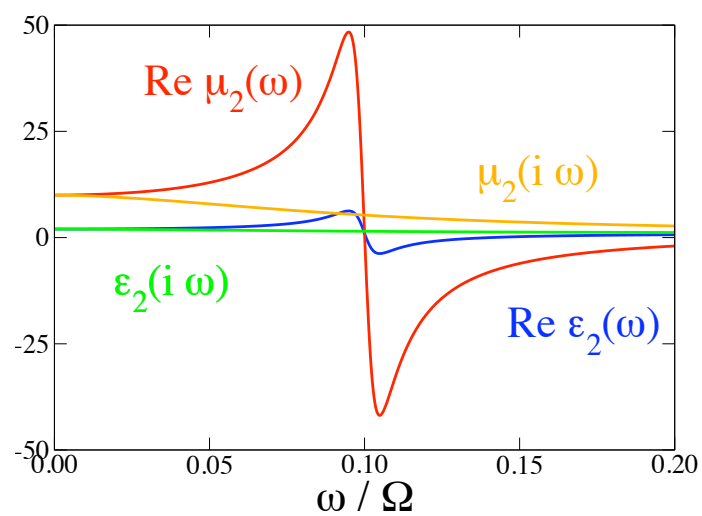
Drude metal (Au)

$\Omega_E = 9.0$ eV $\quad \Gamma_E = 35$ meV



Metamaterial

$\text{Re } \epsilon_2(\omega) < 0 \quad \text{Re } \mu_2(\omega) < 0$



$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$

$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$

$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$

Attraction-repulsion crossover

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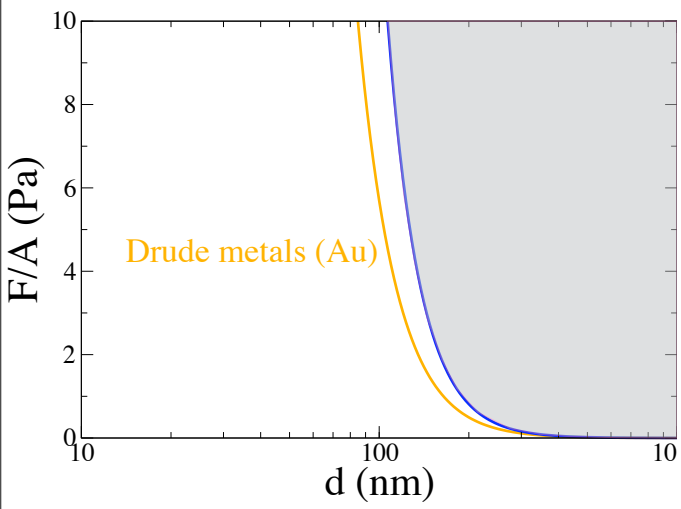
Drude metal (Au)

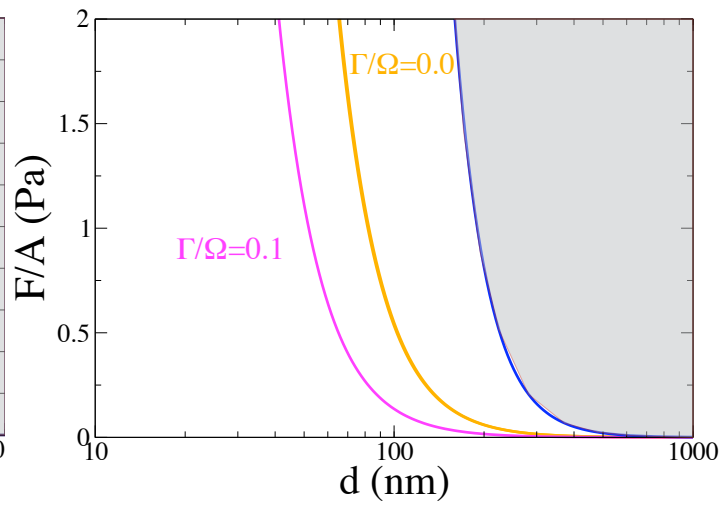
Drude metal (Au)



Only attraction

Metamaterial

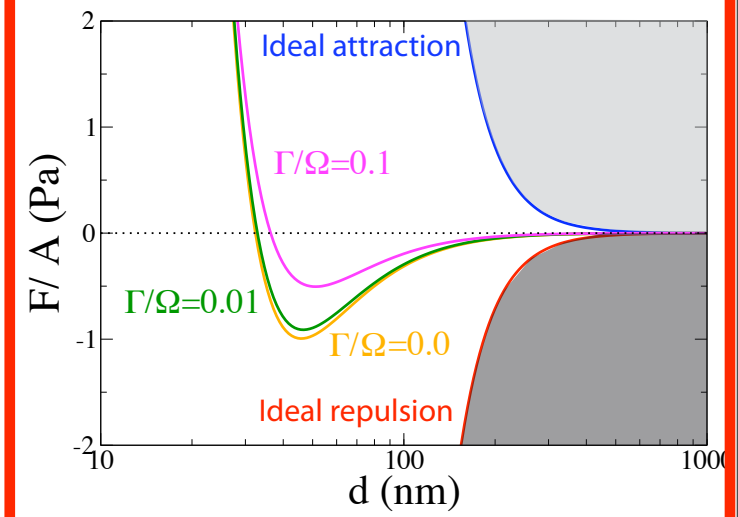
Metamaterial



Only attraction

Drude metal (Au)

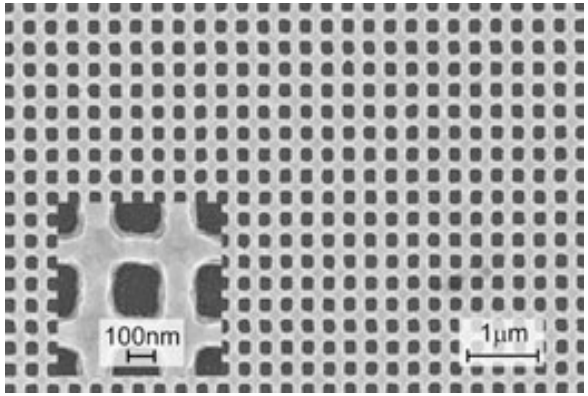
Metamaterial



Repulsion-attraction

Drude background

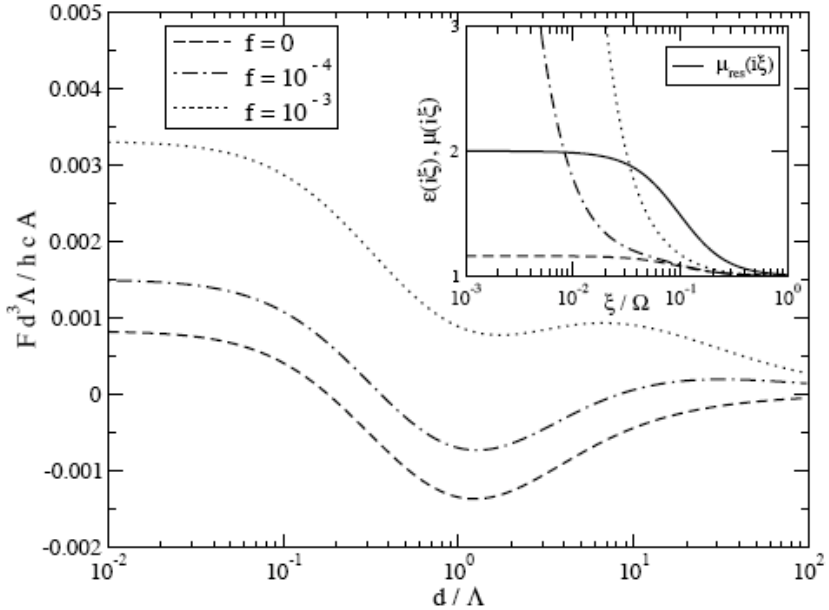
- In some metallic-based MMs, there is a net conductivity due to the metallic structure, like the fishnet design on the right.



$$\epsilon(\omega) = 1 - f\frac{\Omega_D^2}{\omega^2 - i\omega\gamma_D} - (1-f)\frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega}$$

f : filling factor

$$\mu(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}$$



(Rosa, DD, Milonni, PRL 2008)

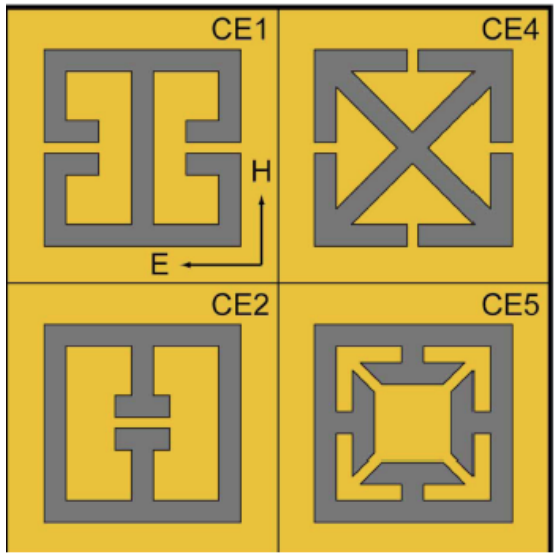
A Drude background is detrimental for Casimir force reduction or repulsion, since it results in an electric response much stronger than the magnetic one

$$\epsilon_2(i\xi) \gg \mu_2(i\xi)$$

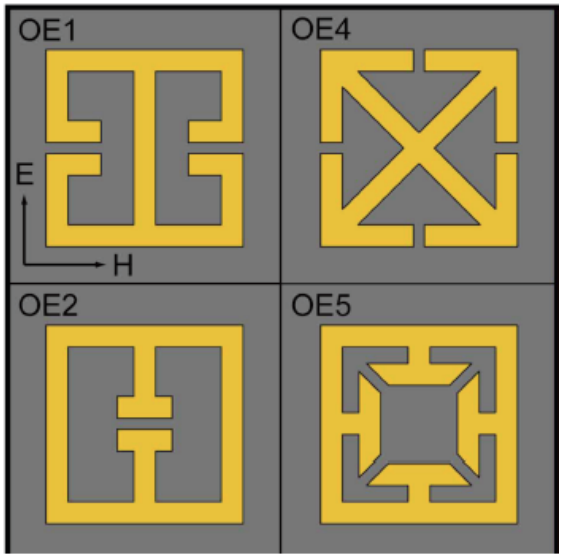
Split ring resonators (SRRs)

SRRs structures provide an opportunity to avoid the large Drude background, since they can be built in two natural complementary ways

Continuously-metallized SSR

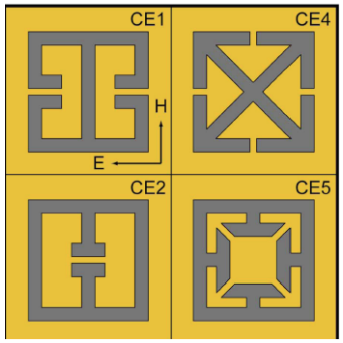


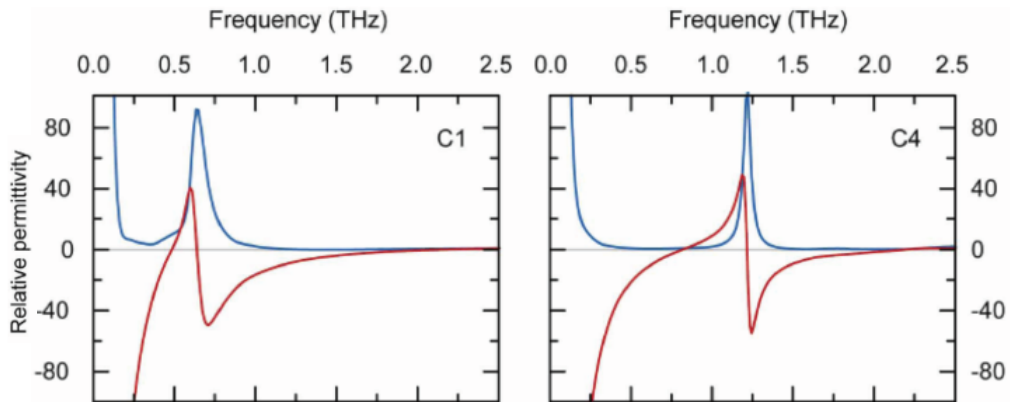
Discontinuously-metallized SSR



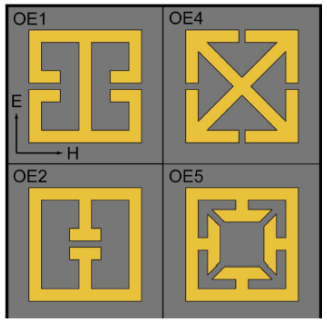
Complementary SRRs + Drude

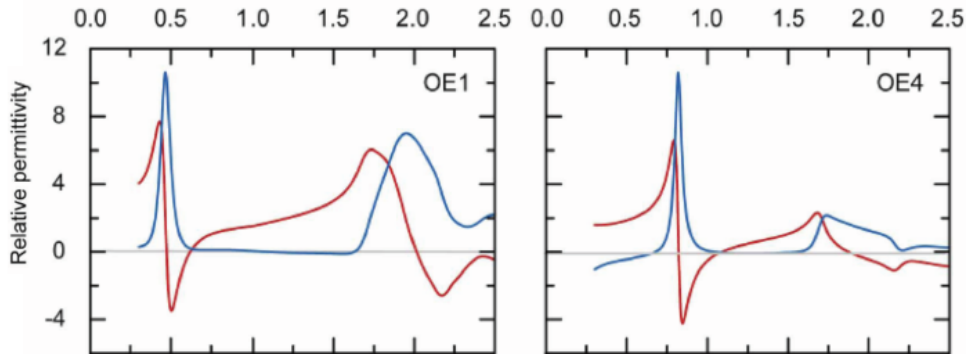
The continuous version presents a considerable Drude background ...





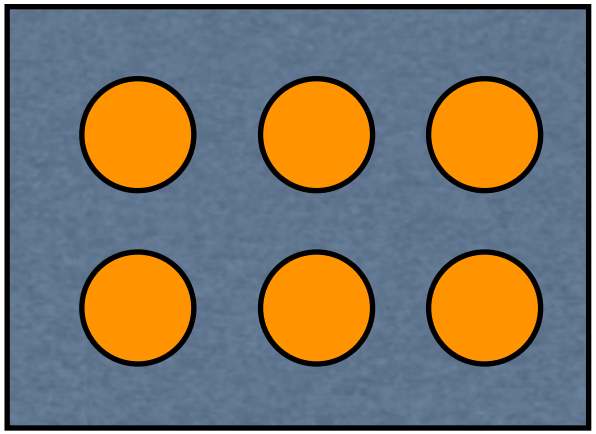
... but the discontinuous one does not!





Maxwell-Garnett $\bar{\epsilon}$

In order to understand how non-connected materials behave, we consider a simple model composed of tiny metallic spheres (radius a, permittivity ϵ_i, filling factor f) in a given host medium (permittivity $\epsilon_{\rm h}$).



Clausius-Mossotti formula

$$\frac{\bar{\varepsilon} - \varepsilon_{\rm h}}{\bar{\varepsilon} + 2\,\varepsilon_{\rm h}} = \frac{f}{a^3}\alpha$$

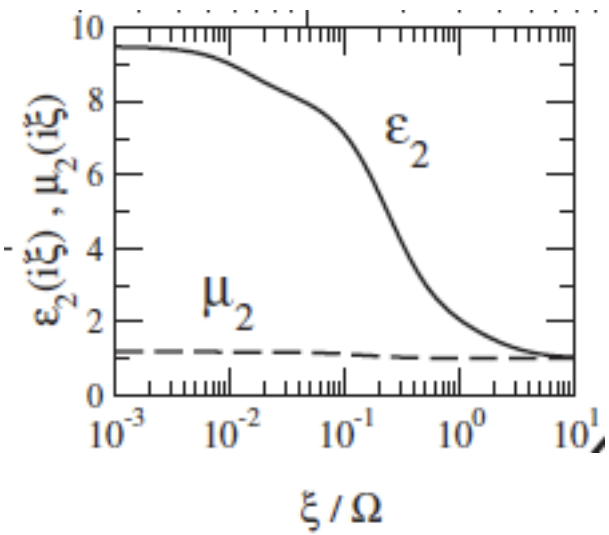
Sphere polarizability

$$\alpha = \frac{\varepsilon_{\rm i} - \varepsilon_{\rm h}}{\varepsilon_{\rm i} + 2\,\varepsilon_{\rm h}} a^3$$

$$\bar{\varepsilon} = \varepsilon_{\rm h} \frac{\varepsilon_{\rm i}(1+2f) + 2\,\varepsilon_{\rm h}(1-f)}{\varepsilon_{\rm i}(1-f) + \varepsilon_{\rm h}(2+f)}$$

Important consequence:

$\bar{\epsilon}(\omega \to 0)$ is finite!



(Rosa, DD, Milonni, PRA 2008)

EMA magnetic activity

Drude-Lorentz for permebility is wrong. The correct expression that results in EMA from Maxwell's equations is (Pendry 1999)

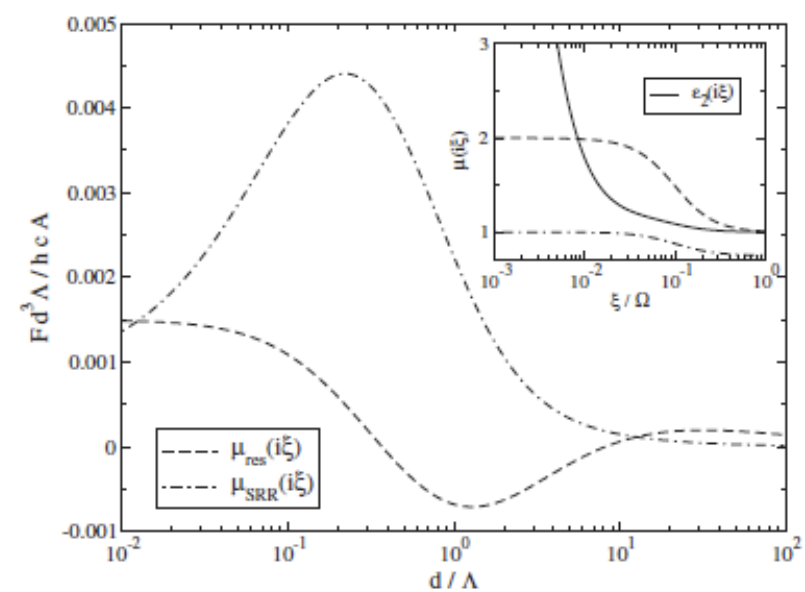
$$\mu_{\text{eff}}(\omega) = 1 - f\frac{\omega^2}{\omega^2 - \omega_m^2 + 2i\gamma_m\omega}$$

The appearance of the ω^2 factor in the numerator is very important:

Although close to the resonance this behaves in the same way as the "standard" EMA permeability, it has a completely different low-frequency behavior

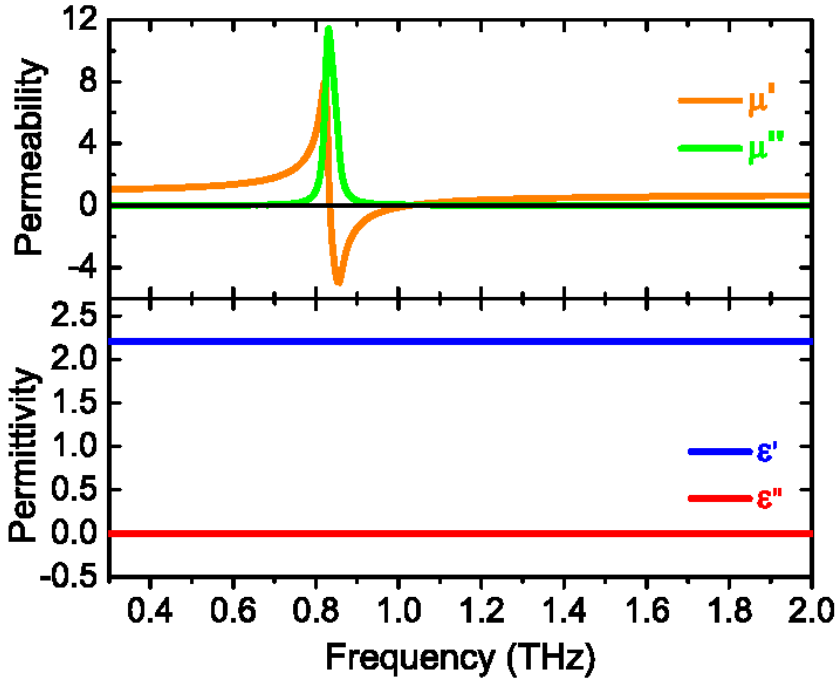
$$\mu_{\text{eff}}(i\xi) < 1 < \epsilon_{\text{eff}}(i\xi)$$

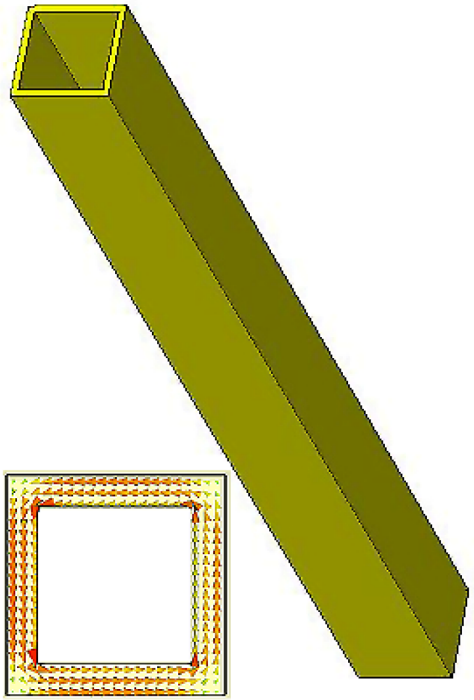
No Casimir repulsion!



All-dielectric metamaterials

- All-dielectric MM → path to magnetism without metal
- No Drude background → low ε
- Requires high index contrast, difficult at high frequencies
- Low indices; however, also means less Casimir attraction to overcome (Question: which dominates?)





Optical anisotropy

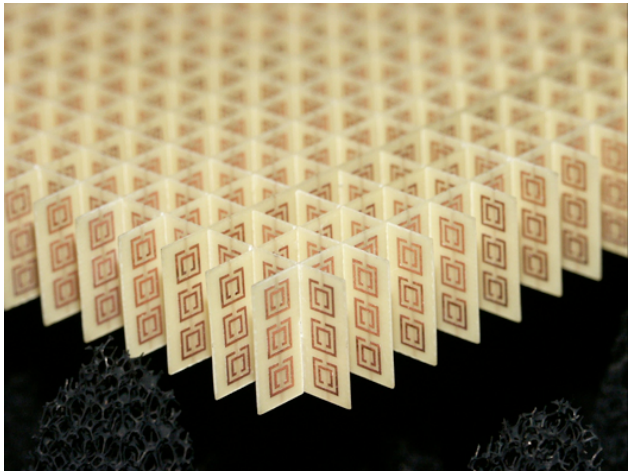
In an anisotropic medium, the constitutive relations between E, D, B, and H are more involved:

$$\mathrm{D} = \epsilon \cdot \mathrm{E} \qquad \mathrm{H} = \mu^{-1} \cdot \mathrm{B}$$

due to the tensorial nature of the permittivity and permeability

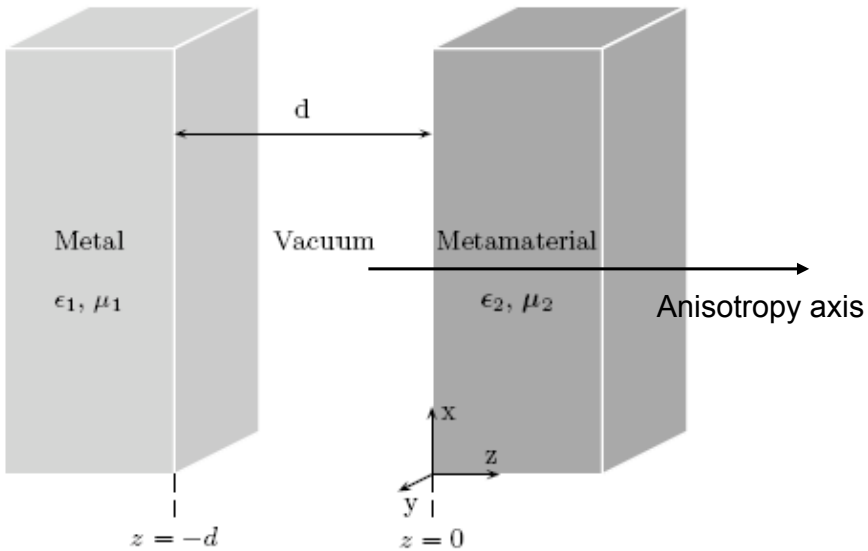
$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}$$

due to the tensorial nature of the permittivity and permeability



Examples of uniaxial anisotropy in stacked MMs

Anisotropy: Uniaxial MMs



$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \; ; \; \mu_{ij} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$

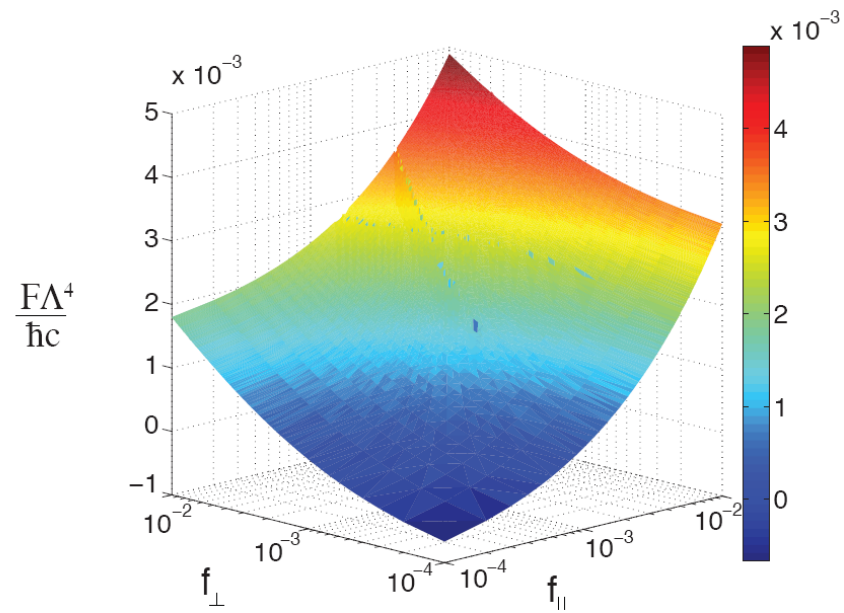
Anisotropy produces polarization mixing (non-diagonal reflection matrices)

$$\epsilon_{xx}(\omega) = \epsilon_{yy}(\omega) = 1 - (1 - f_x)\frac{\Omega_{e,x}^2}{\omega^2 - \omega_{e,x}^2 + i\gamma_{e,x}\omega} - f_x\frac{\Omega_{D,x}^2}{\omega^2 + i\gamma_{D,x}\omega},$$

$$\epsilon_{zz}(\omega) = 1 - (1 - f_z)\frac{\Omega_{e,z}^2}{\omega^2 - \omega_{e,z}^2 + i\gamma_{e,z}\omega} - f_z\frac{\Omega_{D,z}^2}{\omega^2 + i\gamma_{D,z}\omega},$$

$$\mu_{xx}(\omega) = \mu_{yy}(\omega) = 1 - \frac{\Omega_{m,x}^2}{\omega^2 - \omega_{m,x}^2 + i\gamma_{m,x}\omega},$$

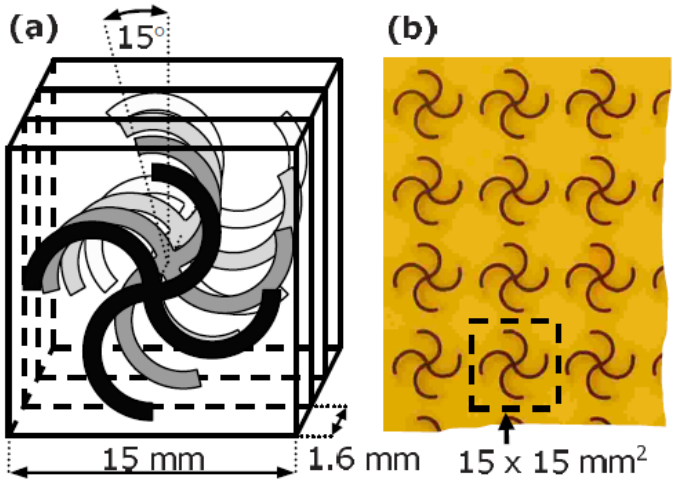
$$\mu_{zz}(\omega) = 1 - \frac{\Omega_{m,z}^2}{\omega^2 - \omega_{m,z}^2 + i\gamma_{m,z}\omega}$$

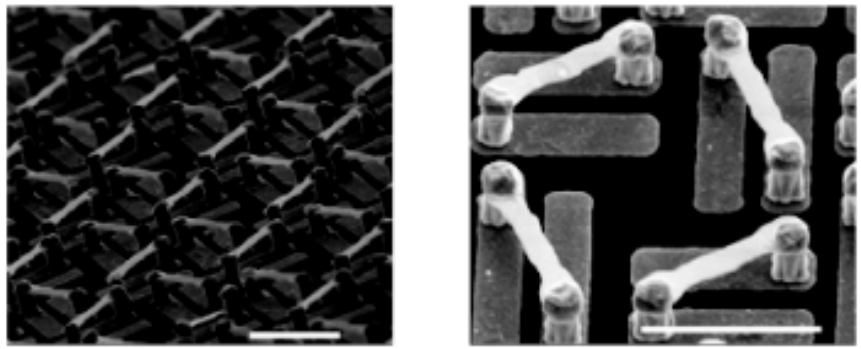


(Rosa, DD, Milonni, PRA 2008)

Chiral MMs and Casimir

The chirality of a MM is defined by the chirality of its unit cell





In a chiral medium, the constitutive relations mix electric and magnetic fields

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r},\omega) - i\kappa(\omega)\mathbf{H}(\mathbf{r},\omega)$$
$$\mathbf{B}(\mathbf{r},\omega) = i\kappa(\omega)\mathbf{E}(\mathbf{r},\omega) + \mu(\omega)\mathbf{H}(\mathbf{r},\omega)$$

dispersive chirality: $\kappa(\omega) = \dfrac{\omega_k \omega}{\omega^2 - \omega_{\kappa R}^2 + i\gamma_k \omega}$

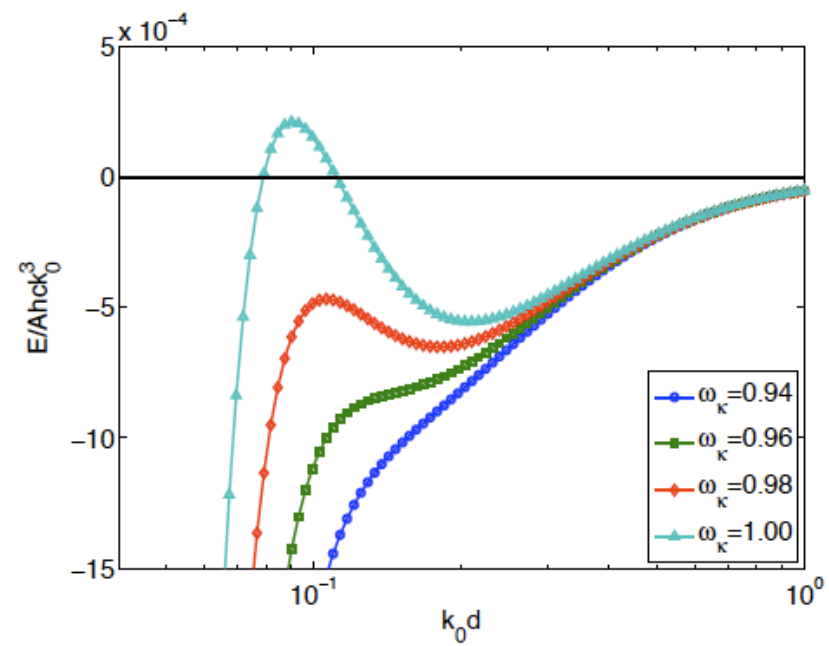
Casimir repulsion with chirality

In chiral MMs the reflection matrix is non-diagonal (mixing of E and H fields).

The integrand of the Casimir-Lifshitz force between two identical chiral MMs has the form:

$$F = \frac{(r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} - 2(r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}{1 - (r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} + (r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}$$

One might achieve repulsive Casimir forces with strong chirality (i.e., large values of r_{sp})



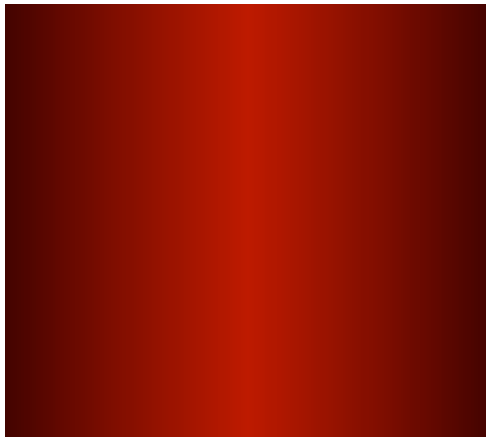
(Soukoulis et al, 2009)

Beyond the EMA

Everything discussed so far is based on the assumption that the effective medium approximation (EMA) holds. We recall that this amounts to treating the MM in the "long-wavelength approximation", i.e., field wavelengths much larger than the typical size of the unit cell of the MM.

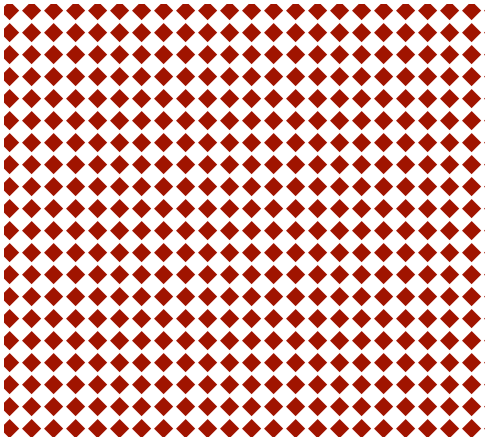
How to calculate Casimir forces when EMA does not hold?

Homogeneous medium



EMA

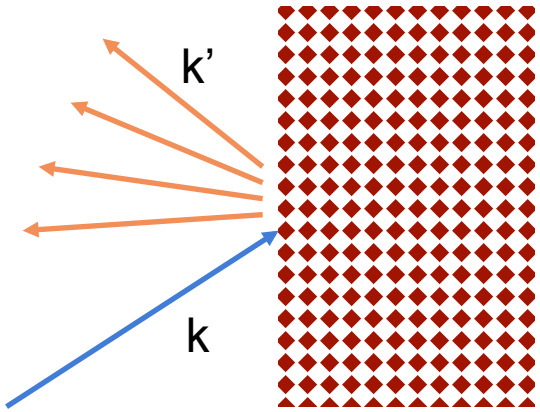
Non-homogeneous medium



beyond EMA

Scattering theory

The Casimir force still may be described in terms of reflections (scattering theory)



Symbolically, we may write the Casimir energy as

$$\boxed{\frac{E(d)}{A} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\left[1 - \mathcal{R}_1 e^{-\mathcal{K}d} \mathcal{R}_2 e^{-\mathcal{K}d}\right]}$$

where $\mathcal{R}_i \equiv \mathcal{R}_i(\mathbf{k}_{||}, \mathbf{k}'_{||}, p, p', i\xi)$

Finding the reflection matrix

The reflection matrix can be obtained with standard methods of numerical electromagnetism. One solves Maxwell equations for the transverse fields

$$-ik\frac{\partial \mathbf{E}_t}{\partial z} = \nabla_t \left[\chi \hat{e}_3 \cdot \nabla \times \mathbf{H}_t\right] - k^2 \mu \hat{e}_3 \times \mathbf{H}_t$$
$$-ik\frac{\partial \mathbf{H}_t}{\partial z} = -\nabla_t \left[\zeta \hat{e}_3 \cdot \nabla \times \mathbf{E}_t\right] + k^2 \epsilon \hat{e}_3 \times \mathbf{E}_t$$

Assuming a two-dimensional periodic structure, we have

$$\mathbf{E}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{E}_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mathbf{H}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{H}_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

where

$$\epsilon(x,y) = \sum_{m,n} \epsilon_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mu(x,y) = \sum_{m,n} \mu_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

Exact reflection matrix

One can then write the equations for the transverse fields as

$$\boxed{-ik\frac{\partial \Psi_{m'n'}}{\partial z} = \sum_{mn} H_{m'n',mn}\Psi_{mn}}$$

$$\Psi_{mn} = \begin{bmatrix} \mathcal{E}^x_{mn} \\ \mathcal{E}^y_{mn} \\ \mathcal{H}^x_{mn} \\ \mathcal{E}^y_{mn} \end{bmatrix} = \begin{bmatrix} \Psi^1_{mn} \\ \Psi^2_{mn} \\ \Psi^3_{mn} \\ \Psi^4_{mn} \end{bmatrix}$$

Here H is a complicated matrix, that encapsulated the coupling of modes in the periodic structure.

By numerically solving this equation and imposing the proper boundary conditions of the field on the vacuum-metamaterial interphase (RCWA or S-matrix techniques), one can find the reflection matrix of the MM.

Ongoing projects at LANL

- Casimir force engineering with metamaterials (LANL, SNL)

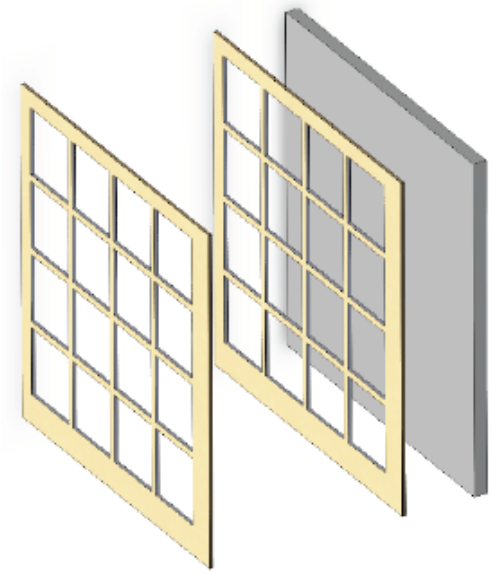
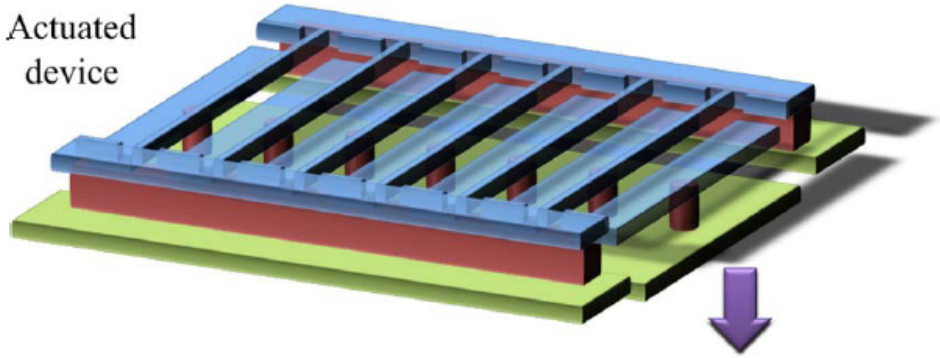


Fig. III.4: Stacked fishnet structure for creating multiple magnetic resonances.

- Controlling the Casimir force with deformable nanostructures (LANL, ANL, NIST, Indiana)



Final remarks

- Metamaterials offer several possibilities to tailor the electromagnetic response for Casimir force engineering.

- Casimir force reduction should be possible with MMs with decreased dielectric response, increased magnetic activity, and possibly with chirality.

- Casimir force repulsion is more challenging, since permeabilities and chiralities have to be much stronger than permittivities over a large range of frequencies.

- Effective medium approximation should break down for gaps of the order or smaller than the size of the MM unit cell. Exact numerical methods that go beyond EMA are needed in those regimes.