Reply to "Comment on 'Anomalies in electrostatic calibrations for the measurement of the Casimir force in a sphere-plane geometry' "

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In a recent Comment, Decca *et al.* [Phys. Rev. A **79**, 026101 (2009)] discussed the origin of the anomalies recently reported by us in Phys. Rev. A **78**, 036102(R) (2008). Here we restate our view corroborated by their considerations that quantitative geometrical and electrostatic characterizations of the conducting surfaces (a topic not discussed explicitly in the literature until very recently) are critical for the assessment of precision and accuracy of the demonstration of the Casimir force and for deriving meaningful limits on the existence of Yukawian components possibly superimposed to the Newtonian gravitational interaction.

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I. INTRODUCTION

In the last decade, various efforts have been focused on demonstrating the Casimir force and exploring hypothetical short-range forces of gravitational origin [1]. Limits to the existence of these forces—or their tentative discovery—in the micrometer range rely on the control at the highest level of accuracy of the Casimir force and the related systematic effects [2,3].

In this context, we have investigated the celebrated sphere-plane geometry in a range of parameters for which the hypothetical Yukawian contribution of gravitational origin should be optimally detected [4]. This implies exploiting a combination of spheres with large radius of curvature, such as the one already used in [5], and small separation gaps between the sphere and the planar surface similar to the ones explored in [6–8] with spheres having an order of 100 μm radius of curvature. Notice that large radius of curvature and relatively large distances as in [5] are not adequately sensitive to Yukawian forces with small interaction range. Conversely, microspheres at small distances as used in [6–8] have small sensitivity to the amplitude of Yukawa forces due to the smaller expected signal arising from the reduced effective surfaces of interaction. In this regard, limits to the Yukawa force based on a formal mapping between an ideal parallel-plate geometry and the sphere-plane configuration actually used in the experimental setup as in [9] are invalid, as the proximity force approximation (PFA)—typically used for forces acting between surfaces [10]—does not hold for forces of volumetric character such as the gravitational force or its hypothetical short-range relatives.

In [4], we reported two anomalies in the electrostatic calibration of our apparatus after discussing and ruling out some systematic effects. This has triggered the interest of the authors of [11] who have added two interesting points: first attempting to explain our first anomaly in terms of a systematic deviation from the ideal single-curvature pattern for the spherical surface and second presenting a distance-independent contact potential in one of their experimental setups. We welcome these different insights and would like to discuss here their implications in the general context of both accurate demonstrations of the Casimir force and precision experiments on Yukawian gravitational forces, as in the following.

II. DEVIATION FROM IDEAL SPHERICAL GEOMETRY

Among the possible reasons for the first anomaly we have briefly discussed in [4], a couple of possibilities arise from geometrical effects, namely, the validity of the PFA approximation in our case and a surface obtained by convolving various spheres with different radii of curvature having in common the point of minimum distance from the plate. The authors of [11] provide a further example of a deviation from geometry that certainly, for an appropriate choice of the additional parameters added to the model, may mimic any desired power-law exponent for the scaling of the electrostatic curvature coefficient k_{el} upon the distance. In particular, we concur that a softer dependence is obtainable by guessing higher curvatures around the point of minimum distance. In general, it is indeed intuitive that, for instance, harder scaling with the distance is obtained in regions at lower curvature—in the extreme case of a flat surface (i.e., infinite radius of curvature) one expects a scaling of k_{el} with distance through an exponent -3—and conversely exponents softer than the expected -2 should be associated with even *sharper* regions.

However, this interpretation in terms of a modified geometry is hard to reconcile with the measurement of the capacitance versus distance that better follows the behavior expected for a surface with a single radius of curvature, as shown in Fig. 1. The electrostatic curvature coefficient k_{el} is related to the second spatial derivative of the capacitance C and the effective mass m_{eff} as

$$k_{el} = \frac{C''}{8\pi^2 m_{eff}}. \quad (1)$$

The model considered in [11] implies a capacitance for the modified sphere-plane configuration expressed by the formula

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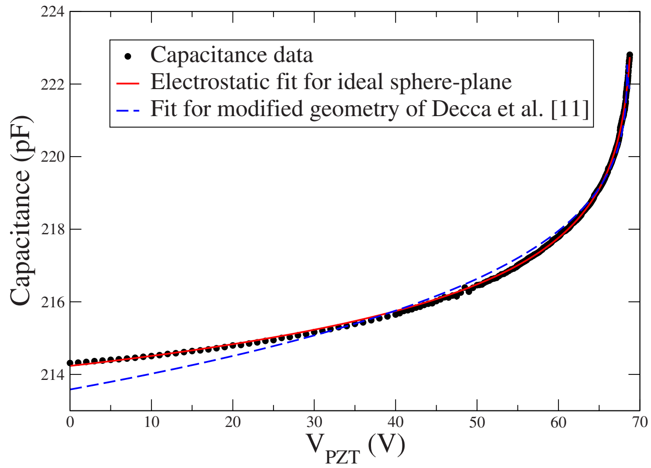

FIG. 1. (Color online) Capacitance versus applied piezoelectric (PZT) voltage data (black circles) and its best fit under the two hypotheses of ideal spherical geometry and modified geometry as in [11]. The modified geometry fit (blue dashed line) is based on Eq. (3), with the distance $d=\beta(V^0_{\mathrm{PZT}}-V_{\mathrm{PZT}})$, where $\beta=(87\pm 2)$ nm/V is the PZT actuation coefficient. The best fit with the constraint of A_2 terms equal to zero yields $A_1^{\mathrm{mod}}=(222.96\pm 0.04)$ pF, $A_3^{\mathrm{mod}}=-(346.2\pm 1)$ pF/m, and $V^0_{\mathrm{PZT}}=(68.43\pm 0.05)$ V, with a reduced $\chi^2=77.4$. The ideal geometry fit (red continuous line) is based on Eq. (4) and the resulting parameters are $A_1^{\mathrm{id}}=(193.9\pm 0.2)$ pF, $A_3^{\mathrm{id}}=-(1.757\pm 0.002)$ pF, and $V^0_{\mathrm{PZT}}=(69.31\pm 0.02)$ V, with a reduced $\chi^2=2.9$. The coefficient A_3^{id} is in agreement within one standard deviation with the less accurate theoretical expectation of $-2\pi\epsilon_0 R=-(1.72\pm 0.02)$ pF. If the A_2 terms are not constrained to zero in both fits, one gets $A_1^{\mathrm{mod}}=(223.8\pm 1.5)$ pF, $A_2^{\mathrm{mod}}=-(359524\pm 13000)$ pF/m, $A_3^{\mathrm{mod}}=-(433\pm 135)$ pF, and $V^0_{\mathrm{PZT}}=(68.59\pm 0.45)$ V, with a reduced $\chi^2=13.6$, and $A_1^{\mathrm{id}}=(193.9\pm 0.2)$ pF, $A_2^{\mathrm{id}}=-(29000\pm 2800)$ pF/m, $A_3^{\mathrm{id}}=-(1.705\pm 0.005)$ pF, and $V^0_{\mathrm{PZT}}=(69.25\pm 0.02)$ V, with a reduced $\chi^2=2.6$.

$$C_{\mathrm{mod}} = 2\pi\epsilon_0\left[R_{CD}\ln(R_{CD}/d) + (R_{AB}-R_{CD})\ln\left(\frac{R_{AB}-R_{CD}}{d+h}\right) - (R_{AB}-R)\ln\left(\frac{R_{AB}-R}{d+h+H}\right)\right] \quad (2)$$

(see [11] for the definition of the various geometrical quantities) up to a term of the type A_1+A_2d arising from the double integration of Eq. (1). This expression may be approximated, in the distance range discussed in [11], as

$$\tilde{C}_{\mathrm{mod}} = A_1^{\mathrm{mod}} + A_2^{\mathrm{mod}}d + A_3^{\mathrm{mod}}d^{0.3}. \quad (3)$$

For the ideal sphere-plane capacitance, we obtain instead

$$C_{\mathrm{id}} = A_1^{\mathrm{id}} + A_2^{\mathrm{id}}d + A_3^{\mathrm{id}}\ln(R/d), \quad (4)$$

with $A_3^{\mathrm{id}}=-2\pi\epsilon_0 R$. The linear terms A_2^{id} and A_2^{mod} may represent the effect of a constant electric field, to which our ac capacitance meter should not be sensitive (see comment below). We have fitted our capacitance data with both Eqs. (3) and (4), supposing that $A_2^{\mathrm{id}}=A_2^{\mathrm{mod}}=0$. Fitting with the modified geometry [Eq. (3)] yields a best fit with a reduced χ^2 that is significantly larger than that of the ideal geometry [Eq. (4)], as detailed in the caption of Fig. 1. Moreover, by using the parameters provided in [11] ($R_{AB}=1.6$ $R=49.4$ mm, $R_{CD}=30$ μm, $H=250$ nm, $h=8$ nm) chosen to reproduce the anomalous scaling power law observed by us in [4], we find that the best fit with Eq. (2) gives a discrepancy of about 20% in the expected coefficient for the distance dependence, while the discrepancy is 2.1% using the ideal sphere-plane formula, quite close to the nominal relative error of the radius of curvature of the sphere $R=(30.9\pm 0.15)$ mm. This suggests that the capacitance data are better explained by using an idealized geometry rather than the sophisticated geometry proposed in [11] that, moreover, should have been evidenced by the dedicated atomic force microscopy (AFM) imaging of the sphere that we performed after the runs. Notice that in our setup the capacitance is measured *dynamically* by using an ac bridge, while k_{el} is measured by looking at the frequency shift related to the gradient of spatial dependent but *static* force. By supposing that static or slowly changing charges—not affecting the dynamical measurement of the capacitance—are present on the two surfaces, an ideal capacitance and an anomalous scaling of k_{el} are mutually consistent.

III. DISTANCE DEPENDENCE OF THE CONTACT POTENTIAL

With regard to the observation of the dependence on distance of the contact potential we have reported in [4], the authors of [11] show previously unpublished data, in Fig. 3, for their experiment located in Indiana. In this plot no systematic trend of V_0 is observable in the entire explored range of distances. However, an issue arises if the same data—provided to us by R. Decca —are plotted including the error bars for V_0 at one standard deviation, as appearing in Fig. 2. The contact potential is not constant within the error bars, showing scatter over several standard deviations, and perhaps a weak sinusoidal component. By fitting the data with a constant value, one gets a reduced $\chi^2=7.2$ (resulting from an absolute $\chi^2=3603$ and 499 degrees of freedom for the 500 data points); therefore the hypothesis of constant contact potential is highly unlikely, and the use of a constant compensating external potential will generate large residuals and related systematic errors. One may argue that the error bars associated to the contact potential have been underestimated. In fact, if the error bars are increased by a factor of $\simeq\sqrt{7.2}$, the reduced χ^2 approaches values of order unity, making the constant contact potential hypothesis realistic. But doing so increases the average error bar to $\simeq 0.35$ mV, with the relative average error in each determination of V_0 increasing from the initial 0.85% to 2.29%. This will invalidate, once propagated through the electrostatic calibration analysis, the precision of 0.2% quoted in [11], and is at variance with the relative error of 0.85% resulting from the standard deviation of 0.13 mV of $V_0=15.29$ mV quoted in the same paper. In other words, this is an example of data already collected requiring a reanalysis of the precision and accuracy, as we have commented in the conclusions of [4].

It is worth pointing out that the plot presented in Fig. 3 of [11] should not be considered representative of the overall

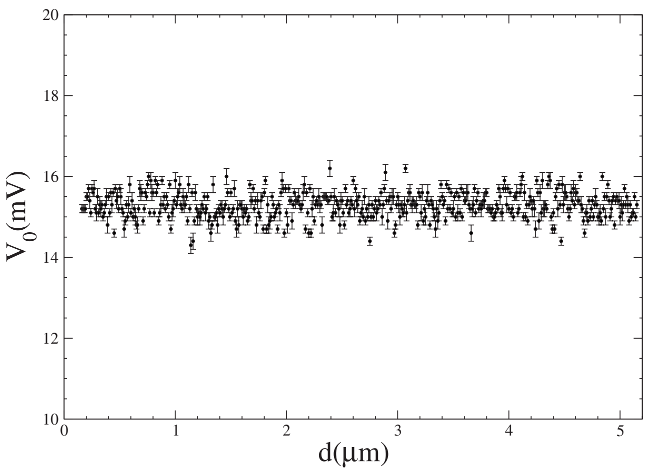

FIG. 2. Minimizing voltage versus sphere-cantilever distance resulting for the data analysis of the electrostatic calibration for the same run as in Fig. 3 of [11], also including the error bars of each determination of V_0 (courtesy of R. Decca). By considering the scattering of the data around their average value of $\langle V_0 \rangle = 15.29$ mV, we obtain a standard deviation of $\Delta V_0 = 0.31$ mV (at variance with the quoted value of 0.13 mV in Fig. 3 of [11]), while the average value of the error bars is $\delta V_0 = 0.13$ mV. See the text for the discussion of the χ^2 analysis under the hypothesis of a constant value for the contact potential.

picture. In addition to our findings in [4], clear evidence of a distance-dependent V_0 has been recently found by the groups operating in Grenoble [12], Amsterdam [13], and Yale [14], and in a reanalysis of the data collected at the Lucent Laboratories [7], as we will report in a future publication. Even in the data by the Riverside group reported in Fig. 4 of [15] a slope seems evident in spite of a rather coarse scale chosen for the vertical axis, although the authors believe that the scattering of the data overwhelms any systematic trend [16]. It is understood that future measurements will clarify under which conditions the contact potential can be considered as constant for two surfaces in close proximity to each other, including the possible role of image charges [17], and further investigations on its time variability—also started recently [18]—will be necessary. We also remark on the fact that the presence of a contact potential depending on the distance requires a proper handling of the electrostatic term, as recently outlined in [19].

IV. GENERAL ISSUES ON PRECISION AND ACCURACY

The authors of [11] also comment about the precision and accuracy of the experiments in the sphere-plane configuration and on the negligible role of the patch potentials in their experimental setups. We believe that various claims on the precision and accuracy in previous papers—and the mixing of experimental facts and theoretical hypotheses supposed to be tested by the data themselves—have generated confusion in the literature and inconsistent methodologies for their quantitative assessments. As a first representative example, we mention [6] in which the precision of the measurement has been assessed by comparing the data with the expected Casimir force. This generates a manifestly model-dependent experimental precision in contradiction with the concept that this quantity is merely a figure of merit of the reproducibility of the measurements. Also, the model for patch potentials in [20] assumes some specific form for the two-dimensional Fourier spectrum of patches and crucially depends on the range of spatial wavelengths contributing to it. Using this phenomenological model and a spectral range arbitrarily chosen based on the size of the grains in the material, in [11] it is concluded that the effects of patches have been investigated in detail in [9] and in [21] and found negligible in those experiments. On the contrary, we believe that the model in [20] does not necessarily describe any realistic experiment (as indeed argued in [20] itself) and that a *detailed* investigation of patch effects in Casimir experiments would require *in situ* measurements, using, for instance, Kelvin probe techniques [22]. A third example of model-dependent experimental claim in the same spirit is the evaluation of the limits to Yukawian forces based on a formal mapping of the sphere-plane configuration to the parallel plane case via PFA [9] already mentioned in Sec. I. All these methodological issues will also require further in depth analysis, both on the data collected so far, and via a third generation of experiments capitalizing on the current discussions.

In conclusion, we believe that the message in our contribution [4] is reinforced by the discussion presented in [11]: a very detailed geometrical and electrical characterization of the sphere-plane setup is necessary prior to assessing the precision and accuracy of the Casimir force measurements. Characterizations of this nature require dedicated efforts, as, for instance, initiated in [23], where systematic measurements have been carried out to study the dependence of the forces upon the shape and roughness of the microspheres or using apparata with unprecedented levels of stability [13]. It is also crucial to declare all the relevant parameters in the entire explored range of distances as first stressed in [24], including their possible time variability, and also quoting the percentage of rejection of samples or data runs.

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[1] E. Fischbach and C. L. Talmadge, *The Search for Non-Newtonian Gravity* (AIP, New York/Springer-Verlag, New York, 1999).
[2] A. Lambrecht and S. Reynaud, Eur. Phys. J. D **8**, 309 (2000).
[3] R. Onofrio, New J. Phys. **8**, 237 (2006).
[4] W. J. Kim, M. Brown-Hayes, D. A. R. Dalvit, J. H. Brownell, and R. Onofrio, Phys. Rev. A **78**, 020101(R) (2008).
[5] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).
[6] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998).
[7] H. B. Chan, V. A. Aksyuk, R. N. Kleinman, D. J. Bishop, and F. Capasso, Science **291**, 1941 (2001).
[8] R. S. Decca, D. Lopez, E. Fischbach, and D. E. Krause, Phys. Rev. Lett. **91**, 050402 (2003).
[9] R. S. Decca, D. Lopez, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, Ann. Phys. (N.Y.) **318**, 37 (2005).
[10] J. Blocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, Ann. Phys. (N.Y.) **105**, 427 (1977).
[11] R. S. Decca, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, D. Lopez, U. Mohideen, and V. M. Mostepanenko, Phys. Rev. A **79**, 026101 (2009).
[12] G. Jourdan, Ph.D. thesis, University Joseph Fourier, Grenoble, 2007.
[13] S. de Man, K. Heeck, and D. Iannuzzi, Phys. Rev. A **79**, 024102(2009).
[14] W. J. Kim, A. O. Sushkov, D. A. R. Dalvit, and S. K. Lamoreaux, e-print arXiv:0812.2798.
[15] H.-C. Chiu, C.-C. Chang, R. Castillo-Garza, F. Chen, and U. Mohideen, J. Phys. A **41**, 164022 (2008).
[16] U. Mohideen (private communication). We were unable to obtain data from Fig. 4 of [15] for an independent analysis. Various attempts to reconstruct the data by scanning that figure were not felt by us as reliable enough to avoid the risk, due to the considerable overlap among many points, of introducing artifacts and biases in their analysis.
[17] B. C. Stipe, H. J. Mamin, T. D. Stowe, T. W. Kenny, and D. Rugar, Phys. Rev. Lett. **87**, 096801 (2001).
[18] S. E. Pollack, S. Schlamminger, and J. H. Gundlach, Phys. Rev. Lett. **101**, 071101 (2008).
[19] S. K. Lamoreaux, e-print arXiv:0808.0885.
[20] C. C. Speake and C. Trenkel, Phys. Rev. Lett. **90**, 160403 (2003).
[21] F. Chen, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Phys. Rev. A **69**, 022117 (2004).
[22] N. A. Robertson, J. R. Blackwood, S. Buchman, R. L. Byer, J. Camp, D. Gill, J. Hanson, S. Williams, and P. Zhou, Class. Quantum Grav. **23**, 2665 (2006).
[23] P. J. van Zwol, G. Palasantzas, M. van de Schootbrugge, J. Th. M. de Hosson, and V. S. J. Craig, Langmuir **24**, 7528 (2008).
[24] D. Iannuzzi, M. Lisanti, and F. Capasso, Proc. Natl. Acad. Sci. U.S.A. **101**, 4019 (2004).