A tutorial on Casimir interactions between nanostructured materials

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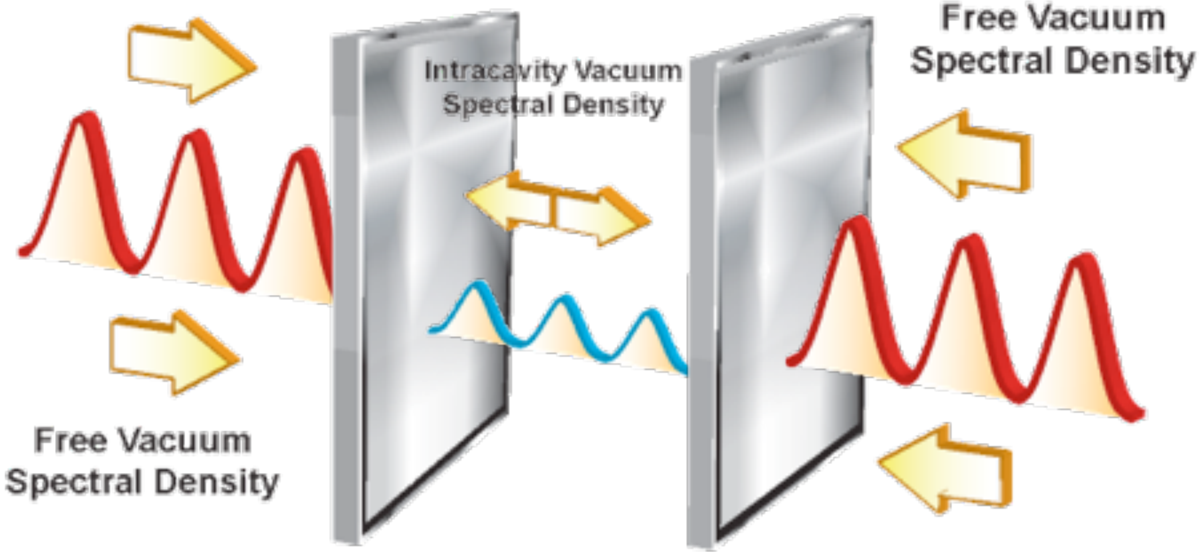
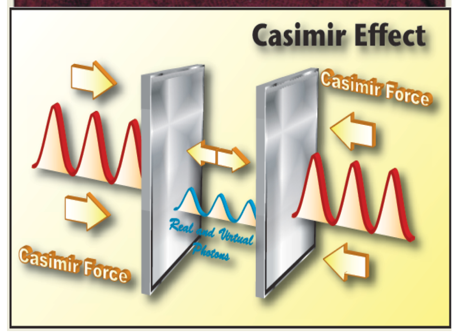
Outline of this Talk

- Brief intro to Casimir physics
  - Basics, relevance, and simple geometries
- Tailoring Casimir forces with metamaterials
  - Effective medium/homogenization in Casimir physics
- Tailoring Casimir forces with nanostructures
  - Metallic gratings for Casimir force manipulation

Brief intro to Casimir physics



The Casimir force

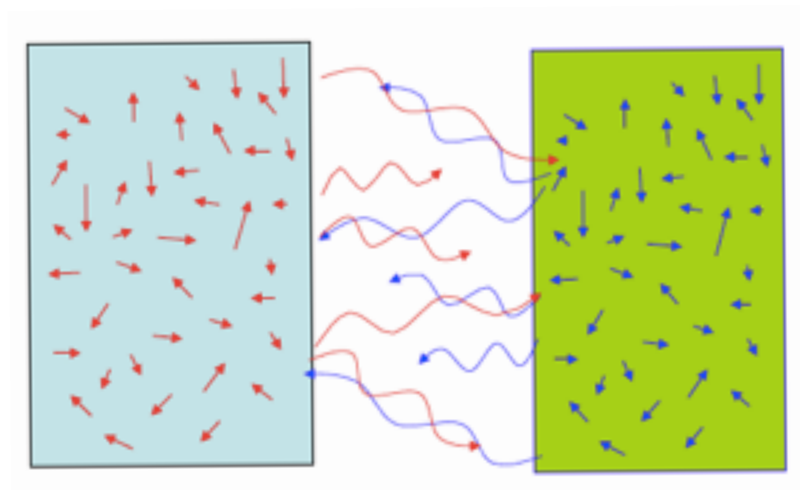


- The Casimir effect is a universal effect from confinement of vacuum fluctuations: it depends only on $\hbar$, c and geometry

$$E = \frac{1}{2}\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \Rightarrow \boxed{\frac{F}{A} = \frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$

$$(130 \mathrm{nN/cm}^2 \text{ @ } d = 1\mu\mathrm{m})$$

- It can also be interpreted as arising from fluctuations of charges and currents within the materials

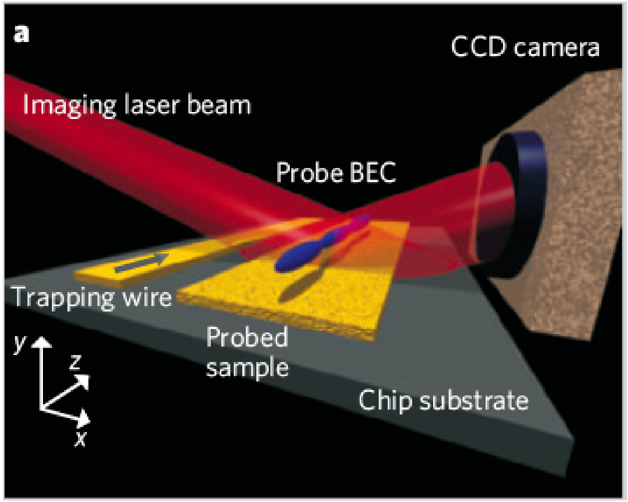


- The magnitude and sign of the force depends on geometry, materials, and temperature

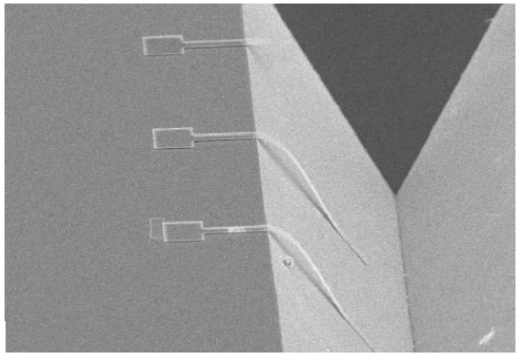
Some relevant applications

- Quantum Science and Technology:

  Atom-surface interactions (e.g., atom chips)



- Nanotechnology: challenges and opportunities



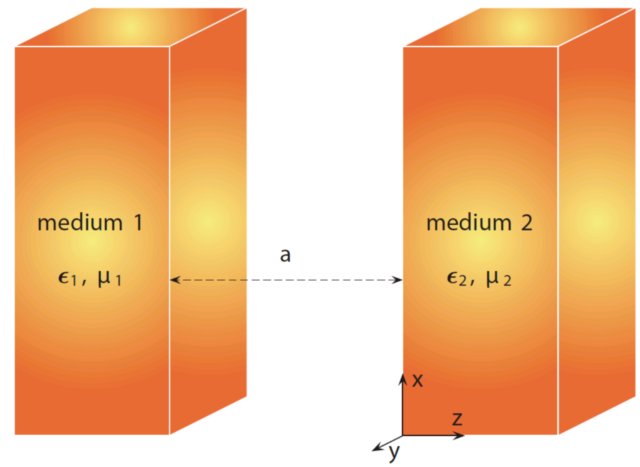
stiction in MEMS

Contactless force transmission

The Lifshitz formula

Force between materials slabs:

$$\boxed{\frac{F}{A} = \mathrm{Im}\int_0^\infty \frac{d\omega}{2\pi}\int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}\hbar\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)K_3\mathrm{Tr}\frac{\mathbf{R}_1\cdot\mathbf{R}_2 e^{2iK_3d}}{1-\mathbf{R}_1\cdot\mathbf{R}_2 e^{2iK_3d}}}$$



$$K_3 = \sqrt{\omega^2/c^2 - k_\parallel^2}$$

Reflection matrices (Fresnel formulas for isotropic media):

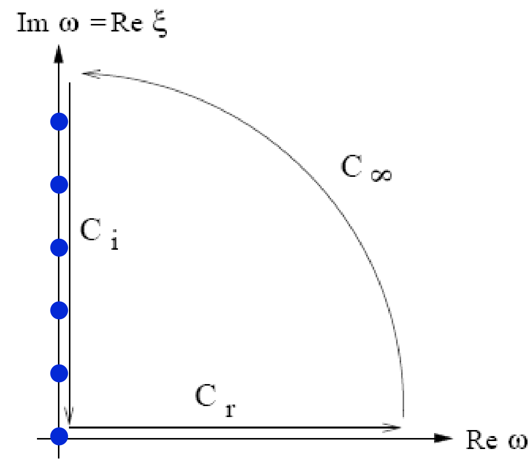
$$r^{\mathrm{TM,TM}}(\omega,\mathbf{k}_\parallel) = \frac{\epsilon(\omega)K_3 - \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}{\epsilon(\omega)K_3 + \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}$$

$$r^{\mathrm{TE,TE}}(\omega,\mathbf{k}_\parallel) = \frac{\mu(\omega)K_3 - \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}{\mu(\omega)K_3 + \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}$$

Going to imaginary frequencies

The function $\coth(\hbar\omega/2k_BT)$ has poles on the imaginary frequency axis at

$$\omega_m = i\xi_m \quad , \quad \xi_m = m\frac{2\pi k_B T}{\hbar}$$



After Wick rotation:

$$\boxed{\frac{F}{A} = 2k_BT\sum_{m=0}^{\infty\,'}\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}K_3(i\xi_m)\mathrm{Tr}\frac{\mathbf{R}_1(i\xi_m)\cdot\mathbf{R}_2(i\xi_m)e^{-2K_3(i\xi_m)d}}{1-\mathbf{R}_1(i\xi_m)\cdot\mathbf{R}_2(i\xi_m)e^{-2K_3(i\xi_m)d}}}$$

Kramers-Kronig (causality) relations:

$$\epsilon(i\xi) = 1 + \frac{2}{\pi}\int_0^\infty\frac{\omega\epsilon''(\omega)}{\omega^2+\xi^2}d\omega \qquad \mu(i\xi) = 1 + \frac{2}{\pi}\int_0^\infty\frac{\omega\mu''(\omega)}{\omega^2+\xi^2}d\omega$$

Casimir physics is a broad-band frequency phenomenon

The sign of the Casimir force

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$

The sign of the force is directly connected to the sign of the product of the reflection coefficients on the two plates, evaluated at imaginary frequencies. As a rule of thumb, we have (p=TE,TM)

$$R_1^p(i\xi) \cdot R_2^p(i\xi) > 0 \; (\forall\, \xi \leq c/d) \Rightarrow \mathrm{Attraction}$$

$$R_1^p(i\xi) \cdot R_2^p(i\xi) < 0 \; (\forall\, \xi \leq c/d) \Rightarrow \mathrm{Repulsion}$$

In terms of permittivities and permeabilities:

$$\epsilon_a(i\xi) \gg \epsilon_b(i\xi)$$

$$\mu_b(i\xi) \gg \mu_a(i\xi)$$

⟶ Repulsion

Ideal attraction-repulsion

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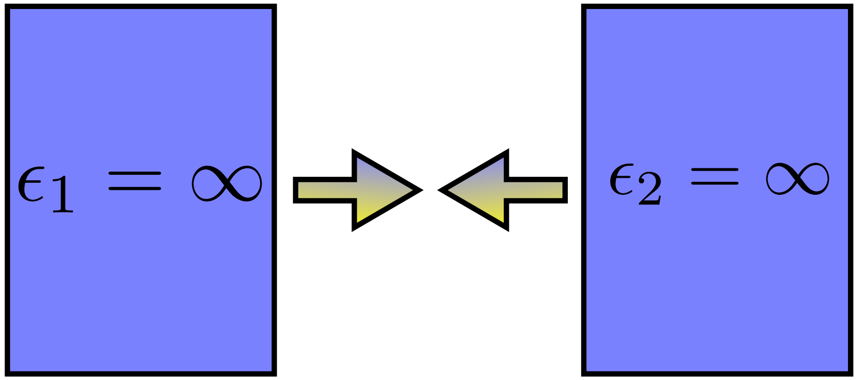
- Ideal attractive limit

Casimir (1948)

$$\boxed{\frac{F}{A} = +\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



Ideal attraction-repulsion

- ## Ideal attractive limit

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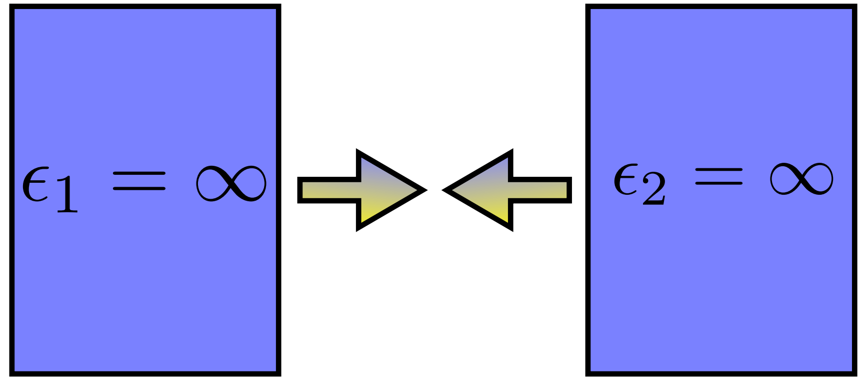
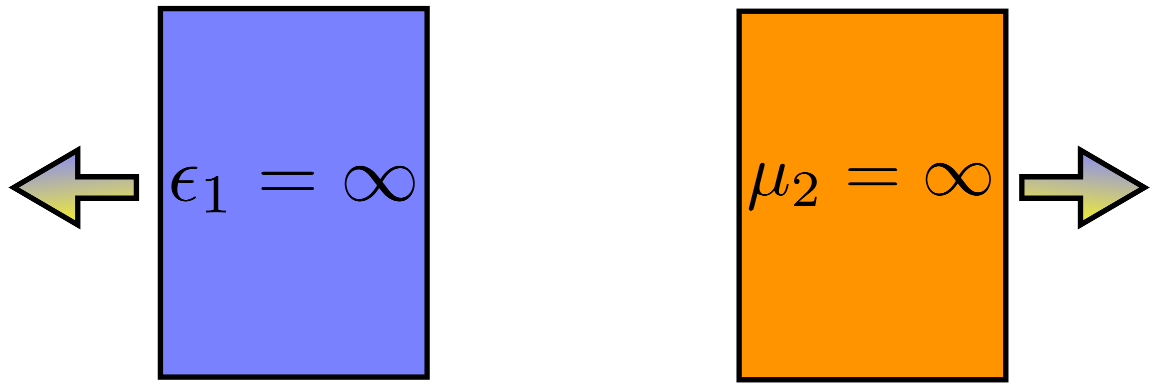


- ## Ideal repulsive limit

Boyer (1974)

$$\boxed{\frac{F}{A} = -\frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



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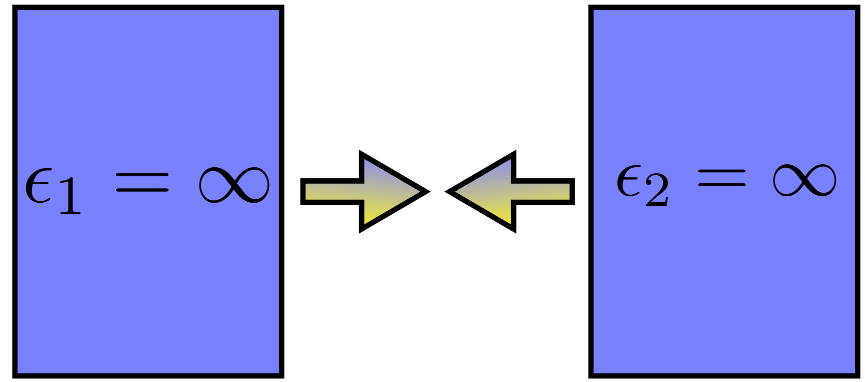

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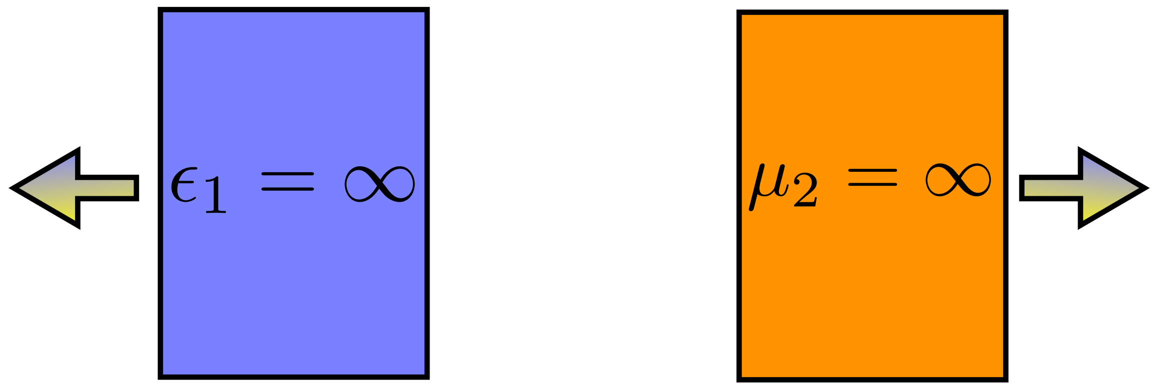

- ## Real repulsive limit

Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$

Ideal attraction-repulsion

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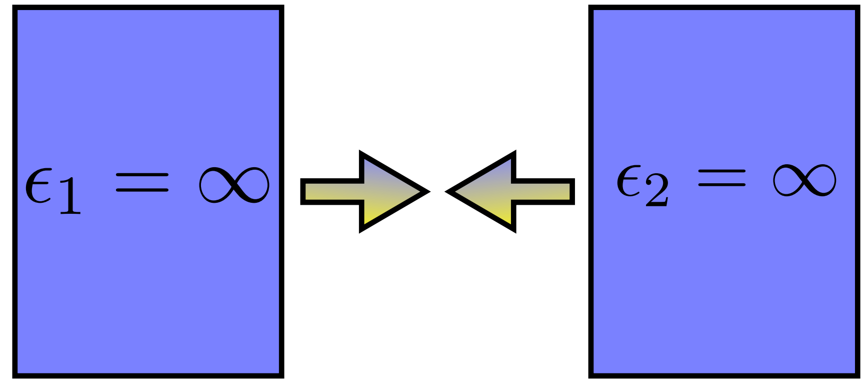

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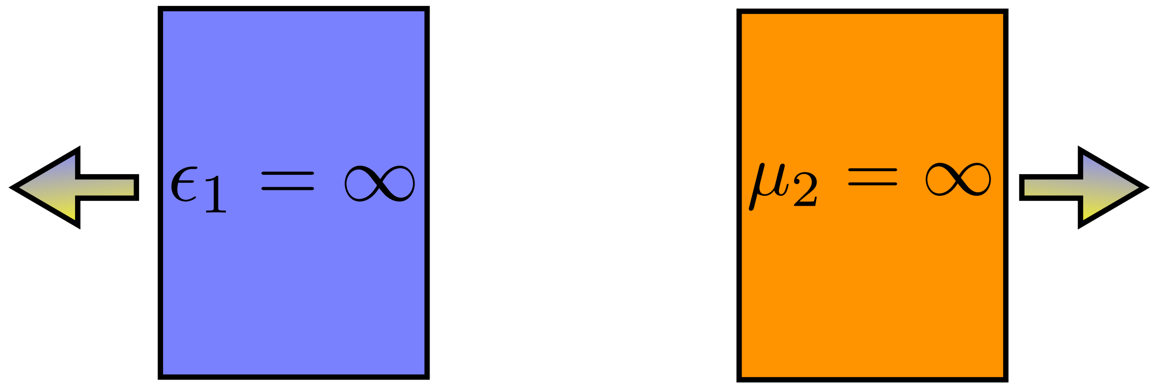

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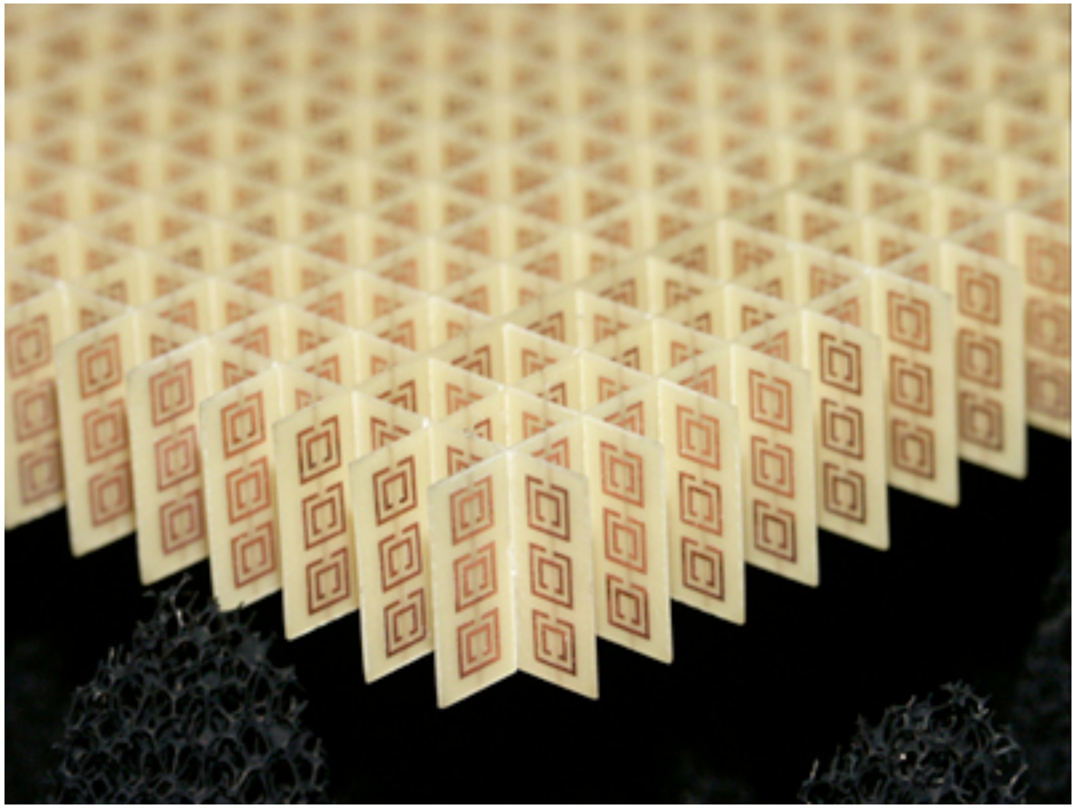
Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$

$\longrightarrow$ **Metamaterials**

Metamaterials and Casimir

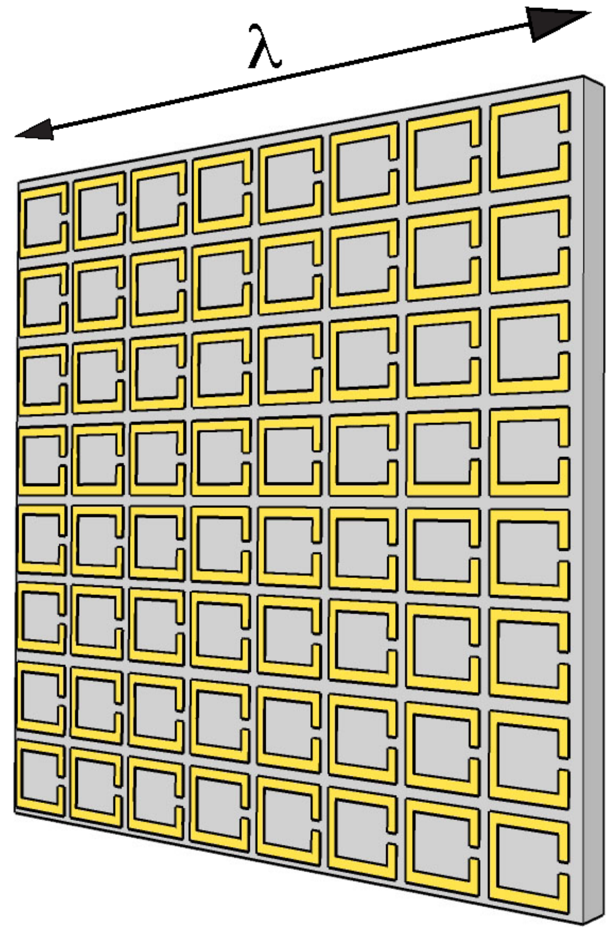


Effective medium approximation

In the effective medium approximation (EMA) one describes the MM with an effective electric permittivity and an effective magnetic permeability.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

$$\mu_{eff} = 1 - \frac{\dfrac{\pi r^2}{a^2}}{1 + \dfrac{2\sigma i}{\omega r \mu_0} - \dfrac{3}{\pi^2 \mu_0 \omega^2 C r^3}}$$



This is an approximation valid when the MM is probed at wavelengths much larger that the average distance between the constituent "particles" of the MM.

EMA: Drude-Lorentz responses

Close to the resonance, both $\epsilon(\omega)$ and $\mu(\omega)$ can be modeled by Drude-Lorentz formulas

$$\epsilon_\alpha(\omega) = 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega}$$

$$\mu_\alpha(\omega) = 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}$$

Typical separations

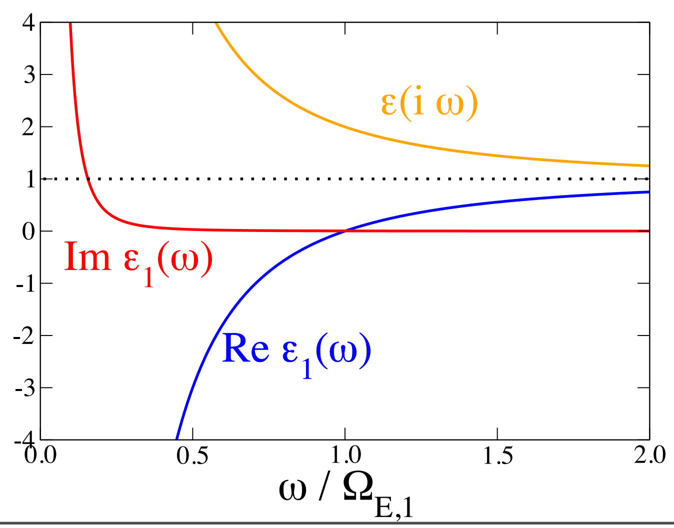
$d = 200 - 1000 \text{ nm}$

Infrared-optical frequencies

$\Omega/2\pi = 5 \times 10^{14}\text{Hz}$

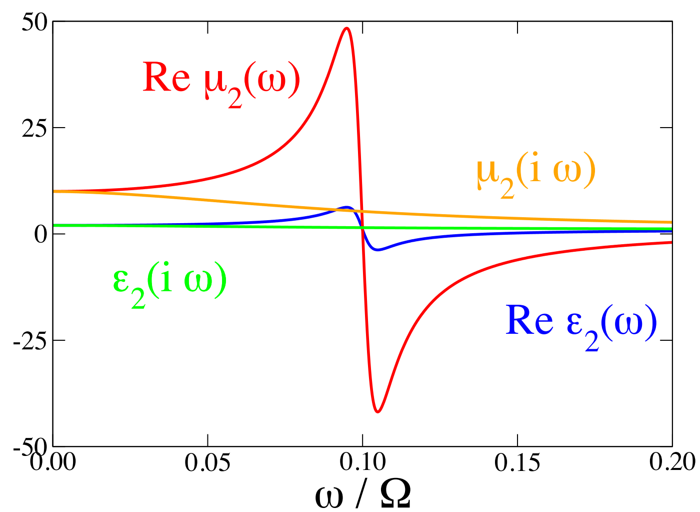
Drude metal (Au)

$\Omega_E = 9.0 \text{ eV} \quad \Gamma_E = 35 \text{ meV}$



Metamaterial

$\text{Re } \epsilon_2(\omega) < 0 \quad \text{Re } \mu_2(\omega) < 0$



$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$

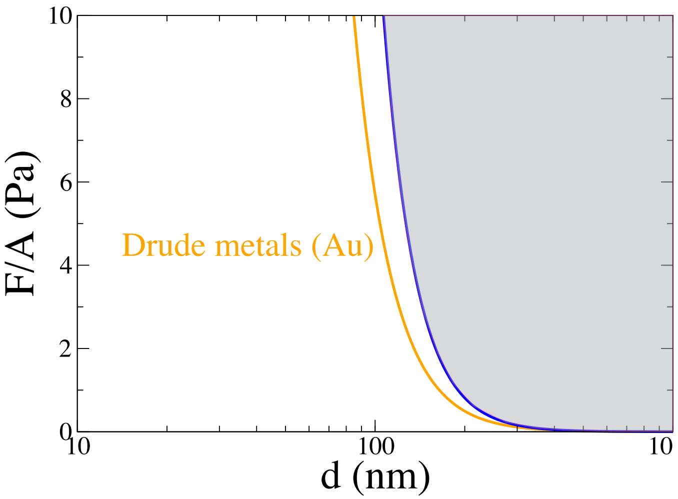
$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$

$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$

Attraction-repulsion crossover

Drude metal (Au)

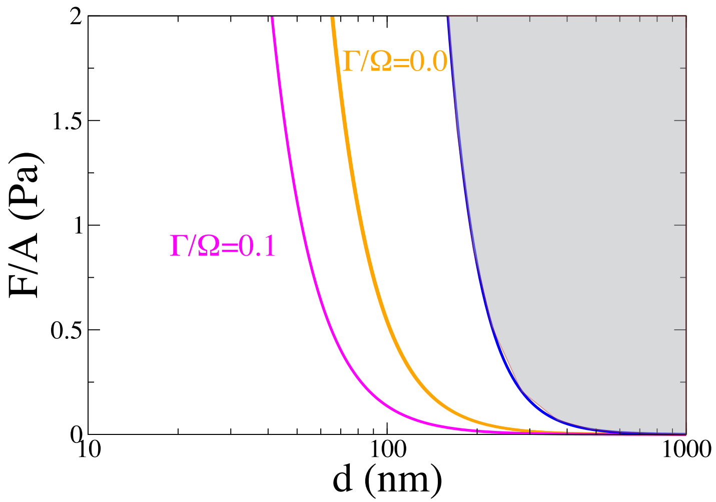
Drude metal (Au)



Only attraction

Metamaterial

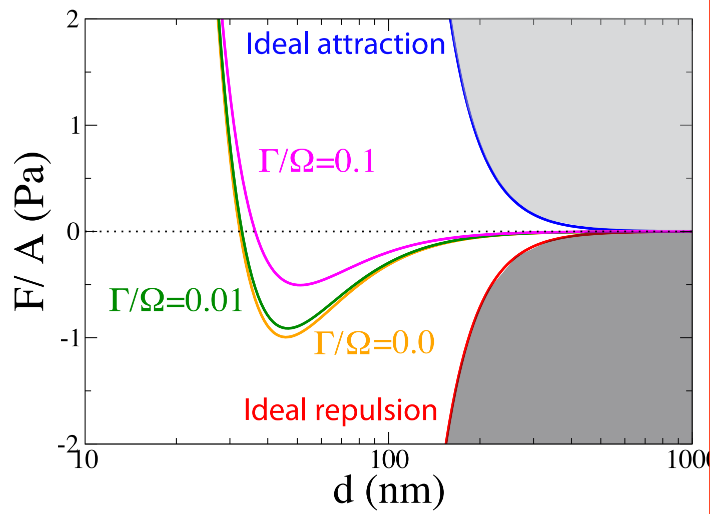
Metamaterial



Only attraction

Drude metal (Au)

Metamaterial



Repulsion-attraction

Drude background

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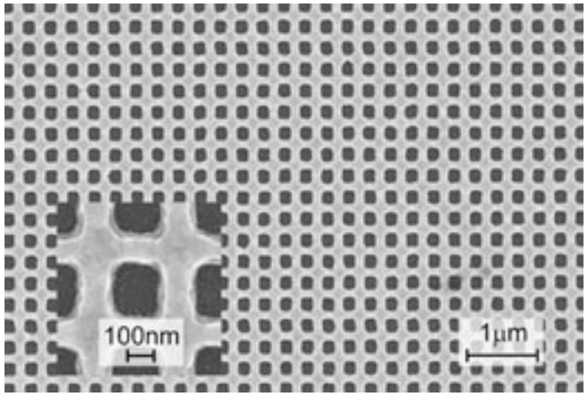
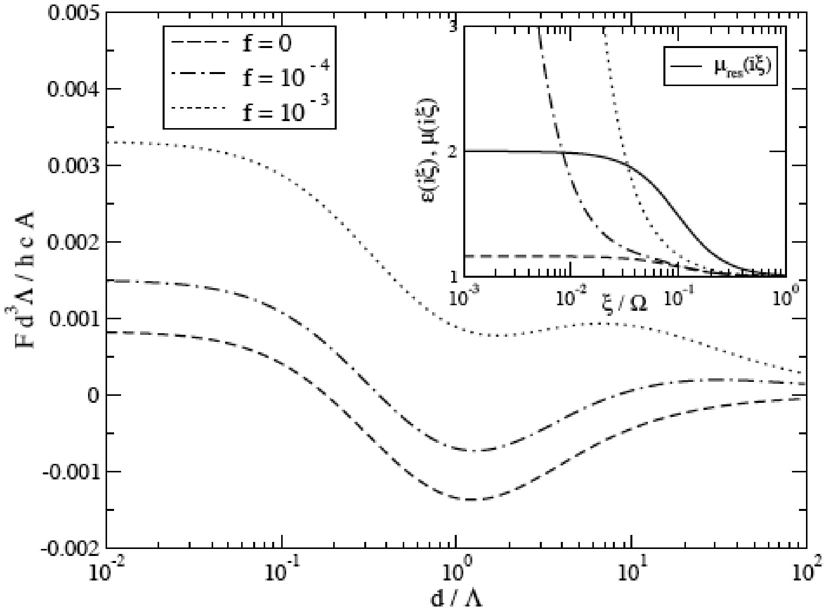
- In some metallic-based MMs, there is a net conductivity due to the metallic structure, like the fishnet design on the right.



$$\epsilon(\omega) = 1 - f\frac{\Omega_D^2}{\omega^2 - i\omega\gamma_D} - (1-f)\frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega}$$

f : filling factor

$$\mu(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}$$



Rosa, DD, Milonni, PRL **100**, 183602 (2008)

A Drude background is detrimental for Casimir force reduction or repulsion, since it results in an electric response much stronger than the magnetic one

$$\epsilon_2(i\xi) \gg \mu_2(i\xi)$$

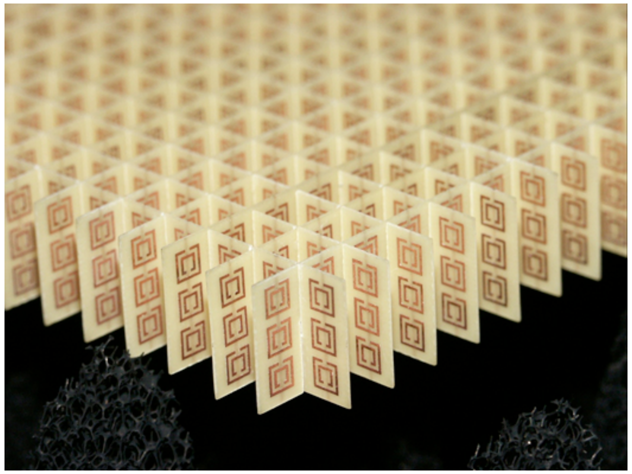
Optical anisotropy

In an anisotropic medium, the constitutive relations between E, D, B, and H are more involved:

$$\mathrm{D} = \epsilon \cdot \mathrm{E} \qquad \mathrm{H} = \mu^{-1} \cdot \mathrm{B}$$

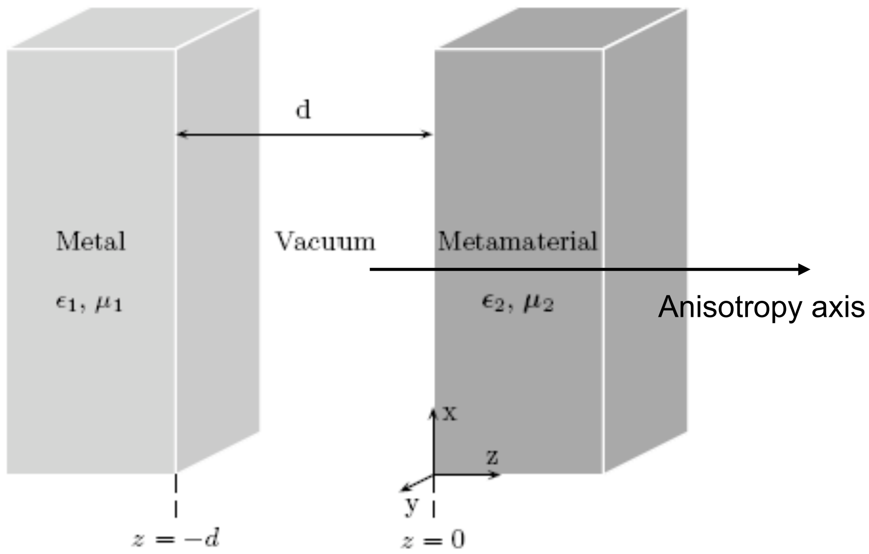
due to the tensorial nature of the permittivity and permeability

$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}$$



Examples of uniaxial anisotropy in stacked MMs

Anisotropy: Uniaxial MMs



$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \quad ; \quad \mu_{ij} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$

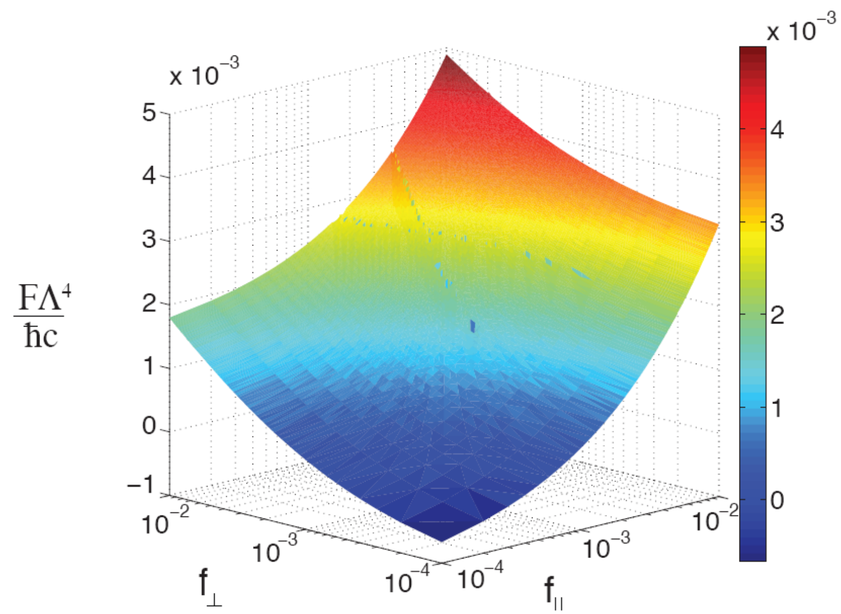
Anisotropy produces polarization mixing (non-diagonal reflection matrices)

$$\epsilon_{xx}(\omega) = \epsilon_{yy}(\omega) = 1 - (1-f_x)\frac{\Omega_{e,x}^2}{\omega^2 - \omega_{e,x}^2 + i\gamma_{e,x}\omega} - f_x \frac{\Omega_{D,x}^2}{\omega^2 + i\gamma_{D,x}\omega},$$

$$\epsilon_{zz}(\omega) = 1 - (1-f_z)\frac{\Omega_{e,z}^2}{\omega^2 - \omega_{e,z}^2 + i\gamma_{e,z}\omega} - f_z \frac{\Omega_{D,z}^2}{\omega^2 + i\gamma_{D,z}\omega},$$

$$\mu_{xx}(\omega) = \mu_{yy}(\omega) = 1 - \frac{\Omega_{m,x}^2}{\omega^2 - \omega_{m,x}^2 + i\gamma_{m,x}\omega},$$

$$\mu_{zz}(\omega) = 1 - \frac{\Omega_{m,z}^2}{\omega^2 - \omega_{m,z}^2 + i\gamma_{m,z}\omega}$$



Rosa, DD, Milonni, PRA **78**, 032117 (2008)

EMA: correct model for μ

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Drude-Lorentz for permeability is wrong. The correct expression that results in EMA from Maxwell's equations is

$$\boxed{\mu_{\text{eff}}(\omega) = 1 - f\frac{\omega^2}{\omega^2 - \omega_m^2 + 2i\gamma_m\omega}}$$

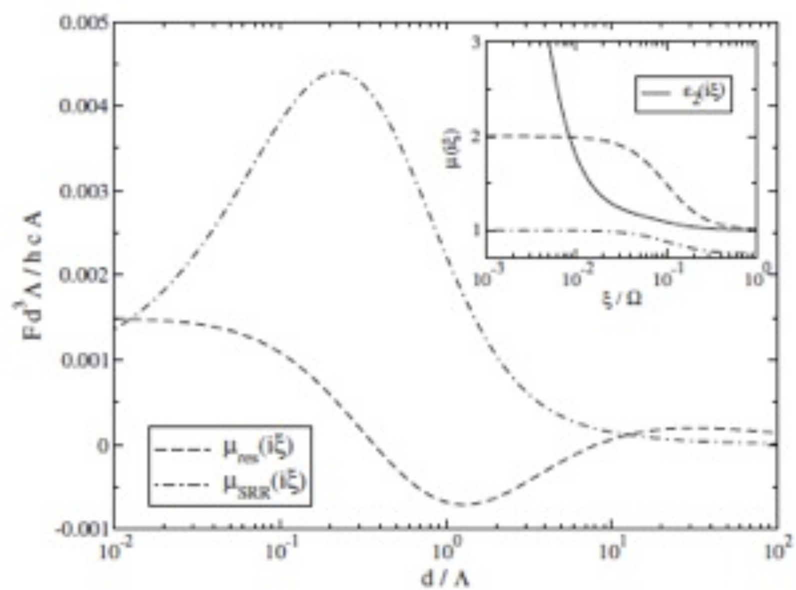
(Pendry 1999)

The appearance of the ω^2 factor in the numerator is very important:

Although close to the resonance this behaves in the same way as the Drude-Lorentz EMA permeability, it has a completely different low-frequency behavior

$$\mu_{\text{eff}}(i\xi) < 1 < \epsilon_{\text{eff}}(i\xi)$$

No Casimir repulsion!



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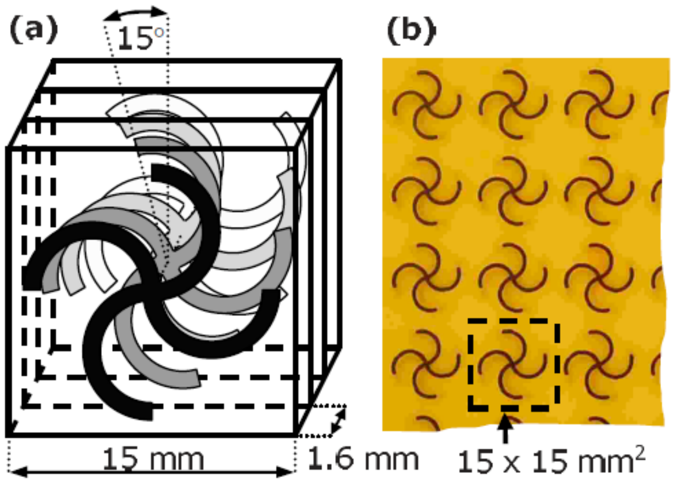
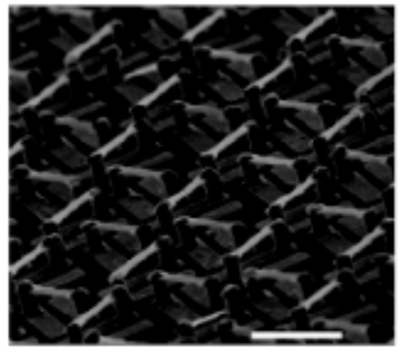
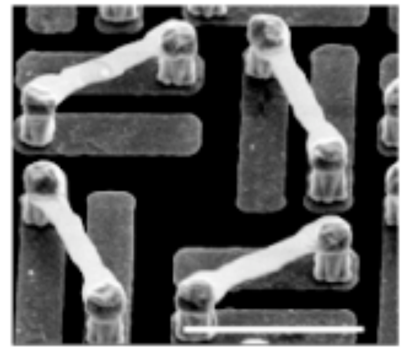
Other Casimir MMs: chirality

The chirality of a MM is defined by the chirality of its unit cell







In a chiral medium, the constitutive relations mix electric and magnetic fields

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r},\omega) - i\kappa(\omega)\mathbf{H}(\mathbf{r},\omega)$$
$$\mathbf{B}(\mathbf{r},\omega) = i\kappa(\omega)\mathbf{E}(\mathbf{r},\omega) + \mu(\omega)\mathbf{H}(\mathbf{r},\omega)$$

dispersive chirality: $\kappa(\omega) = \dfrac{\omega_k \omega}{\omega^2 - \omega_{\kappa R}^2 + i\gamma_k \omega}$

Repulsion and chiral MMs

In chiral MMs the reflection matrix is non-diagonal (mixing of E and H fields).

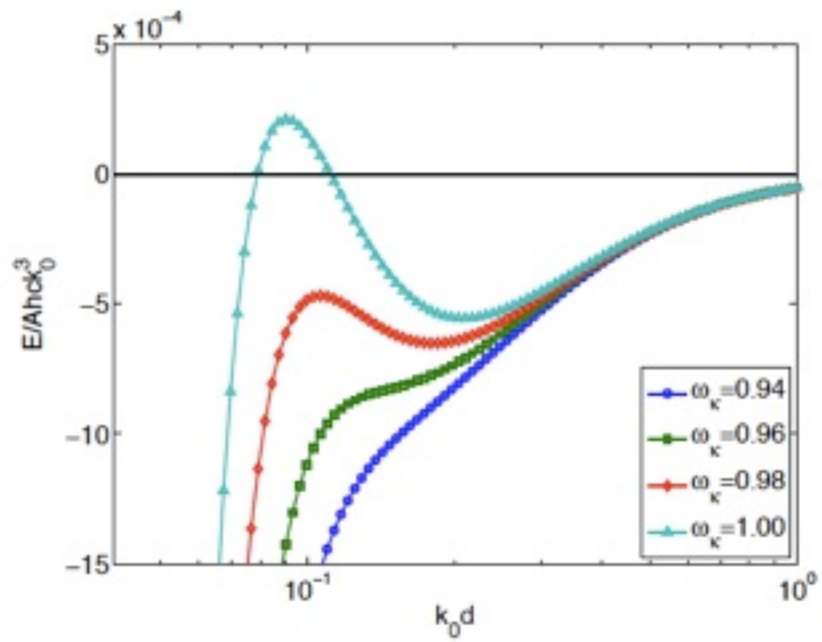
The integrand of the Casimir-Lifshitz force between two identical chiral MMs has the form:

$$F = \frac{(r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} - 2(r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}{1 - (r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} + (r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}$$

One might achieve repulsive Casimir forces with strong chirality (i.e., large values of r_{sp})

Same-chirality (SC) materials: repulsion

Opposite-chirality (OC) materials: repulsion



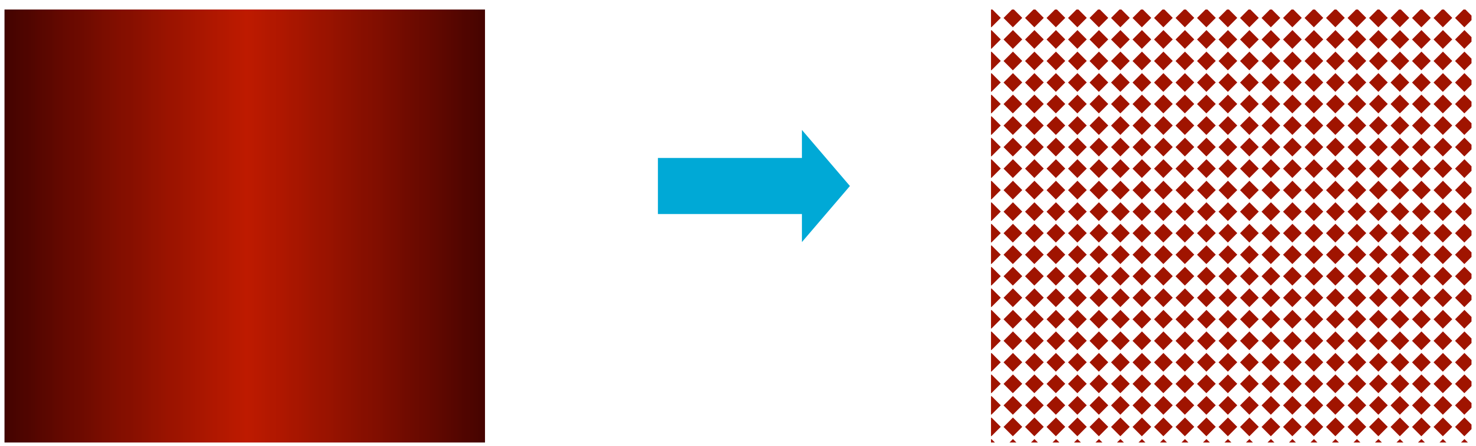
Soukoulis *et al.*, PRL 2009

Going beyond EMA

So far, we have treated the MM in the “long-wavelength approximation”, i.e., field wavelengths much larger than the typical size of the unit cell of the MM.

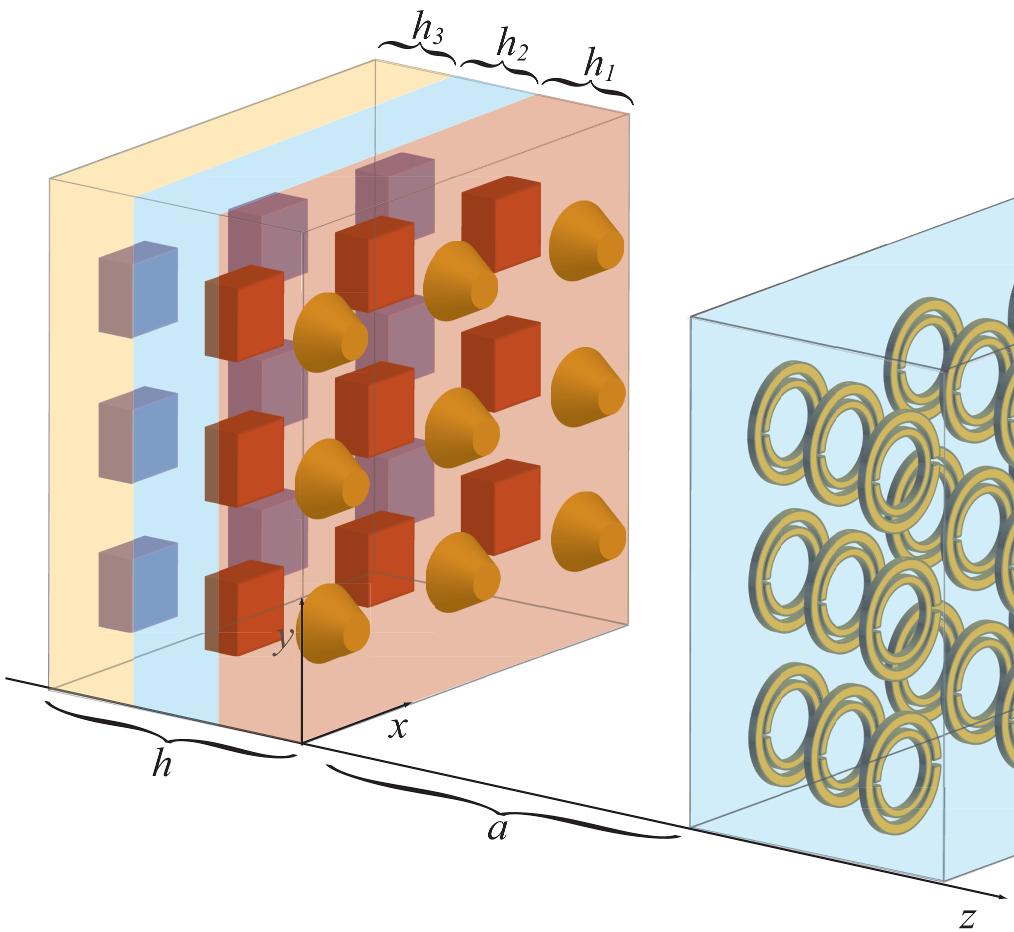
How to calculate Casimir forces when EMA does not hold?
Can one trust predictions of Casimir repulsion with MMs based on EMA?

Homogeneous medium | Non-homogeneous medium

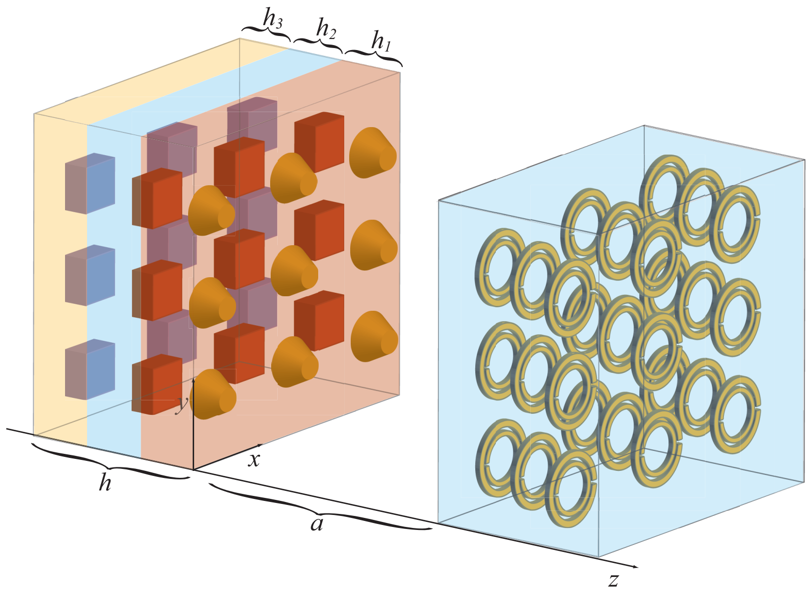


EMA | beyond EMA

Casimir nanostructures

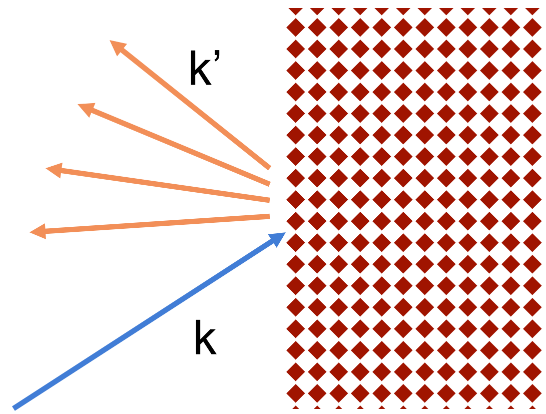


Scattering theory



The Casimir force still may be described in terms of reflections (scattering theory)

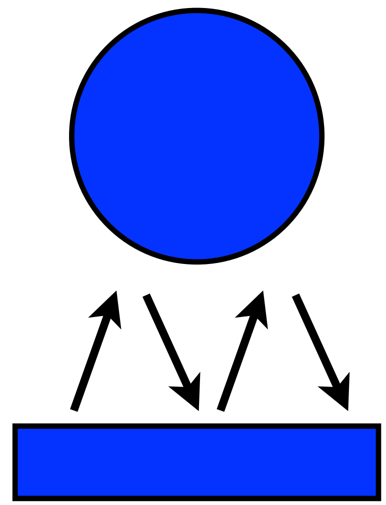
$$\mathcal{R}_i(\omega, \mathbf{k}, \mathbf{k}', p, p')$$



Symbolically, we may write the Casimir energy as

$$\boxed{\frac{E(d)}{A} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\left[1 - \mathcal{R}_1 e^{-\mathcal{K}d} \mathcal{R}_2 e^{-\mathcal{K}d}\right]}$$

$$\propto \sum_{n=1}^{\infty} \frac{1}{n} \left[\mathcal{R}_1(i\xi) e^{-d\mathcal{K}(i\xi)} \mathcal{R}_2(i\xi) e^{-d\mathcal{K}(i\xi)}\right]^n$$



Solving for the reflection matrix

The reflection matrix can be obtained with standard methods of numerical electromagnetism. One way is to solve Maxwell equations for the transverse fields

$$-ik\frac{\partial \mathbf{E}_t}{\partial z} = \nabla_t\left[\chi\hat{e}_3\cdot\nabla\times\mathbf{H}_t\right] - k^2\mu\hat{e}_3\times\mathbf{H}_t$$
$$-ik\frac{\partial \mathbf{H}_t}{\partial z} = -\nabla_t\left[\zeta\hat{e}_3\cdot\nabla\times\mathbf{E}_t\right] + k^2\epsilon\hat{e}_3\times\mathbf{E}_t$$

Assuming a two-dimensional periodic structure, we have

$$\mathbf{E}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}}\sum_{m,n}\mathcal{E}_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mathbf{H}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}}\sum_{m,n}\mathcal{H}_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

where

$$\epsilon(x,y) = \sum_{m,n}\epsilon_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mu(x,y) = \sum_{m,n}\mu_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

Exact reflection matrix

One can then write the equations for the transverse fields as

$$\boxed{-ik\frac{\partial \boldsymbol{\Psi}_{m'n'}}{\partial z} = \sum_{mn} H_{m'n',mn}\boldsymbol{\Psi}_{mn}}$$

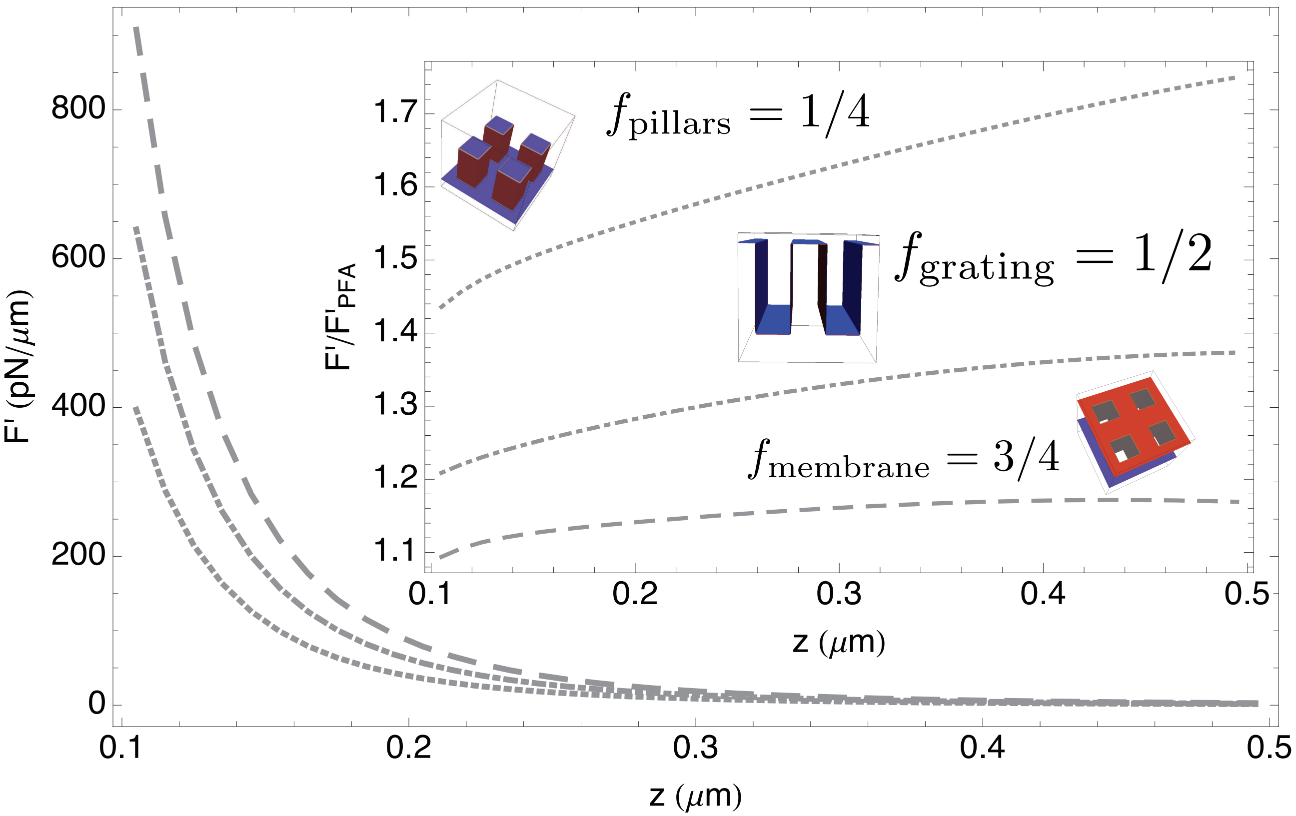
$$\boldsymbol{\Psi}_{mn} = \begin{bmatrix} \mathcal{E}^x_{mn} \\ \mathcal{E}^y_{mn} \\ \mathcal{H}^x_{mn} \\ \mathcal{E}^y_{mn} \end{bmatrix} = \begin{bmatrix} \Psi^1_{mn} \\ \Psi^2_{mn} \\ \Psi^3_{mn} \\ \Psi^4_{mn} \end{bmatrix}$$

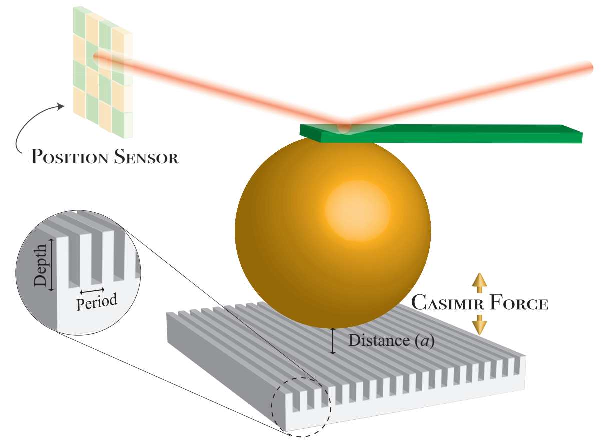
Here H is a complicated matrix, that encapsulates the coupling of modes in the periodic structure.

By numerically solving this equation and imposing the proper boundary conditions of the field on the vacuum-metamaterial interphase (RCWA or S-matrix techniques), one can find the reflection matrix of the MM.

2D periodic structures

Casimir force between a Au plane and Si pillars/grating/membrane @ T=300 K





$$R = 50\mu\text{m}$$

$$\text{period} = 400 \text{ nm}$$

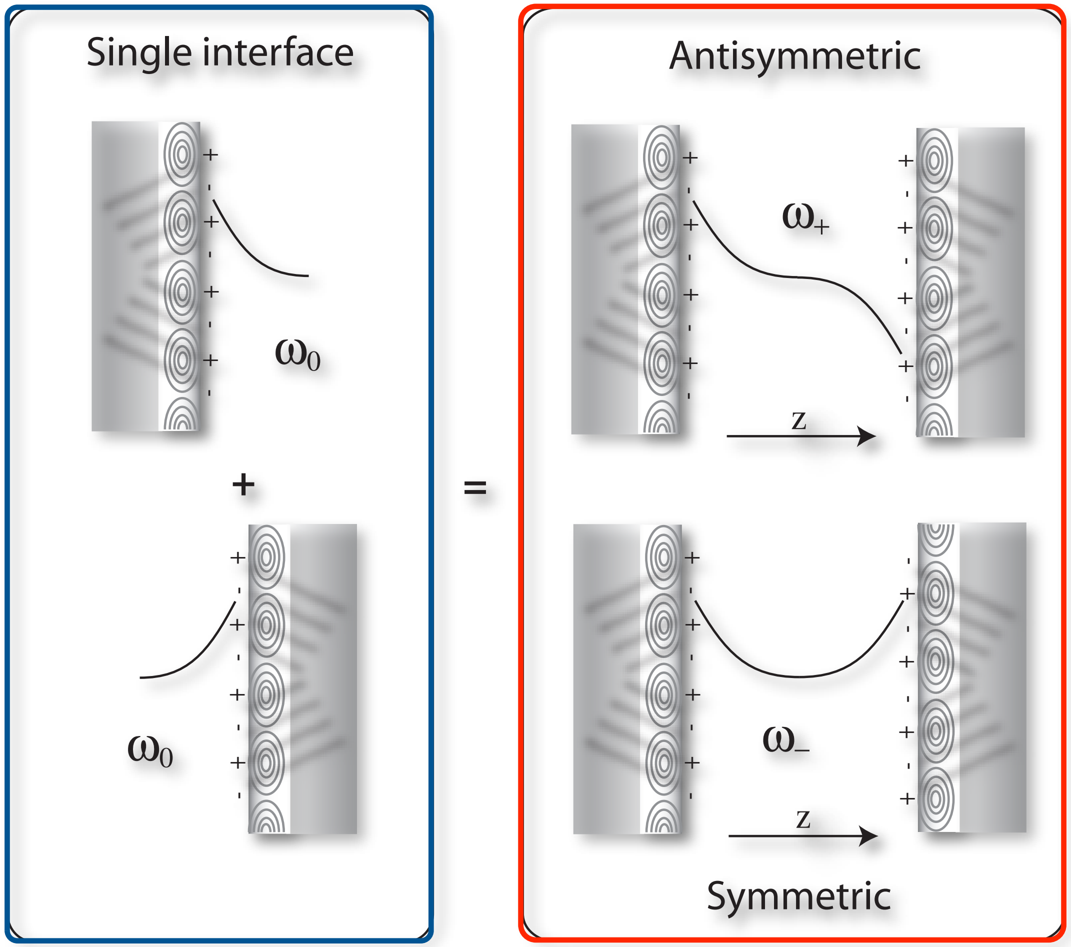
$$\text{depth} = 1070 \text{ nm}$$

Davids, Intravaia, Rosa, DD, PRA **82**, 062111 (2010)

Note: The MIT group has developed much more advanced numerical methods

(previous talk by Alejandro Rodriguez)

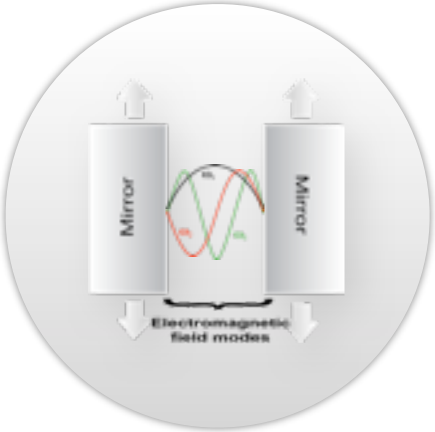
Casimir plasmonics



Mode summation approach

An alternative approach to the scattering formulation is to compute the Casimir energy as a sum over the zero-point energy of the EM in the presence of boundaries

$$E = \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p\right]_L}_{\text{Infinite zero point energy}} \underbrace{- \sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p\right]_{L\to\infty}}_{\text{Setting the zero}}$$



In the case of metallic plates described by the plasma model

$$\left.\begin{aligned} \mu[\omega] &= 1 \\ \epsilon[\omega] &= 1 - \frac{\omega_p^2}{\omega^2} \end{aligned}\right\} \to \qquad E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2} \left[\omega_+ + \omega_-\right]_{L\to\infty}^{L}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m^p\right]_{L\to\infty}^{L}}_{\text{Photonic contribution } (E_{ph})}$$

Surface plasmons interaction

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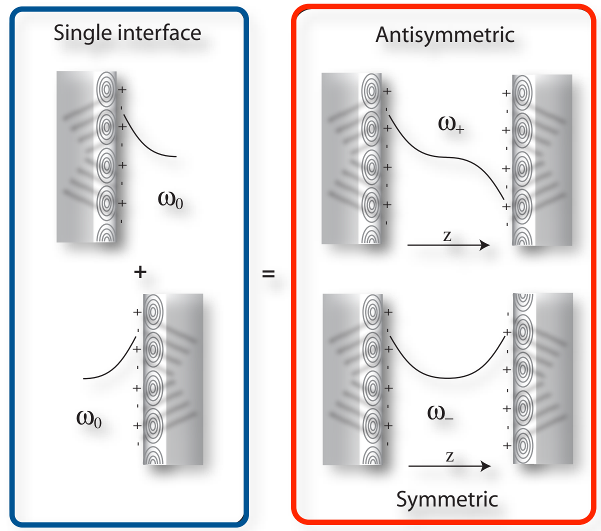


Surface plasmons are evanescent modes of the EM field associated with electronic density oscillations at the metal-vacuum interface.

When the tails of the evanescent fields overlap, the two surface plasmons hybridize

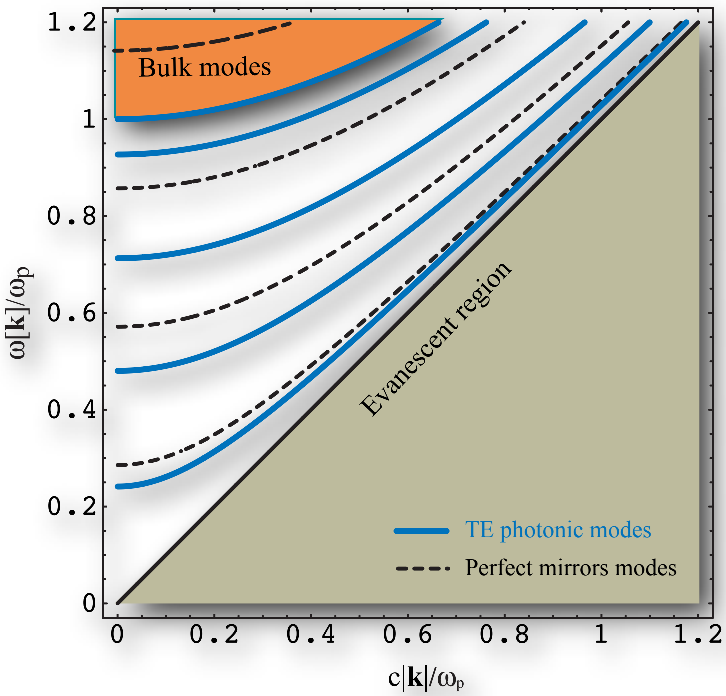
$$2\times\omega_{sp}[\mathbf{k}] \rightarrow \begin{cases} \omega_+[\mathbf{k}] \\ \omega_-[\mathbf{k}] \end{cases}$$

At short distances the Casimir energy is given by the shift in the zero-point energy of the surface plasmons due to their Coulomb (electrostatic) interaction)

$$E_{sp} = A\int \frac{d^2\mathbf{k}}{(2\pi)^2}\left(\frac{\hbar\omega_+}{2} + \frac{\hbar\omega_-}{2} - 2\frac{\hbar\omega_{sp}}{2}\right) = -\frac{\hbar c\alpha\pi^2 A}{580\lambda_p L^2}$$

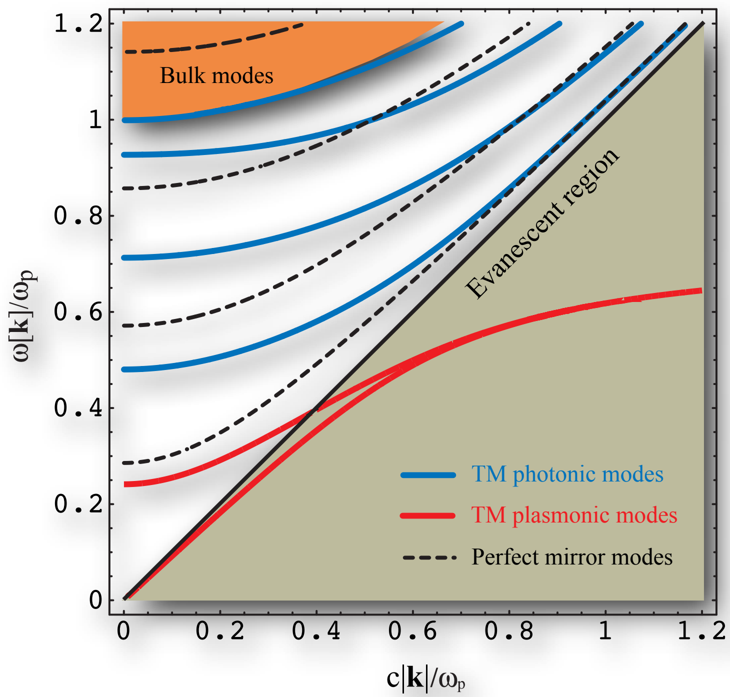
Mode spectrum in a cavity

$$E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2}\left[\omega_+ + \omega_-\right]^{L}_{L\to\infty}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2}\left[\sum_m \omega_m^p\right]^{L}_{L\to\infty}}_{\text{Photonic contribution } (E_{ph})}$$



All the TE-modes belong to the propagative sector

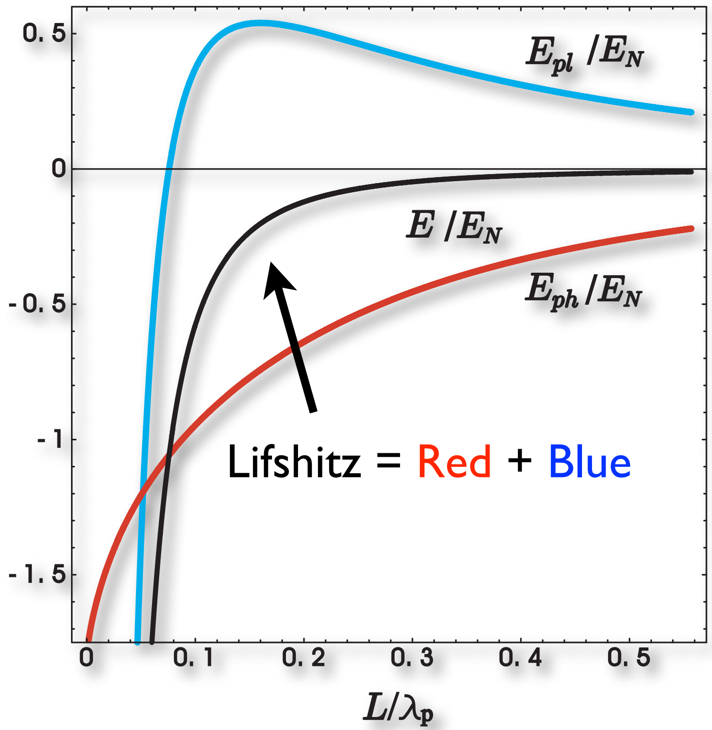
They differ from the perfect mirrors modes because of the dephasing due to the non perfect reflection coefficient.



TM-modes propagative modes look qualitatively like TE modes.

There are only two evanescent modes. They are the generalization to all distances of the coupled plasmon modes.

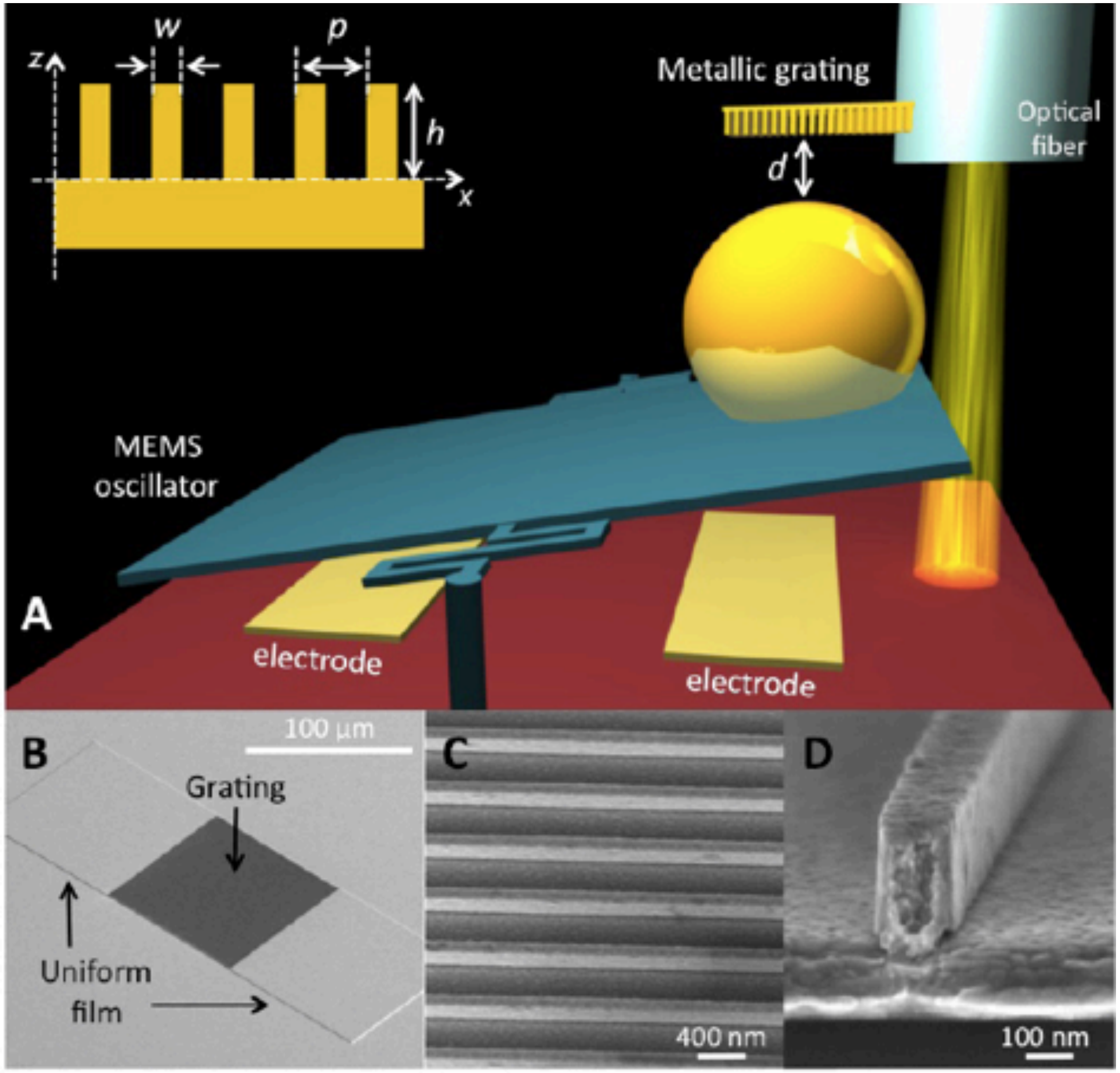
Plasmonic-photonic contributions



- At short distance the plasmonic contribution dominates and is attractive
- At large distance the two contributions are opposite in sign and balance

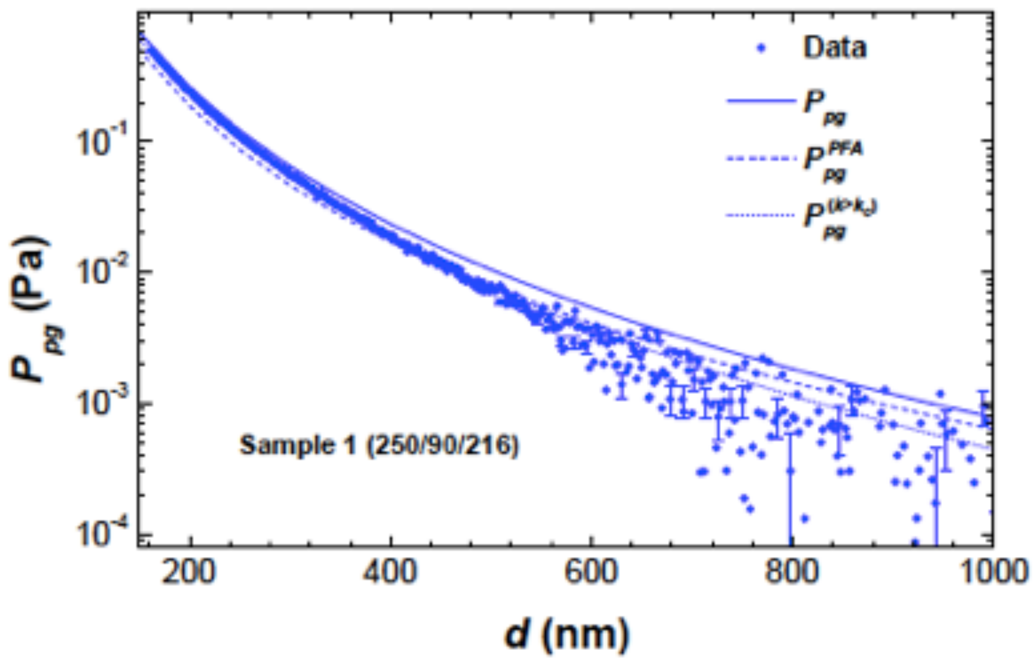
Can one control the Casimir force by changing the balance of the two contributions?

Metallic nano-gratings

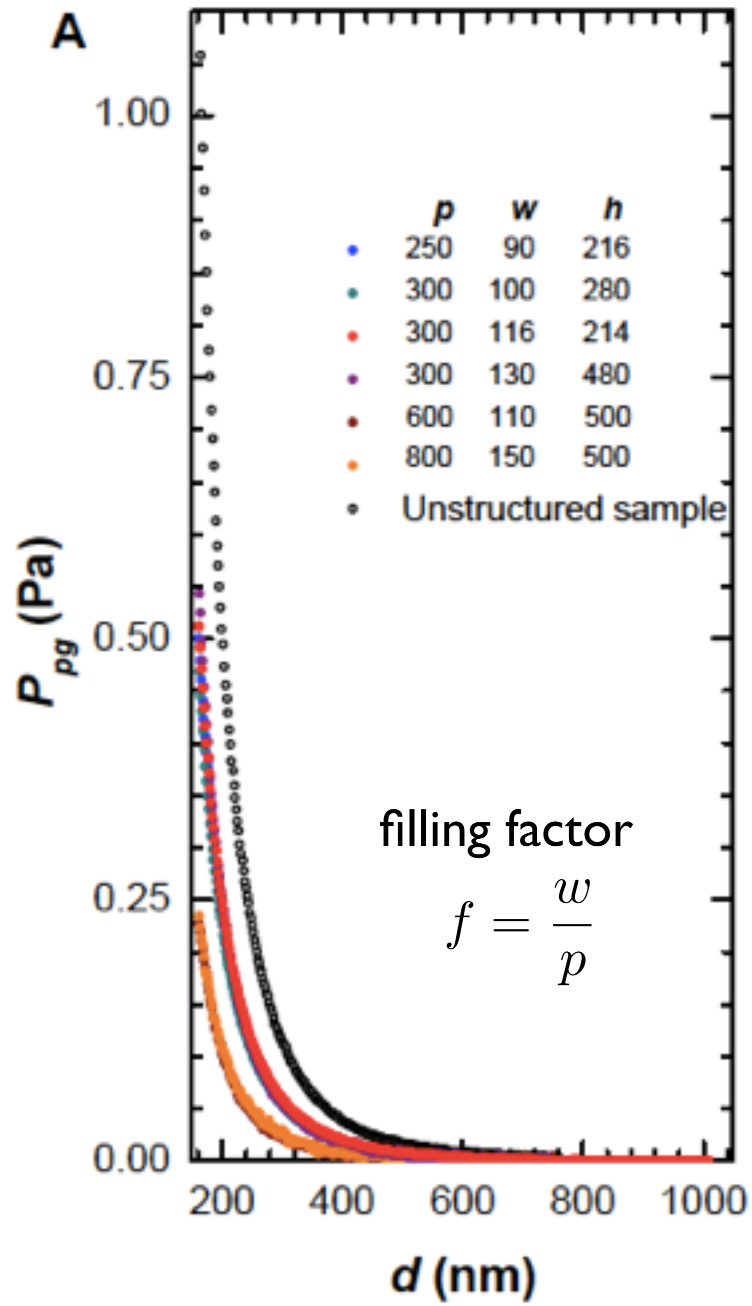


Strong force reduction

- Torsional balance set-up
- Metallic sphere $(R = 150\ \mu\text{m})$
- Metallic nanostructures $w, p, h \approx 100\ \text{nm}$
- Sputtering and electroplating



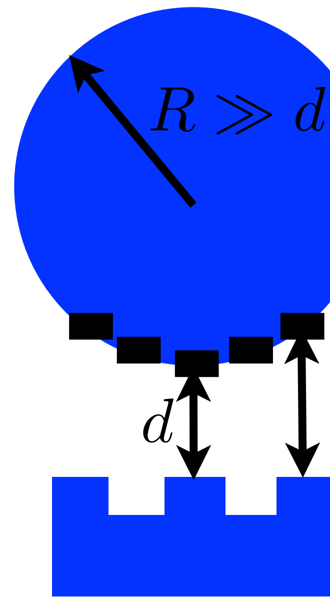
IUPUI, ANL, NIST, LANL collaboration



Modeling and simulation

- Use of standard PFA to treat the sphere's curvature

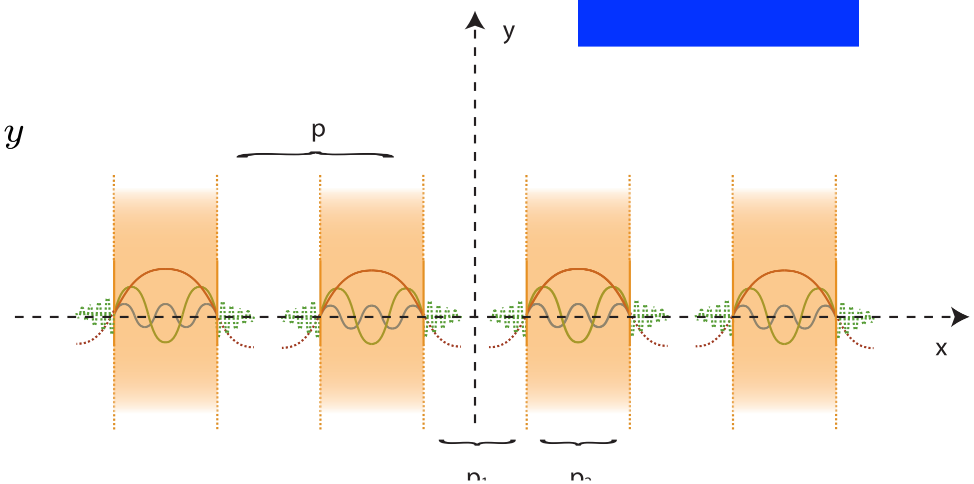
$$F'_{sg} \approx 2\pi R P_{pg} \qquad d/R < 6 \times 10^{-3}$$



- Exact computation of the plane-grating pressure P_{pg}

Scattering approach + modal expansions   Li (1993)

$$\begin{pmatrix} E_z(x,y) \\ E_x(x,y) \\ H_z(x,y) \\ H_x(x,y) \end{pmatrix}_i = \sum_{\nu,s} A_\nu^{(s,i)} \mathbf{Y}^{(s,i)}[x, \eta_\nu^{(s,i)}] e^{\imath\lambda[\eta_\nu^{(s,i)}]y}$$



Analytical expressions for eigenvectors
Transcendental equation for eigenvalues

$$0 = \tilde{D}^{(s)}(\eta) = -\cos(\alpha_0 p) + \cos(p_1\sqrt{\eta}) \cos(p_2\sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}) - \frac{1}{2}\left(\frac{\sqrt{\eta - [\epsilon(i\xi)-1]\xi^2}}{\sigma_2^{(s)}(i\xi)\sqrt{\eta}} + \frac{\sigma_2^{(s)}(i\xi)\sqrt{\eta}}{\sqrt{\eta - [\epsilon(i\xi)-1]\xi^2}}\right) \sin(p_1\sqrt{\eta}) \sin(p_2\sqrt{\eta - [\epsilon(i\xi)-1]\xi^2}),$$

Intravaia *et al.*, PRA **86**, 042101 (2012)

Reflection matrices

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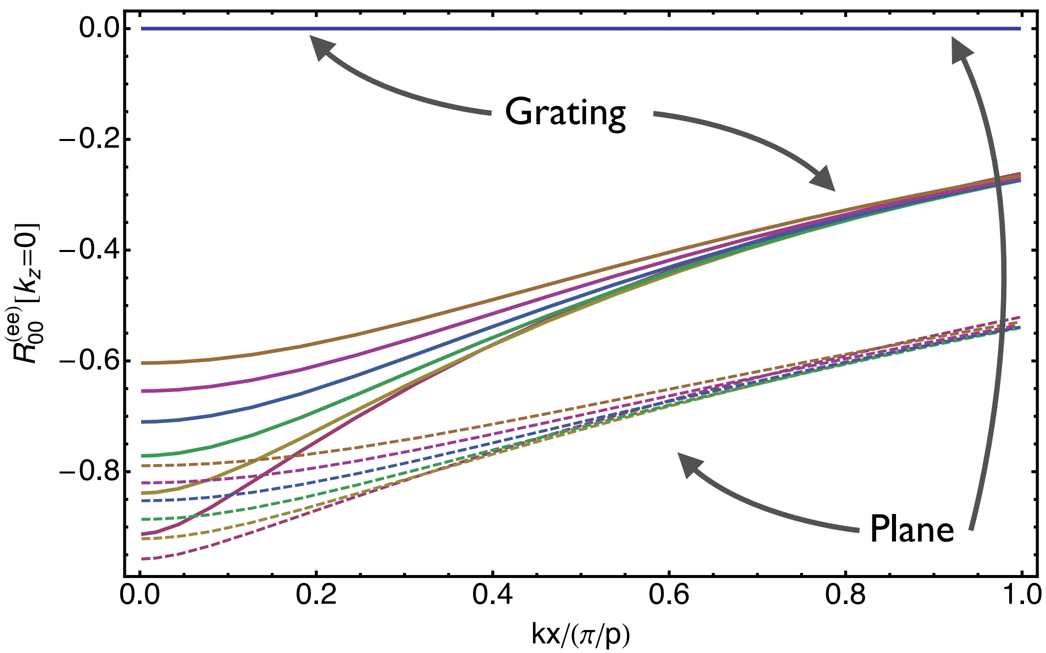
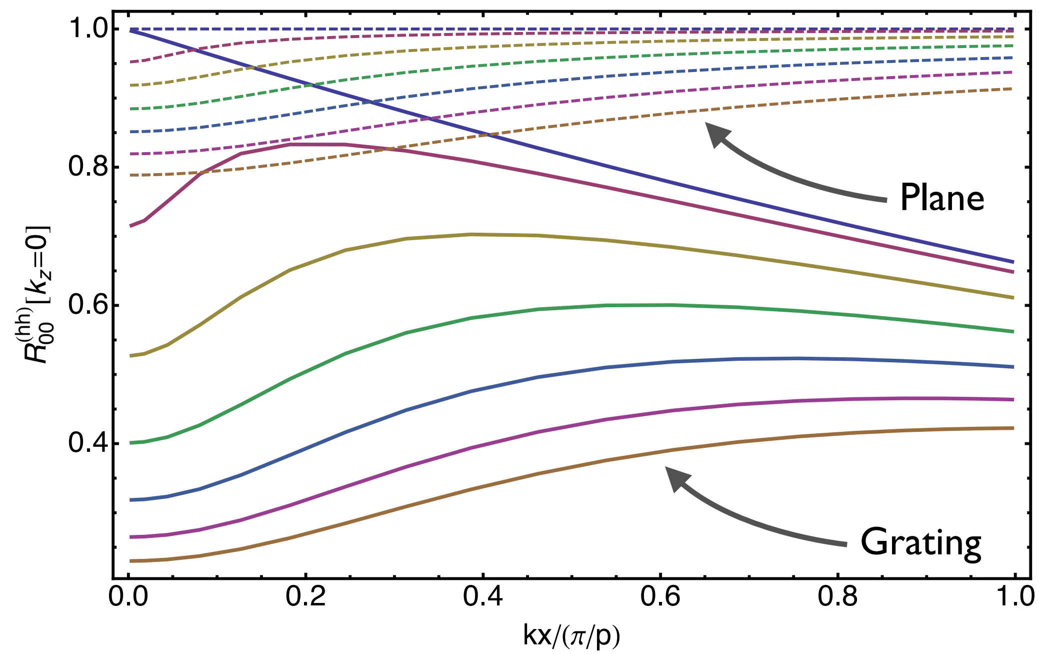
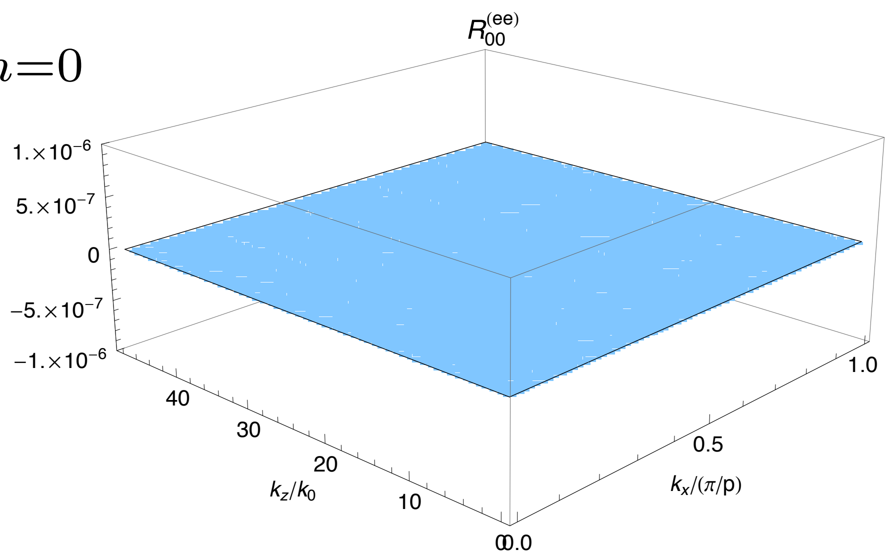
e-polarization (first 7 ξ_n's)



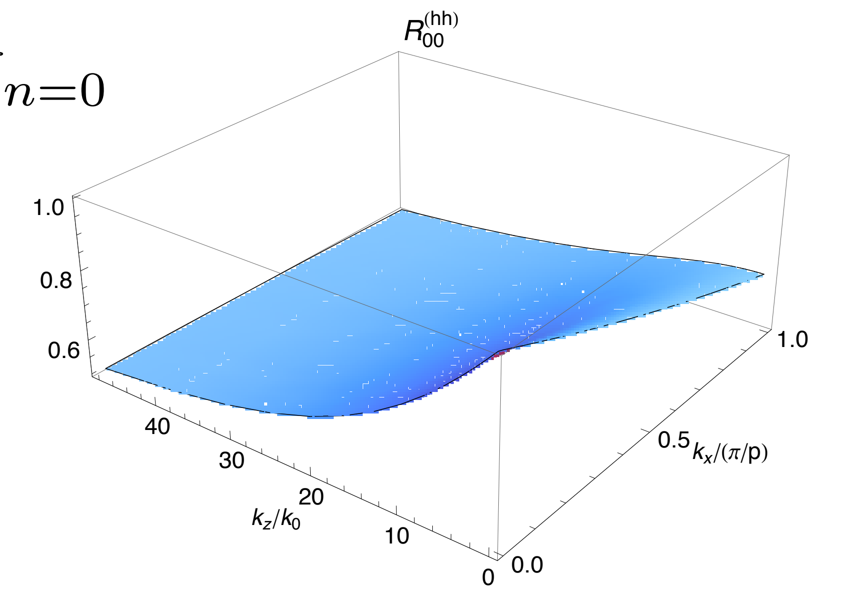
h-polarization (first 7 ξ_n's)



$\xi_{n=0}$

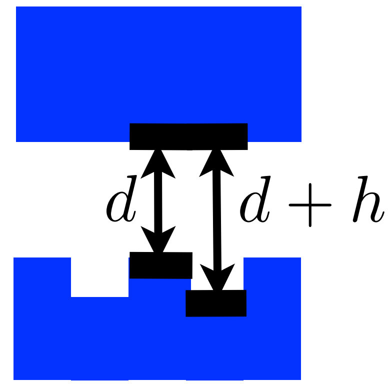


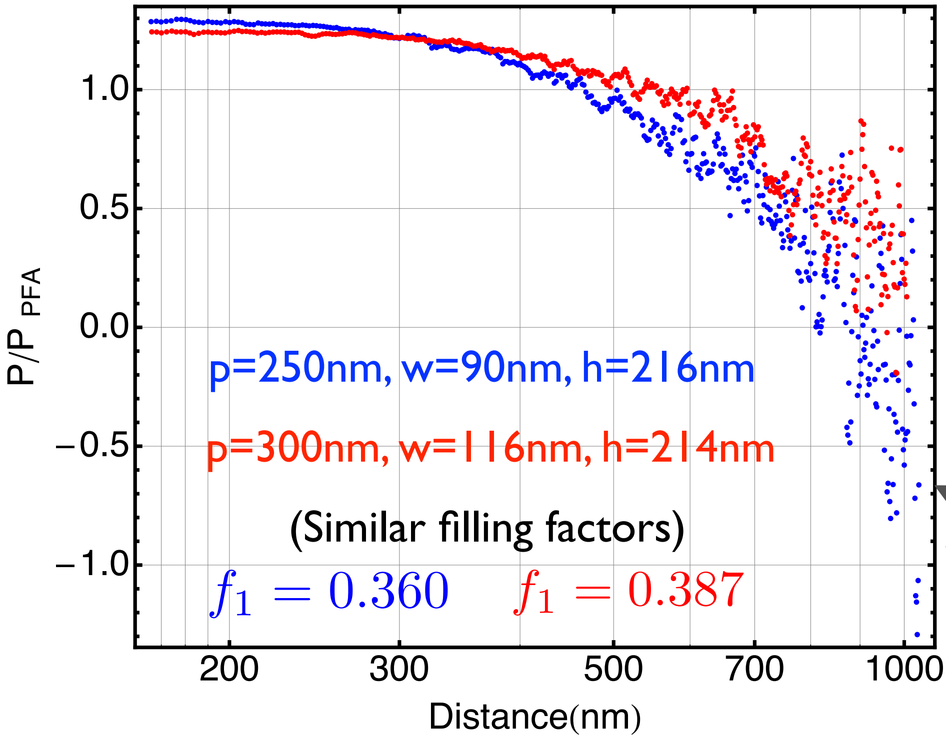
$\xi_{n=0}$



Normalizing to PFA for grating

$$P_{pg}^{\mathrm{PFA}}(d) = f P_{pp}(d) + (1-f) P_{pp}(d+h)$$





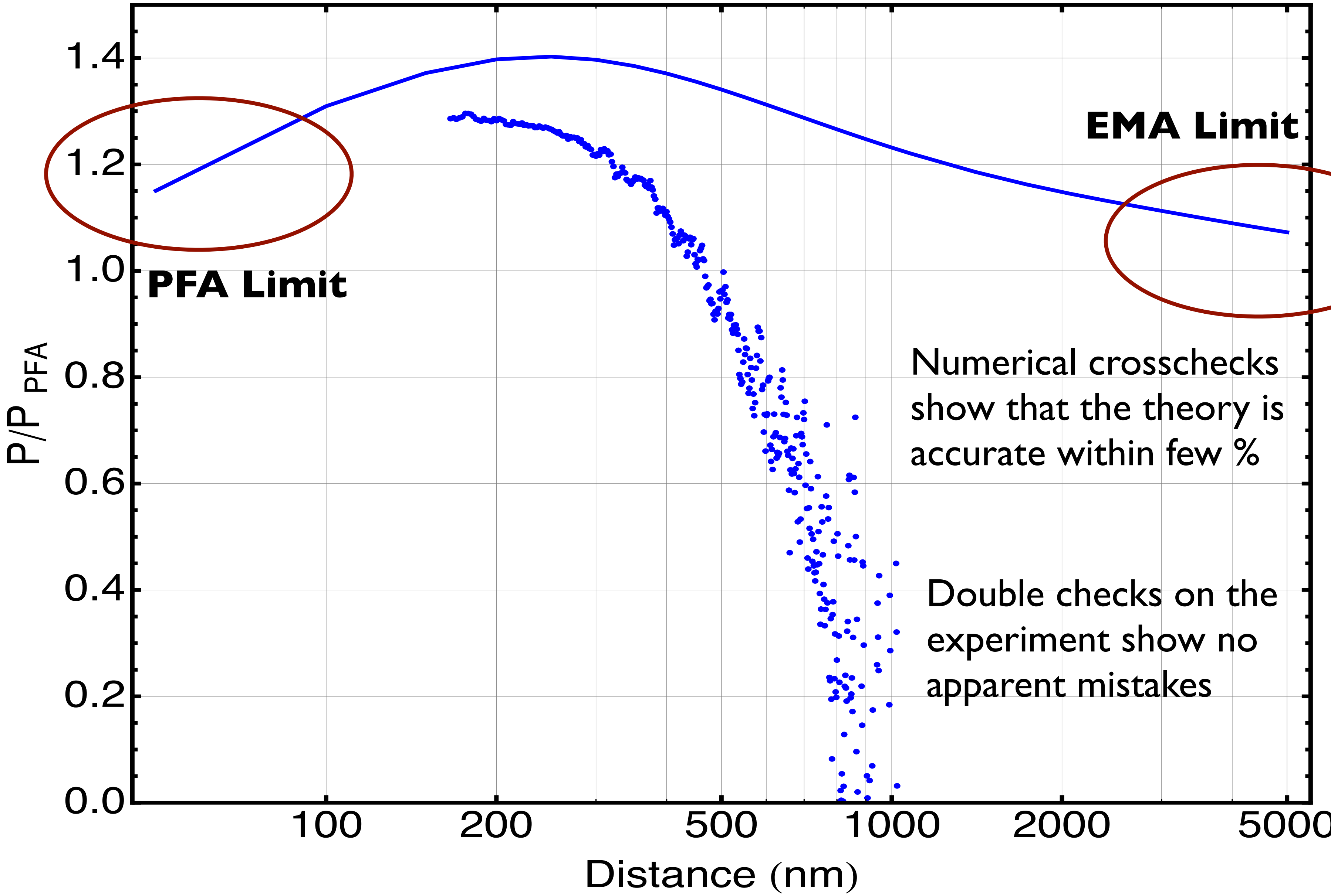
Small separations: *PFA underestimates the total pressure*.

Large separations: *PFA overestimates the exact pressure*.

Pressure is going to zero faster than d^{-4}

- Strong suppression of the Casimir force

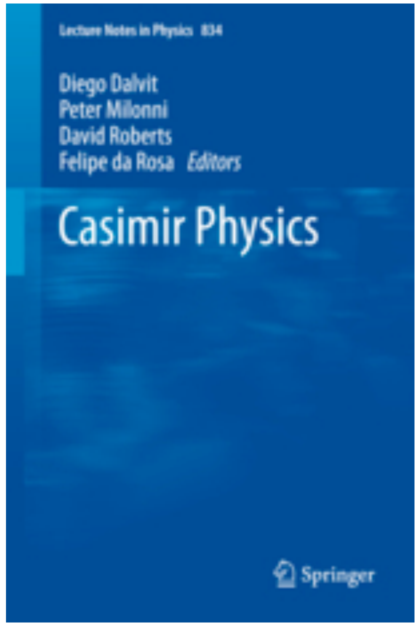
Open problem



Numerical crosschecks show that the theory is accurate within few %

Double checks on the experiment show no apparent mistakes

Intravaia *et al.*, to appear in Nature Communications (2013)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