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One-loop graviton corrections to Maxwell's equations

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We compute the graviton induced corrections to Maxwell's equations in the one-loop and weak field approximations. The corrected equations are analogous to the classical equations in anisotropic and inhomogeneous media. We analyze in particular the corrections to the dispersion relations. When the wavelength of the electromagnetic field is much smaller than a typical length scale of the graviton two-point function, the speed of light depends on the direction of propagation and on the polarization of the radiation. In the opposite case, the speed of light may also depend on the energy of the electromagnetic radiation. We study in detail wave propagation in two special backgrounds: flat Robertson-Walker and static, spherically symmetric spacetimes. In the case of a flat Robertson-Walker gravitational background we find that the corrected electromagnetic field equations correspond to an isotropic medium with a time-dependent effective refractive index. For a static, spherically symmetric background the graviton fluctuations induce a vacuum structure which causes birefringence in the propagation of light.

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I. INTRODUCTION

It is well known that when the QED vacuum is modified by external conditions such as background fields, finite temperature, or boundary conditions, the propagation of the photons can be affected in a non-trivial way. The vacuum behaves as a dispersive medium in which the propagation of light generally depends on the direction of propagation and on the polarization of the radiation. Physically, the effect can be understood as follows: the photon exists for part of the time as a virtual $e^- e^+$ pair, on which the external conditions do act and modify the propagation. In previous works the problem of photon propagation in modified QED vacua has been analyzed for external electromagnetic (EM) fields [1–4], boundary conditions [5,6], external gravitational fields [7,8], and finite temperature [9]. Further references and details can be found in Ref. [10]. There is also an experiment under construction to detect birefringence of the QED vacuum in the presence of a strong magnetic field [11].

The phenomenon is of course quite general, and not restricted to $e^- e^+$ pairs. The interaction of the photon with any other field will produce similar effects: the photon will not follow, in general, a geodesic of spacetime.[1] In this paper we will analyze the effect of the coupling between a classical electromagnetic field and a quantum gravitational field on the propagation of EM radiation waves. We will show that, indeed, the graviton loop leads to effects that are similar to those already studied and calculated for QED vacua.

Even though general relativity is a non-renormalizable theory, the one-loop corrections are meaningful when the quantized gravitational field theory is looked upon as an effective field theory [13]. It is possible to compute, for instance, the leading (long distance) quantum corrections to the Newtonian potential [14,15]. Our calculation provides another example of a quantum gravity effect that can be estimated using general relativity as a low energy effective field theory for quantum gravity. Moreover, it could also be of some interest from a phenomenological point of view. Indeed, Amelino-Camelia *et al.* [16] pointed out that many quantum gravity scenarios predict a frequency-dependent velocity of light that could be observable for (cosmological) gamma-ray bursts. Gambini and Pullin [17] studied the propagation of light in canonical quantum gravity and found that the modified Maxwell's equations imply a frequency and helicity dependent velocity of propagation. We will see that, in principle, similar results can be found by taking into account the interaction between gravitons and EM radiation in the low energy theory.

This paper is organized as follows. In Sec. II we obtain the quantum corrections to the classical Maxwell's equations induced by the graviton loop. We write the corrections in terms of the coincidence limit of the graviton two-point function. In Sec. III we present a qualitative analysis of the quantum corrections to the dispersion relations. We analyze the cases $\lambda \gg L_c$ and $\lambda \ll L_c$, where λ is the wavelength of the classical electromagnetic radiation and L_c is a typical scale

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[1] It is even possible to have superluminal propagation. However, as extensively discussed in the literature [7,10,12], this does not imply causality violations.

of variation of the graviton two-point function. We show that in general the velocity of light depends on the direction of propagation and on the polarization; i.e., we find gravitational birefringence. In Sec. IV we discuss quantitatively two examples: a flat Robertson-Walker (RW) background metric and a static metric with spherical symmetry. Section V contains our conclusions and final remarks.

II. EFFECTIVE EQUATIONS OF MOTION

Consider pure gravity described by the Einstein-Hilbert (classical) action[2]

$$S_{\rm G}=\frac{2}{\kappa^2}\int d^4x\sqrt{-g}R, \tag{1}$$

where $\kappa^2=32\pi G$ and R is the Ricci scalar. The classical action for the EM field is given by

$$S_{\rm EM}=-\frac{1}{4}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}, \tag{2}$$

where $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu$ is the field strength tensor and A_μ the gauge potential. The classical energy-momentum tensor associated with the EM field is given by

$$T^{\sigma\tau}_{\rm EM}=F^\sigma_{\ \mu}F^{\tau\mu}-\frac{1}{4}g^{\sigma\tau}F_{\mu\nu}F^{\mu\nu}. \tag{3}$$

The classical action of the EM field depends on the (classical) gravitational background. It is then natural to ask ourselves what the effects are on Maxwell's equations due to a change in the gravitational background and, in this paper particularly, what the implications are of the one-loop graviton fluctuations.

We define the classical action of the combined system (gravitational field plus classical EM radiation)

$$S_{\rm clas}=S_{\rm EM}+S_G\,. \tag{4}$$

The effect of quantum metric fluctuations can be analyzed by means of the background field method, expanding the total action $S_{\rm clas}$ around a background metric as $g_{\mu\nu}\to g_{\mu\nu}+\kappa h_{\mu\nu}$, and integrating over the graviton field ($h_{\mu\nu}$) degrees of freedom to get an effective action for the background fields $g_{\mu\nu}$ and A_μ. In order to fix the gauge one must choose a gauge-breaking term $\chi^\mu[g,h]$, with its corresponding gauge-breaking action $S_{\rm gauge}[g,h]=-(1/2)\int d^4x\sqrt{-g}\chi^\mu g_{\mu\nu}\chi^\nu$ and its corresponding ghost action $S_{\rm ghost}$ [18]. The complete effective action $S_{\rm eff}$ is obtained by integrating the full action $S\equiv S_{\rm clas}+S_{\rm gauge}+S_{\rm ghost}$ over the graviton and ghost fields. We evaluate this effective action in the one-loop approximation, for which we expand S up to second order in gravitons. The second order term can be shown to have the form [19]

$$S^{(2)}=\int d^4x\sqrt{-g}h_{\mu\nu}(O^{\mu\nu\sigma\tau}+P^{\mu\nu\sigma\tau})h_{\sigma\tau}, \tag{5}$$

where $\hat{O}\equiv O^{\mu\nu\sigma\tau}$ is a second order differential operator that depends on the background metric and is independent of the EM field [19] (we will not need its exact form in what follows), and the tensor $\hat{P}\equiv P^{\mu\nu\sigma\tau}$ arises from the expansion of $S_{\rm EM}$ to second order in gravitons, and reads

$$\begin{aligned}P^{\mu\nu\sigma\tau}=-\frac{\kappa^2}{8}\bigg[&F_{\alpha\beta}F^{\alpha\beta}\left(\frac{1}{4}g^{\mu\nu}g^{\sigma\tau}-\frac{1}{2}g^{\mu\sigma}g^{\nu\tau}\right)\\&+\frac{1}{2}F_{\alpha\beta}g^{\mu\nu}(-g^{\beta\sigma}F^{\alpha\tau}+g^{\alpha\sigma}F^{\beta\tau})\\&+\frac{1}{2}F_{\alpha\beta}g^{\sigma\tau}(-g^{\beta\mu}F^{\alpha\nu}+g^{\alpha\mu}F^{\beta\nu})\\&+2F_{\alpha\beta}(F^{\alpha\tau}g^{\beta\mu}g^{\nu\sigma}+F^{\nu\beta}g^{\alpha\sigma}g^{\mu\tau}+F^{\nu\tau}g^{\alpha\mu}g^{\beta\sigma})\bigg].\end{aligned} \tag{6}$$

There is also a second order term in the ghost fields that for gauge-breaking terms linear in the metric fluctuations decouples from the gravitons and is only coupled to the background metric. This means that the one-loop effective action for the combined system reads [19]

$$S_{\rm eff}[g_{\mu\nu},A_\mu]=S_{\rm clas}+\frac{i}{2}{\rm Tr}\ \ln(\hat{O}+\hat{P})-i\ {\rm Tr}\ln\hat{G}_{\rm ghost}, \tag{7}$$

where $\hat{G}_{\rm ghost}$ is the second order differential operator that comes from integrating over the ghost fields.

It is extremely complicated, in general, to calculate the one-loop effective action. In this paper we will use the weak field approximation, assuming that the EM field is a test field that does not affect the background metric $g_{\mu\nu}$. In this approximation the effective action takes the form

$$\begin{aligned}S_{\rm eff}=S_{\rm clas}+&\frac{i}{2}{\rm Tr}\ln\hat{O}-i\ {\rm Tr}\ln\hat{G}_{\rm ghost}\\&+\int d^4x\sqrt{-g}F_{\mu\nu}(x)F_{\sigma\tau}(x)M^{\mu\nu\sigma\tau}(x,x),\end{aligned} \tag{8}$$

where

[2] Our metric has signature $(-+++)$ and the Riemann and Ricci tensors, and the scalar curvature are defined as $R^\mu_{\ \nu\alpha\beta}=\partial_\alpha\Gamma^\mu_{\nu\beta}-\cdots$, $R_{\alpha\beta}=R^\mu_{\ \alpha\mu\beta}$, and $R=g^{\alpha\beta}R_{\alpha\beta}$, respectively. We use units such that $\hbar=c=1$.

$$M^{\mu\nu\sigma\tau}(x,x) = -\frac{\kappa^2}{16}\langle h_{\alpha\beta}(x)h_{\zeta\eta}(x)+h_{\zeta\eta}(x)h_{\alpha\beta}(x)\rangle \times\Bigg[\left(\frac{1}{8}g^{\alpha\beta}g^{\zeta\eta}-\frac{1}{4}g^{\alpha\zeta}g^{\beta\eta}\right)(g^{\sigma\mu}g^{\tau\nu}-g^{\sigma\nu}g^{\tau\mu}) + \frac{1}{2}g^{\alpha\beta}(g^{\mu\zeta}g^{\sigma\nu}g^{\tau\eta}-g^{\mu\sigma}g^{\nu\zeta}g^{\tau\eta}-g^{\sigma\eta}g^{\mu\zeta}g^{\tau\nu} + g^{\sigma\eta}g^{\mu\tau}g^{\nu\zeta})+g^{\alpha\zeta}(g^{\mu\sigma}g^{\tau\eta}g^{\nu\beta}+g^{\sigma\beta}g^{\tau\nu}g^{\mu\eta} - g^{\nu\sigma}g^{\tau\eta}g^{\mu\beta}-g^{\sigma\beta}g^{\tau\mu}g^{\nu\eta})+g^{\sigma\beta}g^{\tau\eta}g^{\mu\alpha}g^{\nu\zeta} - g^{\sigma\beta}g^{\tau\eta}g^{\nu\alpha}g^{\mu\zeta}\Bigg]. \tag{9}$$

Note that the tensor $M^{\mu\nu\sigma\tau}$ has the following symmetry properties, which are similar to the symmetries of the Riemann tensor: $M^{\mu\nu\sigma\tau}=-M^{\nu\mu\sigma\tau}$, $M^{\mu\nu\sigma\tau}=-M^{\mu\nu\tau\sigma}$, and $M^{\mu\nu\sigma\tau}=M^{\sigma\tau\mu\nu}$. In view of these properties, the only nonvanishing components of this tensor are M^{0i0j}, M^{0ijk}, and M^{ijkl}, where i,j,k,l are spatial indices. This tensor has 21 independent components and depends on the two-point function of gravitons, evaluated in an arbitrary quantum state $|\Psi\rangle$. The two-point function of gravitons can be written as follows:

$$G^{\mu\nu\sigma\tau}(x,x')\equiv\langle\Psi|h^{\mu\nu}(x)h^{\sigma\tau}(x')+h^{\sigma\tau}(x')h^{\mu\nu}(x)|\Psi\rangle, \tag{10}$$

taken at coincidence $(x=x')$. In the following we will assume that the graviton state $|\Psi\rangle$ preserves the symmetries of the background metric, so that the tensor $M^{\mu\nu\sigma\tau}$ will share those same symmetries.

The (one-loop) gravitationally modified equations of motion for the EM field, $\delta S_{\rm eff}/\delta A_\nu=0$, are given by

$$\nabla_\mu G^{\mu\nu}=0, \quad G^{\mu\nu}=F^{\mu\nu}-4M^{\mu\nu}_{\sigma\tau}F^{\sigma\tau}. \tag{11}$$

These equations are analogous to the classical Maxwell's equations in the presence of a linear, anisotropic, and nonhomogeneous media. To see this explicitly, we recall that in a local Lorentz frame $F^{0m}=E^m$ and $F^{mn}=\epsilon^{mn}_kB^k$, so that we can introduce the vectors $G^{0m}=D^m$ and $G^{mn}=\epsilon^{mn}_kH^k$, namely

$$D^j\equiv E^j-8M^{0j}_{0m}E^m-4M^{0j}_{mn}\epsilon^{mn}_kB^k, \tag{12a}$$

$$H^j\equiv B^j-2\epsilon^{jmn}M_{mnik}\epsilon^{ik}_qB^q-4\epsilon^{jki}M_{ki0m}E^m. \tag{12b}$$

The quantum corrected equations in a local Lorentz frame read

$$\nabla\cdot\mathbf{D}=0, \tag{13a}$$

$$\nabla\cdot\mathbf{B}=0, \tag{13b}$$

$$\nabla\times\mathbf{H}=\partial_t\mathbf{D}, \tag{13c}$$

$$\nabla\times\mathbf{E}=-\partial_t\mathbf{B}, \tag{13d}$$

where we have also included the equations that follow from the Bianchi identity $\nabla_\mu F_{\nu\sigma}+\nabla_\nu F_{\sigma\mu}+\nabla_\sigma F_{\mu\nu}=0$. The constitutive relations $\mathbf{D}[\mathbf{E},\mathbf{B}]$ and $\mathbf{H}[\mathbf{E},\mathbf{B}]$ are unusual when $M_{0ijk}\neq0$, since in this case *both* the electric and magnetic fields appear in the definition of **D** and **H**. However, when $M_{0ijk}=0$ the relations are exactly equivalent to those of a linear medium characterized by spacetime dependent electric permitivity and magnetic permeability tensors defined as

$$D^i=\epsilon^i_jE^j, \quad \epsilon^i_j\equiv\delta^i_j-8M^{i0}_{0j}, \tag{14a}$$

$$B^i=\mu^i_jH^j, \quad \mu^i_j\equiv\delta^i_j+2\epsilon^i_{mn}M^{mnab}\epsilon_{abj}. \tag{14b}$$

We emphasize two relevant technical points. On the one hand, the graviton two-point function will depend, in general, on the choice of the gauge-breaking term for the graviton fluctuations. However, the background metric will also depend on the gauge-breaking term through the semiclassical Einstein equation. Both dependences should cancel out in the dynamics of any test field (see Refs. [15] and [20]). We will not consider this problem in what follows; i.e., we will assume that the background metric already contains the back reaction of gravitons, computed with a given gauge-breaking term. On the other hand, the two-point function will diverge at the coincidence limit. Adequate counterterms are needed to absorb the divergences. In the spirit of effective field theories, these counterterms will not contribute in the long distance and low energy limit, which will be dominated by the non-local, non-analytic part of the two-point function [13,15].

III. QUALITATIVE ANALYSIS

Given the quantum corrected equations, we can distinguish two different physical regimes depending on the relative size of the wavelength λ of an EM radiation field described by $F^{\mu\nu}$ and the typical scale of variation of $M^{\mu\nu\sigma\tau}$, L_c.

When $\lambda\ll L_c$, we can take the tensor $M^{\mu\nu\sigma\tau}$ outside the covariant derivative in Eq. (11), as it does not change significantly over the scale λ. The equation of motion can then be written in the form

$$\nabla_\mu F^{\mu\nu}-4M^{\mu\nu}_{\sigma\tau}\nabla_\mu F^{\sigma\tau}=0. \tag{15}$$

We first introduce the variables [7]

$$F_{\mu\nu}=f_{\mu\nu}e^{i\psi}, \tag{16}$$

with $f_{\mu\nu}$ the amplitude and ψ the phase, such that $k_\mu=\nabla_\mu\psi$ is the momentum of the EM wave. We assume that the amplitude $f_{\mu\nu}$ is the slow varying variable and ψ is the fast varying variable, so that from now on we discard any gradients and/or time variations of the amplitude $f_{\mu\nu}$.

We start from Eq. (15) and make use of the new variables to write

$$k_\rho f^{\rho\nu}-4k_\rho M^{\rho\nu\sigma\tau}f_{\sigma\tau}=0. \tag{17}$$

The remaining Maxwell's equation implies the following:

$$k_\mu f_{\nu\sigma}+k_\nu f_{\sigma\mu}+k_\sigma f_{\mu\nu}=0. \tag{18}$$

We now multiply Eq. (17) by k^μ and make use of Eq. (18) to obtain

$$k^2 f^{\mu\nu}+8k^{[\mu}M^{\nu]\sigma\tau\rho}k_\sigma f_{\tau\rho}=0. \tag{19}$$

This equation is similar to the one discussed by Drummond and Hathrell in Ref. [7]. (See also Ref. [8].) In those references the corrections are due to fermion loops and are proportional to the Riemann tensor $R^{\mu\nu\rho\sigma}$ (this is true in the case of empty spacetimes, so that $R^{\mu\nu}$ and R vanish [7]), instead of $M^{\mu\nu\rho\sigma}$.

In the absence of quantum corrections one obtains the usual dispersion relation $k^2=0$. The graviton loop induces modifications to this relation; i.e., light rays do not follow null spacetime geodesics. We will analyze in detail several examples in the following sections. Here we discuss qualitatively some general properties of the modified dispersion relations.

It is easy to see that the tensor $M^{\mu\nu\rho\sigma}$ is dimensionless and proportional to the square of the Planck length $L_P\sim\kappa$. Therefore we expect $M^{\mu\nu\rho\sigma}=(L_P/L_c)^2C^{\mu\nu\rho\sigma}$, with $C^{\mu\nu\rho\sigma}$ a dimensionless, slowly varying tensor. Inserting this in Eq. (19), we see that the modified dispersion relation will have the general form

$$k^2+c_{\mu\nu}k^\mu k^\nu=0, \tag{20}$$

where $c_{\mu\nu}$ is a slowly varying tensor of order $O(L_P^2/L_c^2)$, which depends on the direction of propagation and the polarization of the EM radiation. Therefore, we expect the modifications in the speed of light to be proportional to L_P^2/L_c^2 and gravitational birefringence of the same order of magnitude.

We consider now the opposite case $\lambda\gg L_c$. For simplicity and in order to be able to compare with previous works, we assume that the background metric is flat and that the quantum state for gravitons is such that the two-point function has a random variation on micro-scales (much smaller than any other scale of the system, but still larger than L_P). In other words, we assume that the spacetime looks classical at scales larger than L_c and has a complicated random structure at scales smaller than L_c. This kind of state has been considered before in the context of loop quantum gravity [17,21].

In this situation, the quantum corrected equation (11) reads

$$\partial_\mu F^{\mu\nu}-4M^{\mu\nu}_{\sigma\tau}(\partial_\mu F^{\sigma\tau})-4(\partial_\mu M^{\mu\nu}_{\sigma\tau})F^{\sigma\tau}=0. \tag{21}$$

It is, of course, not possible to neglect the derivatives of the tensor $M_{\mu\nu\rho\sigma}$. The equation describes the propagation of a classical electromagnetic wave in a random media.

As the wavelength is much larger than L_c, in order to obtain a modified dispersion relation we average the field equation over a spacetime domain of size L^4, with $L_c\ll L\ll\lambda$. To compute the average of the products $(\partial M)F$ and $M(\partial F)$ we expand F around a point x_0 in the domain. Schematically $F(x)\approx F(x_0)+\partial F(x_0)(x-x_0)$. Denoting by $\langle\cdots\rangle$ the average over the domain and using that for a random structure $\langle M\rangle=0$ and $\langle\partial M\rangle=0$, we obtain $\langle F\partial M\rangle\approx\partial F(x_0)\langle\partial M(x-x_0)\rangle$ and $\langle M\partial F\rangle\approx\partial^2F(x_0)\langle M(x-x_0)\rangle$. Therefore, in this approximation, the average of Eq. (21) will contain higher derivatives of the EM fields [as long as $\langle M(x-x_0)\rangle\neq 0$]. As a consequence, on dimensional grounds we expect a modified dispersion relation of the form

$$k^2+c_{\mu\nu}k^\mu k^\nu+c_{\mu\nu\rho}k^\mu k^\nu k^\rho=0, \tag{22}$$

where $c_{\mu\nu}=O(L_P^2/L^2)$ and $c_{\mu\nu\rho}=O(L_P^2/L)$.

The quadratic correction modifies the speed of light as in the previous case. The cubic term is qualitatively different, since it produces a variation of the speed of light that increases linearly with the energy of the EM radiation. As already mentioned, this kind of correction may be relevant from a phenomenological point of view, because it induces a non-trivial structure in the arrival time of light rays coming from gamma-ray bursts [16,17].

IV. EXAMPLES

In the following sections we will concentrate on two particular classes of gravitational backgrounds, namely flat RW metrics and static spherically symmetric backgrounds.

A. Flat Robertson-Walker background

We first consider the case of a flat RW background, whose metric in conformal coordinates reads[3]

$$ds^2=a^2(\eta)(-d\eta^2+d\mathbf{x}^2). \tag{23}$$

Under the assumption that the graviton quantum vacuum preserves the symmetries of the background metric, we can conclude that the tensor $M^{\mu\nu\sigma\tau}$ has the same symmetries. For RW spacetimes the metric is invariant under spatial reflections (due to its homogeneity and isotropy), so that $M^{0ijk}=0$. For the remaining two non-vanishing sets of components of the tensor, M^{0i0j} and M^{ijkl}, we use the invariance of the metric under spatial rotations. This implies that they can be written in the form

$$M^{0i0j}=f_1(\eta)g^{ij}, \tag{24a}$$

$$M^{ijkl}=f_2(\eta)(g^{ik}g^{jl}-g^{il}g^{jk}), \tag{24b}$$

where $f_1(\eta)$ and $f_2(\eta)$ are functions of time. Note that the non-vanishing components of the tensor have the same form as the components of the Riemann tensor in RW spacetimes, apart from the global factors f_1 and f_2. To determine these two functions, we recall that the tensor $M_{\mu\nu\sigma\tau}$ is proportional to the two-point function of gravitons, which in RW backgrounds can be expressed in terms of the two-point function of a massless minimally coupled scalar field [22,23]. Therefore the functions $f_1(\eta)$ and $f_2(\eta)$ must be

[3]For simplicity we have considered the case of a flat RW background. Our results can be easily generalized to the closed and open RW spacetimes.

proportional to $\langle \phi^2(\eta)\rangle$. In order to calculate the constants of proportionality exactly, we need to go beyond symmetry arguments and we must face the exact evaluation of the graviton two-point function $G^{\mu\nu\rho\sigma}(x,x')$ of Eq. (10) in a RW gravitational background. The result can be found in the literature (see for example Ref. [22]). We only need to quote the final result of that reference. In the traceless ($h^\mu_\mu=0$), transverse ($\nabla_\mu h^{\mu\nu}=0$), and synchronous gauge (hence $h^{0\mu}=0$), and assuming the graviton vacuum state to be homogeneous and isotropic, the coincidence limit of the two-point function has only spatial non-vanishing components, and reads[4] [22]

$$G^{ijkl}(x,x)=2\sum_{\mathbf{k}}\ (m^i m^j m^{k*} m^{l*}+m^{i*}m^{j*}m^k m^l)|F(x,\mathbf{k})|^2. \tag{25}$$

The sum over $\mathbf{k}$ denotes a sum over a three dimensional set of spatial wave vectors. The complex (spacelike) vector $m^i(\mathbf{k})$ is defined as $m^i(\mathbf{k})=(1/\sqrt{2})[e^i_1(\mathbf{k})+ie^i_2(\mathbf{k})]$. The vectors $\mathbf{e}_1(\mathbf{k})$ and $\mathbf{e}_2(\mathbf{k})$ are spacelike vectors, such that the set $\{\mathbf{e}_1(\mathbf{k}),\mathbf{e}_2(\mathbf{k}),\hat{\mathbf{k}}\}$ forms an orthonormal basis in the three dimensional spacelike hypersections [here $\hat{\mathbf{k}}=\mathbf{k}/k$ and $k=(\mathbf{k}\cdot\mathbf{k})^{1/2}$]. The mode function F is given by $F(x,\mathbf{k})=F(\eta,\mathbf{x},\mathbf{k})\equiv f(\eta,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}/\sqrt{32\pi^3 kV}$, where V is a constant comoving volume.[5] The mode functions $F(x,\mathbf{k})$ and $f(\mathbf{k},\eta)$ are a solution to the equations

$$\Box F(x,\mathbf{k})=0, \tag{26a}$$

$$\ddot{f}(\eta,\mathbf{k})+\frac{2\dot{a}}{a}\dot{f}(\eta,\mathbf{k})+k^2 f(\eta,\mathbf{k})=0, \tag{26b}$$

respectively, i.e., correspond to the dynamical equation of a massless minimally coupled scalar field in a RW background. Here the overdot denotes derivation with respect to the conformal time variable η.

The two-point function, given in Eq. (25), can be simplified by making use of the identity $e^i_1 e^j_1+e^i_2 e^j_2+k^{-2}k^i k^j=g^{ij}$. We can then perform the sum over momenta as $|F(x,\mathbf{k})|^2$ depends only on the modulus of $\mathbf{k}$. The final result reads

$$G^{ijkl}(x,x)=\frac{4}{15}\left(\frac{3}{2}g^{ik}g^{jl}+\frac{3}{2}g^{il}g^{jk}-g^{ij}g^{kl}\right)\langle\phi^2(\eta)\rangle, \tag{27}$$

where $\langle\phi^2(\eta)\rangle\equiv\Sigma_{\mathbf{k}}|F(x,\mathbf{k})|^2$ is the coincidence limit of the two-point function of a massless minimally coupled scalar field in a RW background. We can now insert this formula in Eq. (9) in order to read off the expressions for $f_1(\eta)$ and $f_2(\eta)$ in Eqs. (24a) and (24b). This procedure yields

$$f_1(\eta)=\frac{\kappa^2}{48a^2(\eta)}\langle\phi^2(\eta)\rangle,$$

$$f_2(\eta)=-\frac{\kappa^2}{16}\langle\phi^2(\eta)\rangle. \tag{28}$$

The quantum correction to the classical EM action due to the coupling with the graviton degrees of freedom can now be obtained from Eq. (8) by making use of the above results. We get

$$\langle S^{(2)}_{\mathrm{EM}}\rangle=\int d^4x\sqrt{-g}[-4a^2(\eta)f_1(\eta)F_{0i}F^{0i}+2f_2(\eta)F_{ij}F^{ij}]. \tag{29}$$

Note that the one-loop effective action for the electromagnetic field is $S_{\mathrm{EM}}+\langle S^{(2)}_{\mathrm{EM}}\rangle$, which is a divergent quantity and has to be suitably renormalized. This is accomplished by the renormalization of $\langle\phi^2(\eta)\rangle$, for example, by means of adiabatic regularization [24].

Having the EM effective action and assuming that one first solves the pure gravity part in order to get the corrected background metric after graviton back reaction [that is, we assume one solves the pure gravitational part and gets the new scale factor $a(\eta)$], one can get the corrected Maxwell's equations from the variation of the effective action, namely, $\delta S_{\mathrm{eff}}/\delta A_\mu=0$. We will assume that $f_1(\eta)$ and $f_2(\eta)$ have a time variation much slower than that associated with the EM field, so that we can approximate the equations of motion as in Eq. (15). The source-free equations are the usual ones

$$\frac{1}{a(\eta)}\nabla\cdot\mathbf{B}=0, \tag{30a}$$

$$\frac{1}{a(\eta)}\dot{\mathbf{B}}+\frac{2\dot{a}(\eta)}{a(\eta)}\mathbf{B}=-\frac{1}{a(\eta)}\nabla\times\mathbf{E}, \tag{30b}$$

where the overdot denotes $\partial/\partial\eta$, and ∇ is vector notation for $\partial/\partial\mathbf{x}$.

The other two equations read

$$\frac{1}{a(\eta)}\epsilon_{\mathrm{eff}}(\eta)\nabla\cdot\mathbf{E}=0, \tag{31a}$$

$$\frac{1}{a(\eta)}\epsilon_{\mathrm{eff}}(\eta)\dot{\mathbf{E}}+\frac{2\dot{a}(\eta)}{a(\eta)}\epsilon_{\mathrm{eff}}(\eta)\mathbf{E}=\frac{1}{a(\eta)}\mu^{-1}_{\mathrm{eff}}(\eta)\nabla\times\mathbf{B}. \tag{31b}$$

In the first term on the left hand side of the last equation we have discarded a contribution proportional to the time derivative of the effective electric permittivity $\epsilon_{\mathrm{eff}}(\eta)$ since, as

[4]The choice of vacuum corresponds to that used in Ref. [22]. This graviton vacuum is homogeneous, isotropic, and the same for the two helicity states of the gravitons $(+2,-2)$.

[5]The normalization of the mode function F differs from that of Allen [22] in a κ^{-1} factor. The reason is that we have defined the graviton fluctuations via $g_{\mu\nu}\to g_{\mu\nu}+\kappa h_{\mu\nu}$ and our graviton two-point function [see Eq. (10)] is given in terms of this $h^{\mu\nu}$, whereas Allen has $g_{\mu\nu}\to g_{\mu\nu}+\tilde{h}_{\mu\nu}$, and defines the graviton two-point function in terms of $\tilde{h}_{\mu\nu}$.

already discussed in Eq. (15), we are assuming that it does not change significantly over the wavelength of the EM field. The effective electric permittivity and magnetic permeability tensors in RW backgrounds are proportional to the identity (3×3) matrix, namely, $\epsilon^i_j=\epsilon_{\rm eff}(\eta)\delta^i_j$ and $\mu^i_j=\mu_{\rm eff}(\eta)\delta^i_j$, with

$$\epsilon_{\rm eff}(\eta)\equiv 1+8a^2(\eta)f_1(\eta), \tag{32a}$$

$$\mu_{\rm eff}(\eta)\equiv 1+8f_2(\eta). \tag{32b}$$

Hence the presence of gravitons introduces a time dependent effective refraction index $n_{\rm eff}(\eta)=\sqrt{\epsilon_{\rm eff}(\eta)\mu_{\rm eff}(\eta)}$ for a traveling EM wave, and therefore implies a time dependent speed of light in the medium[6]

$$v_{\rm eff}(\eta)=1-4[a^2(\eta)f_1(\eta)+f_2(\eta)]=1+\frac{\kappa^2}{6}\langle\phi^2(\eta)\rangle. \tag{33}$$

This effective speed of light is the same for all directions of propagation and for all polarizations of the EM radiation field, in agreement with the isotropy and homogeneity of RW spacetimes. In Appendix A we give an alternative derivation of this result based on a direct study of the dispersion relation for light in the graviton modified medium.

The renormalized two-point function $\langle\phi^2(\eta)\rangle$ does not have a definite sign (see, for example, Refs. [19,25]). The effective speed of light in the graviton vacuum can be greater or smaller than that in free space, depending on the particular form of the scaling parameter $a(\eta)$. In any case, the correction is extremely small, typically proportional to the ratio of the spacetime scalar curvature and Planck's curvature, $R/R_{\rm P}$. As we go back in time towards the big bang singularity, the modulus of the correction to the phase velocity increases. Of course, we cannot trust this calculation for such early times since the correction would become too large and since general relativity would not be valid as an effective low energy and large distance theory in that regime.

It is worth mentioning that similar results are obtained due to QED vacuum polarization [7]. The QED effects are generically much larger than the graviton corrections. However, there are situations in which the QED correction vanishes, while the graviton correction does not. To show an explicit example, let us consider de Sitter spacetime. Virtual e^-e^+ pairs modify Maxwell's equations as follows [7]:

$$\left(1+\frac{7\alpha R}{1080\pi m^2}\right)D_\mu F^{\mu\nu}=0, \tag{34}$$

where m is the mass of the electron and α the fine structure constant. The corrected equations coincide with Maxwell's equations up to a trivial normalization and the dispersion relation is the classical one. However, graviton vacuum corrections in de Sitter spacetime do affect the propagation of individual light rays.

B. Static and spherically symmetric backgrounds

In this section we consider a static and spherically symmetric spacetime described by the metric

$$ds^2=-A(r)dt^2+B(r)dr^2+r^2d\Omega^2. \tag{35}$$

In order to compute the tensor $M^{\mu\nu\sigma\tau}$ one needs to calculate the graviton two-point function, evaluated in an arbitrary quantum state. It would be a rather formidable task to explicitly calculate such an object. Instead, we will use symmetry arguments and assume that the graviton quantum state preserves the symmetries of the background metric to derive the basic structure of the tensor $M^{\mu\nu\sigma\tau}$. It is shown in Appendix B that the tensor $M^{\mu\nu\sigma\tau}$ can be written as

$$\begin{aligned}M^{\mu\nu\sigma\tau}(r)=&f_1(r)U^{\mu\nu}U^{\sigma\tau}+f_2(r)V^{\mu\nu}V^{\sigma\tau}\\&+f_3(r)(X^{\mu\nu}X^{\sigma\tau}+Y^{\mu\nu}Y^{\sigma\tau})\\&+f_4(r)(W^{\mu\nu}W^{\sigma\tau}+Z^{\mu\nu}Z^{\sigma\tau}),\end{aligned} \tag{36}$$

where the functions $f_i(r)$ depend on the particular choice of vacuum. The antisymmetric order two tensors $U^{\mu\nu},V^{\mu\nu},W^{\mu\nu},X^{\mu\nu},Y^{\mu\nu}$, and $Z^{\mu\nu}$ are defined in Appendix B. Just as in the case of RW backgrounds, the only nonvanishing components are M^{0i0j} and M^{ijkl}, with i,j,k,l spatial indices. In general, the structure of the tensor $M^{\mu\nu\sigma\rho}$ in Eq. (36) is much more complicated than that for the Riemann tensor corresponding to the metric (35). However, this form for the tensor $M^{\mu\nu\sigma\tau}$ is good enough to carry an analysis parallel to that of Drummond and Hathrell [7].

Starting from Eq. (19), which describes the propagation of an EM wave in the presence of gravitons, we show in Appendix B that it has non-trivial solutions only when the wave momentum k_μ satisfies the following determinantal condition:

$$\begin{aligned}&k^2[(1+8f_3)k^2-8(f_3+f_4)k_r^2-8(f_2+f_3)(k_\theta^2+k_\phi^2)]\\&\times[(1+8f_3)(1+8f_1)k^2-8(f_3+f_4)(1+8f_1)k_r^2\\&-8(f_1+f_4)(1+8f_3)(k_\theta^2+k_\phi^2)]=0.\end{aligned} \tag{37}$$

The solution $k^2=0$ corresponds to the usual dispersion relation, in which the light ray follows the null geodesics of the background metric. Apart from this (trivial) case, the previous equation admits new dispersion relations, depending on the direction of propagation and polarization of the EM radiation field.

When the light ray moves radially ($k_\theta=k_\phi=0$) the determinantal condition [see Eq. (37)] has two possible solutions

$$(1+8f_3)(-k_t^2+k_r^2)-8(f_3+f_4)k_r^2=0, \tag{38a}$$

$$(1+8f_3)(1+8f_1)(-k_t^2+k_r^2)-8(f_3+f_4)(1+8f_1)k_r^2=0. \tag{38b}$$

If we assume that $(1+8f_1)\neq0$, the two dispersion relations that follow from the above equations are the same, which

[6]This velocity corresponds to the phase velocity $v_{\rm phase}=c/n_{\rm eff}$.

agrees with the fact that as the gravitational background is spherically symmetric, a radial EM wave should not be affected by birefringence. We can conclude that for radial light rays the absolute value of the quantum corrected velocity is given by

$$\left|\frac{k_t}{k_r}\right| = 1 - 4(f_4 + f_3). \tag{39}$$

When the EM wave moves transversally ($k_r = k_\theta = 0$), we get the following two possible solutions (for $1 + 8f_3 \neq 0$):

$$-(1+8f_3)k_t^2 + (1-8f_2)k_\phi^2 = 0, \tag{40a}$$

$$-(1+8f_1)k_t^2 + (1-8f_4)k_\phi^2 = 0. \tag{40b}$$

As opposed to the previous case, the two dispersion relations that follow from these two equations are different. Light rays propagate with different velocities

$$\left|\frac{k_t}{k_\phi}\right| = 1 - 4(f_2 + f_3), \tag{41a}$$

$$\left|\frac{k_t}{k_\phi}\right| = 1 - 4(f_1 + f_4), \tag{41b}$$

depending on their polarization. Similarly, given the symmetry under the exchange of ϕ to θ, azimuthal moving EM waves ($k_r = k_\phi = 0$) have the same dispersion relations as transverse light rays. Both for traverse and azimuthal propagation we obtain gravitational birefringence due to graviton vacuum fluctuations. Similar results are obtained from corrections due to QED vacuum effects.

It is worth mentioning that the previous results are applicable to any static and spherically symmetric background. We will now consider two particular examples, the Schwarzschild background and the Reissner-Nordström background.[7] The Schwarzschild spacetime is described by the metric

$$\mathrm{d}s^2 = -\left(1 - \frac{2MG}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2MG}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \tag{42}$$

In this case, using dimensional analysis, we can write the functions $f_i(r)$ $(i = 1, \ldots, 4)$ as $f_i(r) = (M_{\rm P}/M)^2 \mathcal{F}_i(r/2MG)$ with $\mathcal{F}_i$ four dimensionless functions and $M_{\rm P}$ the Planck mass. As we have already mentioned, the exact form of these functions is unknown, since it is not possible to calculate explicitly the graviton two-point function for the Schwarzschild background. However, near the horizon $r \approx 2MG$, we can approximate

$$M^{\mu\nu\sigma\tau}(r \approx 2MG) \approx \left(\frac{M_P}{M}\right)^2 [\mathcal{F}_1(1)U^{\mu\nu}U^{\sigma\tau} + \mathcal{F}_2(1)V^{\mu\nu}V^{\sigma\tau} + \mathcal{F}_3(1)(X^{\mu\nu}X^{\sigma\tau} + Y^{\mu\nu}Y^{\sigma\tau}) + \mathcal{F}_4(1)(W^{\mu\nu}W^{\sigma\tau} + Z^{\mu\nu}Z^{\sigma\tau})]. \tag{43}$$

Hence, near the horizon the birefringence effects induced by gravitons are of order $O(M_P^2/M^2)$.

The Reissner-Nordström spacetime describes the metric of a static and charged black hole:

$$\mathrm{d}s^2 = -\left(1 - \frac{2MG}{r} + \frac{Q^2 G}{4\pi r^2}\right)\mathrm{d}t^2 + \left(1 - \frac{2MG}{r} + \frac{Q^2 G}{4\pi r^2}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \tag{44}$$

This spacetime has in general two event horizons $r_\pm$. The exterior one at $r_+ = MG[1 + (1 - Q^2/4\pi M^2)^{1/2}]$ coincides with that of the Schwarzschild metric in the limit $Q^2 \ll M^2$. We can carry out an analysis similar to the previous one, and write $f_i(r) = (M_P/M)^2 \mathcal{G}_i(r/2MG, r/Q\sqrt{G})$ with $\mathcal{G}_i$ four new dimensionless functions. Near the exterior event horizon ($r = r_+$) we can approximate

$$M^{\mu\nu\sigma\tau}(r \approx r_+) \approx \left(\frac{M_P}{M}\right)^2 \Bigg[\mathcal{G}_1\left(\frac{r_+}{2MG}, \frac{r_+}{Q\sqrt{G}}\right)U^{\mu\nu}U^{\sigma\tau} + \mathcal{G}_2\left(\frac{r_+}{2MG}, \frac{r_+}{Q\sqrt{G}}\right)V^{\mu\nu}V^{\sigma\tau} + \mathcal{G}_3\left(\frac{r_+}{2MG}, \frac{r_+}{Q\sqrt{G}}\right)(X^{\mu\nu}X^{\sigma\tau} + Y^{\mu\nu}Y^{\sigma\tau}) + \mathcal{G}_4\left(\frac{r_+}{2MG}, \frac{r_+}{Q\sqrt{G}}\right) \times (W^{\mu\nu}W^{\sigma\tau} + Z^{\mu\nu}Z^{\sigma\tau})\Bigg]. \tag{45}$$

Hence, near the outer event horizon the birefringence effects induced by one-loop gravitons are again of order $O(M_P^2/M^2)$.

V. CONCLUSIONS

In this paper we have computed the quantum corrections to (classical) Maxwell's equations due to the interaction between the EM field and the fluctuations of the spacetime metric. The modified equations look like Maxwell's equations in the presence of a linear medium, with electric permittivity and magnetic permeability tensors proportional to the coincidence limit of the graviton two-point function. From the modified equations we have found the quantum corrections to the dispersion relations. In general, as for the

[7] As we already mentioned at the end of Sec. II, the background metric should be a solution of the semi-classical Einstein equation. This is not the case for the particular examples we will be considering in this section. We include them only for illustrative purposes.

case of linear media in classical electrodynamics, the speed of light depends on the direction of propagation and on the state of polarization. The quantum corrections we have computed should be considered along with the graviton back reaction on the background metric (this is crucial, for example, for the cancellation of the gauge-breaking dependence; see Refs. [15,19]). Throughout the paper we have assumed that the metric does contain such (back reaction) corrections, and have focused on the interaction of the gravitons with the electromagnetic field.

We have shown that, when the EM field wavelength λ is small compared to the typical scale L_c of variation of the permittivity and permeability tensors, the corrections to the speed of light are proportional to $(L_P/L_c)^2$. We have described in detail the quantum corrections in both RW gravitational backgrounds and static spherically symmetric spacetimes.

For RW spacetimes we have computed the quantum corrections by two different methods: the analysis of the modified Maxwell's equations in a coordinate basis and the study of the dispersion relations in a local Lorentz frame. The corrections we found are similar to those of Ref. [19], where the analysis was based on the effect of gravitons on the spacetime null geodesics. The results agree qualitatively but not quantitatively (as is to be expected), since the coupling of gravitons to point particles is in general different to the coupling to massless fields.

For spherically symmetric spacetimes we have been able to estimate the quantum corrections using symmetry and dimensional arguments (see Appendix B), avoiding the explicit computation of the graviton two-point function.

The quantum corrections for small wavelengths are also qualitatively similar to those produced by virtual e^-e^+ in non-trivial backgrounds. However, for some particular cases (such as the de Sitter background), the QED vacuum does not affect the propagation of photons, whereas the graviton vacuum does induce a modification on the propagation of light rays.

The opposite limit, $\lambda \gg L_c$, is more interesting from a phenomenological point of view. Assuming a non-trivial and random spacetime structure at scales of order L_c, the modified field equations are similar to the ones describing the propagation of classical waves in random media. To lowest order, it is possible to describe the wave propagation in terms of an "effective medium." The average corrections to the speed of light are independent of the wavelength and proportional to $(L_P/L_c)^2$. The "effective medium" is only an averaged description. Stochastic fluctuations are expected to occur, for example, in the arrival time of photons coming from point sources [26,27]. To next order, the corrections are proportional to $L_P^2 E/(L_c\lambda)$, where E is the photon energy. In the extreme case $L_P \sim L_c$ (which we cannot reach within our effective field theory approach) the corrections would be proportional to E/E_P, where E_P is the Planck energy. In this regime quantum gravity effects increase with energy, while other medium effects should decrease with energy. They can be distinguished by this property and could be relevant in cosmological situations [16].

In the effective field theory approach to quantum gravity one also expects classical, stochastic fluctuations of the spacetime metric. Its dynamics should be described by a "semi-classical Einstein-Langevin equation" [28]. These classical fluctuations will also affect the propagation of photons (see for example [29]).

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APPENDIX A: STRUCTURE OF THE TENSOR *M* FOR HOMOGENEOUS AND ISOTROPIC BACKGROUNDS

We consider a flat homogeneous and isotropic spacetime described by the metric (conformal coordinates)

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2). \tag{A1}$$

In order to compute the tensor $M^{\mu\nu\sigma\tau}$ one needs to calculate the graviton two-point function, evaluated in an arbitrary quantum state. We will assume that the graviton state preserves the symmetries of the background metric. The aim of this appendix is to use those symmetries to find the structure of the tensor $M^{\mu\nu\sigma\tau}$ given in Eq. (9). In order to obtain this structure we choose an orthonormal basis of the spacetime under consideration. The orthonormal basis is defined by the vector fields

$$\mathbf{e}_\eta = \frac{1}{a(\eta)} \frac{\partial}{\partial \eta}, \tag{A2a}$$

$$\mathbf{e}_x = \frac{1}{a(\eta)} \frac{\partial}{\partial x}, \tag{A2b}$$

$$\mathbf{e}_y = \frac{1}{a(\eta)} \frac{\partial}{\partial y}, \tag{A2c}$$

$$\mathbf{e}_z = \frac{1}{a(\eta)} \frac{\partial}{\partial z}. \tag{A2d}$$

We also introduce the following set of antisymmetric tensors:

$$U_x^{\mu\nu} = \mathbf{e}_\eta^\mu \mathbf{e}_x^\nu - \mathbf{e}_\eta^\nu \mathbf{e}_x^\mu, \tag{A3a}$$

$$U_y^{\mu\nu} = \mathbf{e}_\eta^\mu \mathbf{e}_y^\nu - \mathbf{e}_\eta^\nu \mathbf{e}_y^\mu, \tag{A3b}$$

$$U_z^{\mu\nu} = \mathbf{e}_\eta^\mu \mathbf{e}_z^\nu - \mathbf{e}_\eta^\nu \mathbf{e}_z^\mu, \tag{A3c}$$

$$V_x^{\mu\nu} = \mathbf{e}_y^\mu \mathbf{e}_z^\nu - \mathbf{e}_y^\nu \mathbf{e}_z^\mu, \tag{A3d}$$

$$V_y^{\mu\nu} = \mathbf{e}_z^\mu \mathbf{e}_x^\nu - \mathbf{e}_z^\nu \mathbf{e}_x^\mu, \tag{A3e}$$

$$V_z^{\mu\nu} = \mathbf{e}_x^\mu \mathbf{e}_y^\nu - \mathbf{e}_x^\nu \mathbf{e}_y^\mu. \tag{A3f}$$

Given the fact that the **e**'s form an orthonormal basis, this set of six tensors constitutes a basis for the antisymmetric order two tensors.

It is easy to see that the symmetries of the tensor $M^{\mu\nu\sigma\tau}$ (antisymmetric in the first two indices, and the last two, and symmetric under the exchange of the two pair of indices) imply that the tensor must be a linear combination of the following kind (with coefficients that may depend only on η):

$$
\begin{aligned}
M^{\mu\nu\sigma\tau}= & a_1(\eta)U_x^{\mu\nu}U_x^{\sigma\tau}+a_2(\eta)(U_x^{\mu\nu}U_y^{\sigma\tau}+U_y^{\mu\nu}U_x^{\sigma\tau})+a_3(\eta)(U_x^{\mu\nu}U_z^{\sigma\tau}+U_z^{\mu\nu}U_x^{\sigma\tau})\\
&+a_4(\eta)(U_x^{\mu\nu}V_x^{\sigma\tau}+V_x^{\mu\nu}U_x^{\sigma\tau})+a_5(\eta)(U_x^{\mu\nu}V_y^{\sigma\tau}+V_y^{\mu\nu}U_x^{\sigma\tau})+a_6(\eta)(U_x^{\mu\nu}V_z^{\sigma\tau}+V_z^{\mu\nu}U_x^{\sigma\tau})\\
&+a_7(\eta)U_y^{\mu\nu}U_y^{\sigma\tau}+a_8(\eta)(U_y^{\mu\nu}U_z^{\sigma\tau}+U_z^{\mu\nu}U_y^{\sigma\tau})+a_9(\eta)(U_y^{\mu\nu}V_x^{\sigma\tau}+V_x^{\mu\nu}U_y^{\sigma\tau})+a_{10}(\eta)(U_y^{\mu\nu}V_y^{\sigma\tau}+V_y^{\mu\nu}U_y^{\sigma\tau})\\
&+a_{11}(\eta)(U_y^{\mu\nu}V_z^{\sigma\tau}+V_z^{\mu\nu}U_y^{\sigma\tau})+a_{12}(\eta)U_z^{\mu\nu}U_z^{\sigma\tau}+a_{13}(\eta)(U_z^{\mu\nu}V_x^{\sigma\tau}+V_x^{\mu\nu}U_z^{\sigma\tau})\\
&+a_{14}(\eta)(U_z^{\mu\nu}V_y^{\sigma\tau}+V_y^{\mu\nu}U_z^{\sigma\tau})+a_{15}(\eta)(U_z^{\mu\nu}V_z^{\sigma\tau}+V_z^{\mu\nu}U_z^{\sigma\tau})+a_{16}(\eta)V_x^{\mu\nu}V_x^{\sigma\tau}\\
&+a_{17}(\eta)(V_x^{\mu\nu}V_y^{\sigma\tau}+V_y^{\mu\nu}V_x^{\sigma\tau})+a_{18}(\eta)(V_x^{\mu\nu}V_z^{\sigma\tau}+V_z^{\mu\nu}V_x^{\sigma\tau})\\
&+a_{19}(\eta)V_y^{\mu\nu}V_y^{\sigma\tau}+a_{20}(\eta)(V_y^{\mu\nu}V_z^{\sigma\tau}+V_z^{\mu\nu}V_y^{\sigma\tau})+a_{21}(\eta)V_z^{\mu\nu}V_z^{\sigma\tau}. \qquad \text{(A4)}
\end{aligned}
$$

Since the metric is homogeneous and isotropic, the functions a_2, a_3, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{17}, a_{18}, and a_{20} must vanish identically. Here we should stress once again the fact that the quantum state of the gravitons does not break these symmetries. The background symmetries also imply that $a_1=a_7=a_{12}\equiv\alpha$ and $a_{16}=a_{19}=a_{21}\equiv\beta$.

After these symmetry considerations, we can write the general expression of the tensor $M^{\mu\nu\sigma\tau}$ for a homogeneous and isotropic background:

$$
\begin{aligned}
M^{\mu\nu\sigma\tau}= & \alpha(\eta)(U_x^{\mu\nu}U_x^{\sigma\tau}+U_y^{\mu\nu}U_y^{\sigma\tau}+U_z^{\mu\nu}U_z^{\sigma\tau})\\
&+\beta(\eta)(V_x^{\mu\nu}V_x^{\sigma\tau}+V_y^{\mu\nu}V_y^{\sigma\tau}+V_z^{\mu\nu}V_z^{\sigma\tau}). \qquad \text{(A5)}
\end{aligned}
$$

This is all the information we can obtain regarding the tensor structure of $M^{\mu\nu\sigma\tau}$ by making use of the symmetries of the gravitational background. The functions $\alpha(\eta)$ and $\beta(\eta)$ will depend on the particular choice of vacuum and are not known *a priori*.

We now make use of the newly obtained form for $M^{\mu\nu\sigma\tau}$ [see Eq. (A5)] to solve the equation of motion for the EM field in the presence of graviton one-loop quantum fluctuations in flat homogeneous and isotropic backgrounds [see Eq. (19)]. Following the very same steps as described in Ref. [7], we introduce the six (dependent) functions

$$
\begin{aligned}
u_x&=f_{\mu\nu}U_x^{\mu\nu}, \quad v_x=f_{\mu\nu}V_x^{\mu\nu},\\
u_y&=f_{\mu\nu}U_y^{\mu\nu}, \quad v_y=f_{\mu\nu}V_y^{\mu\nu},\\
u_z&=f_{\mu\nu}U_z^{\mu\nu}, \quad v_z=f_{\mu\nu}V_z^{\mu\nu}. \qquad \text{(A6)}
\end{aligned}
$$

From Eq. (18) it follows that

$$f_{\mu\nu}=k_\mu a_\nu-k_\nu a_\mu, \qquad \text{(A7)}$$

for some gauge potential a_μ. Hence, for a given EM wave momentum k_μ, $f_{\mu\nu}$ has three independent components (one amplitude and two polarizations), as we still have the choice of gauge for the EM field. Since $f_{\mu\nu}$ is gauge invariant, without loss of generality we consider the Coulomb gauge and choose $a_\eta=0$. With this choice the three non-vanishing components for the gauge potential are a_x, a_y, and a_z, so that

$$
\begin{aligned}
f_{\eta x}&=k_\eta a_x, \quad f_{xy}=k_x a_y-k_y a_x,\\
f_{\eta y}&=k_\eta a_y, \quad f_{yz}=k_y a_z-k_z a_y,\\
f_{\eta z}&=k_\eta a_z, \quad f_{zx}=k_z a_x-k_x a_z, \qquad \text{(A8)}
\end{aligned}
$$

and we can then write

$$
\begin{aligned}
u_x&=2f_{\eta x}, \quad v_x=2f_{yz},\\
u_y&=2f_{\eta y}, \quad v_y=2f_{zx},\\
u_z&=2f_{\eta z}, \quad v_z=2f_{xy}, \qquad \text{(A9)}
\end{aligned}
$$

so that in terms of the independent set $\{u_x,u_y,u_z\}$ the three dependent ones can be written as

$$v_x=\frac{1}{k_\eta}(k_y u_z-k_z u_y), \qquad \text{(A10a)}$$

$$v_y=\frac{1}{k_\eta}(k_z u_x-k_x u_z), \qquad \text{(A10b)}$$

$$v_z=\frac{1}{k_\eta}(k_x u_y-k_y u_x). \qquad \text{(A10c)}$$

With all these definitions in mind we project Eq. (19) onto the three tensors U_x, U_y, and U_z (which yield u_x, u_y, and u_z, respectively), to obtain the following set of equations:

$$0 = k^2 u_x + 8\alpha[u_x(-k_\eta^2 + k_x^2) + k_x(k_y u_y + k_z u_z)] + 8\beta k_\eta(k_y v_z - k_z v_y), \tag{A11a}$$

$$0 = k^2 u_y + 8\alpha[u_y(-k_\eta^2 + k_y^2) + k_y(k_x u_x + k_z u_z)] + 8\beta k_\eta(k_z v_x - k_x v_z), \tag{A11b}$$

$$0 = k^2 u_z + 8\alpha[u_z(-k_\eta^2 + k_z^2) + k_z(k_x u_x + k_y u_y)] + 8\beta k_\eta(k_x v_y - k_y v_x). \tag{A11c}$$

We point out that the components of the vector k correspond to the orthonormal basis given in Eqs. (A2), so that $k = k^\eta \mathbf{e}_\eta + k^x \mathbf{e}_x + k^y \mathbf{e}_y + k^z \mathbf{e}_z$. If we write the components (v_x, v_y, v_z) as functions of (u_x, u_y, u_z), we have

$$0 = k^2 u_x + 8\alpha[u_x(-k_\eta^2 + k_x^2) + k_x(k_y u_y + k_z u_z)] + 8\beta[k_y(k_x u_y - k_y u_x) - k_z(k_z u_x - k_x u_z)], \tag{A12a}$$

$$0 = k^2 u_y + 8\alpha[u_y(-k_\eta^2 + k_y^2) + k_y(k_x u_x + k_z u_z)] + 8\beta[k_z(k_y u_z - k_z u_y) - k_x(k_x u_y - k_y u_x)], \tag{A12b}$$

$$0 = k^2 u_z + 8\alpha[u_z(-k_\eta^2 + k_z^2) + k_z(k_x u_x + k_y u_y)] + 8\beta[k_x(k_z u_x - k_x u_z) - k_y(k_y u_z - k_z u_y)]. \tag{A12c}$$

Imposing the condition that the determinant of this set of equations vanish, so that we do not obtain the trivial solution $(u_x = u_y = u_z = 0)$, we obtain the following determinantal restriction:

$$k^2(1+8\alpha)[(1+8\alpha)k^2 - 8(\alpha+\beta)(k_x^2 + k_y^2 + k_z^2)]^2 = 0. \tag{A13}$$

Let us assume that the EM radiation is characterized by a three-dimensional momentum $\mathbf{k}$, such that $k_x^2 + k_y^2 + k_z^2 = \mathbf{k}\cdot\mathbf{k}$. The non-trivial dispersion relation becomes then

$$(1+8\alpha)(-k_\eta^2 + \mathbf{k}\cdot\mathbf{k}) - 8(\alpha+\beta)\mathbf{k}\cdot\mathbf{k} = 0. \tag{A14}$$

The EM radiation waves will propagate with the dispersion relation

$$\frac{k_\eta^2}{\mathbf{k}\cdot\mathbf{k}} = 1 - 8(\alpha+\beta), \tag{A15}$$

or, equivalently,

$$\left|\frac{k_\eta}{\mathbf{k}}\right| = 1 - 4(\alpha+\beta). \tag{A16}$$

We can now compare with the previous formulation of this problem in terms of the two-point function of the gravitons (see Sec. IV A). We already know from symmetry considerations that the tensor $M^{\mu\nu\sigma\tau}$ can be written as

$$M^{\mu\nu\sigma\tau} = \alpha(\eta)(U_x^{\mu\nu}U_x^{\sigma\tau} + U_y^{\mu\nu}U_y^{\sigma\tau} + U_z^{\mu\nu}U_z^{\sigma\tau}) + \beta(\eta)(V_x^{\mu\nu}V_x^{\sigma\tau} + V_y^{\mu\nu}V_y^{\sigma\tau} + V_z^{\mu\nu}V_z^{\sigma\tau}). \tag{A17}$$

Let us now calculate the only non-vanishing components of this tensor ($M^{\eta i \eta j}$ and M^{ijmn}) in the coordinate basis defined by the vector fields $\partial_\eta, \partial_x, \partial_y$ and ∂_z. We get

$$M^{\eta i \eta j} = \alpha(\eta)(U_x^{\eta i}U_x^{\eta j} + U_y^{\eta i}U_y^{\eta j} + U_z^{\eta i}U_z^{\eta j}) = \frac{\alpha(\eta)}{a^2(\eta)} g^{ij}, \tag{A18}$$

$$M^{ijmn} = \beta(\eta)(V_x^{ij}V_x^{mn} + V_y^{ij}V_y^{mn} + V_z^{ij}V_z^{mn}) = \beta(\eta)(g^{im}g^{jn} - g^{in}g^{jm}), \tag{A19}$$

which means in particular that the functions $\alpha(\eta)$ and $\beta(\eta)$ introduced in this appendix are related to the functions defined in Eqs. (24a) and (24b) as $f_1(\eta) = \alpha(\eta)/a^2(\eta)$ and $f_2(\eta) = \beta(\eta)$. Hence, we conclude that

$$\left|\frac{k_\eta}{\mathbf{k}}\right| = 1 - 4(\alpha+\beta) = 1 - 4[a^2(\eta)f_1(\eta) + f_2(\eta)] = 1 + \frac{\kappa^2}{6}\langle\phi^2(\eta)\rangle, \tag{A20}$$

which agrees with our previous result obtained from Maxwell's equations and the two-point function of the gravitons [see Eq. (33)].

APPENDIX B: STRUCTURE OF THE TENSOR M FOR STATIC AND SPHERICALLY SYMMETRIC BACKGROUNDS

We assume a static and spherically symmetric spacetime described by the metric

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2. \tag{B1}$$

In order to compute the general form of the tensor $M^{\mu\nu\sigma\tau}$ we will follow the same steps as in Appendix A. We choose an orthonormal basis of the spacetime given by the vectors $\mathbf{e}_t$, $\mathbf{e}_r$, $\mathbf{e}_\theta$, and $\mathbf{e}_\phi$. This orthonormal basis is defined by

$$\mathbf{e}_t = [A(r)]^{-1/2}\frac{\partial}{\partial t}, \tag{B2a}$$

$$\mathbf{e}_r = [B(r)]^{-1/2}\frac{\partial}{\partial r}, \tag{B2b}$$

$$\mathbf{e}_\theta = \frac{1}{r}\frac{\partial}{\partial\theta}, \tag{B2c}$$

$$\mathbf{e}_\phi = \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}. \tag{B2d}$$

We also introduce the following basis of antisymmetric order two tensors:

$$U^{\mu\nu}=\mathbf{e}_t^\mu\mathbf{e}_r^\nu-\mathbf{e}_t^\nu\mathbf{e}_r^\mu, \tag{B3a}$$

$$V^{\mu\nu}=\mathbf{e}_\theta^\mu\mathbf{e}_\phi^\nu-\mathbf{e}_\theta^\nu\mathbf{e}_\phi^\mu, \tag{B3b}$$

$$X^{\mu\nu}=\mathbf{e}_t^\mu\mathbf{e}_\theta^\nu-\mathbf{e}_t^\nu\mathbf{e}_\theta^\mu, \tag{B3c}$$

$$Y^{\mu\nu}=\mathbf{e}_t^\mu\mathbf{e}_\phi^\nu-\mathbf{e}_t^\nu\mathbf{e}_\phi^\mu, \tag{B3d}$$

$$W^{\mu\nu}=\mathbf{e}_r^\mu\mathbf{e}_\theta^\nu-\mathbf{e}_r^\nu\mathbf{e}_\theta^\mu, \tag{B3e}$$

$$Z^{\mu\nu}=\mathbf{e}_r^\mu\mathbf{e}_\phi^\nu-\mathbf{e}_r^\nu\mathbf{e}_\phi^\mu. \tag{B3f}$$

The tensor $M^{\mu\nu\sigma\tau}$ must be a linear combination of the following kind (with coefficients that may depend only on r):

$$\begin{aligned}M^{\mu\nu\sigma\tau}=&a_1(r)U^{\mu\nu}U^{\sigma\tau}+a_2(r)(U^{\mu\nu}V^{\sigma\tau}+V^{\mu\nu}U^{\sigma\tau})+a_3(r)(U^{\mu\nu}X^{\sigma\tau}+X^{\mu\nu}U^{\sigma\tau})\\&+a_4(r)(U^{\mu\nu}Y^{\sigma\tau}+Y^{\mu\nu}U^{\sigma\tau})+a_5(r)(U^{\mu\nu}W^{\sigma\tau}+W^{\mu\nu}U^{\sigma\tau})+a_6(r)(U^{\mu\nu}Z^{\sigma\tau}+Z^{\mu\nu}U^{\sigma\tau})\\&+a_7(r)V^{\mu\nu}V^{\sigma\tau}+a_8(r)(V^{\mu\nu}X^{\sigma\tau}+X^{\mu\nu}V^{\sigma\tau})+a_9(r)(V^{\mu\nu}Y^{\sigma\tau}+Y^{\mu\nu}V^{\sigma\tau})\\&+a_{10}(r)(V^{\mu\nu}W^{\sigma\tau}+W^{\mu\nu}V^{\sigma\tau})+a_{11}(r)(V^{\mu\nu}Z^{\sigma\tau}+Z^{\mu\nu}V^{\sigma\tau})+a_{12}(r)X^{\mu\nu}X^{\sigma\tau}\\&+a_{13}(r)(X^{\mu\nu}Y^{\sigma\tau}+Y^{\mu\nu}X^{\sigma\tau})+a_{14}(r)(X^{\mu\nu}W^{\sigma\tau}+W^{\mu\nu}X^{\sigma\tau})+a_{15}(r)(X^{\mu\nu}Z^{\sigma\tau}+Z^{\mu\nu}X^{\sigma\tau})\\&+a_{16}(r)Y^{\mu\nu}Y^{\sigma\tau}+a_{17}(r)(Y^{\mu\nu}W^{\sigma\tau}+W^{\mu\nu}Y^{\sigma\tau})+a_{18}(r)(Y^{\mu\nu}Z^{\sigma\tau}+Z^{\mu\nu}Y^{\sigma\tau})\\&+a_{19}(r)W^{\mu\nu}W^{\sigma\tau}+a_{20}(r)(W^{\mu\nu}Z^{\sigma\tau}+Z^{\mu\nu}W^{\sigma\tau})+a_{21}(r)Z^{\mu\nu}Z^{\sigma\tau}.\end{aligned} \tag{B4}$$

Since the metric is static, we can make use of the time inversion invariance to show that the terms in Eq. (B4) involving the functions a_2, a_5, a_6, a_8, a_9, a_{14}, a_{15}, a_{17}, and a_{18} must vanish identically. Here we should stress once again that the quantum state of the gravitons does not break time inversion invariance. (This is not true in general. For example, the Unruh vacuum state in Schwarzschild spacetime does break the time inversion symmetry.)

Because of the spherical symmetry of the metric (spatial inversion as well), the following coefficients have to vanish: a_3, a_4, a_{10}, a_{11}, a_{13}, and a_{20}. Furthermore, the coefficients a_{12} and a_{16} must be equal as they have to be invariant under spatial rotations. The same is true for the pair a_{19} and a_{21}.

After these symmetry considerations, we can finally write the general expression for the tensor $M^{\mu\nu\sigma\tau}$ in a static, spherically symmetric background:

$$\begin{aligned}M^{\mu\nu\sigma\tau}=&f_1(r)U^{\mu\nu}U^{\sigma\tau}+f_2(r)V^{\mu\nu}V^{\sigma\tau}\\&+f_3(r)(X^{\mu\nu}X^{\sigma\tau}+Y^{\mu\nu}Y^{\sigma\tau})\\&+f_4(r)(W^{\mu\nu}W^{\sigma\tau}+Z^{\mu\nu}Z^{\sigma\tau}).\end{aligned} \tag{B5}$$

The functions $f_i(r)$ (with $i=1,\dots,4$) will depend on the particular choice of graviton state and are not known *a priori*.

As in Apendix A we introduce the six functions

$$\begin{aligned}&u=f_{\mu\nu}U^{\mu\nu},\quad v=f_{\mu\nu}V^{\mu\nu},\\&x=f_{\mu\nu}X^{\mu\nu},\quad y=f_{\mu\nu}Y^{\mu\nu},\\&w=f_{\mu\nu}W^{\mu\nu},\quad z=f_{\mu\nu}Z^{\mu\nu}.\end{aligned} \tag{B6}$$

In the Coulomb gauge, the three non-vanishing components for the gauge potential are a_r, a_θ, and a_ϕ, so that

$$\begin{aligned}&f_{tr}=k_ta_r,\quad f_{r\theta}=k_ta_\theta,\\&f_{t\theta}=k_ta_\phi,\quad f_{r\phi}=k_ra_\theta-k_\theta a_r,\\&f_{t\phi}=k_ra_\phi-k_\phi a_r,\quad f_{\theta\phi}=k_\theta a_\phi-k_\phi a_\theta,\end{aligned} \tag{B7}$$

and we can then write

$$\begin{aligned}&u=2f_{tr},\quad w=2f_{r\theta},\\&x=2f_{t\theta},\quad z=2f_{r\phi},\\&y=2f_{t\phi},\quad v=2f_{\theta\phi},\end{aligned} \tag{B8}$$

so that in terms of the independent set $\{u,x,y\}$, the three dependent ones can be written as

$$w=\frac{1}{k_t}(k_rx-k_\theta u), \tag{B9a}$$

$$z=\frac{1}{k_t}(k_ry-k_\phi u), \tag{B9b}$$

$$v=\frac{1}{k_t}(k_\theta y-k_\phi x). \tag{B9c}$$

We now project Eq. (19) onto the three tensors U, X, and Y (which yield u, x, and y, respectively), to obtain the following set of equations:

$$0=k^2u-8f_1u(k_t^2-k_r^2)+8f_3k_r(xk_\theta+yk_\phi)$$
$$+8f_4k_t(wk_\theta+zk_\phi), \tag{B10a}$$

$$0=k^2x+8f_1uk_rk_\theta+8f_2vk_tk_\phi-8f_3x(k_t^2-k_\theta^2)$$
$$+8f_3yk_\theta k_\phi-8f_4wk_tk_r, \tag{B10b}$$

$$0=k^2y+8f_1uk_rk_\phi-8f_2vk_tk_\theta+8f_3xk_\theta k_\phi$$
$$-8f_3y(k_t^2-k_\phi^2)-8f_4zk_tk_r. \tag{B10c}$$

If we write the components (v,w,z) as functions of (u,x,y), we have

$$0=[k^2(1+8f_1)-8(f_1+f_4)(k_\theta^2+k_\phi^2)]u+8(f_3+f_4)k_rk_\theta x$$
$$+8(f_3+f_4)k_rk_\phi y, \tag{B11a}$$

$$0=8(f_1+f_4)k_rk_\theta u+[k^2(1+8f_3)-8(f_2+f_3)k_\phi^2$$
$$-8(f_3+f_4)k_r^2]x+8(f_3+f_4)k_\theta k_\phi y, \tag{B11b}$$

$$0=8(f_1+f_4)k_rk_\phi u+8(f_2+f_3)k_\theta k_\phi x$$
$$+[k^2(1+8f_3)-8(f_2+f_3)k_\theta^2-8(f_3+f_4)k_r^2]y. \tag{B11c}$$

In this case the components of the vector k in the orthonormal basis are given by $k=k^t\mathbf{e}_t+k^r\mathbf{e}_r+k^\theta\mathbf{e}_\theta+k^\phi\mathbf{e}_\phi$.

Imposing that the determinant of the set of three equations vanishes, we obtain the following determinantal condition:

$$k^2[(1+8f_3)k^2-8(f_3+f_4)k_r^2-8(f_2+f_3)(k_\theta^2+k_\phi^2)]$$
$$\times[(1+8f_3)(1+8f_1)k^2-8(f_3+f_4)(1+8f_1)k_r^2$$
$$-8(f_1+f_4)(1+8f_3)(k_\theta^2+k_\phi^2)]=0. \tag{B12}$$

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