Probing geometry effects of quantum vacuum with cold atoms

Diego A. R. Dalvit
Theoretical Division
Los Alamos National Laboratory

Collaborators

Paulo Maia Neto (Rio de Janeiro)

Astrid Lambrecht (LKB, Paris)

Serge Reynaud (LKB, Paris)

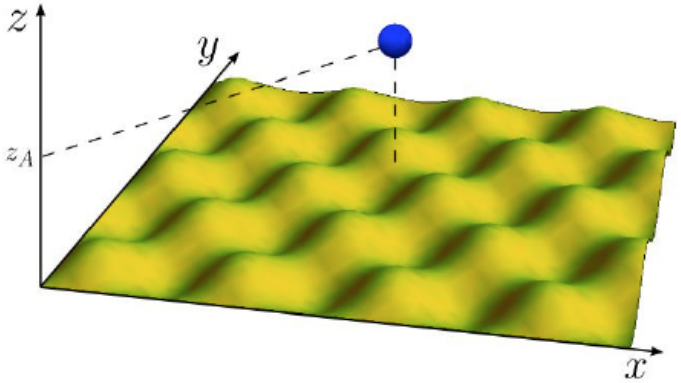
Riccardo Messina (Palermo)

Gustavo Moreno (Buenos Aires)

Esteban Calzetta (Buenos Aires)

Outline of this talk

- Review of theory and experiment on Casimir atom-surface interactions



- Casimir-Polder forces within scattering theory
- Cold atoms for probing lateral Casimir-Polder forces
  - BEC "cantilever" to measure Casimir frequency shifts
  - BEC Bragg spectroscopy to measure Casimir energy profile

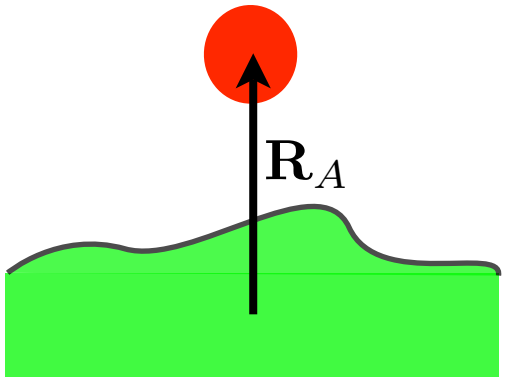
The Casimir-Polder force

vdW - CP interaction

Casimir and Polder (1948)

The interaction energy between a ground-state atom and a surface is given by

$$U_{\mathrm{CP}}(\mathbf{R}_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\,\mathrm{Tr}\,\mathbf{G}(\mathbf{R}_A,\mathbf{R}_A,i\xi)$$



Atomic polarizability: $\alpha(\omega) = \lim_{\epsilon\to 0}\frac{2}{3\hbar}\sum_k \frac{\omega_{k0}|\mathbf{d}_{0k}|^2}{\omega_{k0}^2-\omega^2-i\omega\epsilon}$

Scattering Green tensor: $\left(\nabla\times\nabla\times-\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\right)\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)=\delta(\mathbf{r}-\mathbf{r}')$

The Casimir-Polder force

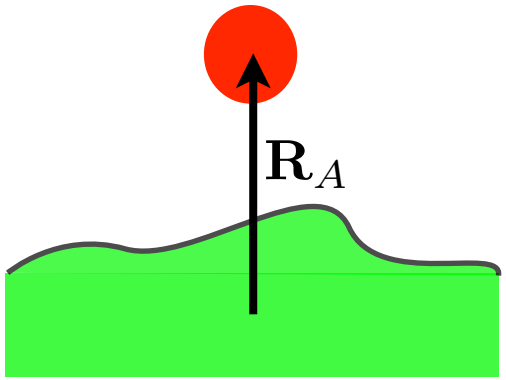
Los Alamos NATIONAL LABORATORY

vdW - CP interaction

Casimir and Polder (1948)

The interaction energy between a ground-state atom and a surface is given by

$$U_{\rm CP}(\mathbf{R}_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\,{\rm Tr}\,\mathbf{G}(\mathbf{R}_A,\mathbf{R}_A,i\xi)$$



Atomic polarizability: $\alpha(\omega) = \lim_{\epsilon\to 0}\frac{2}{3\hbar}\sum_k \frac{\omega_{k0}|\mathbf{d}_{0k}|^2}{\omega_{k0}^2-\omega^2-i\omega\epsilon}$

Scattering Green tensor: $\left(\nabla\times\nabla\times - \frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\right)\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}')$

Eg: Ground-state atom near planar surface @ T=0

Non-retarded (vdW) limit $z_A \ll \lambda_A$

$$U_{\rm vdW}(z_A) = -\frac{\hbar}{8\pi\epsilon_0}\frac{1}{z_A^3}\int_0^\infty \frac{d\xi}{2\pi}\alpha(i\xi)\frac{\epsilon(i\xi)-1}{\epsilon(i\xi)+1}$$

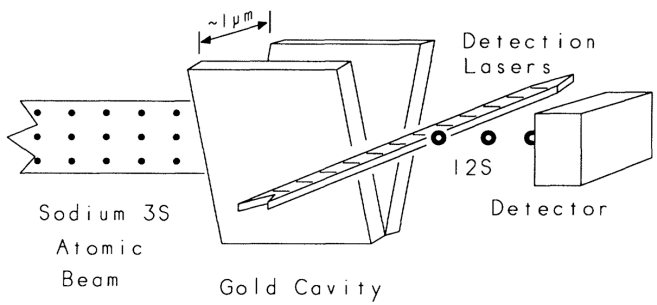
Retarded (CP) limit $z_A \gg \lambda_A$

$$U_{\rm CP}(z_A) = -\frac{3\hbar c\alpha(0)}{8\pi}\frac{1}{z_A^4}\frac{\epsilon_0-1}{\epsilon_0+1}\phi(\epsilon_0)$$

Modern CP experiments

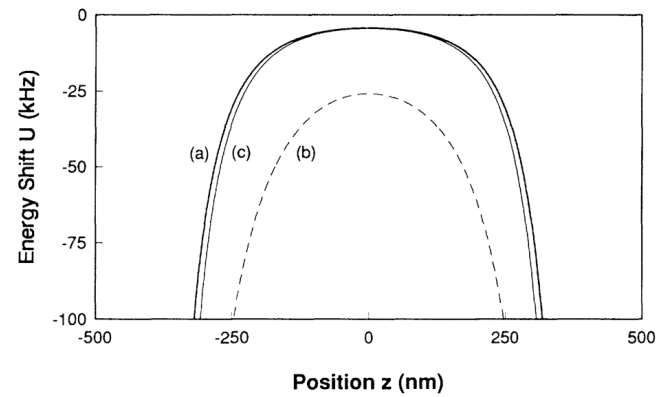
- Deflection of atoms

Hinds et al (1993)



L= 0.7-1.2 um

Exp-Th agreement @ 10%

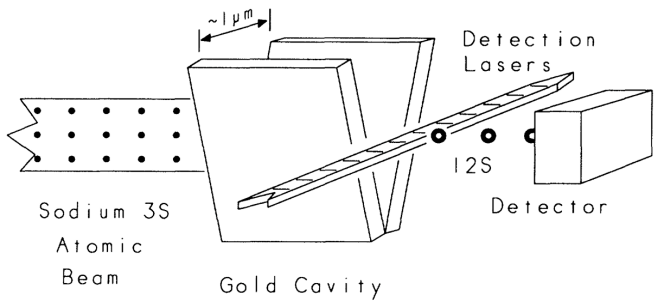


$$U_{CP} = -\frac{1}{4\pi\epsilon_0}\frac{\pi^3\hbar c\alpha(0)}{L^4}\left[\frac{3-2\cos^2(\pi z/L)}{8\cos^4(\pi z/L)}\right]$$

Modern CP experiments

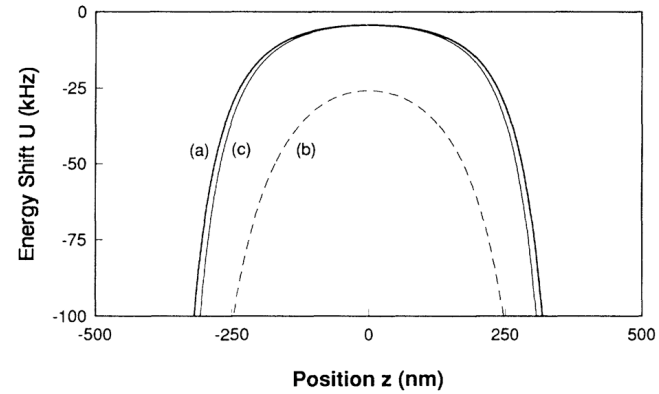
Deflection of atoms

Hinds et al (1993)



L= 0.7-1.2 um

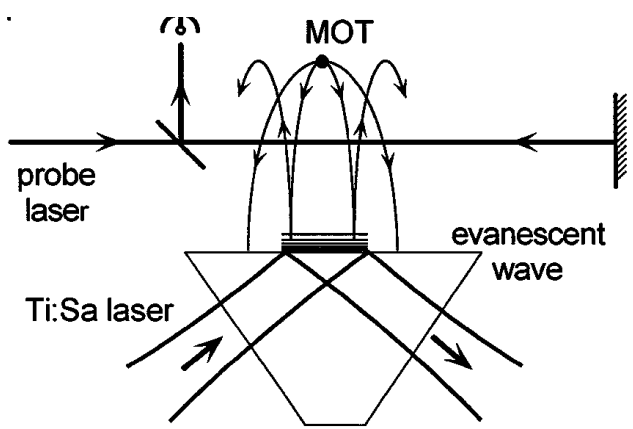
Exp-Th agreement @ 10%



$$U_{CP} = -\frac{1}{4\pi\epsilon_0}\frac{\pi^3\hbar c\alpha(0)}{L^4}\left[\frac{3-2\cos^2(\pi z/L)}{8\cos^4(\pi z/L)}\right]$$

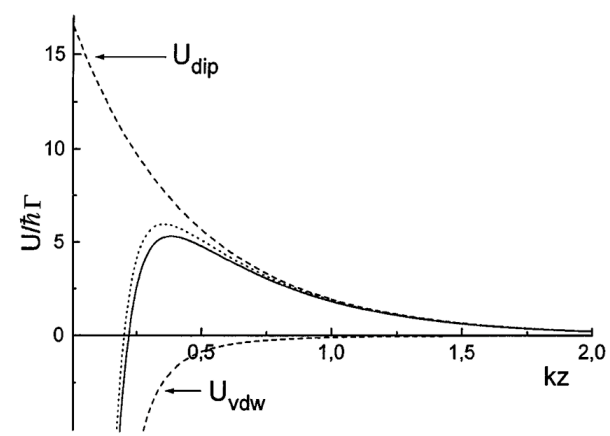
Classical reflection on atomic mirror

Aspect et al (1996)



$$U_{\rm dip} = \frac{\hbar}{4}\frac{\Omega^2}{\Delta}e^{-2kz}$$

$$U_{\rm vdW} = -\frac{\epsilon-1}{\epsilon+1}\frac{1}{48\pi\epsilon_0}\frac{D^2}{z^3}$$

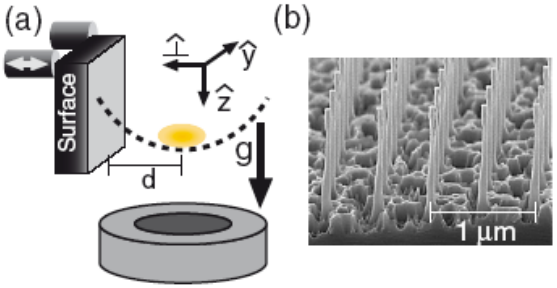


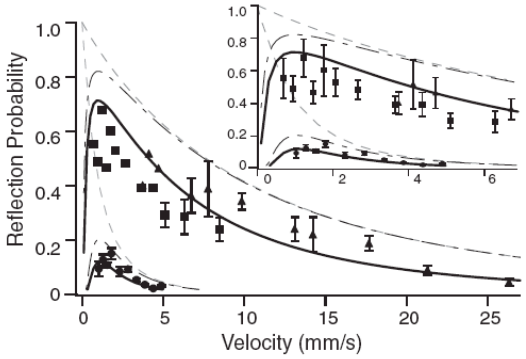
Exp-Th agreement @ 30%

Modern experiments (cont'd)

Quantum reflection

Wave-nature of atoms implies that slow atoms can reflect from purely attractive potentials





$$k = \sqrt{k_0^2 - 2mU/\hbar^2} \qquad \phi = \frac{1}{k^2}\frac{dk}{dr} > 1$$

$$U = -C_n/r^n \ (n > 2)$$

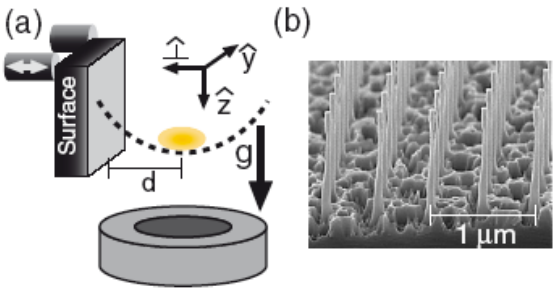
Shimizu (2001) Ketterle et al (2006)

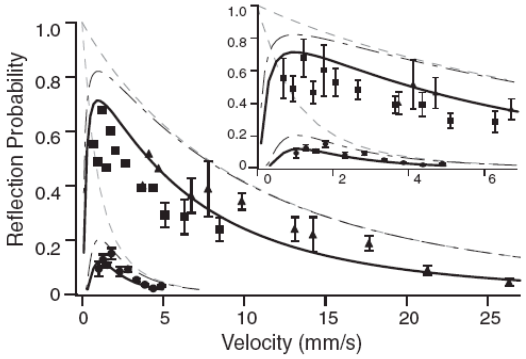
DeKievet et al (2003)

Modern experiments (cont'd)

Quantum reflection

Wave-nature of atoms implies that slow atoms can reflect from purely attractive potentials





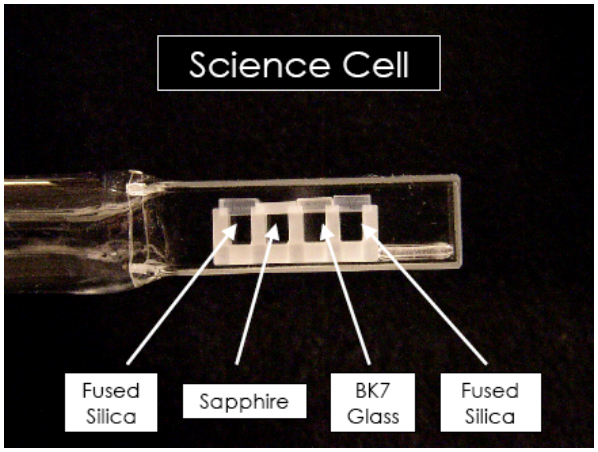
$$k = \sqrt{k_0^2 - 2mU/\hbar^2} \qquad \phi = \frac{1}{k^2}\frac{dk}{dr} > 1$$

$$U = -C_n/r^n \ (n > 2)$$

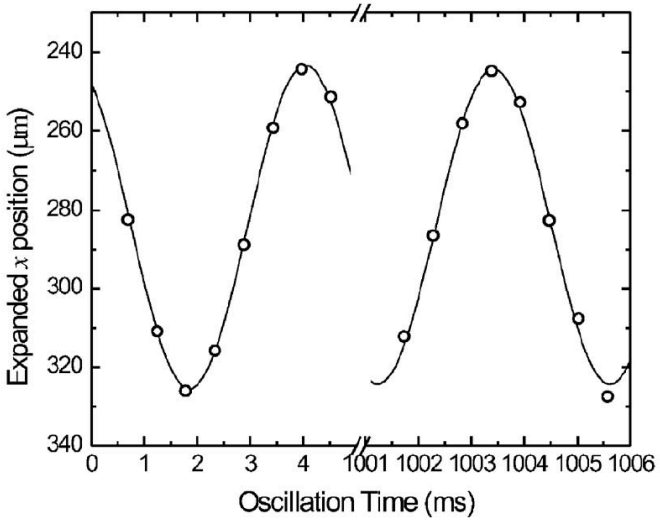
Shimizu (2001) Ketterle et al (2006)
DeKievet et al (2003)

BEC oscillator

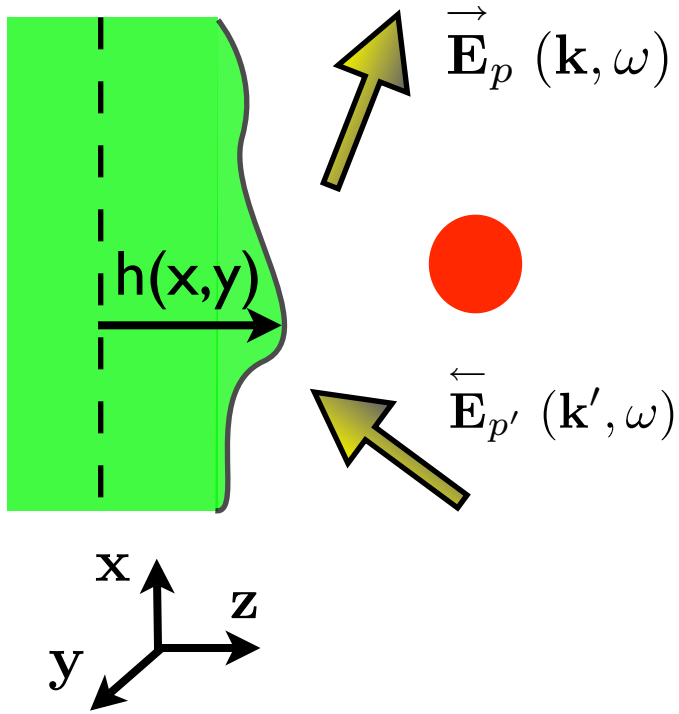
Cornell et al (2007)



$$\gamma_x \equiv \frac{\omega_x - \omega_x'}{\omega_x} \simeq -\frac{1}{2\omega_x^2 m}\partial_x^2 U^*$$



CP within scattering theory



Output fields: $\vec{\mathbf{E}}\ (\mathbf{R},\omega) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{-i\mathbf{k}\cdot\mathbf{r}}\ \vec{\mathbf{E}}\ (\mathbf{k},z,\omega)$

$$\vec{\mathbf{E}}\ (\mathbf{k},z,\omega) = [\vec{E}_{\rm TE}\ (\mathbf{k},\omega)\,\hat{\epsilon}^+_{\rm TE}(\mathbf{k}) + \vec{E}_{\rm TM}\ (\mathbf{k},\omega)\,\hat{\epsilon}^+_{\rm TM}(\mathbf{k})]\ e^{ik_z z}$$

$$\hat{\epsilon}^+_{\rm TE}(\mathbf{k}) = \mathbf{z}\times\mathbf{k} \qquad \hat{\epsilon}^+_{\rm TM}(\mathbf{k}) = \hat{\epsilon}^+_{\rm TE}(\mathbf{k})\times\mathbf{K} \qquad (\mathbf{K} = \mathbf{k} + k_z\mathbf{z})$$

Input fields: idem with $k_z \to -k_z$

Input and output fields related via reflection operators

$$\boxed{\vec{\mathbf{E}}_p\ (\mathbf{k},\omega) = \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \sum_{p'} \langle \mathbf{k},p|\mathcal{R}(\omega)|\mathbf{k}',p'\rangle\ \ \overleftarrow{\mathbf{E}}_{p'}\ (\mathbf{k}',\omega)}$$

Casimir-Polder force:

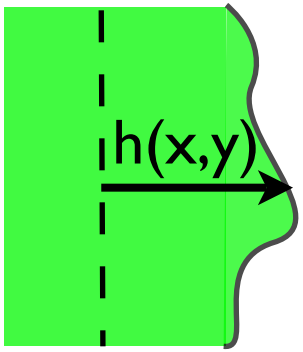
$$\boxed{U_{\rm CP}(\mathbf{R}_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int\frac{d^2\mathbf{k}'}{(2\pi)^2}e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_A}e^{-(\kappa+\kappa')z_A}\frac{1}{2\kappa'}\sum_{p,p'}\hat{\epsilon}^+_p(\mathbf{k})\cdot\hat{\epsilon}^-_{p'}(\mathbf{k}')\,R_{p,p'}(\mathbf{k},\mathbf{k}')}$$

with $\kappa \equiv \sqrt{\xi^2/c^2 + k^2}$ and $R_{p,p'}(\mathbf{k},\mathbf{k}')$ dependent on material properties at freq. $i\xi$

Specular/non specular scattering

In order to treat a general rough or corrugated surface, we make a perturbative expansion in powers of h(x,y)

$$\mathcal{R} = \mathcal{R}^{(0)} + \mathcal{R}^{(1)} + \dots$$



Specular reflection:

$$\langle \mathbf{k}, p | \mathcal{R}^{(0)} | \mathbf{k}', p' \rangle = (2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{k}')\, \delta_{p,p'}\, r_p(\mathbf{k}, \xi)$$

Fresnel coefficients $r_{\mathrm{TE}} = \dfrac{\kappa - \kappa_t}{\kappa + \kappa_t}$ $\quad r_{\mathrm{TM}} = \dfrac{\epsilon(i\xi)\kappa - \kappa_t}{\epsilon(i\xi)\kappa + \kappa_t}$ $\quad (\kappa_t = \sqrt{\epsilon(i\xi)\xi^2/c^2 + k^2})$

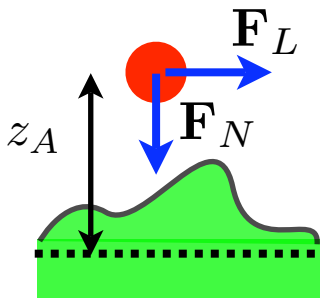
Non-specular reflection:

$$\langle \mathbf{k}, p | \mathcal{R}^{(1)} | \mathbf{k}', p' \rangle = R_{p,p'}(\mathbf{k}, \mathbf{k}')\, H(\mathbf{k} - \mathbf{k}')$$

← Fourier transform of h(x,y)

The non-specular reflection matrices depend on the geometry and material properties.

Lateral Casimir-Polder force



$$U_{\rm CP} = U^{(0)}_{\rm CP}(z_A) + U^{(1)}_{\rm CP}(z_A, x_A)$$

- Normal CP force: $$U^{(0)}_{\rm CP}(z_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{1}{2\kappa}\sum_p \hat{\epsilon}^+_p\cdot\hat{\epsilon}^-_p\; r_p(\mathbf{k},\xi)\; e^{-2\kappa z_A}$$

- Lateral CP force: $$\boxed{U^{(1)}_{\rm CP}(z_A, x_A) = \int\frac{d^2\mathbf{k}}{(2\pi)^2}\; e^{i\mathbf{k}\cdot\mathbf{r}_A}\; g(\mathbf{k}, z_A)\; H(\mathbf{k})}$$

Response function g:

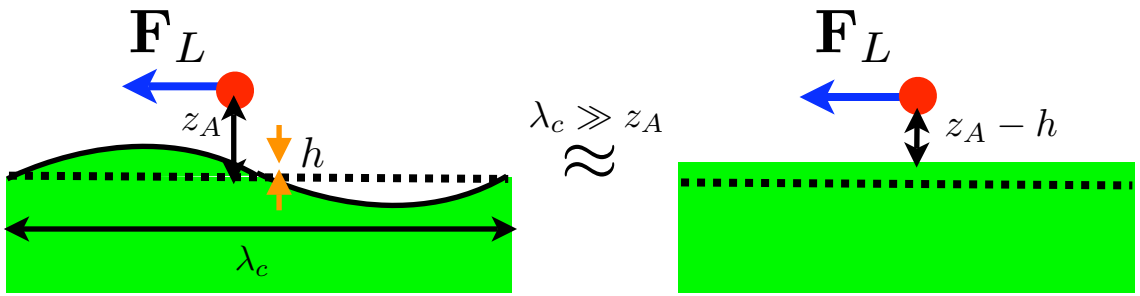
$$g(\mathbf{k}, z_A) = \frac{\hbar}{c^2\epsilon_0}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\int\frac{d^2\mathbf{k}'}{(2\pi)^2}\; a_{\mathbf{k}',\mathbf{k}'-\mathbf{k}}(z_A,\xi)$$

$$a_{\mathbf{k}',\mathbf{k}''} = \sum_{p',p''}\hat{\epsilon}^+_{p'}\cdot\hat{\epsilon}^-_{p''}\;\frac{e^{-(\kappa'+\kappa")z_A}}{2\kappa''}\; R_{p',p''}(\mathbf{k}',\mathbf{k}'')$$

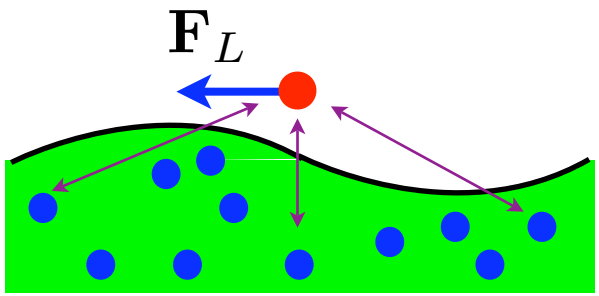
Our approach is perturbative in h(x,y), which should be the smallest length scale in the problem $h \ll z_A, \lambda_c, \lambda_A, \lambda_0$

Approx. methods: PFA & PWS

- Proximity Force Approximation (PFA)

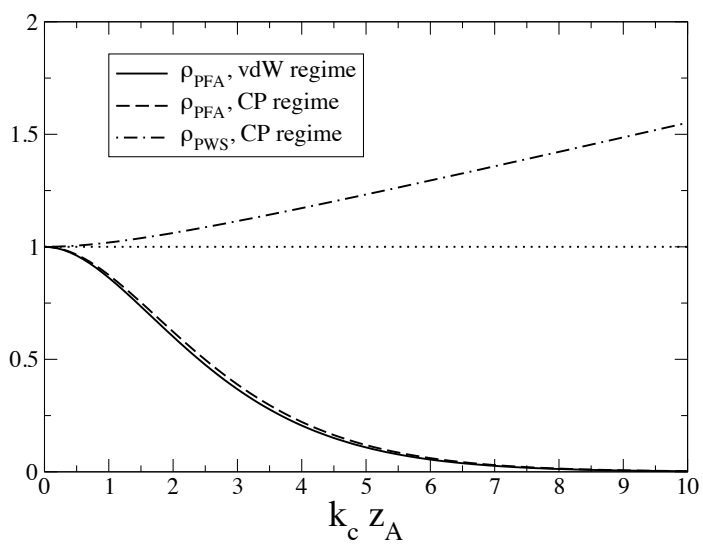


- Pair-wise Summation (PWS)



- Deviations from PFA and PWS

$$\rho_{\rm PFA} = \frac{g(k_c, z_A)}{g(0, z_A)} \qquad \rho_{\rm PWS} \equiv \frac{g(k_c, z_A)}{g_{\rm PWS}(k_c, z_A)}$$



Example:

atom-surface distance $z_A = 2\mu\text{m} \gg \lambda_A$

corrugation wavelength $\lambda_c = 3.5\mu\text{m}$

$\Rightarrow$ $\rho_{\rm PFA} \approx 30\%$ $\quad \rho_{\rm PWS} \approx 115\%$

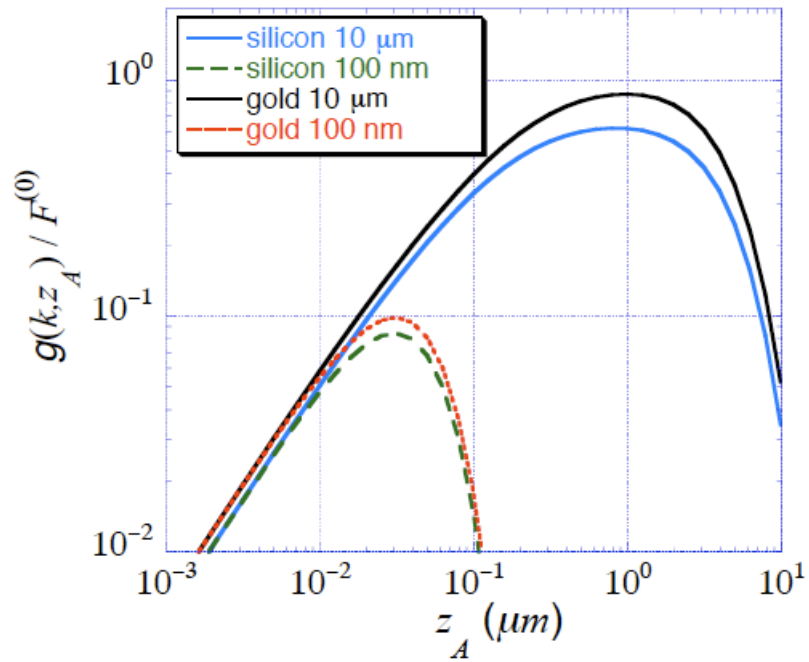
PFA largely overestimates the lateral CP force
PWS underestimates the lateral CP force

Disentangling geometry and real materials effects

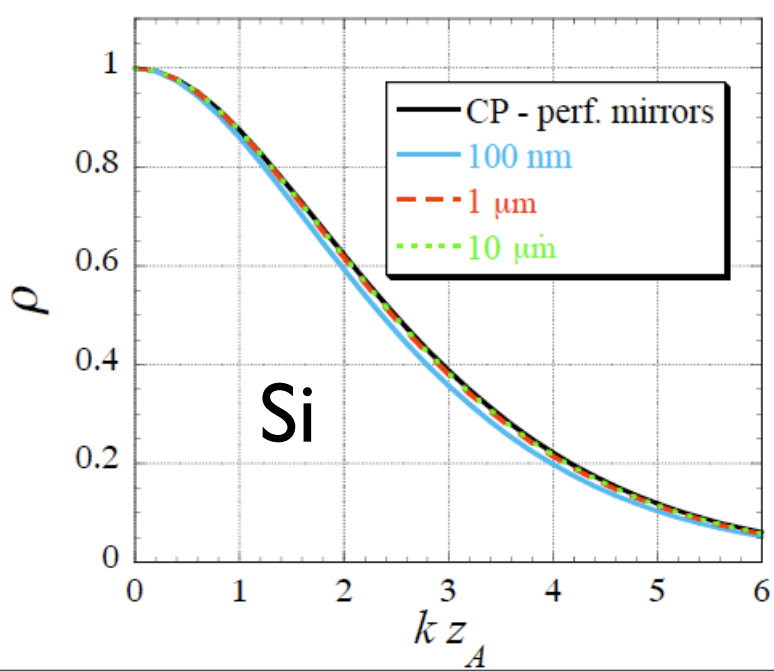
$$g(k,z) = \rho(k,z)\ \eta_F\ F_{\rm CP}^{(0)}$$

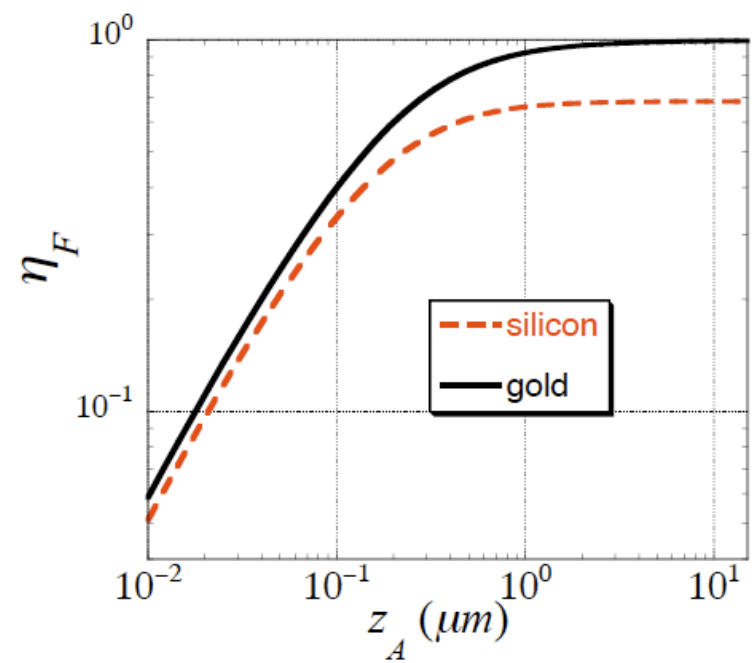
effects of materials

effects of geometry



$$\rho_{\rm CP}^{\rm perf}(k,z_A) = e^{-kz_A}\left[1 + kz_A + \frac{16(kz_A)^2}{45} + \frac{(kz_A)^3}{45}\right]$$

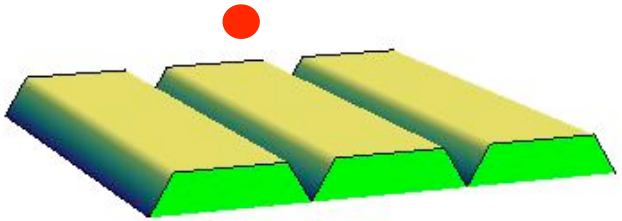




Atoms as local probes

In contrast to the case of the lateral Casimir force between corrugated surfaces, an atom is a local probe of the lateral Casimir-Polder force. Deviations from the PFA can be much larger than for the force between two surfaces!

☑ Even larger deviations from PFA can be obtained for a periodically grooved surface.



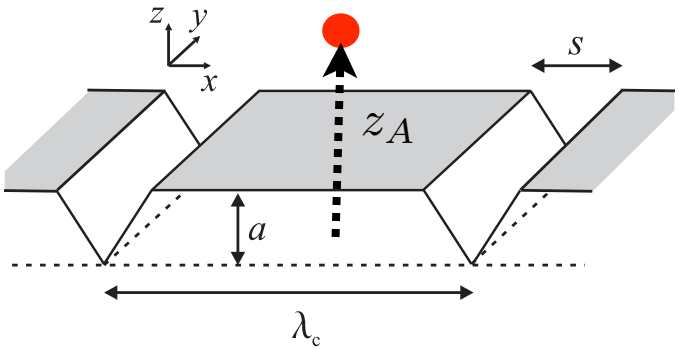
- If the atom is located above one plateau, the PFA predicts that the lateral Casimir-Polder force should vanish, since the energy is thus unchanged in a small lateral displacement.

- A non-vanishing force appearing when the atom is moved above the plateau thus clearly signals a deviation from PFA!

CP energy for grooved surface

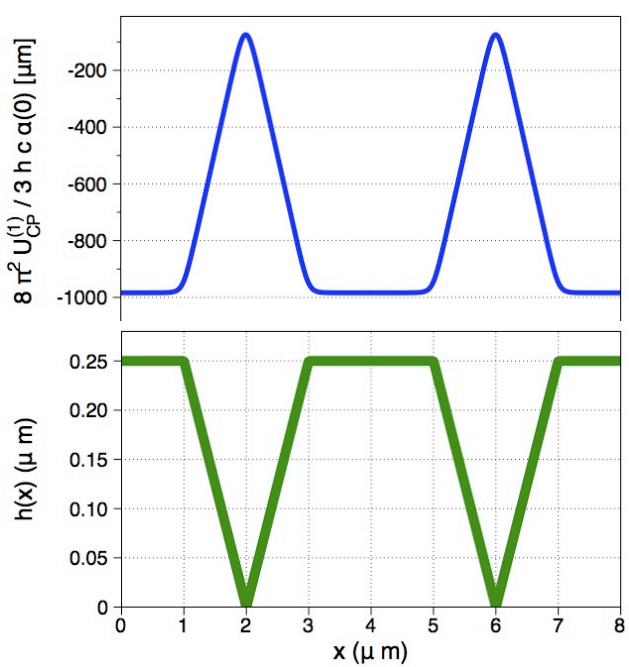
- Surface profile for periodical grooved corrugation

$$h(x) = a\left(1 - \frac{s}{2\lambda_c}\right) + \frac{2a\lambda_c}{\pi^2 s}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1-\cos(n\pi s/\lambda_c)}{n^2}\cos\left(\frac{2\pi n x}{\lambda_c}\right)$$

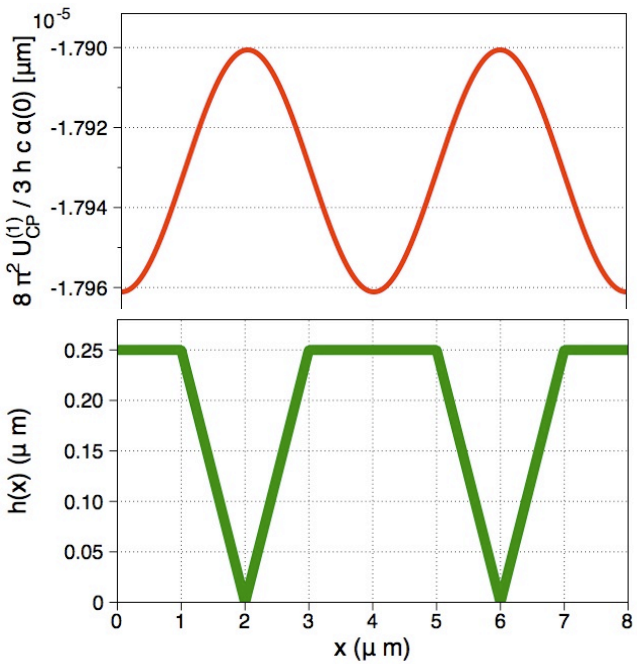


- Single-atom lateral CP energy: it can be easily calculated using that the first order lateral CP energy $U^{(1)}_{\rm CP}(\mathbf{R}_A) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}_A}\, g(\mathbf{k}, z_A)\, H(\mathbf{k})$ is linear in $H(\mathbf{k})$

$k_c z_A = 0.3$



$k_c z_A = 10$



Experimental proposals

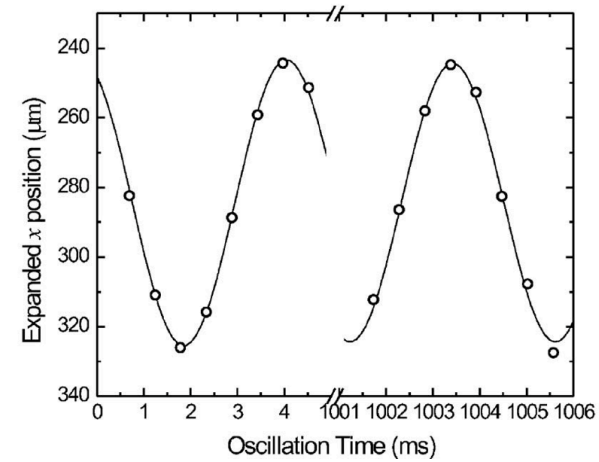
- BEC "cantilever" to measure Casimir frequency shifts
- BEC as an AFM "scanner" of the Casimir energy profile
- BEC Bragg spectroscopy to measure the Casimir lateral energy profile

BEC as a "cantilever"

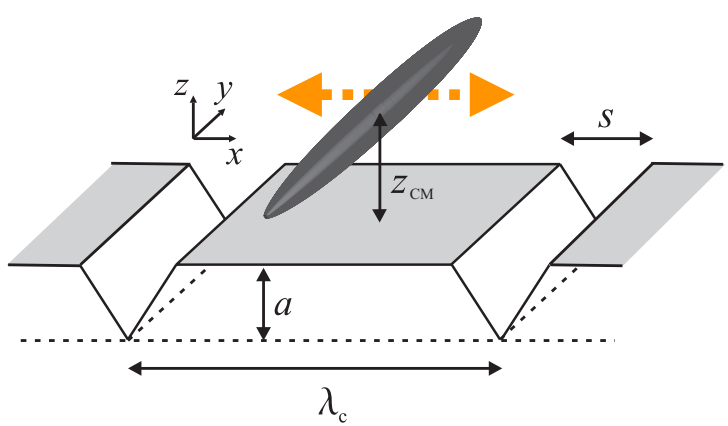
BEC oscillator

- The normal component of Casimir-Polder force $U_{\rm CP}^{(0)}(z)$ shifts the normal dipolar oscillation frequency of a BEC trapped above a surface

Antezza et al (2004) Cornell et al (2005, 2007)



- In order to measure the lateral component $U_{\rm CP}^{(1)}(x,z)$, a cigar-shaped BEC could be trapped parallel to the corrugation lines, and the lateral dipolar oscillation measured as a function of time



$$V(\mathbf{r}) = V_{\rm ho}(\mathbf{r}) + U_{\rm CP}(\mathbf{r})$$

$$V_{\rm ho}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \quad \omega_y \ll \omega_x = \omega_z$$

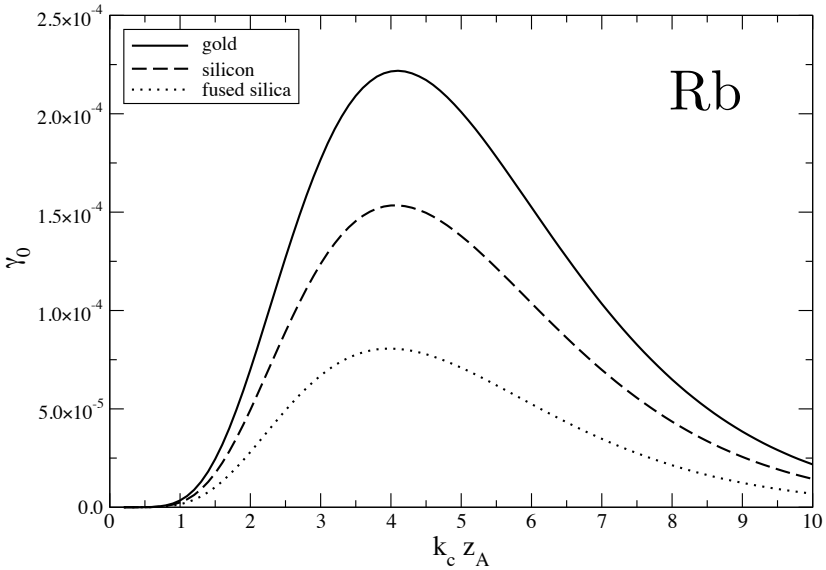
Lateral frequency shift:

$$\omega_{x,\rm CM}^2 = \omega_x^2 + \frac{1}{m}\int dxdz\, n_0(x,z)\frac{\partial^2}{\partial x^2}U_{\rm CP}^{(1)}(x,z)$$

Single-atom/BEC frequency shift

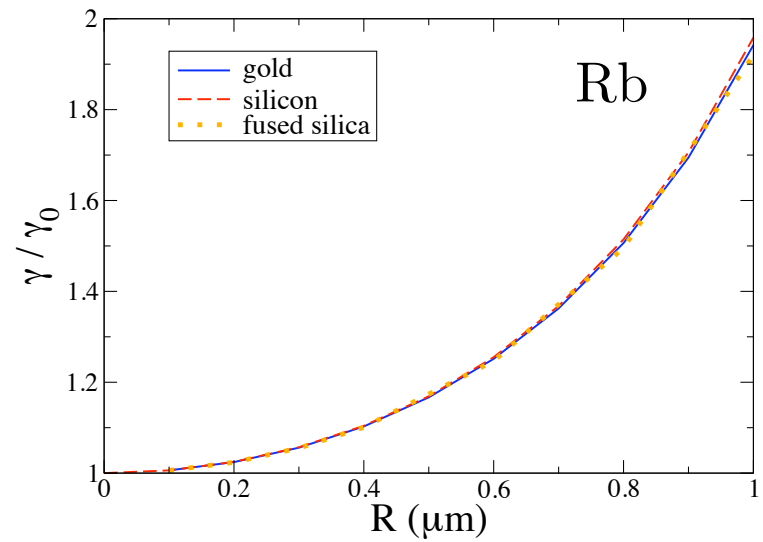
$$\gamma_0 \equiv \frac{\omega_{x,\mathrm{CM}} - \omega_x}{\omega_x}$$

- Single-atom lateral freq. shift



$$\omega_x/2\pi = 229\,\mathrm{Hz}$$
$$z_{\mathrm{CM}} = 2\mu\mathrm{m}$$
$$\lambda_c = 4\mu\mathrm{m}$$
$$a = 250\mathrm{nm}$$
$$s = \lambda_c/2$$

- Single-atom / BEC comparison



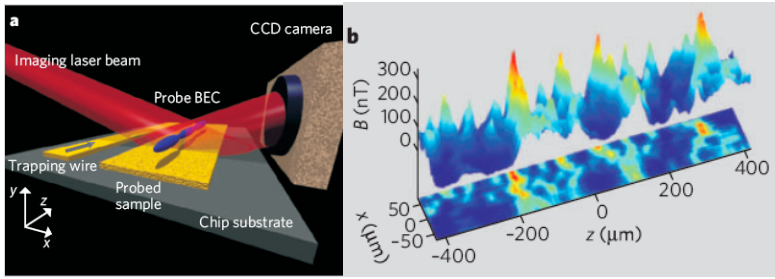
Given the reported sensitivity $\gamma = 10^{-5} - 10^{-4}$ for relative frequency shifts from E. Cornell's experiment, we expect that beyond-PFA lateral CP forces on a BEC above a plateau of a periodically grooved silicon surface should be detectable for distances $z_{\mathrm{CM}} < 3\mu\mathrm{m}$, groove period $\lambda_c = 4\mu\mathrm{m}$, groove amplitude $a = 250\mathrm{nm}$, and a BEC radius of, say, $R \approx 1\mu\mathrm{m}$

BEC as an AFM "scanner"

Density variations of a BEC above an atom chip

- For a quasi one-dimensional BEC, the potential is related to the 1D density profile as

$$V_{\rm ho}(x) + U_{\rm CP}(x) = -\hbar\omega_x\sqrt{1+4a_{\rm scat}n_{1d}(x)}$$



Measurement of the magnetic field variations along a current-carrying wire

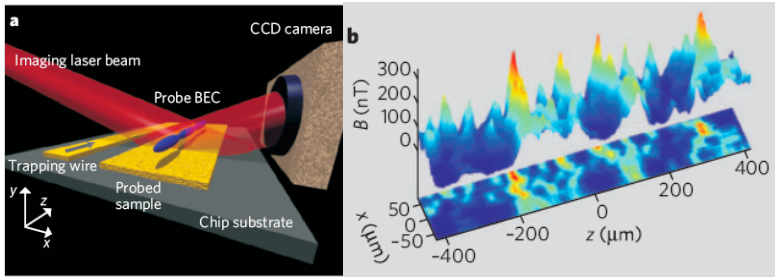
Schmiedmayer et al (2005)

BEC as an AFM "scanner"

Density variations of a BEC above an atom chip

For a quasi one-dimensional BEC, the potential is related to the 1D density profile as

$$V_{\rm ho}(x) + U_{\rm CP}(x) = -\hbar\omega_x\sqrt{1+4a_{\rm scat}n_{1d}(x)}$$



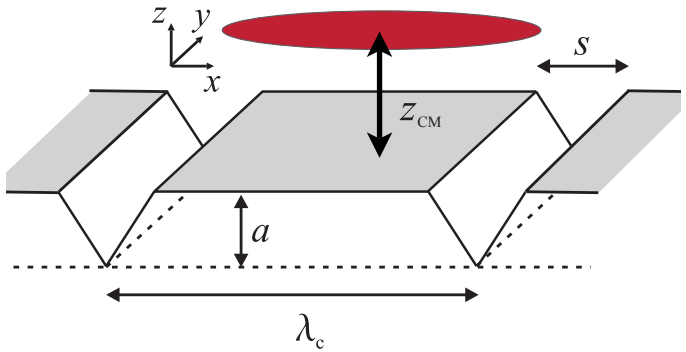
Measurement of the magnetic field variations along a current-carrying wire

Schmiedmayer et al (2005)

For the lateral CP force, perfect conductor, sinusoidal corrugation ($a = 100\mathrm{nm}$), distance $z_A = 2\mu\mathrm{m}$, PFA limit $(k_c z_A \ll 1)$

$$\Delta U_{\rm CP}^{(1)} \simeq 10^{-14}\,\mathrm{eV}$$

To measure the lateral CP force, the elongated BEC should be aligned along the x-direction, and a density modulation along this direction above the plateau would be a signature of a nontrivial (beyond-PFA) geometry effect.

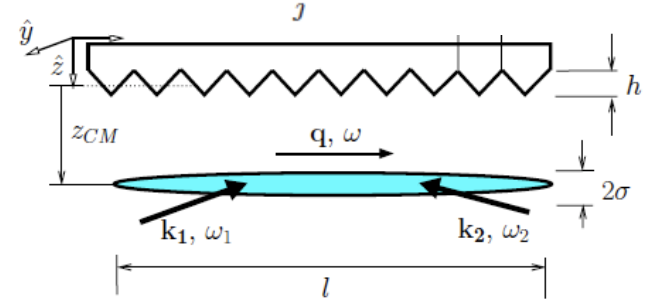


Casimir-modified BEC low energy spectrum

- Mean field BEC dynamics given by the Gross-Pitaevskii equation for the condensate wave-function φ

$$\boxed{\begin{aligned} i\hbar\partial_t\varphi &= -(\hbar^2/2m)\nabla^2\varphi + [U_N(z) + U_L(x,z)]\varphi \\ &\quad +(m/2)(\omega_r^2 r^2 + \omega_x^2 x^2)\varphi + g|\varphi|^2\varphi, \end{aligned}}$$

$$h(x) = \sum_j h_j \cos(jk_c x)$$



$$U(x,y,z) = U_N(z) + U_L(x,z)$$

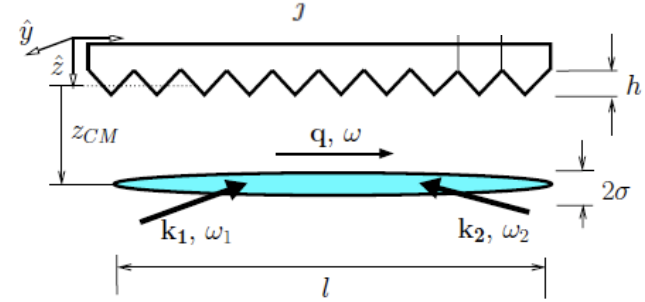
$$U_L^{(1)}(x_A, z_A) = \sum_{j=0}^{\infty} h_j \cos(jk_c x_A) g(jk_c, z_A)$$

Casimir-modified BEC low energy spectrum

- Mean field BEC dynamics given by the Gross-Pitaevskii equation for the condensate wave-function φ

$$\boxed{\begin{aligned} i\hbar\partial_t\varphi &= -(\hbar^2/2m)\nabla^2\varphi + [U_N(z) + U_L(x,z)]\varphi \\ &\quad +(m/2)(\omega_r^2 r^2 + \omega_x^2 x^2)\varphi + g|\varphi|^2\varphi, \end{aligned}}$$

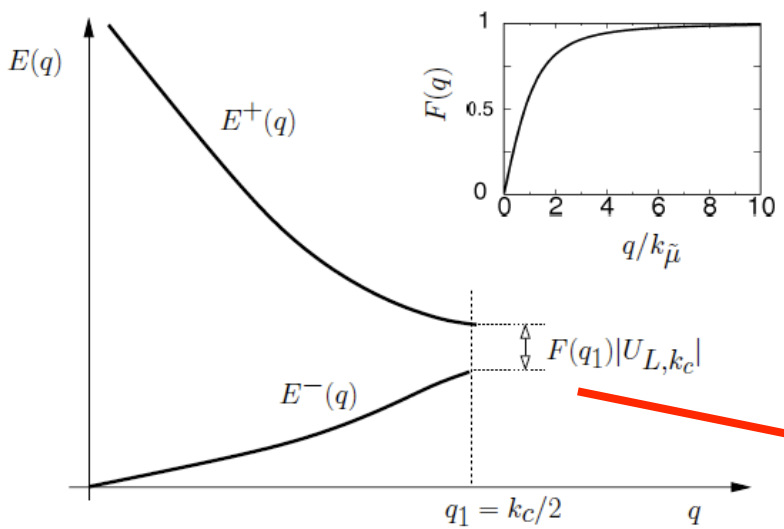
$$h(x) = \sum_j h_j \cos(jk_c x)$$



$$U(x,y,z) = U_N(z) + U_L(x,z)$$

$$U_L^{(1)}(x_A, z_A) = \sum_{j=0}^{\infty} h_j \cos(jk_c x_A) g(jk_c, z_A)$$

- Quantum fluctuations given by the Bogoliubov spectrum + Casimir modifications:



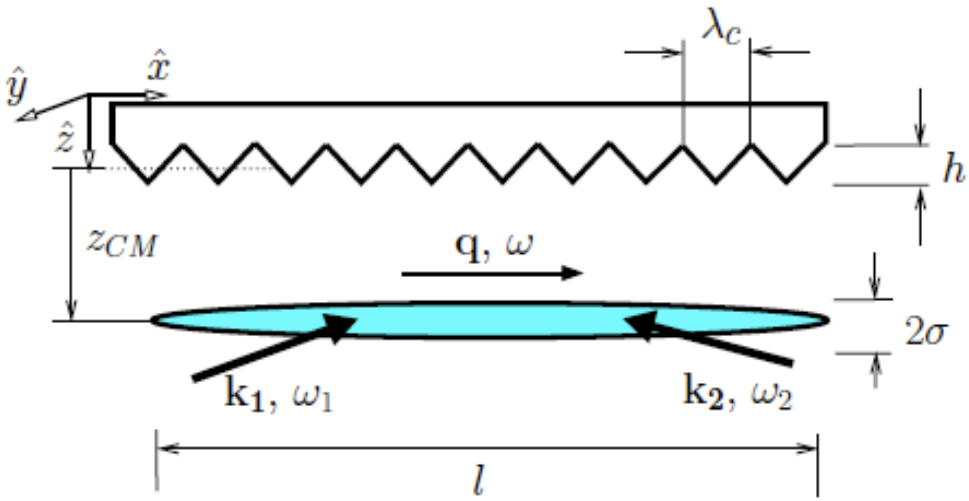
$$\boxed{E(q) = E_{\rm B}(q) + \delta E_{\rm CP}(q)}$$

$$E_{\rm B}(q) = \sqrt{(\hbar^2 q^2/2m)(\hbar^2 q^2/2m + 2\tilde{\mu})}$$

$$\tilde{\mu} = \mu - \hbar\omega_r - U_N(z_{\rm cm})$$

CP opens energy gaps

BEC Bragg spectroscopy



- Two Bragg photons set-up

$$\mathbf{q} = q\hat{\mathbf{x}} = \mathbf{k}_1 - \mathbf{k}_2 \; ; \; \omega = \omega_1 - \omega_2.$$

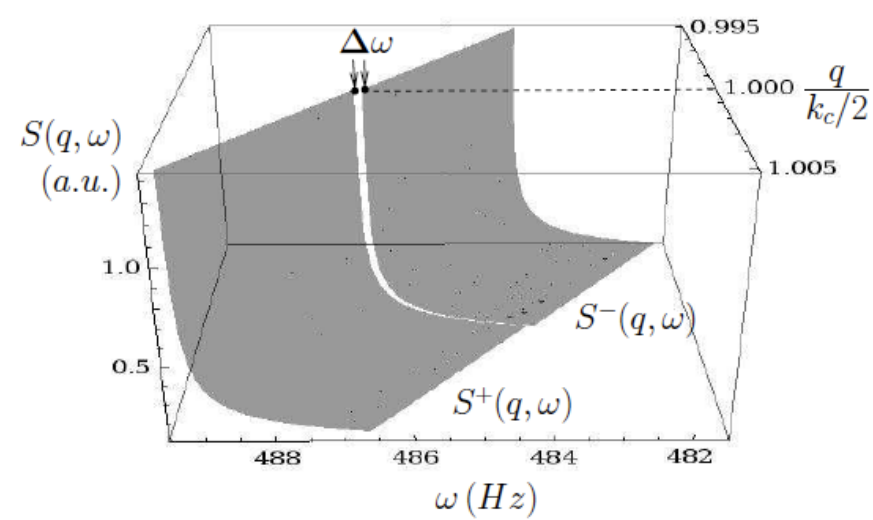
$$\frac{dP_X}{dt} = -m\omega_x^2 X + \sum_{n,i} U_{L,nk_c}(nk_c)\langle \sin(nk_c x_i)\rangle + \\ + \frac{N\hbar q V_{\mathrm{B}}^2}{2} \int d\omega' \, [S(q,\omega') - S(-q,-\omega')] \\ \times \frac{\sin([\omega-\omega']t)}{\omega-\omega'}.$$

Observable: total momentum P_X transfered to the BEC

Dynamic structure factor:

$$S(q,\omega) = \frac{N\hbar^2 q^2}{2mE_{\mathrm{B}}(q)} \, \delta(\hbar\omega - E_{\mathrm{B}}(q))$$ [without CP force]

$$S \to S + \delta S_{\mathrm{CP}}(q,\omega)$$ [with CP force]



Some numbers

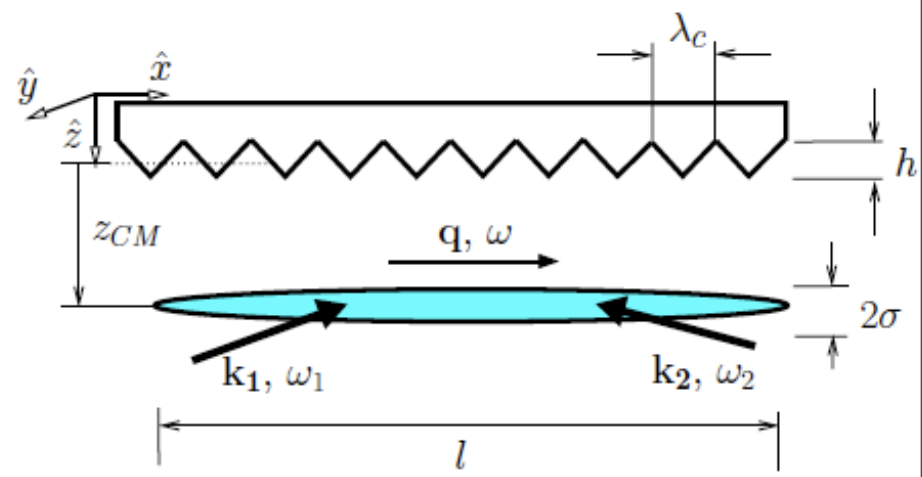
- Parameters of the BEC (^{87}Rb)

$$N = 10^4 \text{ atoms}$$

$$\omega_r/2\pi = 2.7\text{kHz} \qquad \omega_x/2\pi = 0.83 \text{ Hz}$$

$$\sigma = 0.2\mu\text{m} \qquad l/2 = 408\mu\text{m}$$

$$\boxed{E_B(q_1) = \hbar 485\text{Hz}}$$



$$\lambda_c = 2\pi/k_c = 9.75\mu\text{m}$$

$$h = 1\mu\text{m} \qquad z_{\text{cm}} = 3\mu\text{m}$$

- Parameters of the surface

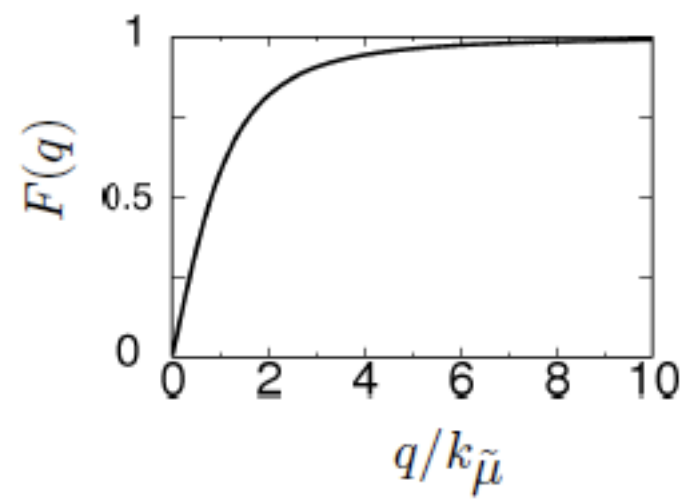
$$g_{\text{CP}}^{\text{perf}}(k,z) = -\frac{3\hbar c\alpha(0)}{8\pi^2\epsilon_0 z^5} e^{-\mathcal{Z}}(1 + \mathcal{Z} + 16\mathcal{Z}^2/45 + \mathcal{Z}^3/45)$$

$$\mathcal{Z} = k_c z \qquad \alpha(0)/\epsilon_0 = 47.3 \times 10^{-30}\text{m}^3$$

$$U^{(1)}_{L,k_c} = h g_{\text{CP}}^{\text{perf}}(k_c, z_{\text{cm}}) \approx \begin{matrix} \hbar\, 1.26\text{Hz} & \text{[Au]} \\ \hbar\, 0.98\text{Hz} & \text{[Si]} \end{matrix}$$

$$\boxed{U^{(1)}_{L,k_c} F(k_c/2) \approx \hbar 0.1\,\text{Hz}}$$

$$F(k_c/2) = 0.08$$



Summary

- Novel cold atoms techniques open a promising way of investigating nontrivial geometrical effects on quantum vacuum
- Important feature of atoms: they can be used as local probes of quantum vacuum fluctuations
- Non-trivial, beyond-PFA effects should be measurable using a BEC as a vacuum field sensor with available technology

For more details see:

Phys. Rev. Lett. 100, 040405 (2008)
J. Phys. A 41, 164028 (2008)
arXiv: 0902.3235 (to appear in PRA)
arXiv: 0904.0238