

Quantum friction and fluctuation theorems

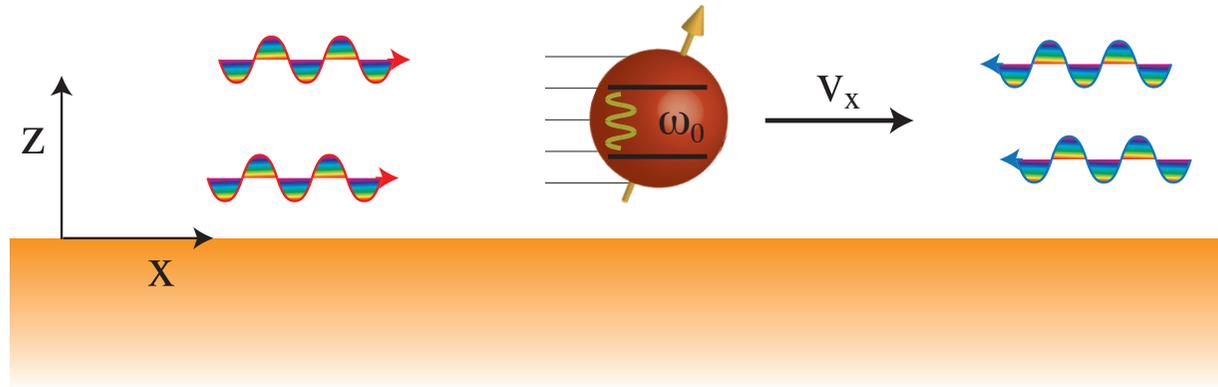
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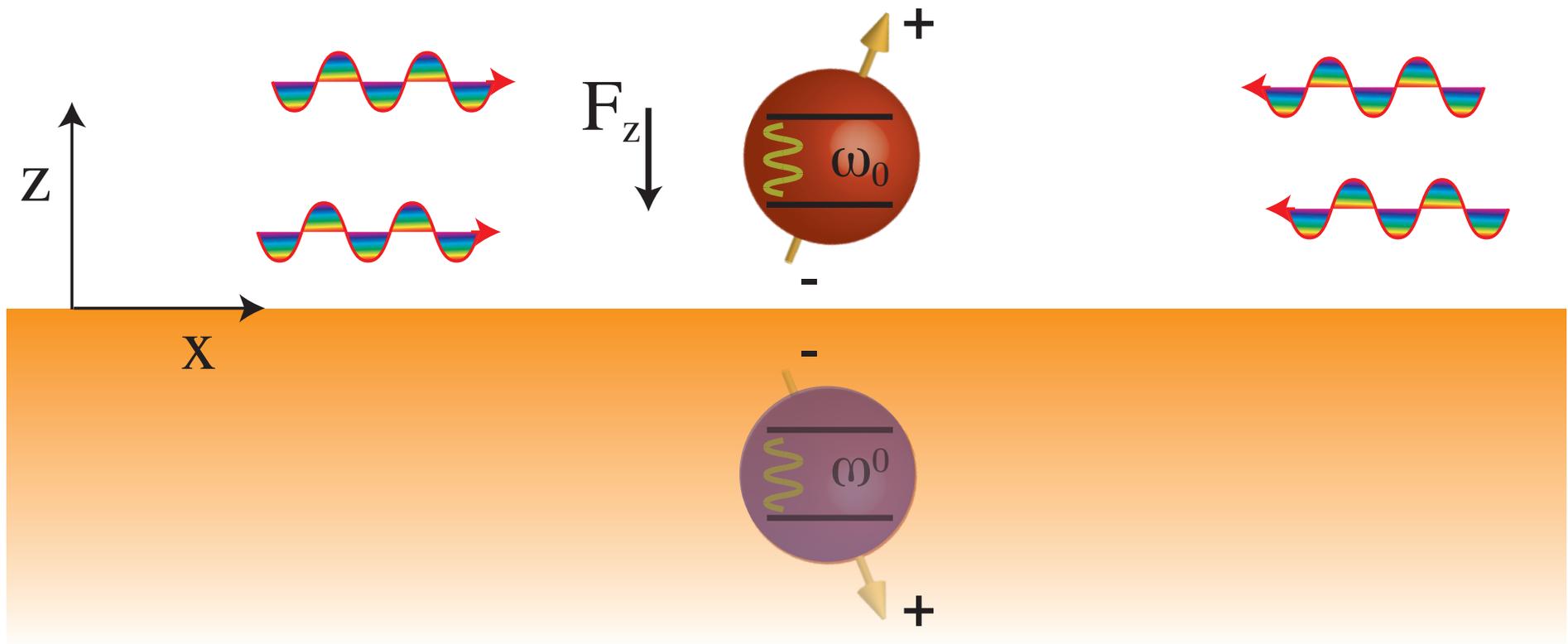


Outline of this Talk



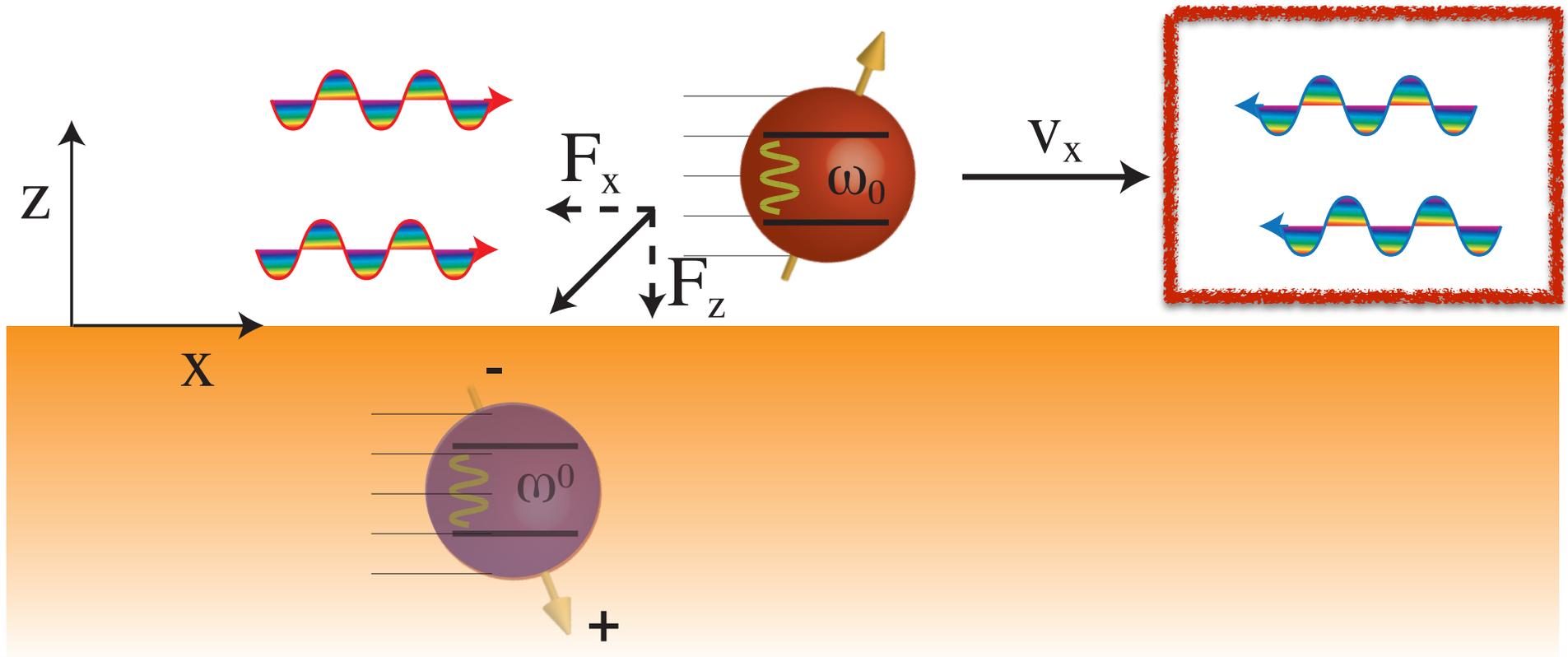
- What is quantum friction?
- Atom-surface interaction: equilibrium
- Atom-surface interaction: non-equilibrium
 - Fluctuation-dissipation vs quantum regression
 - Moving oscillator
 - Moving two-level atom
- Revisiting other approaches

An intuitive picture



An intuitive picture

Photons and plasmon field
perceived with a Doppler
shifted frequency



A variety of predictions

Zero temperature atom-surface quantum friction

Authors	Low velocity dependency	Distance dependency	Comments
Mahanty 1980	\mathbf{v}	\mathbf{z}^{-5}	Approach similar to the calculations of vdW forces but with mistakes
Schaich and Harris 1981	\mathbf{v}	\mathbf{z}^{-10}	Two-state atom with a transition dipole moment normal to a metal surface
Scheel and Buhmann 2009	\mathbf{v}	\mathbf{z}^{-8}	Master-equation approach for multilevel atoms and quantum regression “theorem”.
Barton 2010	\mathbf{v}	\mathbf{z}^{-8}	Perturbation theory using Fermi’s golden rule. Harmonic oscillator.
Philbin and Leonhardt 2009	$\mathbf{0}$	$\mathbf{0}$	Relativistic calculations and analytical/numerical evaluation of the Green’s tensor
Dedkov and Kyasov 2012	\mathbf{v}^3	\mathbf{z}^{-7}	Fluctuation-dissipation theorem applied to the dipole atom as well as to the electric field

Warm-up: Static atom

● **vdW-Casimir force on a static atom:** $F_z(t) = \langle \hat{\mathbf{d}}(t) \cdot \partial_{z_a} \hat{\mathbf{E}}(\mathbf{r}_a, t) \rangle$

$$F_z(t) = \text{Re} \left\{ \frac{2i}{\pi} \int_0^\infty d\omega \int_0^t d\tau e^{-i\omega\tau} \text{Tr} \left[\langle \hat{\mathbf{d}}(t) \hat{\mathbf{d}}(t - \tau) \rangle \cdot \partial_z \underline{G}_I(\mathbf{r}_a, \mathbf{r}_a, \omega) \right] \right\}$$

● **Two-time correlation tensor:** $\underline{C}_{ij}(t, t - \tau) \equiv \langle \hat{d}_i(t) \hat{d}_j(t - \tau) \rangle$

Initial state of atom+field+matter: $\hat{\rho}(0) = \hat{\rho}_a(0) \otimes \hat{\rho}_{\text{fm}}(0)$

Full evolution: $\hat{H} = \hat{H}_a + \hat{H}_{\text{fm}} + \hat{H}_{\text{int}}$

How to compute the two-time dipole-dipole correlator?

- Solve the exact dynamics (when possible!)
- Time-dependent perturbation theory
- Large-time limit: stationary (dressed) states, equilibrium

Fluctuation-dissipation

Stationary density matrix

$$\hat{\rho}(\infty) = \hat{\rho}_{\text{KMS}} \propto e^{-\beta \hat{H}}$$

(Kubo-Martin-Schwinger)

Large time correlator

$$\underline{C}_{ij}(\tau) = \text{tr} \left\{ \hat{d}_i(0) \hat{d}_j(-\tau) \hat{\rho}_{\text{KMS}} \right\}$$

Fluctuation-dissipation (FDT) (Callen & Welton 1951)

- Linear response
- Exact result

power spectrum

$$\underline{S}(\omega) = \frac{\hbar}{\pi} \theta(\omega) \underline{\alpha}_I(\omega)$$

polarizability

$$\underline{S}(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \underline{C}(\tau)$$

$$\underline{\alpha}(\tau) = (i/\hbar) \theta(\tau) \text{tr} \{ [\hat{\mathbf{d}}(0), \hat{\mathbf{d}}(-\tau) \hat{\rho}_{\text{KMS}}] \}$$

Stationary vdW-CP force

$$F_{\text{CP}} = \frac{\hbar}{\pi} \int_0^{\infty} d\xi \text{Tr} \{ \underline{\alpha}(i\xi) \cdot \partial_z \underline{G}(\mathbf{r}_a, \mathbf{r}_a, i\xi) \}$$

Quantum regression

● **Onsager regression theorem:** *The average regression of fluctuations obeys the same laws as the corresponding irreversible process* (Onsager 1931)

● **Quantum regression hypothesis** (aka “theorem”, QRT) (Lax 1963)

$$\underline{C}(t, t - \tau) \equiv \langle \mathbf{d}(t) \mathbf{d}(t - \tau) \rangle = \langle \mathbf{d}^2(t) \rangle e^{-i(\omega_a - i\gamma_a/2)\tau} \quad \begin{array}{l} \text{-weak coupling} \\ \text{- Markov approx} \end{array}$$

● **FDT and QRT predict different decays**

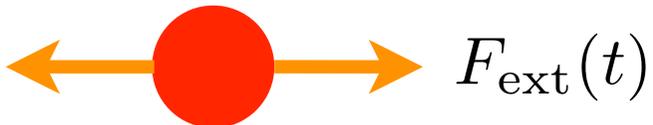
- “Short” times ($\tau\gamma_a \ll 1$): exponential decay QRT = FDT

- “Large” times ($\tau\gamma_a \gg 1$): power-law decay QRT \neq FDT

$$\underline{C}_{ij}(t, t - \tau) = \mathbf{d}_i \mathbf{d}_j e^{-i(\omega_a - i\gamma_a/2)\tau} + \left[\tau \rightarrow \infty \propto \frac{\gamma_a}{\omega_a} (\omega_a \tau)^{-n} \right]$$

QRT predicts the wrong vdW/CP force

Quantum friction (T=0)

$$F_{\text{fric}}(t) = \langle \hat{\mathbf{d}}(t) \cdot \partial_x \hat{\mathbf{E}}(\mathbf{r}_a(t), t) \rangle$$


Prescribed motion

$$\mathbf{r}_a(t) = \begin{cases} (x_0, y_a, z_a) & \text{for } t < t_a \\ (x_{\text{accel}}(t), y_a, z_a) & \text{for } t_a < t < 0 \\ (x_a + v_x t, y_a, z_a) & \text{for } t > 0 \end{cases}$$

$$m_a \ddot{x}_a(t) = F_{\text{ext}}(t) + F_x(t)$$

Stationary ($t \rightarrow \infty$) frictional force

$$F_{\text{fric}} = \text{Re} \left\{ \frac{2}{\pi} \int \frac{d^2 \mathbf{k}}{(2\pi)^2} k_x \int_0^\infty d\omega \int_0^\infty d\tau e^{-i(\omega - k_x v_x) \tau} \text{Tr}[\underline{C}(\tau; v_x) \cdot \underline{G}_I(\mathbf{k}, z_a, \omega)] \right\}$$

$$\underline{C}_{ij}(\tau; v_x) = \text{tr} \{ \hat{d}_i(0) \hat{d}_j(-\tau) \hat{\rho}(\infty) \}$$

No general results as in the equilibrium case $\hat{\rho}(\infty) = ???$

NEQ FT and q-friction

It is still possible to draw general conclusions about the frictional force in the low-velocity limit.

• **Non-equilibrium power spectrum** $\underline{S}(\omega; v_x) = (2\pi)^{-1} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \underline{C}(\tau; v_x)$

$$F_{\text{fric}} = -2 \int \frac{d^2\mathbf{k}}{(2\pi)^2} k_x \int_0^{\infty} d\omega \text{Tr}[\underline{S}(k_x v_x - \omega; v_x) \cdot \underline{G}_I(\mathbf{k}, z_a, \omega)]$$

• **Small velocity analysis: no linear-in-v terms**

- Contributions from $\underline{S}_R(-\omega; v_x)$ cancel upon integration over k_x
- Contributions from $\underline{S}_R(k_x v_x - \omega; 0) \longrightarrow$ equilibrium FDT!

vdW regime:

$$F_{\text{fric}} \approx -\frac{45\hbar}{256\pi^2\epsilon_0} \alpha'_I(z_a, 0) \Delta'_I(0) \frac{v_x^3}{z_a^7}$$

$$\Delta(\omega) = \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 1}$$
$$\alpha'_I(z_a, 0) \propto z_a^{-3}$$

FDT \longrightarrow $\underline{S}(k_x v_x - \omega; 0) = \frac{\hbar}{\pi} \theta(k_x v_x - \omega) \underline{\alpha}_I(k_x v_x - \omega)$

Nonzero for $0 \leq \omega \leq k_x v_x$ **(relevance of low freq)**

$$F_{\text{fric}} \propto \frac{v_x^3}{z_a^{10}}$$

QRT \longrightarrow $\underline{C}_{ij}(\tau; 0) = \mathbf{d}_i \mathbf{d}_j e^{-i(\omega_a - i\gamma_a/2)\tau}$ $\underline{S}(\omega; 0) = \int_{-\infty}^{\infty} \frac{d\tau}{2\pi} e^{i\omega\tau} \underline{C}(\tau; 0)$

Neglects low frequencies

$$F_{\text{fric}} \propto \frac{v_x}{z_a^8}$$

**Second order
perturbation theory**

$$\gamma_a \rightarrow 0$$

FDT = QRT

$$F_{\text{fric}} \propto e^{-a/v_x}$$

**Harmonic
oscillator model**



Exactly solvable model

$$F_{\text{fric}} \propto \frac{v_x^3}{z_a^{10}}$$

Moving harmonic oscillator

● Dipole moment $\hat{\mathbf{d}} = \mathbf{d}\hat{q}$ $\ddot{\hat{q}}(t) + \omega_a^2 \hat{q}(t) = \frac{2\omega_a}{\hbar} \mathbf{d} \cdot \hat{\mathbf{E}}(\mathbf{r}_a(t), t)$

● Dynamic polarizability of moving atom

$$\underline{\alpha}_{ij}(\omega; v_x) = \frac{2\omega_a}{\hbar} \mathbf{d}_i \mathbf{d}_j \left[-\omega^2 + \omega_a^2 - \frac{2\omega_a}{\hbar} \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \mathbf{d} \cdot \underline{G}(\mathbf{k}, \omega + k_x v_x) \cdot \mathbf{d} \right]^{-1}$$

● An exact, non-equilibrium fluctuation-dissipation relation

$$\underline{S}(\omega; v_x) = \frac{\hbar}{\pi} \theta(\omega) \underline{\alpha}_I(\omega; v_x) - \frac{\hbar}{\pi} \underline{J}(\omega; v_x)$$

Non-equilibrium FDT in classical models have the same form

(Chetrite et al. 2008)

$$\underline{J}(\omega; v_x) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} [\theta(\omega) - \theta(\omega + k_x v_x)] \underline{\alpha}(\omega; v_x) \cdot \underline{G}_I(\mathbf{k}, \omega + k_x v_x) \cdot \underline{\alpha}^*(\omega; v_x).$$

● Using $\underline{S}(\omega; v_x)$ one can reobtains

$$F_{\text{fric}} \approx -\frac{45\hbar}{256\pi^2 \epsilon_0} \alpha'_I(z_a, 0) \Delta'_I(0) \frac{v_x^3}{z_a^7}$$

Orders of magnitude

● Near-field quantum friction

$$F_{\text{fric}} \approx - \frac{45\hbar\rho^2\alpha_0^2}{512\pi^3} \frac{v_x^3}{z_a^{10}}$$

surface's electrical resistivity

static atomic polarizability

● Example: ground state ^{87}Rb flying over a silicon surface

$$\alpha_0 = 5.26 \times 10^{-39} \text{ Hz}/(\text{V}/\text{m})^2$$

$$\rho = 6.4 \times 10^2 \text{ } \Omega \text{ m}$$

$$v_x = 340 \text{ m/s}$$

$$z_a = 10 \text{ nm}$$

$$\Rightarrow F_{\text{fric}} \approx -1.3 \times 10^{-20} \text{ N}$$



● How to enhance it? How to measure it?

- excited atomic states?
- higher velocities?
- materials with higher resistivities?
- macroscopic bodies?
- ???
- atomic interferometry?
- near-field AFM?
- ???

Revisiting other approaches

Previous calculations using **master equations, Markovian approx, and QRT** obtained in the large-time limit $F_{\text{fric}}^{\text{QRT}} \propto v_x$ (Buhmann+Scheel 2009)

- Issues:
- Markov / QRT give the wrong large-t decay of correlators
 - Quantum friction is a low frequency phenomenon
 - Results valid only small-t limit, where FDT=QRT

Previous calculations using **perturbation theory** predicted $F_{\text{fric}}^{\text{pert}} \propto v_x$
(Barton 2010)

- Issues:
- short times, weak coupling
 - instantaneous boost of the atom

Note: This part of the work also in collaboration with Carsten Henkel (Postdam)

- Power dissipated into pairs of real plasmons excited in the surface

$$P = \lim_{t \rightarrow \infty} \frac{1}{2} \int d^2\mathbf{k} \int d^2\mathbf{k}' \int_0^\infty d\omega \int_0^\infty d\omega' \hbar(\omega + \omega') \frac{|\langle 0; \mathbf{k}\omega, \mathbf{k}'\omega' | \Psi(t) \rangle|^2}{t}$$

$$F_{\text{fric}} = P/v_x$$

- Second-order time-dependent perturbation theory for $|\Psi(t)\rangle$

atomic transition frequency \rightarrow $g(\omega) = \omega \sqrt{2\Gamma/\pi}$

$$c_{|0; \mathbf{k}\omega, \mathbf{k}'\omega'\rangle}^{(2)}(t) = -\frac{\alpha_0 \Omega^2 \omega_s^2}{8\pi} \frac{\sqrt{kk'} g_\omega g_{\omega'} e^{-(k+k')z_a}}{\sqrt{\omega\omega'} F^*(\omega) F^*(\omega')} \mathcal{M}(t)$$

$$F(\omega) = \omega_P^2 - \omega^2 - i\omega\Gamma$$

(Drude-like dielectric permittivity)

$$\mathcal{M}(t) = \int_{-\infty}^t dt' \int_{-\infty}^{t'} dt'' \left[e^{i(-\Omega+\omega)t'} e^{i(\Omega+\omega')t''} e^{-i\mathbf{k}\cdot\mathbf{r}(t')} e^{-i\mathbf{k}'\cdot\mathbf{r}(t'')} + \{\mathbf{k}\omega\} \leftrightarrow \{\mathbf{k}'\omega'\} \right]$$

Conclusions

- Atom-surface quantum friction from general non-equilibrium stat.
- Non-equilibrium FDT predicts a cubic-in- v frictional force
- Identified issues with previous approaches
 - Correct low frequency behavior missed by Markov, QRT
 - Perturbative calculations depend on boost history
- At high temperatures (classical limit), QRT = FDT, and linear-in- v friction
- Same analysis possible for quantum friction between macroscopic bodies