

## Unconditional Pointer States from Conditional Master Equations

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When part of the environment responsible for decoherence is used to extract information about the decohering system, the preferred *pointer states* remain unchanged. This conclusion—reached for a specific class of models—is investigated in a general setting of conditional master equations using suitable generalizations of predictability sieve. We also find indications that the einselected states are easiest to infer from the measurements carried out on the environment.

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*Introduction.*—Open quantum systems undergo environment-induced superselection (einselection) which leads to a preferred set of quasiclassical pointer states [1]. They entangle least with the environment—and, therefore, lose the least information. Hence, they can be found using the predictability sieve, which seeks states minimizing entropy production [2].

However, the information lost to the environment could be, in principle, intercepted and recovered. Will the preferred states remain at least approximately the same when the environment is monitored in this fashion? This is a serious concern, as decoherence is caused by the entanglement between the system  $S$  and the environment  $\mathcal{E}$ . It is well known that a pair of entangled quantum systems suffers from the basis ambiguity: One can find out about the different states of one of them (e.g.,  $S$ ) by choosing a different measurement of the other (e.g.,  $\mathcal{E}$ ) [1].

This issue was pointed out, for example, by Carmichael *et al.* [3], who used complete monitoring of the photon environment to develop a trajectory approach to quantum dynamics [4]. Reference [3] demonstrated that—when *all* of  $\mathcal{E}$  can be intercepted—any basis of  $S$  can be inferred from the appropriate measurement on  $\mathcal{E}$ , so at least in that limit substantial ambiguity is inevitable. This concern is further underscored by the realization [5] that nearly all of our information comes not from direct observation of the system, but, rather, by intercepting a small fraction of (e.g., photon) environment.

Here we use the predictability sieve in combination with the conditional master equation (CME) [6] (which is obtained when only a part of the environment—and not all of it—is traced out). We show—using specific models—that even when the additional data are taken into account, the pointer states are unchanged. We demonstrate, using fidelity, that even when all of  $\mathcal{E}$  is intercepted pointer states are unchanged. Moreover, using specific models we find indications that—for an observer who acquires the data about the system indirectly by monitoring the environment—pointer states are easiest to discover.

*An example of CME.*—The master equation for a driven two-level atom whose emitted radiation is measured by

homodyne detection [4,6] is an example of CME,

$$d\rho = d\rho^{\text{UME}}[\rho] + d\rho_{\text{st}}[\rho, N]; \quad (1)$$

$$d\rho^{\text{UME}} = -idt[\Omega\sigma_x, \rho] + dt\left(c\rho c^\dagger - \frac{1}{2}c^\dagger c\rho - \frac{1}{2}\rho c^\dagger c\right), \quad (2)$$

$$d\rho_{\text{st}} = (dN - \overline{dN}) \times \left(\frac{(c + \gamma)\rho(c^\dagger + \gamma^*)}{\text{Tr}[(c + \gamma)\rho(c^\dagger + \gamma^*)]} - \rho\right),$$

where we set the spontaneous emission time to 1. We use the Itô version of stochastic calculus.  $\rho$  is a  $2 \times 2$  density matrix of the atom,  $\Omega$  is a frequency of transitions between the excited and the ground state driven by a laser beam,  $\gamma = Re^{i\phi}$  is the amplitude of the local oscillator in the homodyne detector, and  $c = (\sigma_x - i\sigma_y)/2$  is an annihilation operator.  $N_t$  is the number of photons detected until time  $t$ . Its increment  $dN \in \{0, 1\}$  is a dichotomic stochastic process with the average

$$\overline{dN} = \eta dt \text{Tr}[\rho(c^\dagger + \gamma^*)(c + \gamma)] = \eta dt [R^2 + \langle\sigma_x\rangle R \cos\phi - \langle\sigma_y\rangle R \sin\phi + \langle c^\dagger c \rangle] \quad (3)$$

and  $\overline{dN^2} = \overline{dN}$ . The parameter  $\eta \in [0, 1]$  describes the efficiency of the measurement. A fully efficient measurement ( $\eta = 1$ ) occurs when the observer is continuously projecting the environment onto a pure state, so that an initial pure state of the system remains pure after the measurement [7]. In the fully inefficient  $\eta = 0$  case, the observer has no measurement records or ignores them completely. In this case, the equation reduces to the unconditional master equation (UME),  $d\rho = d\rho^{\text{UME}}$ .

The average over realizations  $\overline{d\rho} = d\rho^{\text{UME}}$ , because  $dN - \overline{dN} = 0$  and  $\overline{d\rho_{\text{st}}} = 0$ .  $\overline{d\rho} = d\rho^{\text{UME}}$  is not a special property of Eq. (1) and (2) but an axiomatic property of any CME. The noise average means that we ignore any knowledge about the state of environment so the state of the system cannot be conditioned by this knowledge.

For  $R = 0$  the measurement scheme is simply a photodetection:  $\overline{dN}$  is proportional to the probability that the atom is in the excited state. Every click of the photodetector ( $dN = 1$ ) brings the atom to the ground state, from where it is excited again by the laser beam. For  $R \gg 1$  the homodyne photodetector current is a linear function of  $\langle \sigma_x \rangle \equiv \text{Tr}(\rho \sigma_x)$  for  $\phi = 0$  ( $x$  measurement) or of  $\langle \sigma_y \rangle$  for  $\phi = -\pi/2$  ( $y$  measurement). These homodyne measurements drive the conditional state of the atom towards  $\sigma_x$  and  $\sigma_y$  eigenstates, respectively.

*Conditional pointer states are unconditional.*—Are the pointer states of a stochastic CME the same as pointer states of its corresponding deterministic UME? An affirmative answer requires the assumption that there is only one type of environment coupled to the system and the detector, but to nothing else, so that the detector can, in principle, be fully efficient ( $\eta = 1$ ) in continuously projecting the environment onto pure states. This assumption is standard in quantum optics [7].

According to the predictability sieve [2], pointer states minimize the increase of von Neumann entropy, or, equivalently, the decrease of purity  $P = \text{Tr}(\rho^2)$  due to the interaction with an environment. Suppose that we prepare a system in a pure state  $\rho_0 = \rho_0^2$ . The noise-averaged initial rate of purity loss is

$$\overline{dP}_0 \equiv \text{Tr}(2\rho_0 \overline{d\rho}_0) + \text{Tr}(\overline{d\rho}_0 d\rho_0). \quad (4)$$

Any CME can be written in the form of Eq. (1), where  $N_t$  would represent a general stochastic process. The stochastic process feeds the information from measurements of  $\mathcal{E}$  into the conditional state of  $S$ . The noise-averaged  $\overline{d\rho}_0 = d\rho_0^{\text{UME}}$  depends neither on the efficiency  $\eta$  nor on the kind of measurement we make on  $\mathcal{E}$ . For a deterministic UME the second term on the right-hand side of Eq. (4) would be  $O(dt^2)$ . For a stochastic CME this second term gives a contribution proportional to  $\eta dt$  which comes from  $\text{Tr}[\overline{d\rho}_{\text{st}} d\rho_{\text{st}}]$ . The manifestly positive second term reduces the rate of purity loss because a measurement of  $\mathcal{E}$  tends to purify the conditional state. For  $\eta = 1$  the observer gains full knowledge about the environmental state, the conditional state of the system remains pure all the time, and  $dP_t = 0$ . Thus we see that for  $\eta = 1$  the two terms of Eq. (4) should cancel each other. Given that the first and second terms cancel for  $\eta = 1$  and that the second term is linear in  $\eta$ , we can write the initial purity loss rate as

$$\overline{dP}_0 = (1 - \eta)\text{Tr}(2\rho_0 d\rho_0^{\text{UME}}). \quad (5)$$

Up to the prefactor of  $(1 - \eta)$  this expression is the same as the corresponding one for the UME. Except for  $\eta = 1$  we can conclude that pointer states are the same as those for the UME no matter what the efficiency is or what type of measurement is being made.

When  $\eta = 1$  we have  $\overline{dP}_0 = 0$  and no preferred pointer states can be distinguished with the predictability sieve, in accordance with [3]. However, even the conditional pure state can drift away from the free unitary evolution due

to the coupling with  $\mathcal{E}$  which is measured completely ( $\eta = 1$ ). The faster it drifts away the less predictable is the state of the system. The fidelity with respect to the initial state is defined as  $F_t = \text{Tr}(\rho_0 \rho_t^{\text{int}})$ , where the superscript “int” refers to interaction picture. For any  $\eta$  the noise-averaged initial decrease of fidelity is

$$\overline{dF}_0 \equiv \text{Tr}(\rho_0 \overline{d\rho}_0) = \text{Tr}(\rho_0 d\rho_0^{\text{UME}}). \quad (6)$$

Thus, UME pointer states maximize fidelity.

Fidelity and purity provide a basis for two physically different criteria which lead to the same unconditional pointer states [8]. UME is an average over CME's, so for linear predictability criteria the pointer states should not change. We have, however, seen that the same holds for purity, which is nonlinear.

The expressions (5) and (6) can be worked out for the example of the two-level atom master equation. For  $\Omega \ll 1$  there is one pointer state: the ground state. An atom in the ground state cannot change its state by photoemission and the external driving is slow. In the limit  $\Omega \gg 1$  the externally driven oscillations are much faster than photoemission. In fact it would be misleading to use Eq. (5) and (6) as they stand. It is more accurate to average them over one period of oscillation:  $\overline{dF}_0 \approx \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt \text{Tr}[\rho_0 c_{\text{int}}^\dagger \rho_0 c_{\text{int}} - \rho_0 c_{\text{int}}^\dagger c_{\text{int}}] = (-3 + x_0^2)/8$ . Here the density matrix is parametrized by  $\rho_t = [I + x_t \sigma_x + y_t \sigma_y + z_t \sigma_z]/2$  with  $x_t^2 + y_t^2 + z_t^2 \leq 1$ . Given the last constraint, the states with  $x = \pm 1$  (eigenstates of the self-Hamiltonian  $\Omega \sigma_x$ ) are pointer states [9]. It should be noted that  $\overline{dP}_0$  or  $\overline{dF}_0$  for, say,  $y = \pm 1$  states ( $\sigma_y$  eigenstates) is only 50% worse than for the pointers, for reasons that are specific to our small system.

*Pointer states are the easiest to find.*—Given the assumptions of the argument above, we have seen that pointer states do not depend on the kind of measurement carried out by the observer or on its efficiency. This robustness of pointer states might convey the wrong impression that all types of measurements are equivalent from the point of view of the observer trying to find out about the system by monitoring its environment. In what follows we give two examples which strongly suggest that the measurement of the environment states correlated with the pointer basis of the system is the most efficient one in gaining information about the state of the system.

We begin with the two-level atom. In the limit of  $\Omega \gg 1$ , the pointer states are eigenstates of the driving self-Hamiltonian  $\Omega \sigma_x$ . For  $\eta = 0$  the UME has a stationary mixed state  $\rho_s = I/2 + O(1/\Omega)$ . Suppose that we start monitoring the environment of the atom at  $t = 0$  [detectors are turned on at  $t = 0$ , and  $\eta(t) = \eta \theta(t)$ ]. How fast do we find out about  $S$ ? This can be measured by the purity of the conditional state. For  $\eta \ll 1$  the response of  $\rho$  to the switching-on of  $\eta$  at  $t = 0$  can be described by a small perturbation of the density matrix  $\delta\rho = [\sigma_x \delta x + \sigma_y \delta y + \sigma_z \delta z]/2$  such that  $\rho \approx \rho_s + \delta\rho$ . The evolution of  $\delta\rho$  is described by

$$d\delta x = dt(-\delta x/2) + dn\left(\frac{2R \cos\phi}{1+2R^2}\right),$$

$$d\delta y = dt(-\delta y/2 - 2\Omega\delta z) + dn\left(\frac{-2R \sin\phi}{1+2R^2}\right), \quad (7)$$

$$d\delta z = dt(-\delta z/2 - 2\Omega\delta y) + dn\left(\frac{-1}{1+2R^2}\right),$$

where  $dn = dN - \overline{dN}$  and  $\overline{dN} = \eta dt(R^2 + 1/2)$ . For  $R \gg 1$  a formal solution of these stochastic differential equations leads to a noise-averaged purity

$$\begin{aligned} \overline{P}(t) &\approx \text{Tr}(\rho_s^2) + \text{Tr}(\overline{\delta\rho}^2) \\ &= \frac{1}{2} + \frac{1}{2} \overline{[\delta x_i^2 + \delta y_i^2 + \delta z_i^2]} \\ &= \frac{1}{2} + \eta \frac{R^2 \cos^2\phi}{1+2R^2} (1 - e^{-t}) \\ &\quad + \eta \frac{1+4R^2 \sin^2\phi}{6(1+2R^2)} (1 - e^{-3t/2}). \end{aligned} \quad (8)$$

For any time  $t > 0$  the highest purity is obtained for homodyne ( $R \gg 1$ ) measurement of  $\langle\sigma_x\rangle$  ( $\phi = 0$ ). As anticipated, this is the measurement in the basis of environmental states correlated with the pointer states of the system. The purity saturates for  $t \gg 1$  at

$$P_\infty = \frac{1}{2} + \frac{\eta}{6} (3 \cos^2\phi + 2 \sin^2\phi), \quad (9)$$

for  $R \gg 1$ . The small  $\eta$  measurements in the pointer state  $x$  basis ( $\phi = 0$ ) are only 50% better than in the  $y$  basis ( $\phi = \pi/2$ ) (see Fig. 1). As mentioned before, in the two-level atom, pointer states are not well distinguished from the chaff by the predictability sieve.

To try with an example known for well-distinguished pointer states, let us pick the quantum Brownian motion at zero temperature. We can think of the environment quanta as phonons. The CME obtains from Eqs. (1) and (2) (by a formal replacement)  $c \rightarrow a$ , where  $a, a^\dagger$  are bosonic annihilation/creation operators,

$$\begin{aligned} d\rho &= dt\left(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a\right) \\ &\quad + (dN - \overline{dN})\left(\frac{(\alpha + \gamma)\rho(a^\dagger + y^*)}{\text{Tr}[(\alpha + \gamma)\rho(a^\dagger + y^*)]} - \rho\right). \end{aligned} \quad (10)$$

This equation is valid in the rotating wave approximation. After the replacement  $c \rightarrow a$  in Eq. (3), we get

$$\overline{dN} = \eta dt[R^2 + \langle a \rangle R e^{-i\phi} + \langle a^\dagger \rangle R e^{+i\phi} + \langle a^\dagger a \rangle].$$

For  $R = 0$  the measurement drives the conditional state to the ground state of the harmonic oscillator. In the homodyne limit ( $R \rightarrow \infty$ ) the phonodetector current gives information about the coherent amplitude  $\langle a \rangle$  of the state; the conditional state tends to be localized around coherent states.

For the pointer states fidelity loss  $\overline{dF}_0 = \text{Tr}[\rho_0 \times d\rho_0^{\text{UME}}] = \text{Tr}[\rho_0 a^\dagger \rho_0 a - \rho_0 a^\dagger a]$  is the least. It van-

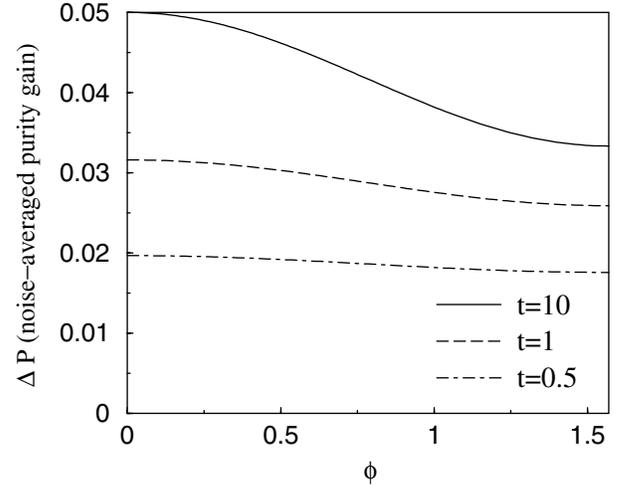


FIG. 1. Purity gain,  $\Delta P = P - 1/2$ , as a function of the homodyne phase  $\phi$  according to the formula (8) for times: 0.5, 1, and 10. The parameters were chosen as  $\eta = 0.1$ ,  $R = 100$  (homodyne limit).

ishes if  $\rho_0$  is a coherent state,  $\rho_0 = |z\rangle\langle z|$  ( $a|z\rangle = z|z\rangle$ ): Coherent states are perfect pointers. In contrast to other states such as, say, number eigenstates, initially they do not lose either purity ( $\overline{dP}_0 = 0$ ) or fidelity ( $\overline{dF}_0 = 0$ ) (but see [10]).

We expect homodyne measurement to provide more information than phonodetection. To support this, pick a coherent state  $|z\rangle$ . If  $r = |z| \gg 1$ , then  $\langle +z | -z \rangle \approx 0$  and  $a^\dagger|z\rangle \approx z^*|z\rangle$ . In this approximation, a general density matrix, in the subspace spanned by  $|\pm z\rangle$ , is

$$\begin{aligned} \rho &= \frac{1+A}{2} |+\rangle\langle +| + \frac{1-A}{2} |-\rangle\langle -| \\ &\quad + C|+\rangle\langle -| + C^*|-\rangle\langle +|. \end{aligned} \quad (11)$$

Here  $A \in [-1, +1]$ . Substitution of Eq. (11) into

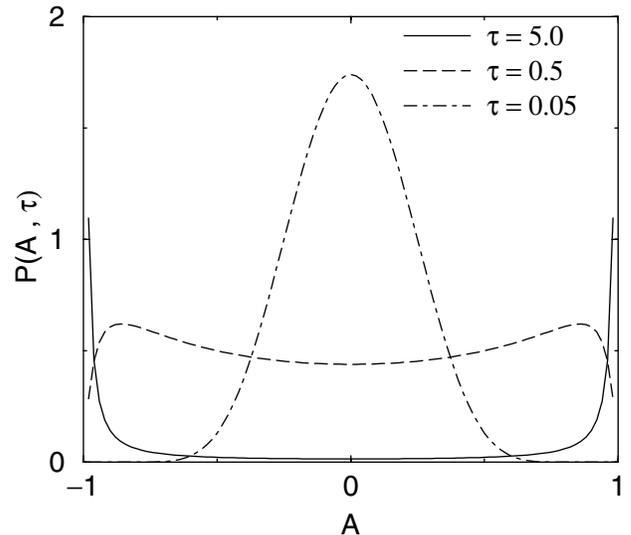


FIG. 2. The probability distribution Eq. (13) for early ( $\tau = 0.05$ ), intermediate ( $\tau = 0.5$ ), and late ( $\tau = 5$ ) times.

Eq. (10), and subsequent left and right projections on  $|\pm z\rangle$ , gives stochastic differential equations for  $A$  and  $C$ . These equations are most interesting in two limits. In the *phonodetection limit* ( $R = 0$ ) they are  $dA = 0$ ,  $dC = -C[2r^2 dt + (dN - \overline{dN})]$ . The off-diagonal  $C$  decays after the decoherence time of  $1/r^2 \ll 1$ .  $A$  does not change; phonodetection does not produce any purity. Phonodetection is a very poor choice: By this measurement we learn nothing about the system. In the opposite *homodyne detection limit* ( $R \rightarrow \infty$ ) the noise  $dN - \overline{dN}$  can be replaced (up to a constant) by a white noise  $dW$  such that  $\overline{dW} = 0$  and  $\overline{dW^2} = dt$  [11]. Again,  $C$  decays after the decoherence time of  $1/r^2$ . Introducing  $B = \tanh^{-1}(A)$ , defining a time scale  $\tau \equiv 4t\eta r^2 \cos^2(\phi - \theta)$  [here  $\theta$  is the phase of the coherent state,  $z = r \exp(i\theta)$ ], and a noise  $d\zeta \equiv 2\sqrt{\eta} r \cos(\phi - \theta)dW$  ( $d\zeta^2 = d\tau$ ), we get the following Stratonovich stochastic equation:

$$\frac{dB}{d\tau} = \tanh B + \frac{d\zeta}{d\tau}. \quad (12)$$

Suppose that at  $t = 0$  we had  $A = B = 0$  and  $C = 0$ . This is the most mixed state possible in our subspace. The probability distribution for  $A$  at time  $\tau > 0$  is

$$p(\tau, A) = \frac{(2\pi\tau)^{-1/2}}{(1 - A^2)^{3/2}} \exp\left(-\frac{\tau}{2} - \frac{\ln^2\left(\frac{1+A}{1-A}\right)}{8\tau}\right). \quad (13)$$

This distribution is localized at  $A = 0$  for  $\tau = 0$  but after a time scale  $\tau \approx 1$  it becomes concentrated at  $A = \pm 1$  (see Fig. 2). By these times the conditional state is almost certainly one of the coherent states  $|\pm z\rangle$  and purity is 1. The asymptotic bimodal distribution is obtained the fastest for a homodyne tuned to the phase of the coherent states,  $\phi = \theta$ . This result is in sharp contrast to the nil result for phonodetection. In Fig. 3 we plot three realizations of a stochastic trajectory  $A(\tau)$ .

For any  $\eta < 1$ , purity becomes 1 after a time proportional to  $1/\eta$ . A patient observer gets the full information about the system monitoring only a small part of the environment: Information about pointer states is recorded by the environment in a redundant way [12,13].

In the above example we assumed that  $r = |z| \gg 1$  so that  $\langle +z | -z \rangle \approx 0$  and  $a^\dagger |z\rangle \approx z^* |z\rangle$ . This convenient assumption also naturally separates the decoherence and purification time scales ( $\sim 1/r^2$ ) from the time scale for decay towards the ground state ( $\sim 1$ ). On the fast time scales  $\sim 1/r^2$  we can neglect the decay and that is why our system remains in the  $|\pm z\rangle$  subspace. In this sense our calculation is self-consistent.

*Concluding remarks.*—The aim of our paper was to study the issue of the preferred states in the context of conditional master equations using the predictability sieve. We have shown under reasonable, but not completely general, conditions that the most classical states of a system which is being monitored are independent both of the type of measurement and of the detector efficiency. Furthermore, we have found indications that the best measurements of

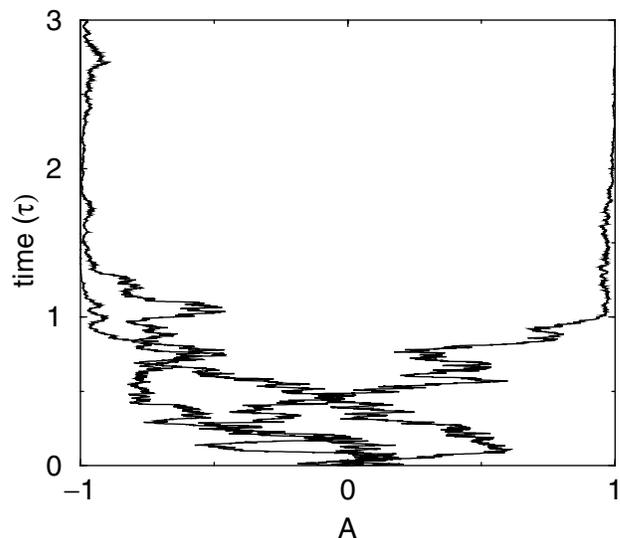


FIG. 3. Three stochastic trajectories of  $A(\tau)$  evolving according to Eq. (12). For any time  $\tau$  an average over such trajectories gives a probability distribution like that in Fig. 2. For late times the trajectories settle down at  $A = \pm 1$ ; the frequency of jumps between  $A = \pm 1$  decays like  $\exp(-\tau/2)/\sqrt{\tau}$ .

the environment for gaining information about a system extract data about its pointer basis.

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