

A tutorial on Casimir interactions between nanostructured materials

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Outline of this Talk

■ Brief intro to Casimir physics

- Basics, relevance, and simple geometries

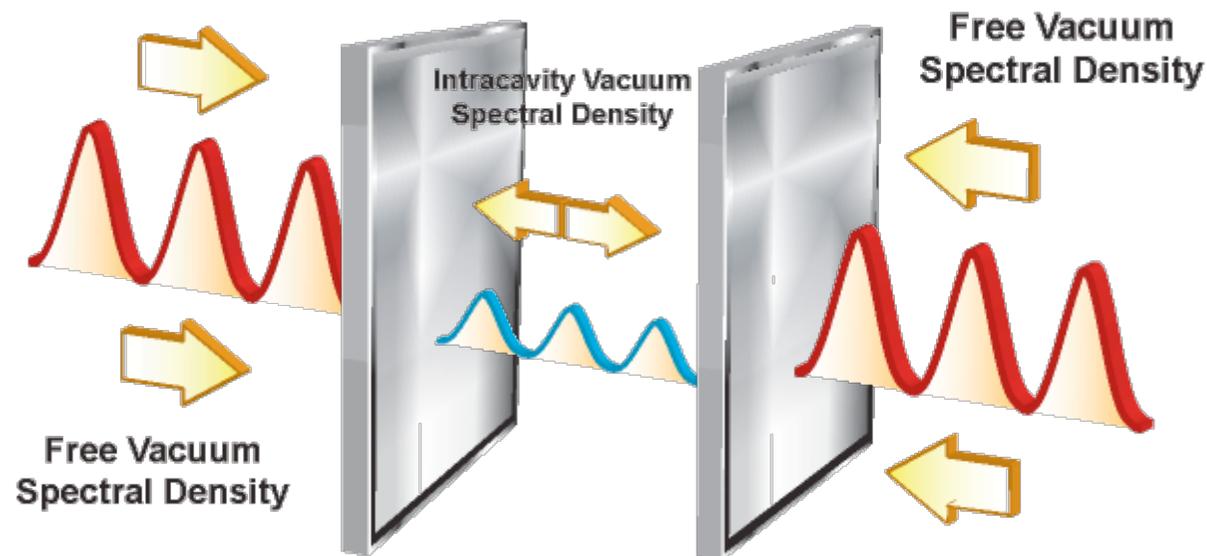
■ Tailoring Casimir forces with metamaterials

- Effective medium/homogenization in Casimir physics

■ Tailoring Casimir forces with nanostructures

- Metallic gratings for Casimir force manipulation

Brief intro to Casimir phys.



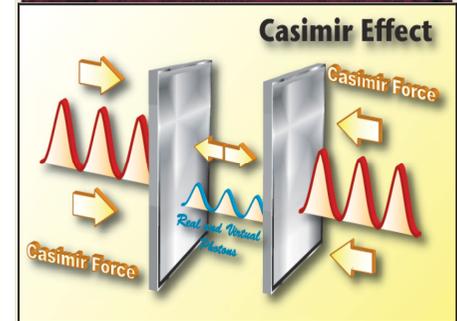
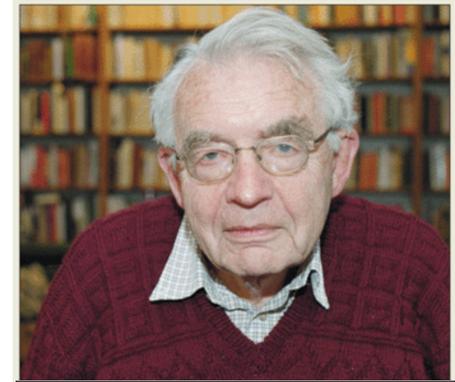
The Casimir force

- **Universal effect from confinement of vacuum fluctuations**

- Depends only on \hbar , c , and geometry

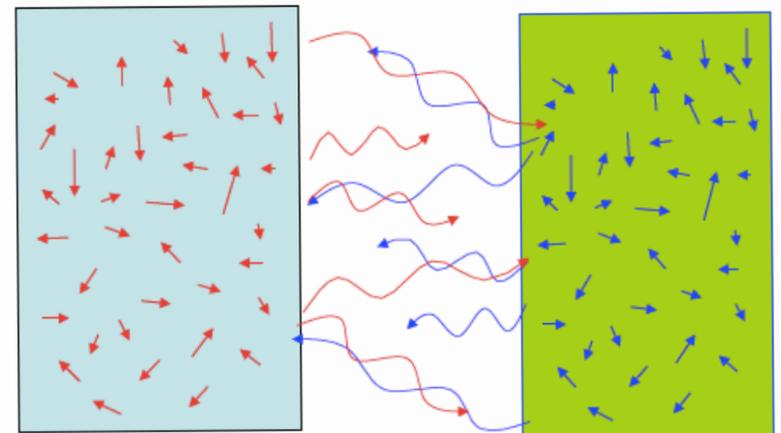
$$E = \frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \Rightarrow \boxed{\frac{F}{A} = \frac{\pi^2}{240} \frac{\hbar c}{d^4}}$$

(130nN/cm² @ $d = 1\mu\text{m}$)



- Alternative interpretation: **fluctuating charges and currents**

- The magnitude and sign of the force depends on geometry, materials, and temperature



The Lifshitz formula

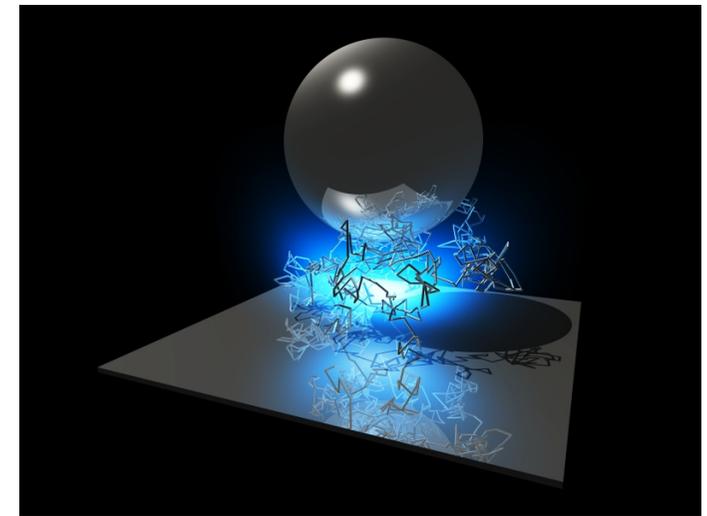
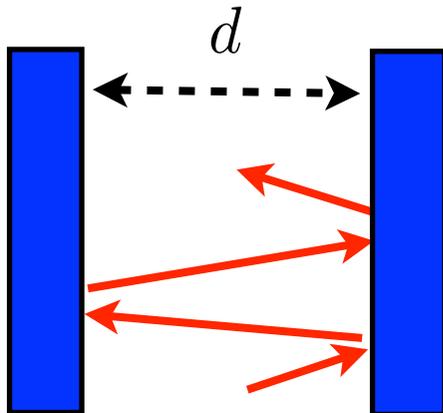
Casimir interaction energy between materials slabs (Lifshitz 1956)

$$\frac{E(d)}{A} = \hbar \sum_p \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \coth\left(\frac{\hbar\omega}{2k_B T}\right) \text{Im} \log[1 - R_{1,p}(\omega, k) R_{2,p}(\omega, k) e^{2id\sqrt{\omega^2/c^2 - k^2}}]$$

Fresnel reflection coefficients $R_{\text{TE}} = \frac{k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}$ $R_{\text{TM}} = \frac{\epsilon(\omega)k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{\epsilon(\omega)k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}$

The log factor can be re-written as

$$\propto \sum_{n=1}^{\infty} \frac{1}{n} [R_{1,p} e^{idk_z} R_{2,p} e^{idk_z}]^n$$



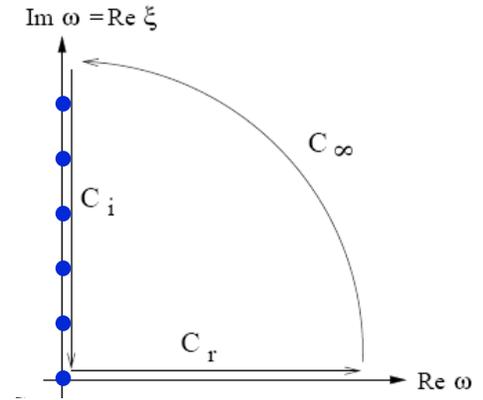
Scattering theory

Going to imaginary freq.

The function $\coth(\hbar\omega/2k_B T)$ has poles on the imaginary frequency axis at

$$\omega_m = i\xi_m, \quad \xi_m = m \frac{2\pi k_B T}{\hbar}$$

After Wick rotation:



$$\frac{F}{A} = -2k_B T \sum_p \sum_{m=0}^{\infty'} \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \sqrt{\xi_m^2/c^2 + k^2} \frac{R_{1,p}(i\xi_m, k) R_{2,p}(i\xi_m, k) e^{-2d\sqrt{\xi_m^2/c^2 + k^2}}}{1 - R_{1,p}(i\xi_m, k) R_{2,p}(i\xi_m, k) e^{-2d\sqrt{\xi_m^2/c^2 + k^2}}}$$

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^{\infty} \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega$$

Kramers-Kronig (causality)

Some limiting cases:

- $F \propto d^{-3}$ (non-retarded limit, small distances)
- $F \propto d^{-4}$ (retarded limit, larger distances)
- $F \propto T d^{-3}$ (classical limit, very large distances)

 **Casimir physics is a broad-band frequency phenomenon**

The sign of the Casimir force

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \text{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$

The sign of the force is directly connected to the **sign of the product of the reflection coefficients** on the two plates, **evaluated at imaginary frequencies**. As a rule of thumb, we have (p=TE, TM)

$$R_1^p(i\xi) \cdot R_2^p(i\xi) > 0 \quad (\forall \xi \leq c/d) \Rightarrow \text{Attraction}$$

$$R_1^p(i\xi) \cdot R_2^p(i\xi) < 0 \quad (\forall \xi \leq c/d) \Rightarrow \text{Repulsion}$$

In terms of permittivities and permeabilities:

$$\begin{array}{l} \epsilon_a(i\xi) \gg \epsilon_b(i\xi) \\ \mu_b(i\xi) \gg \mu_a(i\xi) \end{array} \longrightarrow \text{Repulsion}$$

Ideal attraction-repulsion

● Ideal attractive limit

(Casimir 1948)

$$\frac{F}{A} = + \frac{\pi^2}{240} \frac{\hbar c}{d^4}$$

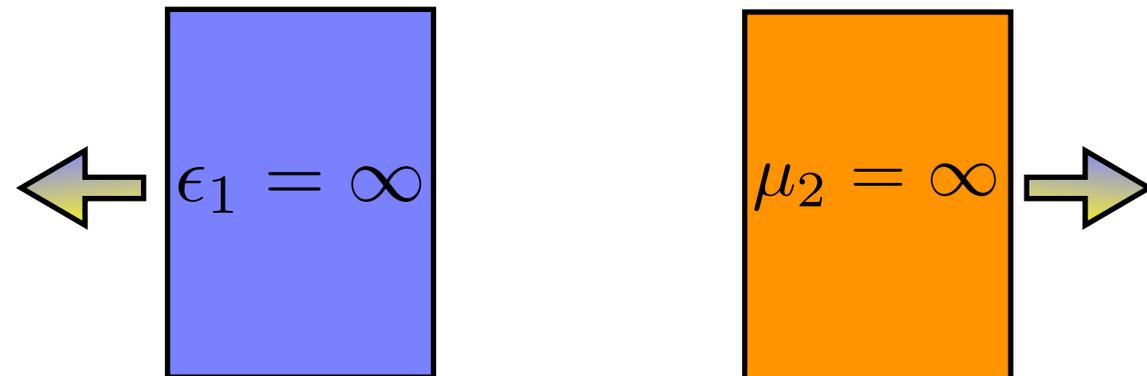
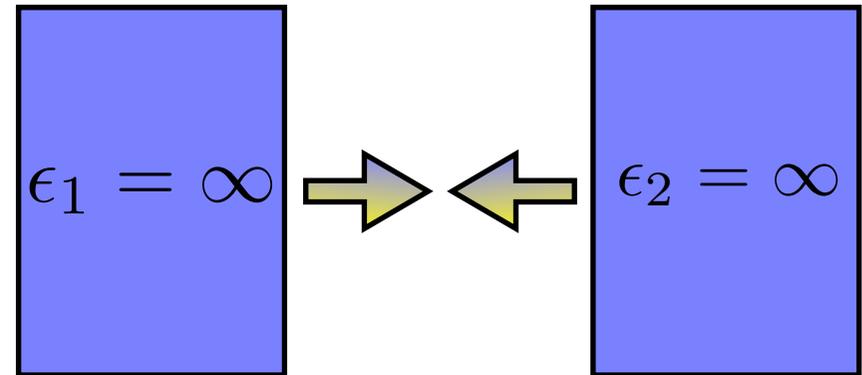
● Ideal repulsive limit

(Boyer 1974)

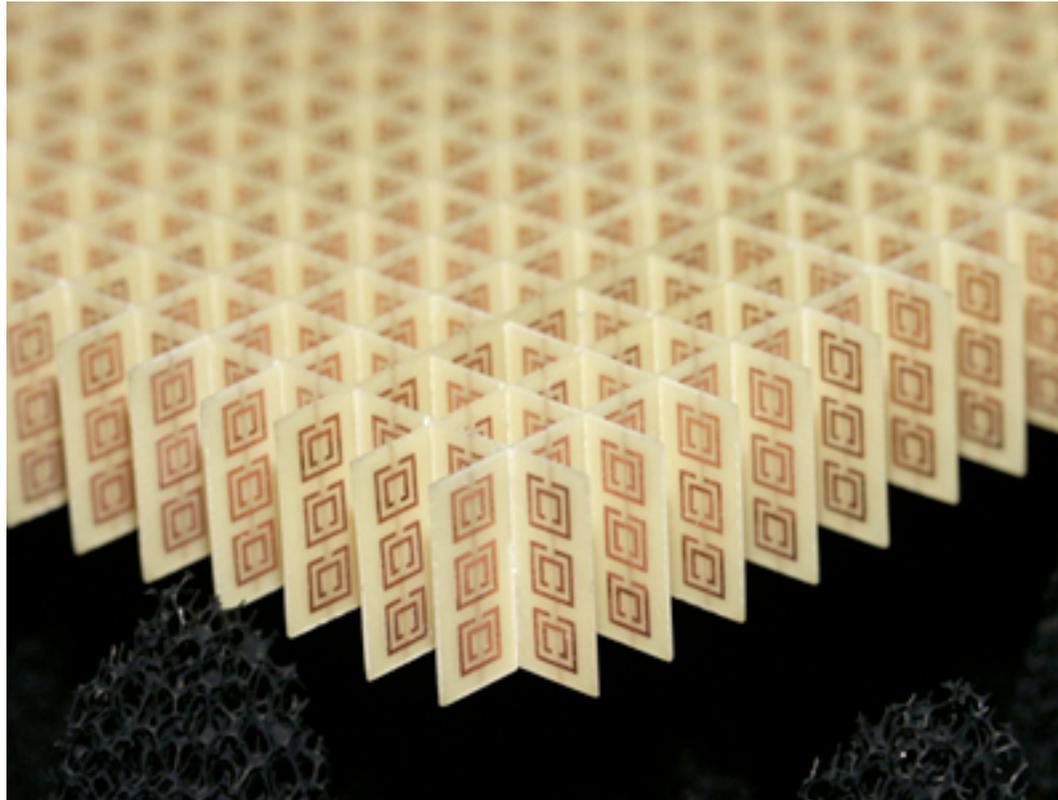
$$\frac{F}{A} = - \frac{7}{8} \frac{\pi^2}{240} \frac{\hbar c}{d^4}$$

● Real repulsion

Natural occurring materials do **NOT** have strong magnetic response in the optical region, i.e. $\mu = 1$ \longrightarrow **Metamaterials**



Metamaterials and Casimir

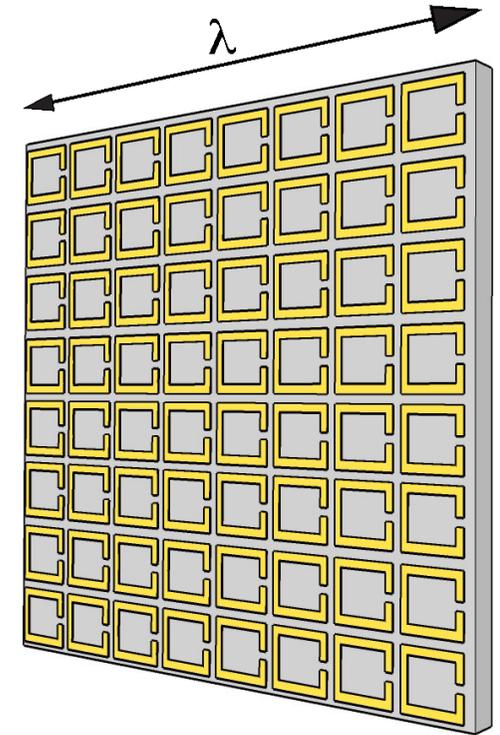


Effective medium approx.

Imagine that the metamaterial is probed at wavelengths much larger than the average distance between the constituent “meta-atoms”

In this situation the MM is effectively a **continuous medium**, whose optical response can be characterized by an effective electric permittivity and an effective magnetic permeability.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$
$$\mu_{eff} = 1 - \frac{\frac{\pi r^2}{a^2}}{1 + \frac{2\sigma i}{\omega r \mu_0} - \frac{3}{\pi^2 \mu_0 \omega^2 C r^3}}$$



Optical response

Close to the resonance, both $\epsilon(\omega)$ and $\mu(\omega)$ can be modeled by Drude-Lorentz formulas

$$\epsilon_{\alpha}(\omega) = 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega}$$

$$\mu_{\alpha}(\omega) = 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}$$

Typical separations

$$d = 200 - 1000 \text{ nm}$$

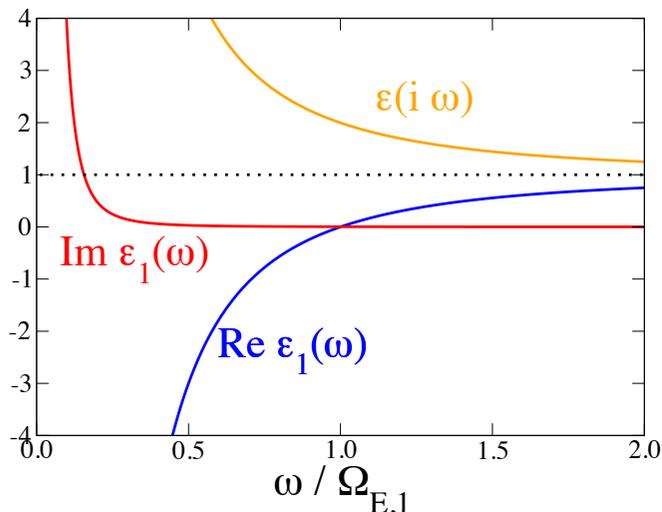


Infrared-optical frequencies

$$\Omega/2\pi = 5 \times 10^{14} \text{ Hz}$$

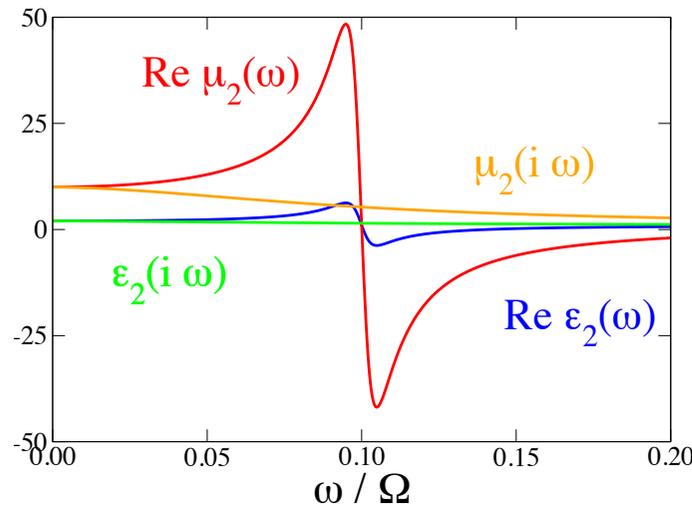
Drude metal (Au)

$$\Omega_E = 9.0 \text{ eV} \quad \Gamma_E = 35 \text{ meV}$$



Metamaterial

$$\text{Re } \epsilon_2(\omega) < 0 \quad \text{Re } \mu_2(\omega) < 0$$



$$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$$

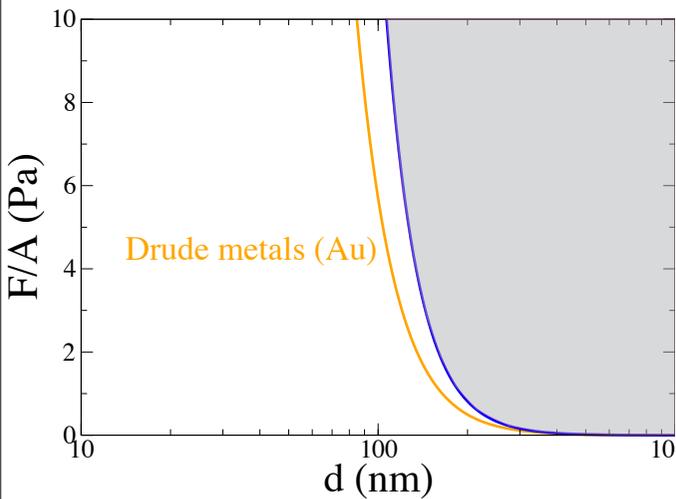
$$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$$

$$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$$

Attraction-repulsion crossover

Drude metal (Au)

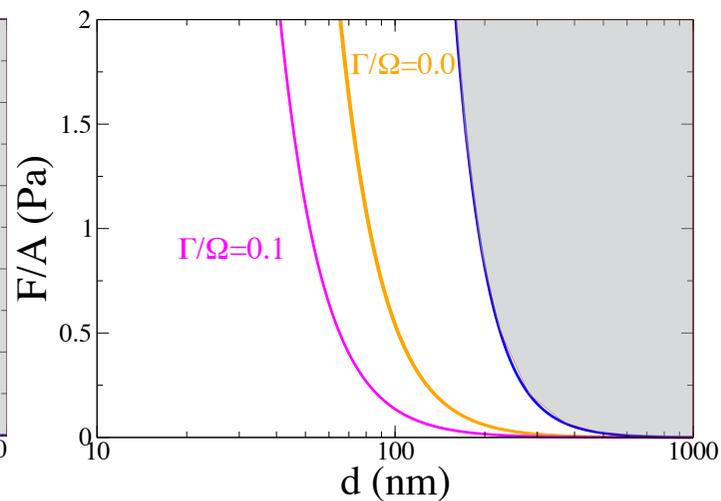
Drude metal (Au)



Only attraction

Metamaterial

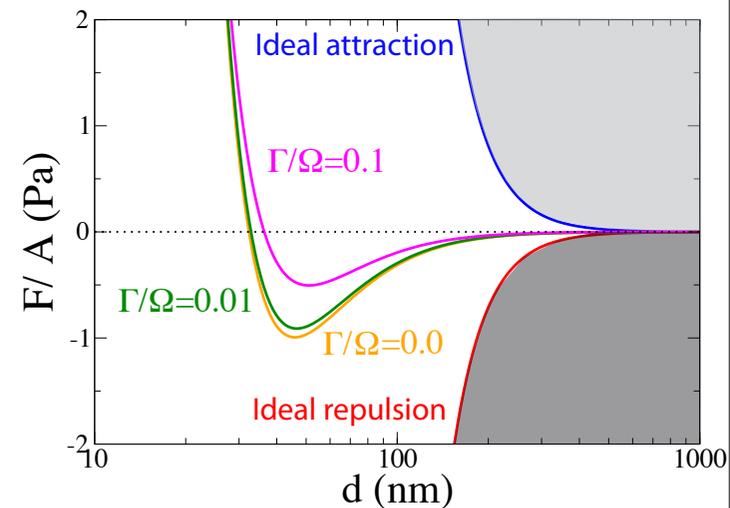
Metamaterial



Only attraction

Drude metal (Au)

Metamaterial



Repulsion-attraction

EMA: correct model for μ

● Drude-Lorentz model for permeability is wrong!

● The correct expression for $\mu_{\text{eff}}(\omega)$ from Maxwell's equations

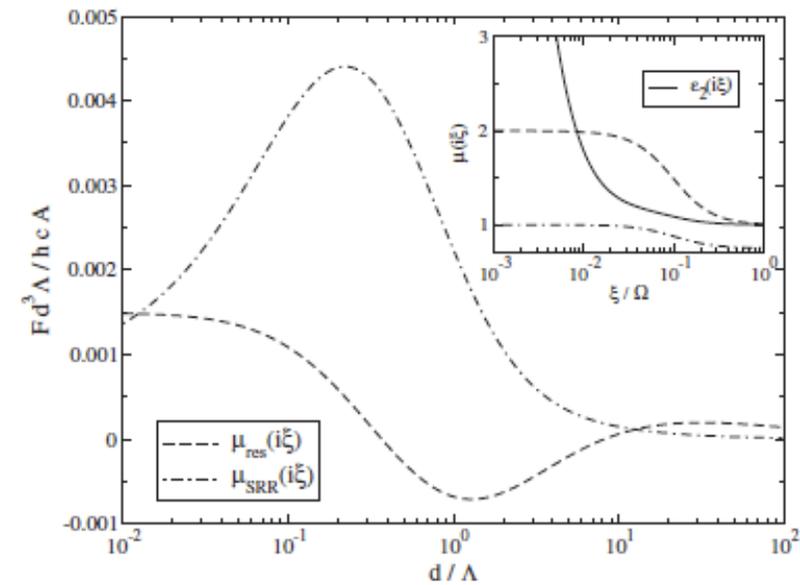
$$\mu_{\text{eff}}(\omega) = 1 - f \frac{\omega^2}{\omega^2 - \omega_m^2 + 2i\gamma_m\omega}$$

(Pendry 1999)

● Correct low frequency behavior very different from Drude-Lorentz model

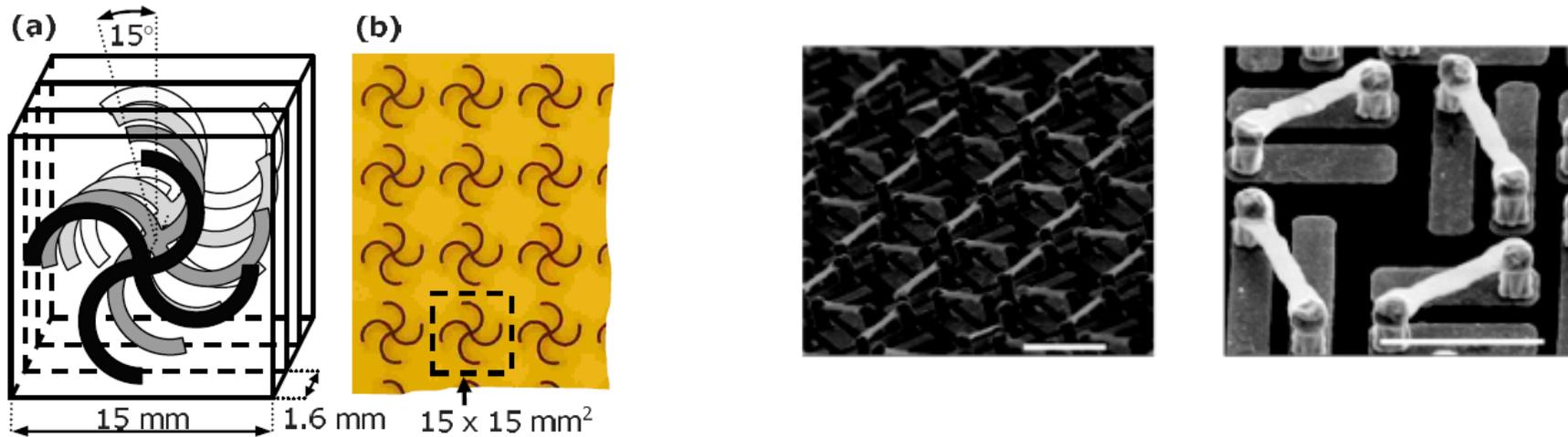
$$\mu_{\text{eff}}(i\xi) < 1 < \epsilon_{\text{eff}}(i\xi)$$

No Casimir repulsion!



(Rosa, DD, Milonni, 2008)

Other Casimir MMs: chirality



● Constitutive relations mix electric and magnetic fields

$$\begin{aligned} \mathbf{D}(\mathbf{r}, \omega) &= \epsilon(\omega)\mathbf{E}(\mathbf{r}, \omega) - i\kappa(\omega)\mathbf{H}(\mathbf{r}, \omega) \\ \mathbf{B}(\mathbf{r}, \omega) &= i\kappa(\omega)\mathbf{E}(\mathbf{r}, \omega) + \mu(\omega)\mathbf{H}(\mathbf{r}, \omega) \end{aligned}$$

dispersive chirality: $\kappa(\omega) = \frac{\omega_k \omega}{\omega^2 - \omega_{\kappa R}^2 + i\gamma_k \omega}$

Reflection matrices become non-diagonal

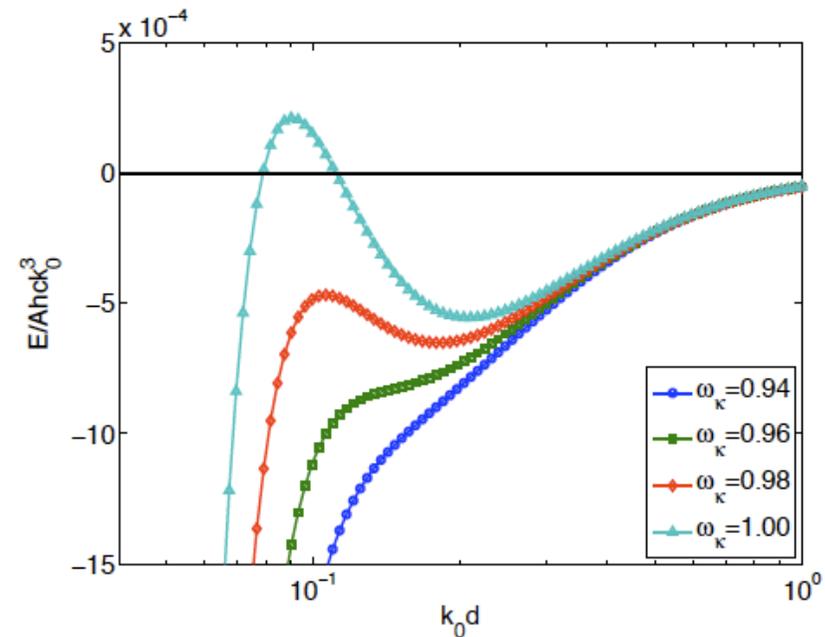
Repulsion and chiral MMs

- Casimir force between two chiral materials

$$F = \frac{(r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} - 2(r_{sp}^2 + r_{ss}r_{pp})^2e^{-4Kd}}{1 - (r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} + (r_{sp}^2 + r_{ss}r_{pp})^2e^{-4Kd}}$$

Repulsion can be achieved with strong chirality, which results in large values of r_{sp}

(Soukoulis *et al.*, 2009)



- However, predictions are based on EMA in a region of parameters where EMA is expected to fail!

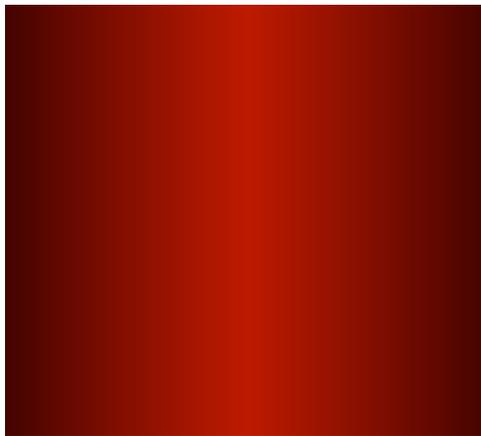
- Exact numerics shows that there is no repulsion

Going beyond EMA

So far, we have treated the MM in the “long-wavelength approximation”, i.e., field wavelengths much larger than the typical size of the unit cell of the MM.

How to calculate Casimir forces when EMA does not hold?

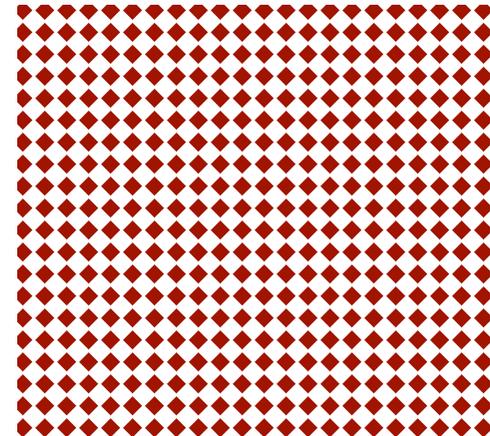
Homogeneous
medium



EMA

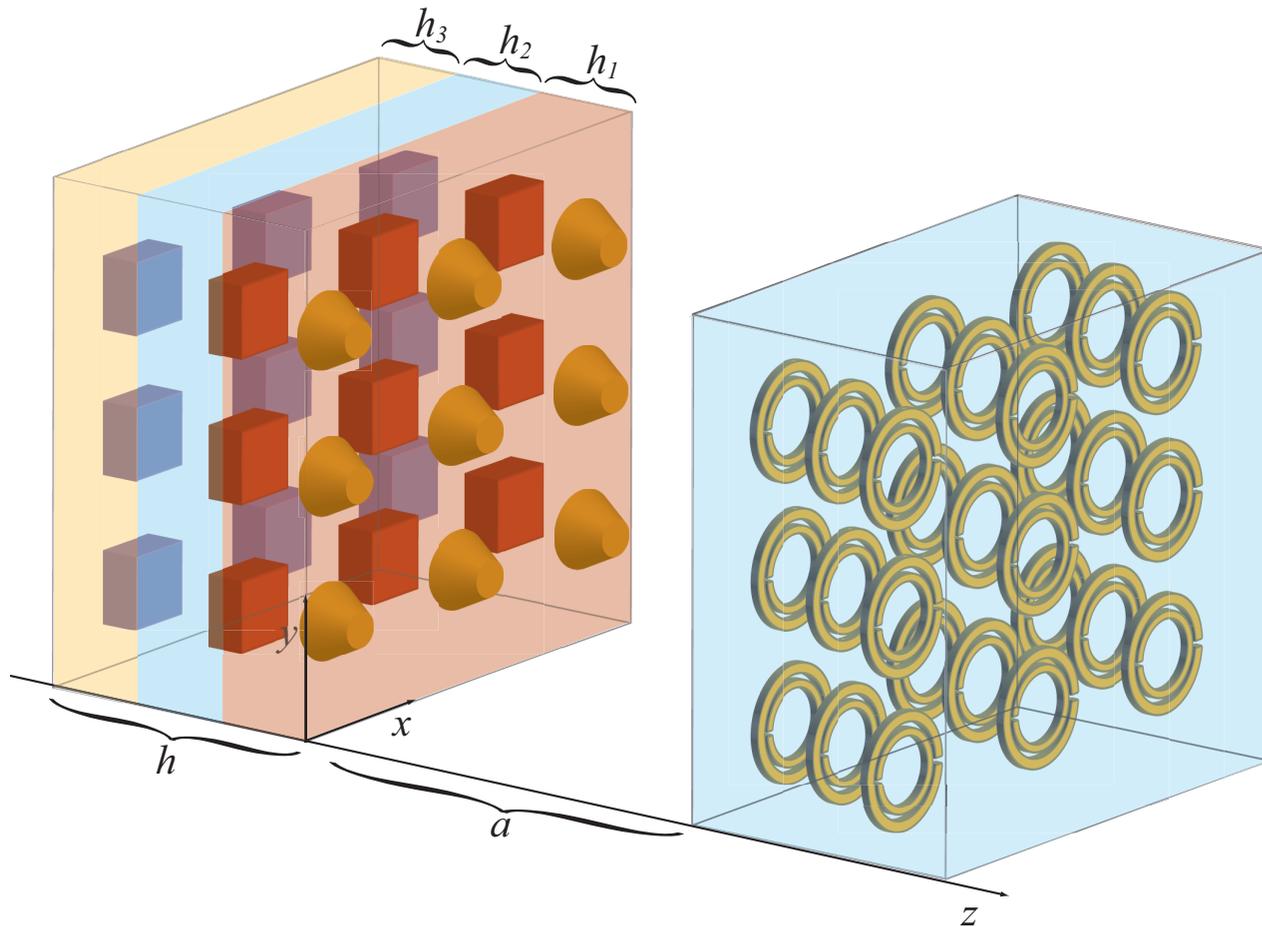


Non-homogeneous
medium

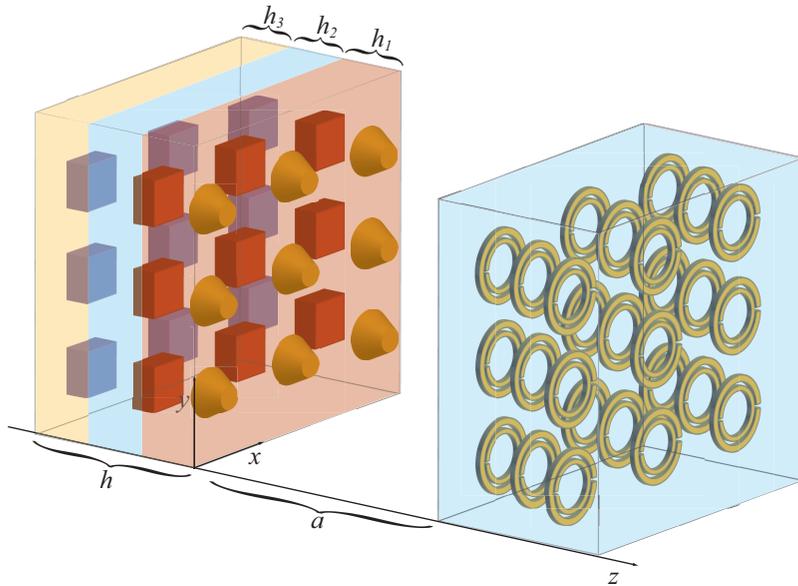


beyond EMA

Casimir nanostructures

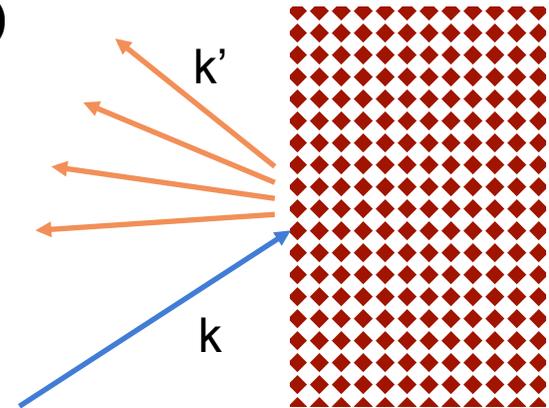


Scattering theory



The Casimir force still may be described in terms of reflections
(scattering theory)

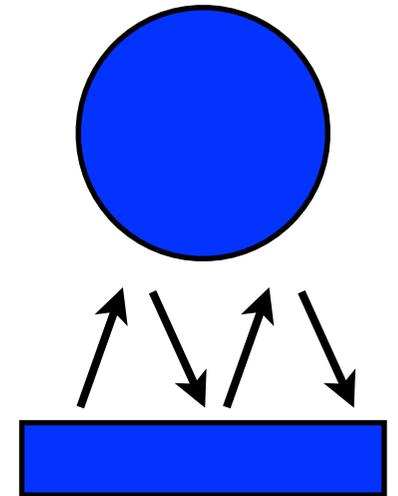
$$\mathcal{R}_i(\omega, \mathbf{k}, \mathbf{k}', p, p')$$



Symbolically, we may write the Casimir energy as

$$\frac{E(d)}{A} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log \det [1 - \mathcal{R}_1 e^{-\mathcal{K}d} \mathcal{R}_2 e^{-\mathcal{K}d}]$$

$$\propto \sum_{n=1}^{\infty} \frac{1}{n} [\mathcal{R}_1(i\xi) e^{-d\mathcal{K}(i\xi)} \mathcal{R}_2(i\xi) e^{-d\mathcal{K}(i\xi)}]^n$$



Finding the reflection matrix

The reflection matrix can be obtained with standard methods of numerical electromagnetism. One way is to solve Maxwell equations for the transverse fields

$$\begin{aligned} -ik \frac{\partial \mathbf{E}_t}{\partial z} &= \nabla_t [\chi \hat{e}_3 \cdot \nabla \times \mathbf{H}_t] - k^2 \mu \hat{e}_3 \times \mathbf{H}_t \\ -ik \frac{\partial \mathbf{H}_t}{\partial z} &= -\nabla_t [\zeta \hat{e}_3 \cdot \nabla \times \mathbf{E}_t] + k^2 \epsilon \hat{e}_3 \times \mathbf{E}_t \end{aligned}$$

Assuming a two-dimensional periodic structure, we have

$$\mathbf{E}_t(x, y) = e^{i\mathbf{k} \cdot \mathbf{r}} \sum_{m,n} \mathcal{E}_{m,n} \exp \left[i \frac{2\pi n}{L_x} x + i \frac{2\pi m}{L_y} y \right]$$

$$\mathbf{H}_t(x, y) = e^{i\mathbf{k} \cdot \mathbf{r}} \sum_{m,n} \mathcal{H}_{m,n} \exp \left[i \frac{2\pi n}{L_x} x + i \frac{2\pi m}{L_y} y \right]$$

where

$$\epsilon(x, y) = \sum_{m,n} \epsilon_{m,n} \exp \left[i \frac{2\pi n}{L_x} x + i \frac{2\pi m}{L_y} y \right]$$

$$\mu(x, y) = \sum_{m,n} \mu_{m,n} \exp \left[i \frac{2\pi n}{L_x} x + i \frac{2\pi m}{L_y} y \right]$$

Exact reflection matrix

One can then write the equations for the transverse fields as

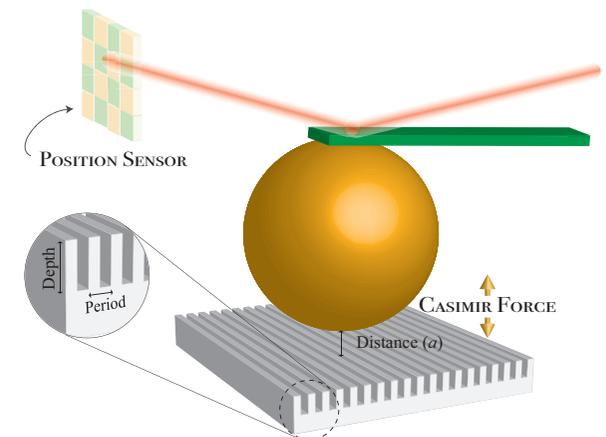
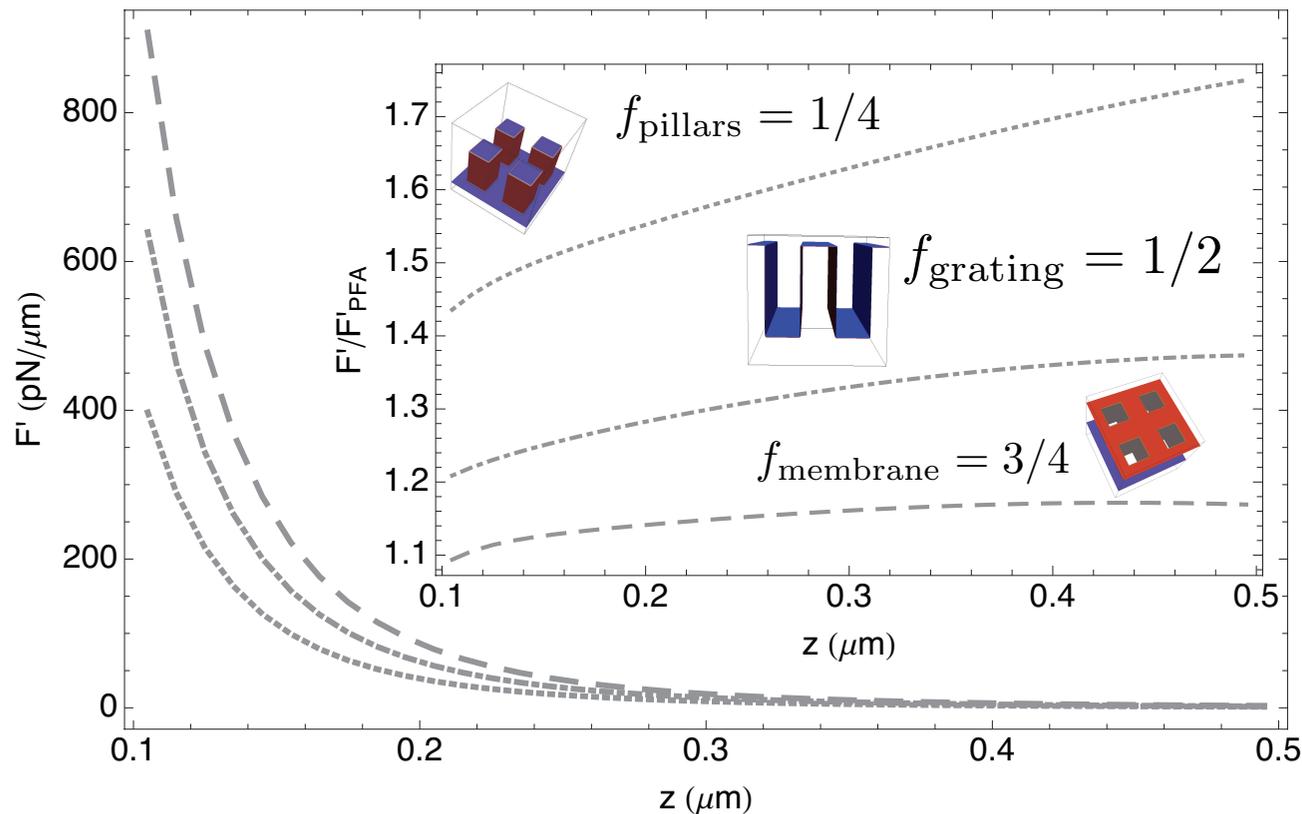
$$\boxed{-ik \frac{\partial \Psi_{m'n'}}{\partial z} = \sum_{mn} H_{m'n',mn} \Psi_{mn}} \quad \Psi_{mn} = \begin{bmatrix} \mathcal{E}_{mn}^x \\ \mathcal{E}_{mn}^y \\ \mathcal{H}_{mn}^x \\ \mathcal{E}_{mn}^y \end{bmatrix} = \begin{bmatrix} \psi_{mn}^1 \\ \psi_{mn}^2 \\ \psi_{mn}^3 \\ \psi_{mn}^4 \end{bmatrix}$$

Here H is a complicated matrix, that encapsulates the coupling of modes in the periodic structure.

By numerically solving this equation and imposing the proper boundary conditions of the field on the vacuum-metamaterial interphase (**RCWA** or **S-matrix techniques**), one can find the reflection matrix of the MM.

2D periodic structures

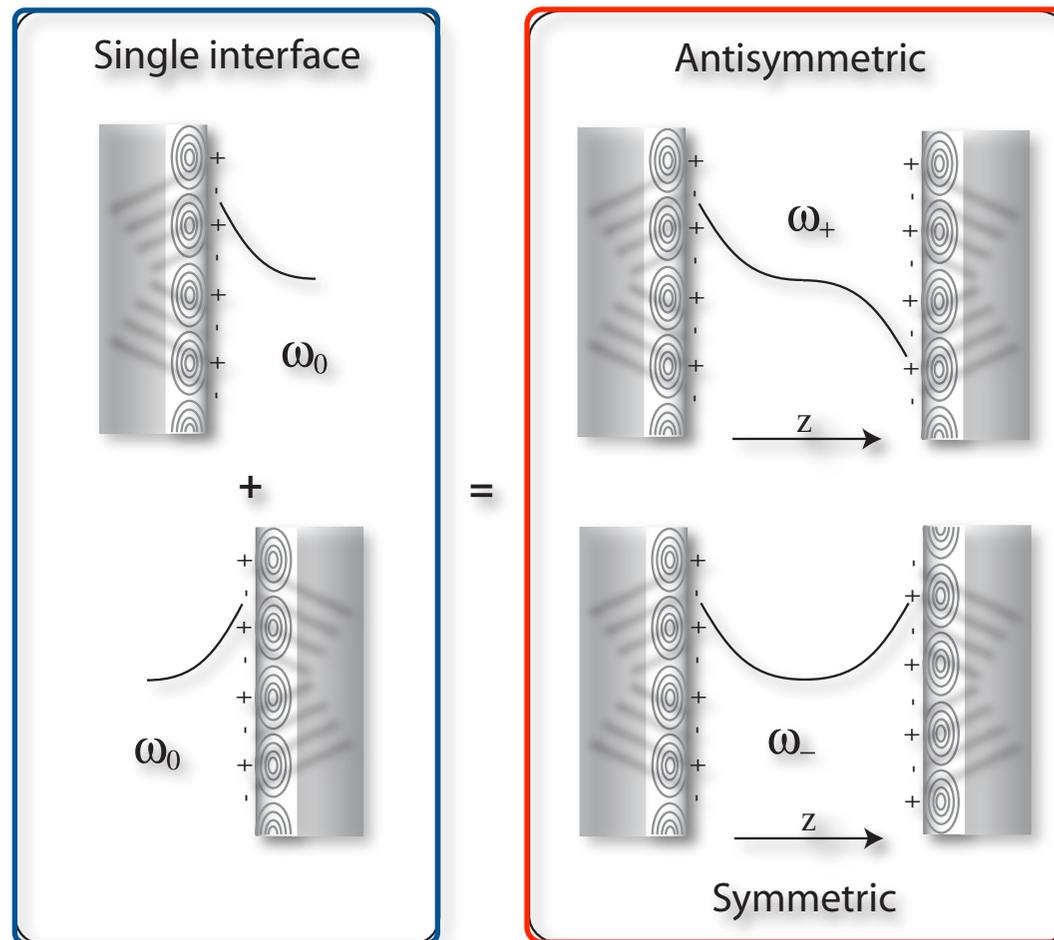
Example: Casimir force between a Au plane and Si pillars / grating / membrane @ $T=300$ K



$R = 50 \mu\text{m}$
period = 400 nm
depth = 1070 nm

(Davids, Intravaia, Rosa, DD, 2010)

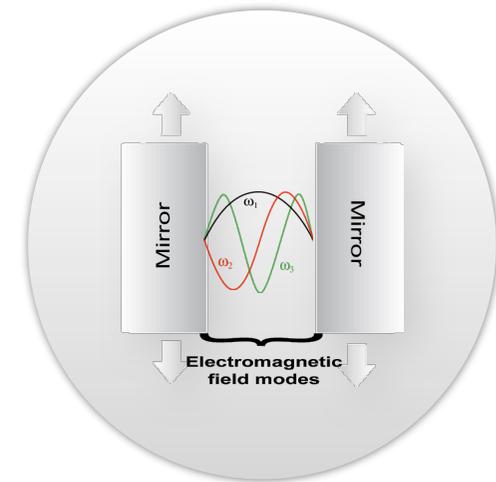
Casimir plasmonics



Mode summation approach

Alternative approach: compute Casimir energy as a sum over zero-point energies

$$E = \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p \right]_L}_{\text{Infinite zero point energy}} - \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p \right]_{L \rightarrow \infty}}_{\text{Setting the zero}}$$



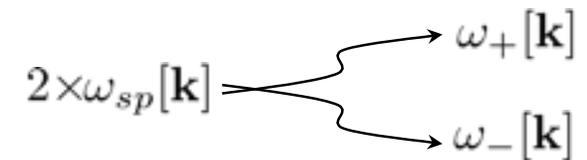
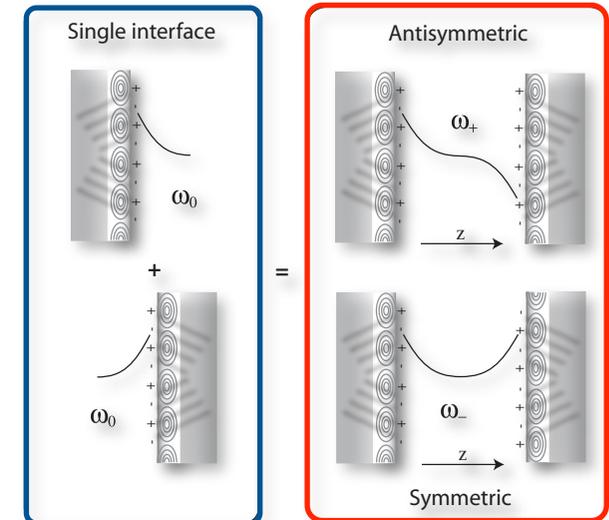
In the case of metallic plates described by the plasma model

$$\left. \begin{array}{l} \mu[\omega] = 1 \\ \epsilon[\omega] = 1 - \frac{\omega_p^2}{\omega^2} \end{array} \right\} \rightarrow E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2} [\omega_+ + \omega_-]_{L \rightarrow \infty}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m^p \right]_{L \rightarrow \infty}}_{\text{Photonic contribution } (E_{ph})}$$

Surface plasmons interaction

● **Surface plasmons:** evanescent modes of the EM field associated with electronic density oscillations at the metal-vacuum interface.

● When the tails of the evanescent fields overlap, the two surface plasmons hybridize

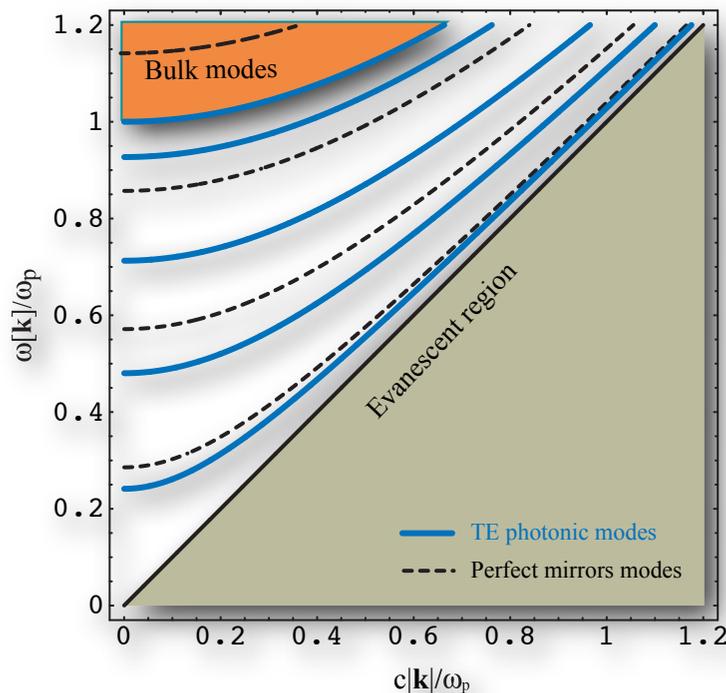


● At short distances the Casimir energy is given by the shift in the zero-point energy of the surface plasmons due to their Coulomb (electrostatic) interaction)

$$E_{sp} = A \int \frac{d^2\mathbf{k}}{(2\pi)^2} \left(\frac{\hbar\omega_+}{2} + \frac{\hbar\omega_-}{2} - 2\frac{\hbar\omega_{sp}}{2} \right) = -\frac{\hbar c \alpha \pi^2 A}{580 \lambda_p L^2}$$

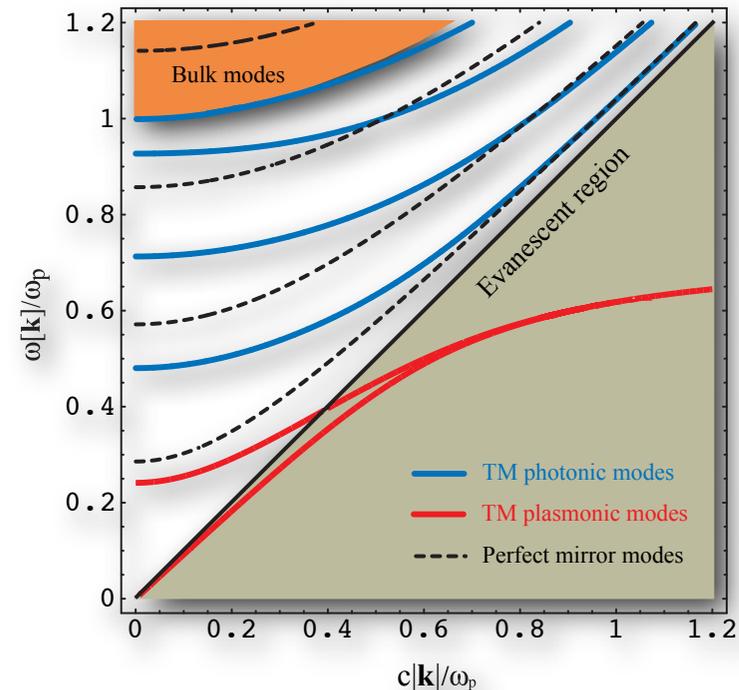
Mode spectrum in a cavity

$$E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2} [\omega_+ + \omega_-]_{L \rightarrow \infty}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p, \mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m^p \right]_{L \rightarrow \infty}^L}_{\text{Photonic contribution } (E_{ph})}$$



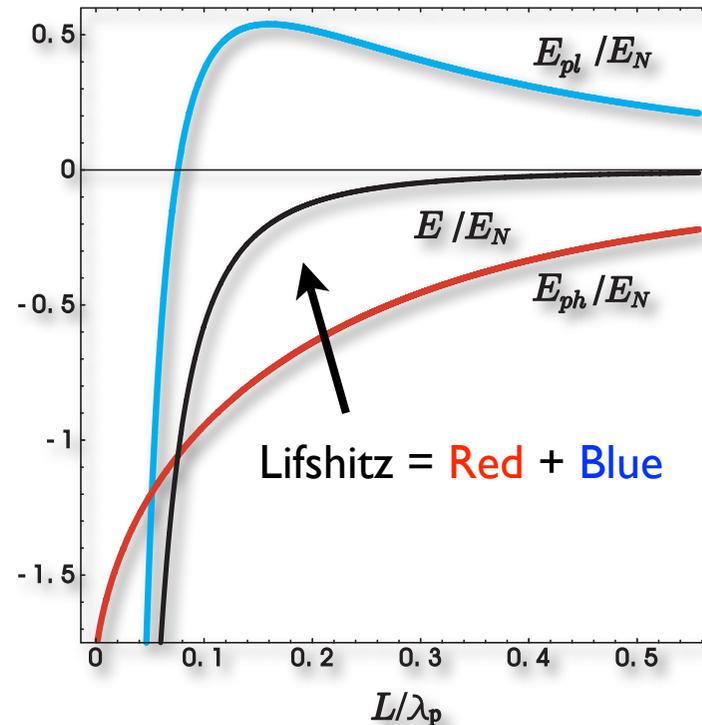
All the TE-modes belong to the propagative sector

They differ from the perfect mirrors modes because of the dephasing due to the non perfect reflection coefficient.



TM-modes propagative modes look qualitatively like TE modes.

There are only two evanescent modes. They are the generalization to all distances of the coupled plasmon modes.

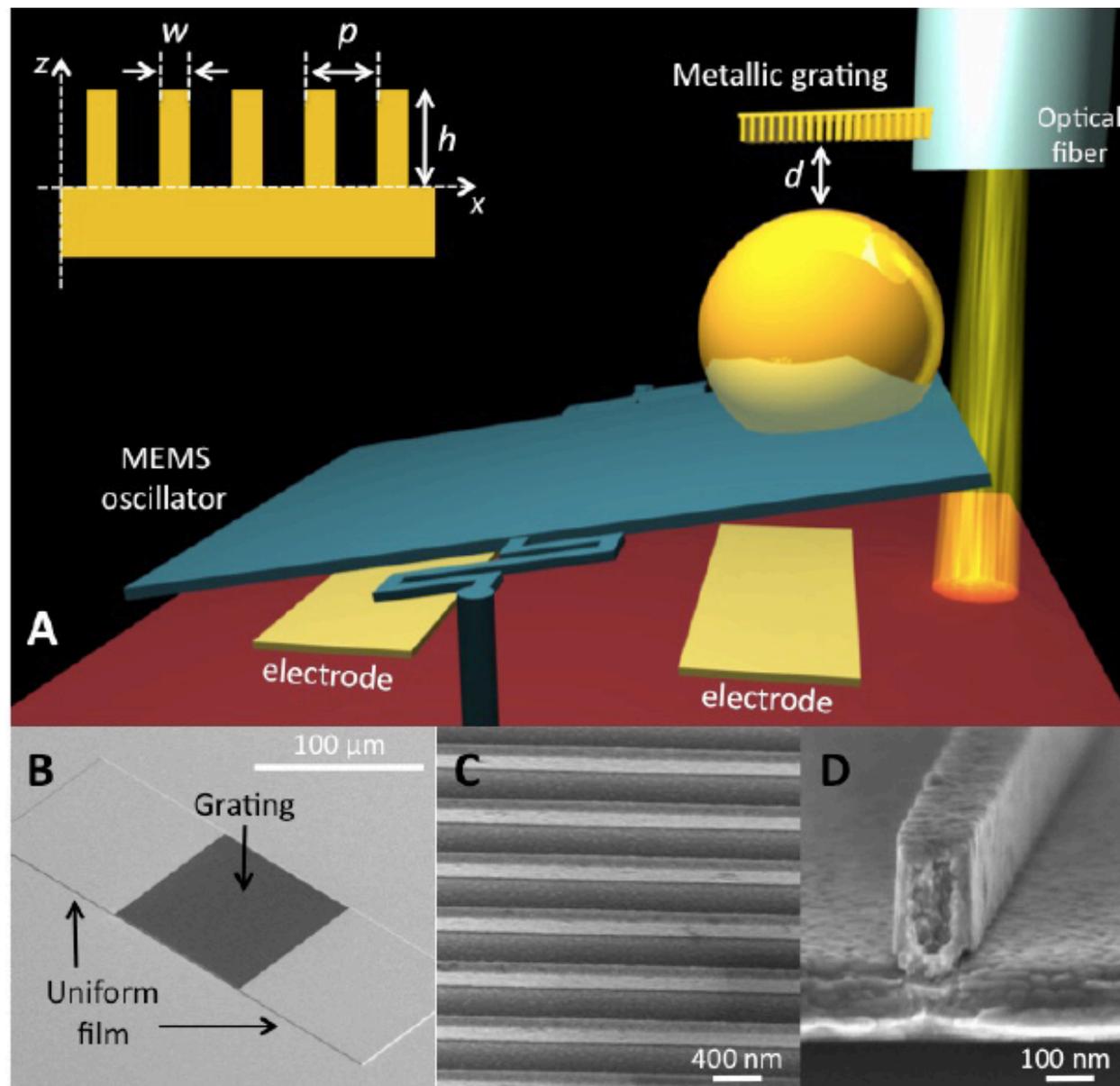


(Intravaia *et al*, 2005)

- The photonic contribution is always attractive
- The plasmonic contribution is repulsive at large distances, and attractive at short distances
- Their sum is always attractive due to a delicate cancellation

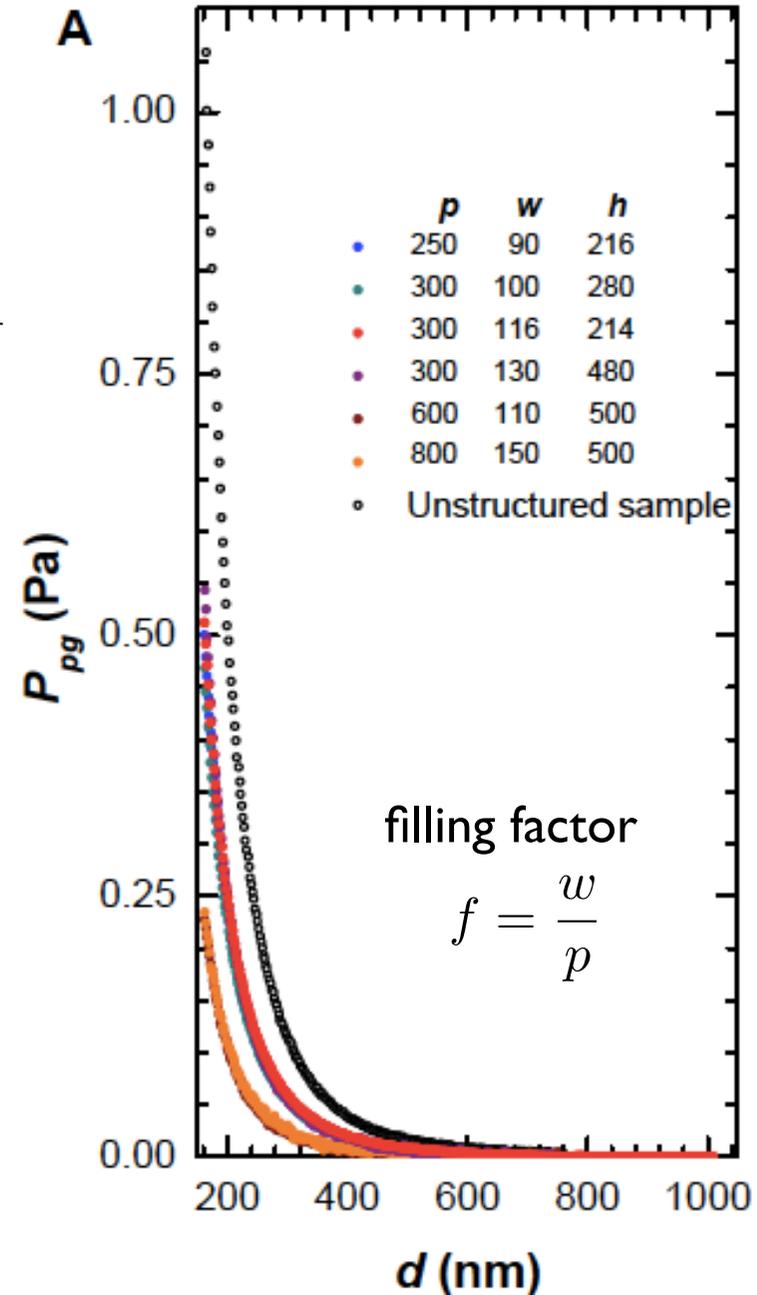
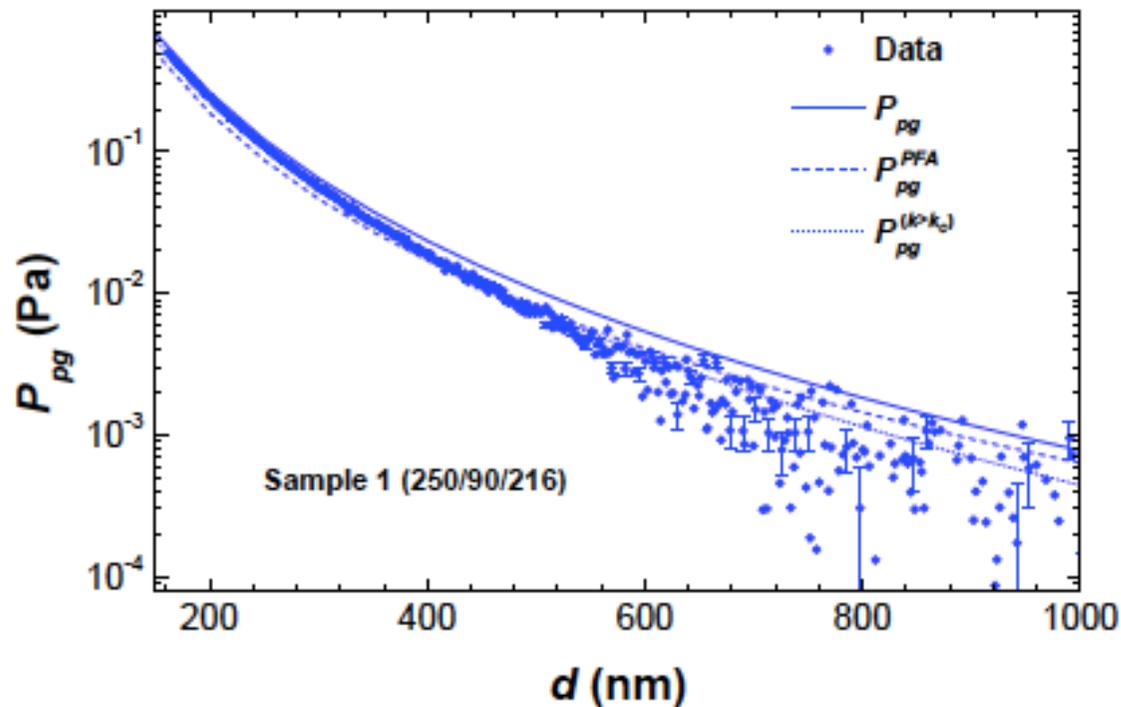
Can one control the Casimir force by changing the balance of the two contributions?

Metallic nano-gratings



Strong force reduction

- Torsional balance set-up
- Metallic sphere ($R = 150 \mu\text{m}$)
- Metallic nanostructures $w, p, h \approx 100 \text{ nm}$
- Sputtering and electroplating



Modeling and simulation

- Use of standard PFA to treat the sphere's curvature

$$F'_{sg} \approx 2\pi R P_{pg} \quad d/R < 6 \times 10^{-3}$$

- Exact plane-grating pressure P_{pg}

Scattering approach + modal expansions (Li 1993)

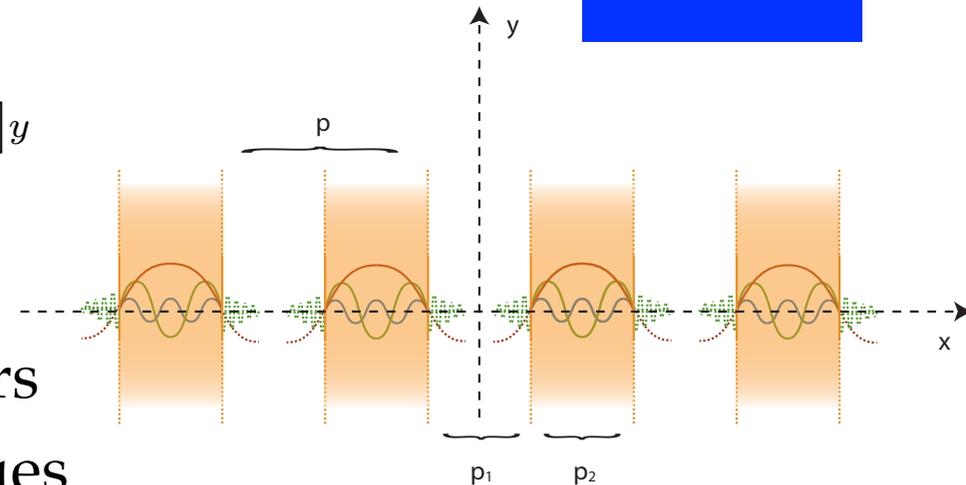
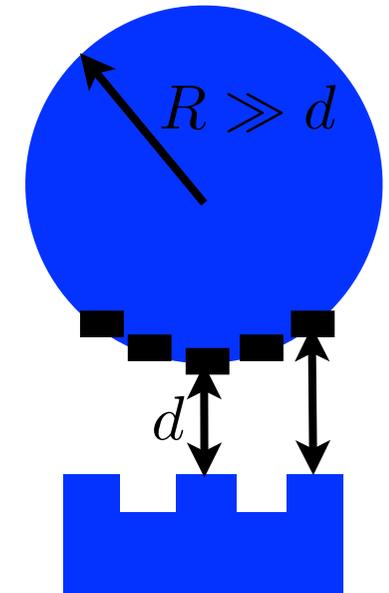
$$\begin{pmatrix} E_z(x, y) \\ E_x(x, y) \\ H_z(x, y) \\ H_x(x, y) \end{pmatrix}_i = \sum_{\nu, s} A_{\nu}^{(s, i)} \mathbf{Y}^{(s, i)} [x, \eta_{\nu}^{(s, i)}] e^{i\lambda[\eta_{\nu}^{(s, i)}] y}$$

Analytical expressions for eigenvectors

Transcendental equation for eigenvalues

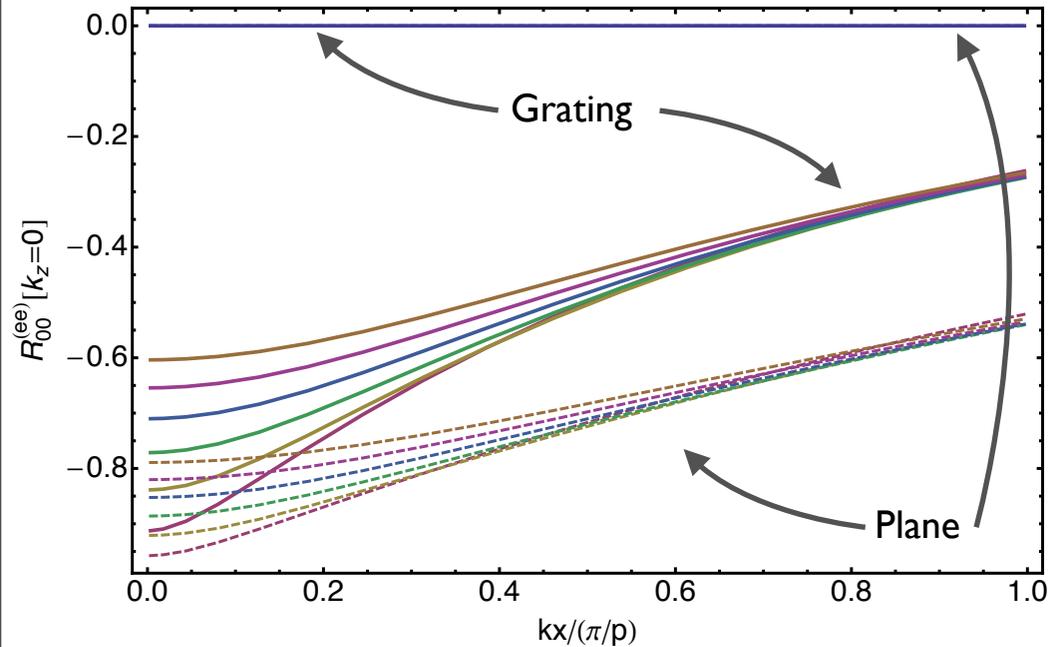
$$0 = \tilde{D}^{(s)}(\eta) = -\cos(\alpha_0 p) + \cos(p_1 \sqrt{\eta}) \cos(p_2 \sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}) - \frac{1}{2} \left(\frac{\sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}}{\sigma_2^{(s)}(i\xi)\sqrt{\eta}} + \frac{\sigma_2^{(s)}(i\xi)\sqrt{\eta}}{\sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}} \right) \sin(p_1 \sqrt{\eta}) \sin(p_2 \sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}),$$

(Intravaia *et al.* 2012)

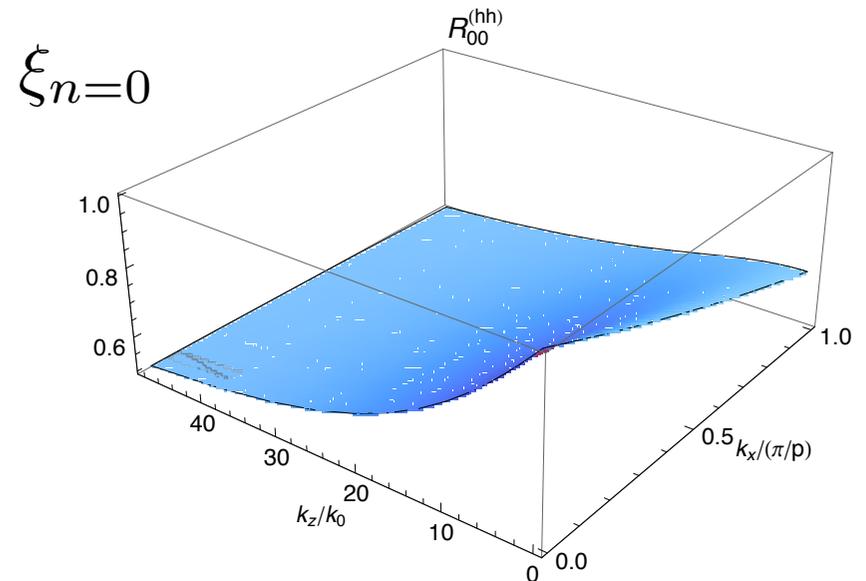
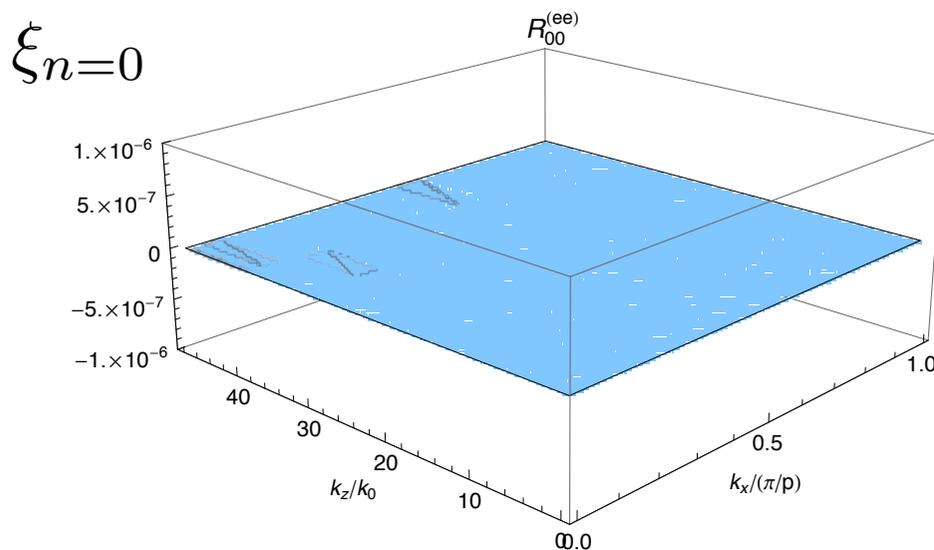
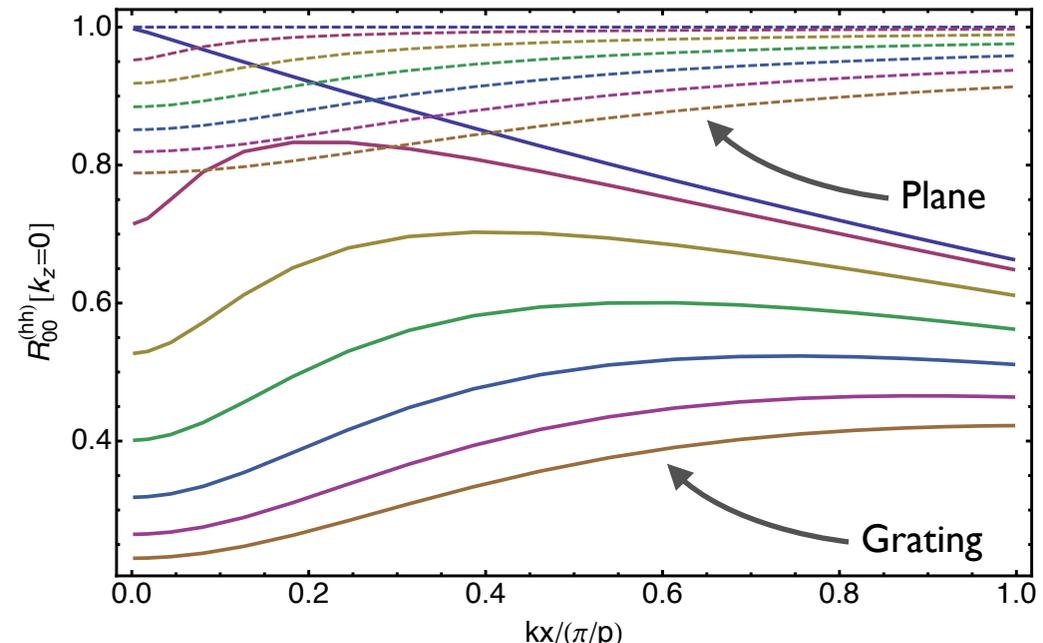


Reflection matrices

e-polarization (first 7 ξ_n 's)

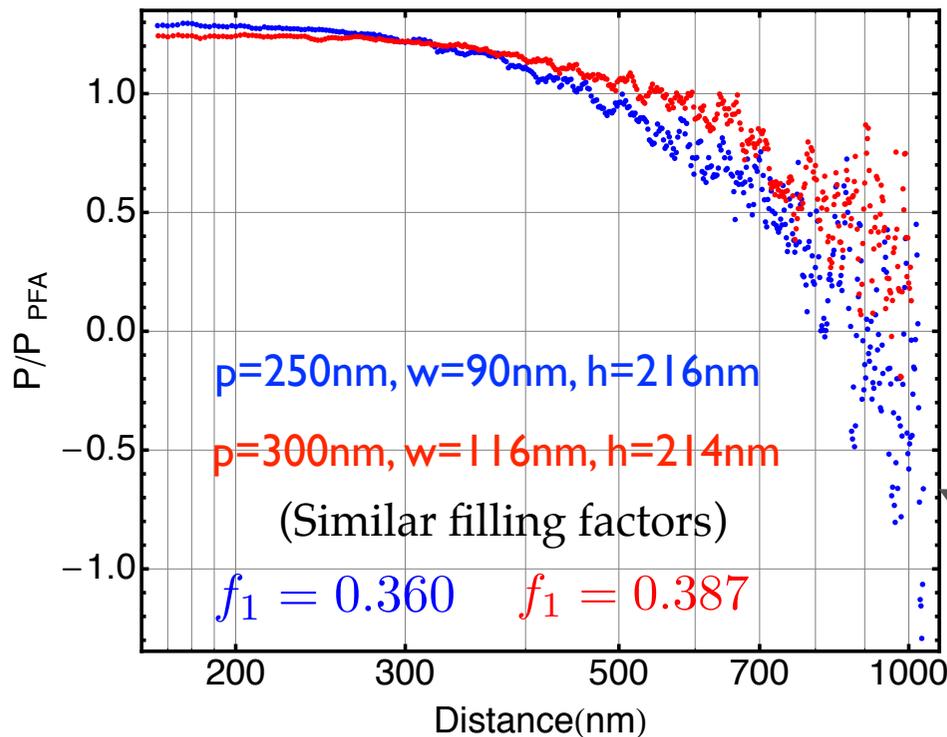
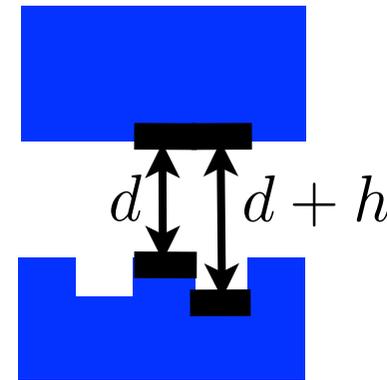


h-polarization (first 7 ξ_n 's)



Normalizing to grating's PFA

$$P_{pg}^{\text{PFA}}(d) = f P_{pp}(d) + (1 - f) P_{pp}(d + h)$$



Small separations: PFA
underestimates the total pressure

Large separations: PFA
overestimates the exact pressure

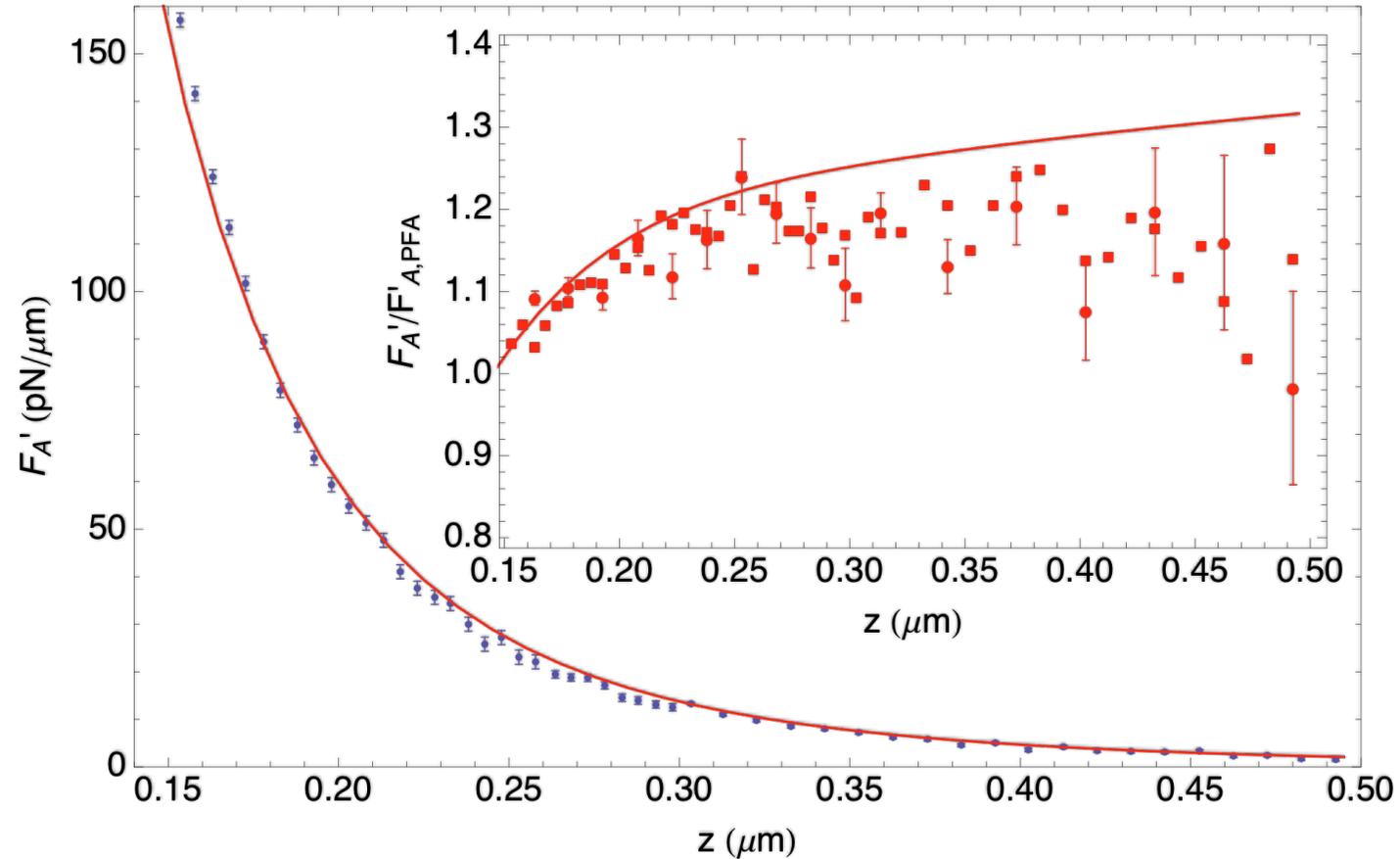
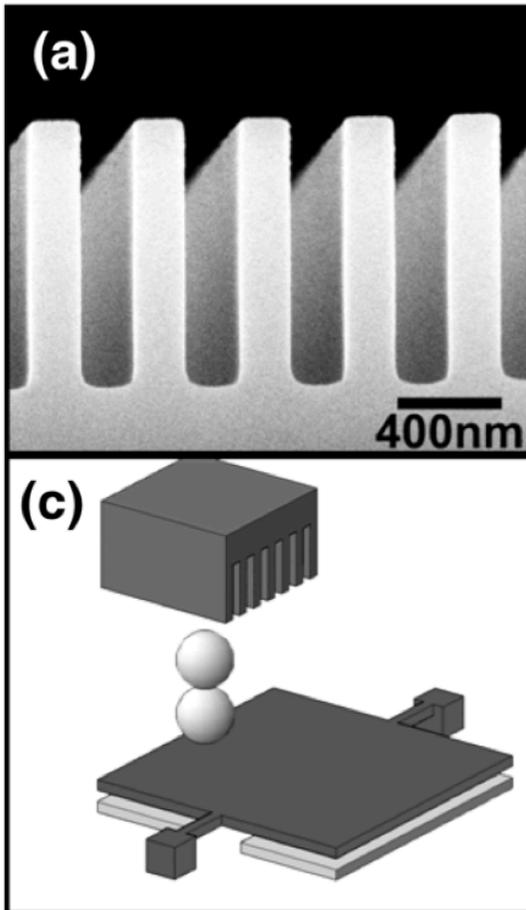
Pressure is going to zero faster than d^{-4}

 **Strong suppression of the Casimir force**

(Intravaia *et al.*, 2013)

Previous works on Si gratings

(Chan *et al*, 2008)

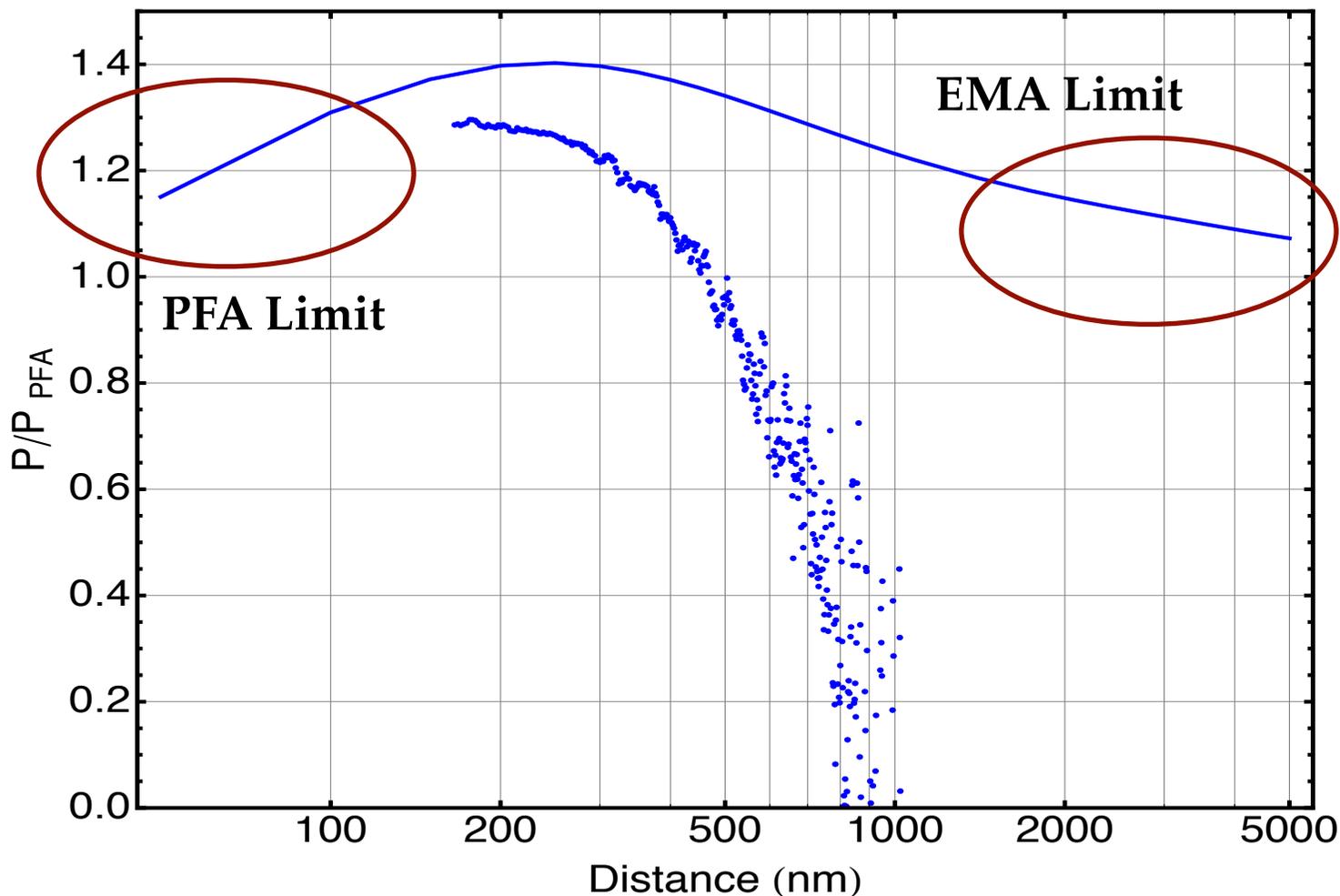


period = $1\mu\text{m}$, depth = 1070 nm, and filling factor = 0.510

Sphere radius of $50\mu\text{m}$

PFA underestimates the real force

Open problem



Numerical crosschecks show that the theory is accurate within few %

Double checks on the experiment show no apparent mistakes

Experiment/theory discrepancy: open problem in Casimir physics



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OPEN

Strong Casimir force reduction through metallic surface nanostructuring

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What is going on?

● Are there problems with the experiment?

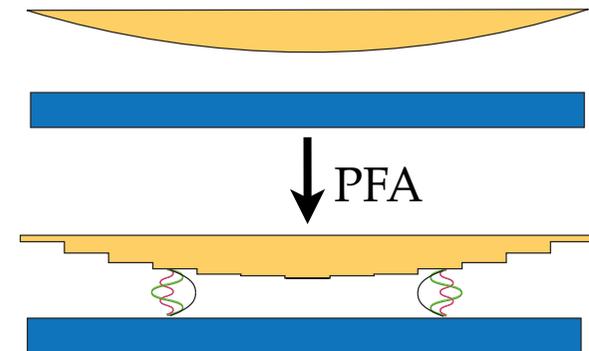
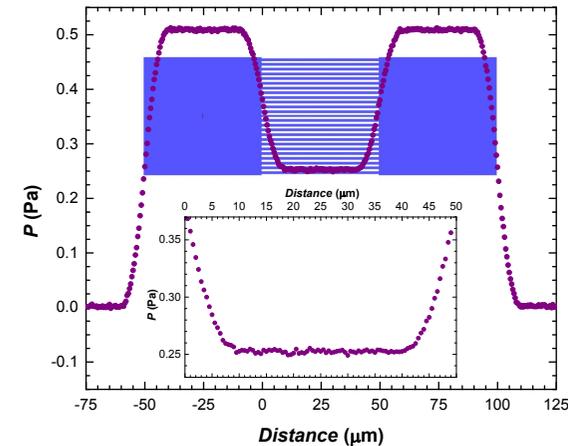
- set-up similar to previous ones
- sphere-plane force re-obtained with new set-up

● Are we correctly describing the experiment?

- finite-size grating
- thermal equilibrium

● Is something wrong with the theory?

- Reflection matrices
- Optical properties
- Surface roughness
- Electrostatic patches
- Validity of PFA for the sphere's curvature
- etc



Final comments

- **Importance of correct description of optical properties**
- **Narrow-band intuition (as in standard photonics) does not always work in Casimir physics**
- **Care must be exercised when using effective medium approximations in Casimir physics**
- **There are still open problems**

Thank you!

