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Decoherence Effects of Motion Induced Radiation

P.A. Maia Neto[1] and D.A.R. Dalvit[2]

[1] Instituto de Física, UFRJ, Caixa Postal 68528
21945-970 Rio de Janeiro, Brazil
[2] T-6, Theoretical Division, MS B288, Los Alamos National Laboratory
Los Alamos, NM 87545, USA

Abstract. The radiation pressure coupling with vacuum fluctuations gives rise to energy damping and decoherence of an oscillating particle. Both effects result from the emission of pairs of photons, a quantum effect related to the fluctuations of the Casimir force. We discuss different alternative methods for the computation of the decoherence time scale. We take the example of a spherical perfectly-reflecting particle, and consider the zero and high temperature limits. We also present short general reviews on decoherence and dynamical Casimir effect.

1 Introduction and Brief Summary of Decoherence Theory

The understanding of the quantum-to-classical transition has been the subject of extensive research[1,2]. The core of the problem is that the Hilbert space of allowed states of a quantum system is huge, whereas the set of states with associated classical properties is a tiny subset of the whole Hilbert space. Some questions that naturally arise are the following: which mechanism is responsible for the classical appearance of macroscopic and mesoscopic quantum systems? How are those few classical states selected from the huge Hilbert space? The common wisdom is that classicality is an emergent property induced on subsystems by their environment.

The interaction between a system S and its environment E creates entanglement (i.e. non reducible correlations) between the states of the system and those of the environment. Imagine that at a given time (say $t = 0$) the state $|\Psi(t)\rangle$ of S+E is a product state, that is, there are no initial correlations. We have $|\Psi(t=0)\rangle = |s\rangle|\epsilon\rangle$, where the first ket corresponds to the system state (assumed for simplicity to be in a pure state), and the second one to the environmental state, also assumed pure. When the two parts begin to interact, entanglement is generally produced. This means that at a later time t, the state $|\Psi(t)\rangle$ will be given by a linear superposition of the form

$$|\Psi(t)\rangle = |s_1\rangle|\epsilon_1\rangle + |s_2\rangle|\epsilon_2\rangle + \dots \tag{1}$$

where $\{|s_i\rangle\}$ and $\{|\epsilon_i\rangle\}$ are states of the system and environment Hilbert spaces, respectively. If the interaction is such that the states $|\epsilon_i\rangle$ become approximately orthogonal ($\langle\epsilon_n|\epsilon_m\rangle \approx \delta_{nm}$), then interference between the system states $|s_n\rangle$ and $|s_m\rangle$ will not be observed. These set of states $\{|s_i\rangle\}$ usually have classical properties. Any quantum superposition of them is a non-classical state, and quickly decays away into a statistical mixture of the states. The coherence of the phase relation between the components of the superposition is lost, and this process is accordingly known as *decoherence*. In other words, the environment monitors the different classical alternatives for the system (the different states $|s_i\rangle$), thereby providing which-way information, even though such information is usually unaccessible to the observer. The set of states $\{|s_i\rangle\}$ are called *pointer states* [3], and they are the states within the huge Hilbert space of the system that become less entangled with the environment. Perfect pointer states are those that produce no entanglement at all, so that an initial product state of S+E will remain a product state throughout the interaction time, which means that those states are robust and stay unperturbed by the interaction. All this will be illustrated in Section 3 in the particular case where the environment is the radiation field at zero temperature (vacuum field) and radiation pressure is responsible for the coupling between system (a mirror) and the environment.

A possible method to identify pointer states is called the 'predictability sieve criterion' [4], which is based on the fact that pointer states are the ones that produce least entropy and remain most pure. Let us explain these concepts. The evolution of the closed combined system S+E is unitary, so that the purity of the whole state $|\Psi\rangle$ is preserved, i.e. $P(\rho_{S+E}) = \mathrm{Tr}\rho^2_{S+E} = 1$ for all times. However, the purity of S is not preserved. To show it one needs to calculate the reduced density matrix of the subsystem S by tracing out the environmental degrees of freedom, $\rho_S = \mathrm{Tr}_E\rho_{S+E}$, and then $P(\rho_S) < 1$. The loss of purity can be associated with a loss of information about the system state. When no measurement involving the environment is made, the density matrix ρ_S contains the state of knowledge of an observer about the system, and purity is a measure of that knowledge. Initially, there is full knowledge of the system state, which is described by a single ket state. Subsequently, the interaction with the environment produces entanglement, and part of the information about the system S is lost to the environment, causing a decrease in the purity of the system. One can also measure the information content of ρ_S through the von Neumann entropy, $S(\rho_S) = -\mathrm{Tr}\rho_S \log \rho_S$. Initially one has full information, and entropy is identically zero; as time goes on, information is lost and entropy is produced. Pointer states are least affected by the environment, so their information content is preserved and hence they produce the least entropy. The idea is then to take every state in the Hilbert space of S, calculate the von Neumann entropy at time t produced via interaction with the environment starting from the given state of the system, and order the initial states in a tower of increasing entropy. The states that lie at the bottom of such a tower are candidates for pointer states. Finally, one must check that those states remain at the bottom when the time t when entropy is calculated is changed, so that the states are robust pointers.

If one can satisfy these conditions, one gets the pointer states of the system S. If not, there are no pointer states for the problem. When these ideas are put into practice, it is more convenient to work with the linear entropy, defined as $S(\rho_S) = 1 - P(\rho_S) = 1 - \mathrm{Tr}\rho_S^2$.

The determination of pointer states for a given problem depends both on the system and environment dynamics, as well as on their interaction. There are three different regimes: 1) When the system Hamiltonian is irrelevant, pointer states are given by the eigenstates of the system's operators contained in the interaction Hamiltonian. A typical example is that of a measuring apparatus (the system) that has no internal dynamics, measuring an external reservoir (the environment), say a photocurrent. 2) When the system dynamics as well as the interaction are relevant, pointers come from an interplay between the two. The most thoroughly studied example is that of quantum Brownian motion (QBM), in which a particle is coupled to a set of harmonic oscillators in a thermal state, thereby suffering decoherence [5]. Although the coupling between the system and the environment is of the type position-position, pointer states are not position eigenstates of the system because the self dynamics of the harmonic oscillator interchanges position and momentum every quarter of a cycle. It turns out that the interplay between interaction and self dynamics leads to coherent states as pointers [6]. This second case is also the relevant one for this paper. 3) Finally, when the environment evolves much slower than the system, pointers may correspond to energy eigenstates of the system's Hamiltonian [7].

In the above we have ignored the information contained in the environmental state, and that is the reason why one traces over the environmental degrees of freedom in order to find the reduced density matrix of the system. However, the information lost to the environment could be, in principle, intercepted and recovered. Performing measurements on the environment one may extract information about the decohering system. In [8] it is shown that the preferred pointer states remain unchanged, even when that information is kept and modifies the dynamical evolution of the system.

Another related way to study the dynamical process underlying in the quantum-to-classical transition is via phase space representations of the reduced quantum dynamics for the system S. Among the many possible representations, one of particular interest is the Wigner function $W(x,p)$, which is defined as a Fourier transform of the reduced density matrix. $W(x,p)$ is a pseudo probability distribution in phase space, and encapsulates the quantum coherence of the system in interference fringes that take both positive and negative values. Imagine that one starts with an initial state for S which is highly non classical, such as a cat state $|\mathrm{cat}\rangle = 1/\sqrt{2}(|\alpha\rangle + |-\alpha\rangle)$, where $|\alpha\rangle$ is a coherent state with large amplitude ($|\alpha| \gg 1$). The corresponding Wigner function will have interference fringes, showing the quantum nature of the state. However, when the system is put in contact with the environment and each component of the state becomes entangled with almost orthogonal states of the environment, the interference fringes will be washed out. In the end the Wigner function becomes positive

defined, with two peaks at the values corresponding to $\pm\alpha$, as a true probability distribution. Decoherence transforms the initial pure state into a mixture of the two coherent states $|\pm\alpha\rangle$.

Until not very long ago the ideas of decoherence were restricted to the theoretical domain. Recent experimental developments have succeeded in studying in real time the process of decoherence in the laboratory, and have tested the predictions of the theory. Here we shall mention a few experiments that have been a hallmark in those developments. First, in the field of cavity QED, superposition states of photons and Rydberg atoms have been created within high-Q microwave cavities. Cat states of around 3 photons have also been produced, and it has been studied how they decay due to decoherence [9]. The coherence of the state was monitored with the help of a measurement of correlations between two consecutive atoms crossing the cavity [10]. Second, in the field of ion trapping, methods for creating superposed motional states of ions were developed, as well as schemes of environment engineering to protect those states from decoherence [11]. Finally, it has been possible to push the size of the cat states further into the macroscopic realm by generating a mesoscopic cat inside a rf-SQUID. The two components of the cat correspond to superconducting currents moving either clockwise or counterclockwise, each containing around 10^9 Cooper pairs [12].

In what we have discussed so far decoherence has been portraited as a "good" effect, in the sense that is responsible for the quantum-classical transition and the appearance of our classical world. Decoherence can also have a "bad" role in the field of quantum computation and quantum information processing. There one performs logical operations making use of the superposition states of quantum mechanics. For such operations to be successful it is very important to maintain the relative phase between the components of the superpositions all along the operations. If decoherence acts, it produces quantum errors that must be somehow corrected. Several methods have been proposed to minimize the effects of decoherence (see [13] for an example in nuclear magnetic resonance).

The prototype calculation of environment induced decoherence is the heuristic position-position interaction Hamiltonian for describing quantum Brownian motion, where the environment is taken to be a collection of harmonic oscillators. Although such a model is quite useful for studying many physical processes associated to dissipation and decoherence of a quantum system, the results that follow from it do not apply to every situation. That is, it is necessary to perform a case by case analysis in order to compute physical observables, such as decoherence and damping rates, how they scale with the parameters of the system, the environment, and their coupling, etc. For usual environments (thermal atoms, thermal light, phonons, etc.) it is in principle possible to design engineering schemes to protect the state of the system from decoherence, for example by reducing the coupling to the environment.

Then, the following question naturally arises: is it possible, at least in principle, to have arbitrarily weak decoherence? In this paper, we consider a fundamental source of decoherence that cannot be 'turned off': the radiation pressure

coupling with the vacuum field [14]. As reviewed in Sec. 2, any particle not completely transparent unavoidably scatters vacuum field fluctuations. This type of coupling is responsible for the Casimir effect. More generally, photons are created out of the vacuum field when moving boundaries are considered, an effect known as dynamical Casimir effect or motion-induced radiation. In Sec. 3, we show how the dynamical Casimir effect engenders decoherence. Our emphasis is on the basic physical ideas, and most of the calculations are referred to [15], but we also briefly discuss a model alternative to the one employed in this reference.

2 Dynamical Casimir Effect

The Casimir effect is perhaps the simplest and most striking effect of the quantum vacuum field (see [17] for reviews). The essential idea is that the boundary conditions modify the spectrum of the radiation field, and thereby its zero-point energy. This modification has direct physical consequences, leading, for example, to an attractive force between two parallel perfectly-reflecting plates (of surfaces A) and at a distance L, given by [18]

$$F = \frac{\pi^2}{240}\frac{\hbar c}{L^4}A.$$

A series of recent experiments [19] reported precision measurements of the Casimir force in agreement with the predictions of Quantum Electrodynamics, although more complete theoretical calculations, taking into account corrections due to finite temperature and conductivity as well as to roughness and geometry of the surfaces are partially yet to be done [20].

The Casimir force may also be computed by taking the average of the Maxwell stress tensor over the field vacuum state [21]. This method suggests that the Casimir force is itself a fluctuating quantity, as noted by Barton. Its fluctuations were first computed for plane perfectly reflecting mirrors [22], and later for spherical and spheroidal particles [23]. More generally, any particle scattering the radiation field is under the action of a fluctuating radiation pressure force exerted by the vacuum field, even in the situations where the *average* force vanishes (for example a single plane mirror at rest). The coupling responsible for those fluctuations also gives rise to a *dissipative* force, when the particle is moving in vacuum. Dissipation of the mirror's mechanical energy is needed to enforce energy conservation, since the motion induces the emission of pairs of photons (for reviews see [27][28]). Because of their common physical origin, fluctuations and dissipation are related by a very general result [29], whose most known application is the Einstein relation between diffusion and friction coefficients for a Brownian particle in the high-temperature limit. This connection provides a very useful tool for deriving the response to an external small perturbation from the fluctuations in the unperturbed case. Linear response theory was employed by Jaekel and Reynaud to infer the vacuum radiation pressure force

on partially-reflecting moving mirrors in the one-dimensional (1D) case [30]. For a single perfect mirror (position $x(t)$) the force is given by

$$F = \frac{\hbar}{6\pi c^2}\frac{d^3x}{dt^3}, \tag{2}$$

a result first obtained by solving the boundary conditions of a moving mirror in the long wavelength approximation, and assuming the effect of the motion to be a small perturbation [27]. Eq. (2) was also derived as the $n \to \infty$ limit of a moving half-space of refractive index n [32]. It also corresponds to the nonrelativistic approximation of the exact result (for a perfect mirror) derived with the help of a conformal coordinate transformation to the co-moving frame [33].

Since the wave equation in three dimensions is not invariant under a general conformal transformation, only approximated methods are used in this case. The dissipative force on a plane mirror was computed within the long wavelength approximation for a scalar [27] and electromagnetic [34] field models. The angular and frequency distributions of the emitted radiation were also computed for a single plane moving mirror [35], a moving dielectric half-space [36] [37] and two parallel plane mirrors [38]. Linear response theory was employed to derive the dissipative force on moving spheres [39]. Small but otherwise arbitrary time-dependent deformations of an initially plane surface were analyzed with the help of different approaches: linear response theory [40], long wavelength approximation [41], and path integrals [42].

The magnitude of the dynamical Casimir effect may be illustrated with the following example, which we shall discuss in detail in Sec. 3. We consider that the 'mirror' is a particle of mass M in a 1D harmonic potential, such that the oscillation frequency is ω_0. From Eq. (2), the equation of motion reads

$$\frac{d^2x}{dt^2} = -\omega_0^2 x + \frac{\hbar}{6\pi M c^2}\frac{d^3x}{dt^3}. \tag{3}$$

For any situation of physical interest, the zero point energy is much smaller than the rest mass energy: $\hbar\omega_0 \ll Mc^2$. In this case, (3) has solutions corresponding to oscillations damped at the rate

$$\Gamma = \frac{\hbar\omega_0}{12\pi M c^2}\omega_0 \ll \omega_0, \tag{4}$$

showing that the dynamical Casimir effect provides a tiny perturbation of the free oscillations.

As could be expected, a larger effect takes place when field modes of a cavity resonator are coupled to the moving boundaries, mainly when the mechanical frequency lies close to a given cavity eigenfrequency. Moore considered a scalar 1D field inside a cavity where one of the mirrors follows a prescribed motion [11]. The field modes were formally built in terms of the solution of a functional equation. This method was later developed [44] and extended to the case where the

two mirrors are set in motion [45]. The case of partially-transmitting mirrors was also calculated, allowing for a reliable estimation of the orders of magnitude for the rate of transmitted photons and the number of photons inside the cavity at steady-state [46]. So far, few three-dimensional (3D) calculations along these lines have been reported. A rectangular cavity made of perfectly-reflecting moving mirrors [47] [48], and a spherical bubble with time-dependent radius [49] were analyzed, the latter motivated by the problem of sonoluminescence.

In this article we only consider a single scatterer, so that no resonant enhancement takes place. In this section, we have shown that the radiation pressure coupling gives rise to energy damping of a particle scattering vacuum fluctuations. In the next section, we show that it also destroys the quantum coherence of the particle.

3 Decoherence and the Casimir Effect

Most treatments of the dynamical Casimir effect consider the particle that scatters the vacuum field (the 'mirror') to follow a prescribed motion (an exception is Ref. [50], which considers fluctuations of position of a particle driven by vacuum radiation pressure). In this article, however, we want to focus on the particle as the dynamical degree of freedom of interest. More specifically, we analyze how the radiation pressure coupling destroys the quantum coherence of an initial superposition state of the particle.

We consider as before that the particle is in a harmonic potential well, corresponding to a frequency of oscillation ω_0. The connection with the previous approaches, where the (classical) particle is assumed to follow a prescribed oscillation, is made by taking a coherent quasi-classical state $|\alpha\rangle$ for the particle, so that the combined particle-field state at $t = 0$ is

$$|\Psi(t=0)\rangle = |\alpha\rangle|0\rangle, \tag{5}$$

where we have assumed that the field is initially in the vacuum state $|0\rangle$. The oscillation gives rise to the emission of photon pairs at time t at the field modes λ_1 and λ_2, with probability amplitudes $b(\lambda_1, \lambda_2, t)$:

$$|\Psi(t)\rangle = |\alpha\rangle \left(B(t)|0\rangle + \sum_{\lambda_1,\lambda_2} b(\lambda_1, \lambda_2, t)|\lambda_1, \lambda_2\rangle \right), \tag{6}$$

where $B(t)$ is such that this state is normalized. As discussed in Sec. 2 (see, in particular, Eq. (4)), the energy damping associated to the dynamical Casimir effect is very small. This effect, and more generally the recoil of the particle, is neglected in (6), where the particle state is assumed not to be modified. Even at this level of approximation, there is decoherence, as we show by taking the initial state of the particle to be the cat state $|\text{cat}\rangle = (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2}$, an example

already mentioned in Sec. 1. It corresponds to the coherent superposition of two wavepackets oscillating out-of-phase in the harmonic potential well. The amplitudes $b(\lambda_1, \lambda_2, t)$ depend on the phase of the oscillation, so that they have an opposite sign when we take the state $|-\alpha\rangle$. Since the evolution operator is linear, the complete state at time t is the superposition of the r.-h.-s. of (6) with the analogous state for $|-\alpha\rangle$. It turns out to be an entangled state of the form discussed in (1):

$$|\Psi(t)\rangle = |\alpha\rangle|\epsilon^{(+)}(t)\rangle + |-\alpha\rangle|\epsilon^{(-)}(t)\rangle, \tag{7}$$

with $|\epsilon^{(\pm)}(t)\rangle = B(t)|0\rangle \pm \sum_{\lambda_1,\lambda_2} b(\lambda_1, \lambda_2, t)|\lambda_1, \lambda_2\rangle$. These field states work as tags for the particle states, providing which-way information about the phase of the oscillation. As time goes on, the information gets better defined, since

$$\langle\epsilon^{(-)}(t)|\epsilon^{(+)}(t)\rangle = 1 - 2\sum_{\lambda_1,\lambda_2} |b(\lambda_1, \lambda_2, t)|^2 \tag{8}$$

decreases as the probability for photon emission increases. When the emitted photons are not detected, all the relevant information about the particle is contained in the reduced matrix $\rho(t) = \mathrm{Tr_F}(|\Psi(t)\rangle\langle\Psi(t)|)$, where the trace is taken over the field states. Since the interference term is gradually washed out as a consequence of the photon emission effect and the corresponding entanglement with the field, $\rho(t)$ decays into the statistical mixture $\rho_m = (|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)/2$. The corresponding time scale t_d may be computed [15] from Eq. (7), and turns out to be proportional to the energy damping time $1/\Gamma$, which is related to the two-photon probabilities by energy conservation:

$$t_d = \frac{1}{4|\alpha|^2}\frac{1}{\Gamma}. \tag{9}$$

Eq. (9) also holds when the coupling with the environment is described by a heuristic master equation in the Lindblad form (derived with the help of the rotating-wave approximation) [51], as well as in the case of position-position coupling to a zero-temperature environment of harmonic oscillators, and has a very simple interpretation [16]: if $1/\Gamma$ is the time needed to damp the energy $2|\alpha|^2\hbar\omega_0$, it corresponds to the emission of $2|\alpha|^2$ pairs of photons (each pair has a total energy equal to $\hbar\omega_0$). On the other hand, coherence is much more delicate than energy, since a single photon provides which-way information that destroys the quantum phase of the cat state. Hence the decoherence time is the time scale for the emission of a single photon. Since $4|\alpha|^2$ photons are emitted during the time interval $1/\Gamma$, the time for a single photon scales as in the r.-h.-s. of (9).

Eq. (9) only holds when $|\alpha| \gg 1$. In this limit, decoherence is much faster than damping, justifying the approach of neglecting the decay of the amplitude α of the coherent states in (6) and (7). This is of course in line with the idea that in the 'macroscopic' limit weird non-classical states are extremely fragile and difficult to observe. For truly macroscopic systems t_d is so short that no experimental monitoring of the decoherence process is possible. However the validity of Eq. (9) is restricted by the additional condition that decoherence is

slower than the free oscillation (this condition is fulfilled by the experiments [9] [11] discussed in Sec. 1). In this regime, the particle oscillates several times in the potential well before coherence is lost, and the r.-h.-s. of Eq. (9) may be written in terms of the distance $\Delta x = 2\sqrt{2\hbar/M\omega_0}|\alpha|$ between the two wavepackets when they are at their turning points (M is the mass of the particle):

$$t_d = 4\left(\frac{\Delta x_0}{\Delta x}\right)^2 \frac{1}{\Gamma}, \tag{10}$$

where $\Delta x_0 = \sqrt{\hbar/(2M\omega_0)}$ is the position uncertainty of the oscillator ground state. Eq. (10) shows more explicitly that the decoherence rate scales as the squared distance in phase space between the two components of the cat state. In Eq. (9), the distance is expressed in terms of the squared difference $\Delta\alpha = 2\alpha$ between the amplitudes of the two coherent states $|\pm\alpha\rangle$. Such dependence, already experimentally observed in [9], was fully verified in [11]. Thus, the decoherence rate is directly connected to the quality of which-way information, for the possibility of resolving the two wavepackets is quantified by the distance between them divided by their width Δx_0.

The second factor entering in the r.-h.-s. of (9) is the damping coefficient Γ. Rather than a phenomenological constant, here Γ quantifies the strength of the radiation pressure coupling to the vacuum field, and is calculated from first principles. As discussed in Sec. 2, it may be obtained directly from the expression for the dissipative radiation pressure force on the particle. In the 1D case, Γ is given by Eq. (4), which jointly with Eq. (9) yields

$$t_d = \frac{3}{(v/c)^2}\frac{2\pi}{\omega_0}, \tag{11}$$

where $v = \sqrt{2\hbar\omega_0/M}\,|\alpha|$ is the velocity of the wavepackets at the moment they cross the bottom of the potential well. Therefore, in the nonrelativistic limit considered in this paper, decoherence is much slower than the free oscillation. The ratio between the two time scales is even larger when considering the real 3D case. If we take a spherical perfectly-reflecting particle of radius R smaller than the range of oscillation, then $\omega_0 R/c < v/c \ll 1$. Since the relevant field modes have frequencies of the order of ω_0, in this limit the particle is much smaller than typical wavelengths (Rayleigh scattering regime), and hence is weakly coupled to the field. The dissipative force in this regime was calculated in [39]; the resulting damping coefficient scales as the squared polarizability of the sphere, leading to an additional factor $(\omega_0 R/c)^6$:

$$t_d = \frac{324}{(v/c)^2}\left(\frac{c}{\omega_0 R}\right)^6 \frac{2\pi}{\omega_0} \gg \left(\frac{c}{v}\right)^8 \frac{2\pi}{\omega_0}. \tag{12}$$

It is also possible to analyze the decoherence effect in a more complete theoretical framework, where the dynamical radiation pressure coupling between

particle and field is fully taken into account. This approach also accounts properly for damping of the particle's energy, as well as for additional effects resulting from the coupling with the field. Moreover, it allows us to analyze decoherence in the more general case of an arbitrary temperature of the field. An ab-initio Hamiltonian model for the particle-field system was derived from first principles in Ref. [26]. This model was the starting point for the discussion of decoherence in Refs. [14] and [15]. The field scattering corresponds to frequency dependent reflection and transmission coefficients that satisfy the passivity requirements discussed in [52]. This means that the dynamics of the particle does not suffer from the instabilities associated to the model of a perfect mirror (as well known from classical electron theory, Eq. (3) is plagued with 'runaway' solutions).

Here we describe the radiation pressure coupling with the alternative, more intuitive model, where the interaction Hamiltonian corresponds to the energy transfer between field and particle:

$$H_{\text{int}} = -xF, \tag{13}$$

where F is the radiation pressure force on the particle, and x its position. This type of model was extensively employed in several contexts associated to the dynamical Casimir effect [27]. Here we focus on the limit where the particle perfectly reflects the (1D) field, but a discussion of partially-reflecting mirrors along these lines is also possible. As shown below, it leads to results for the decoherence and damping rates in agreement with those found in Ref. [15].

Starting from (13), we derive a master equation for the reduced density matrix of the particle. It is similar to the master equation for QBM, derived from the position-position interaction Hamiltonian. Technically, the essential difference arises from the fact that the force operator F in (13) is quadratic in the field operators, which leads to a damping coefficient that depends on the state (and hence temperature) of the field (reservoir). Although the formalism relies on a 1D model, the final results may be generalized to the 3D case.

We write the master equation in terms of the Wigner function $W(x,p,t)$:

$$\partial_t W = -\frac{p}{M}\partial_x W + M\omega_0^{*2} x\partial_p W + 2\Gamma\partial_p(pW) + D_1\frac{\partial^2 W}{\partial p^2} - D_2\frac{\partial^2 W}{\partial x\partial p}. \tag{14}$$

The first two terms in (14) correspond to the harmonic oscillation in the potential well, with a frequency $\omega_0^* = \omega_0 + \delta\omega$ modified by the coupling with the field (on the other hand, when the interaction Hamiltonian is linear in the momentum of the particle, a mass correction appears [26]). The remaining terms describe non-unitary evolution. The damping as well as the diffusion coefficients D_1 and D_2 are time dependent and given in terms of correlation functions of the force operator. The diffusion coefficients are related to the symmetric correlation function:

$$\sigma_{FF}(t) = \langle\{F(t), F(0)\}\rangle, \tag{15}$$

where the brackets denote the anticommutator, and the average is taken over the field state (thermal equilibrium, temperature T).

The term proportional to D_2 in (14) yields a negligible contribution, so that we focus on D_1 :

$$D_1(t) = \frac{1}{2}\int \frac{d\omega}{2\pi}\sigma_{FF}[\omega]\,\mathrm{sync}_t(\omega), \tag{16}$$

where $\sigma_{FF}[\omega]$ is the Fourier transform of $\sigma_{FF}(t)$ and

$$\mathrm{sync}_t(\omega) = \frac{\sin[(\omega-\omega_0)t]}{\omega-\omega_0}$$

is a function peaked around $\omega = \omega_0$ of width $2\pi/t$. Clearly, for a time t long enough, the function $\mathrm{sync}_t(\omega)$ is so sharply peaked that $\sigma_{FF}[\omega]$ is approximately constant over the short frequency interval that contributes in the integral in Eq. (16), and hence may be replaced by its value at $\omega = \omega_0$. In this case, we find

$$D_1(t \to \infty) = \frac{1}{4}\sigma_{FF}[\omega_0]. \tag{17}$$

A sufficient (and also necessary at $T = 0$) condition for the validity of (17) is $\omega_0 t \gg 1$. In other words, for times much longer than the period of oscillation, the field fluctuations at frequency ω_0 provide the dominant contribution to diffusion.

The damping coefficient is likewise connected to the average value of the *commutator* of the force operator taken at different times (anti-symmetric correlation function). When the interaction Hamiltonian is linear in the operators of the environment, as in the position-position model, the commutator is a c-number times a delta function (in time), and as a consequence, the damping coefficient has a constant value that does not depend on the state of the environment. As already mentioned, this is not the case for radiation pressure coupling. In particular, the damping coefficient depends on the temperature of the field, as could be expected having in mind the Stefan-Boltzmann law. At zero temperature, we recover the result given by Eq. (4).

We calculate the pointer states using the predictability sieve criterion, discarding all information about the environment, as discussed in Sec. 1. We start from the master equation, and evaluate the rate of change of linear entropy, assuming an initial pure state. It is straightforward to show that the entropy is minimized for minimum uncertainty Gaussian states, hence the pointer states are the coherent states. This result agrees with the well-known fact that coherent states provide the closest possible realization of a classical state of oscillation, given the constraint imposed by the Heisenberg uncertainty relation. In short, coherent states remain approximately pure because they do not entangle with field states, at least for times shorter than the damping time $1/\Gamma$, as shown by Eq. (6).

On the opposite extreme in Hilbert space, superpositions of coherent states are highly nonclassical and cannot last when the distance between the two components is large. This may be analyzed in detail from Eq. (14). The coherence of the initial state is imprinted on the Wigner function in the form of an interference term W_{int} that oscillates in phase space. When the two state components

are spatially separated by a distance Δx, the oscillation is along the axis of momentum: $W_{\rm int}(x,p) \sim \cos(\Delta x\, p/\hbar)$. Thus, according to Eq. (14), diffusion washes out this oscillatory term, the faster the larger the value of Δx. With an additional factor of 2 to take into account the average over several free rotations of the state in phase space, we find

$$t_d = 2\frac{\hbar^2}{D_1(\Delta x)^2}. \tag{18}$$

To derive the decoherence time when the field is in the vacuum sate, we compute the correlation function $\sigma_{FF}[\omega_0]$ at zero temperature. When replacing the result for D_1 as given by (17) into (18), we obtain the same result already derived in this Section by a more elementary method.

For finite temperatures, the spectrum is approximately constant at low frequencies, so that (17) also holds when $\omega_0 \ll kT/\hbar$ (k is the Boltzmann constant), including the free particle limit $\omega_0 = 0$, provided that the entire frequency interval around ω_0 is contained in the low frequency part of the spectrum, which corresponds to the condition $1/t \ll kT/\hbar$. The damping coefficient may calculated in the high temperature limit as well, and the results are in agreement with Einstein relation

$$D_1 = 2MkT\Gamma. \tag{19}$$

More generally, we may derive a relation between diffusion and damping coefficients valid for arbitrary values of temperature [15], including $T = 0$, starting from the general relation between symmetric and anti-symmetric correlation functions (fluctuation-dissipation theorem).

The decoherence time for high T is derived by replacing (19) into (18). As we discuss below, usually in this limit decoherence is faster than the free oscillation, so that, contrary to the $T = 0$ case, there is no average over many oscillations in this case. To describe the decoherence process, we must evaluate the diffusion coefficient at a time t much shorter than t_d. Hence, we are allowed to use its asymptotic value as given by the Einstein relation (19) only if we assume that $t_d \gg \hbar/(kT)$. The resulting expression is very general [2], and also holds in the free particle case:

$$t_d = \frac{\lambda_T^2}{(\Delta x)^2}\frac{1}{\Gamma}, \tag{20}$$

where $\lambda_T = \hbar/\sqrt{2MkT}$ is the de Broglie wavelength of a particle of mass M in thermal equilibrium. Eq. (20) has a form similar to (10), except that now the reference of distance is set by thermal fluctuations instead of zero point fluctuations.

In order to complete the evaluation of the decoherence time, we need to evaluate the damping coefficient Γ in the high temperature limit. We consider as before a sphere of radius R, which is usually much larger than typical field wavelengths, which are of the order of $\hbar c/(kT)$ (except for very low temperatures or very small spheres). In this short-wavelength regime, the radiation pressure

force may be calculated by replacing the surface of the sphere by a collection of tangent planes, and the final result reads

$$F = -\frac{4\pi^3}{45}\frac{(kT)^4}{\hbar^3 c^4}R^2\frac{dx}{dt}. \tag{21}$$

The force scales with the surface or cross section of the sphere, and is proportional to T^4, in agreement with Stefan-Boltzmann law. As opposed to the vacuum case, here we have a true friction force, i.e. proportional to the velocity of the particle and not to higher-order time derivative as in Eq. (2) (the thermal field is not Lorentz invariant).

In the free case ($\omega_0 = 0$), Γ is simply the coefficient multiplying the velocity in Eq. (21) divided by M. Then, with the help of (20) we find

$$t_d = \frac{45}{8\pi^3}\frac{\hbar^5 c^4}{(kT)^5 R^2 (\Delta x)^2}. \tag{22}$$

Eq. (22) shows that the decoherence time depends strongly on temperature (the same temperature dependence was found in Ref. [54]). Even at the temperature corresponding to the cosmic background radiation, $T = 2.7$K, radiation pressure is a very efficient source of decoherence. As an example, for $R = 1$cm, we have $t_d = 2.7 \times 10^{-21}/(\Delta x[\mathrm{m}])^2$ s, which is in the nanosecond range for a separation $\Delta x = 1\mu$m.

4 Conclusion

The master equation provides a complete description of the particle dynamics when no measurement on the field is made. It accounts for the renormalization of the oscillation frequency, damping, and diffusion and the associated decoherence effect. It also allows for the determination of the pointer states, and all that for any temperature T. On the other hand, the decoherence time scale at $T = 0$ may be calculated by a simpler approach, in which we follow the evolution of the complete particle-field state, calculated with the help of the superposition principle, and trace over the field at the very end. This approach explicitly shows that decoherence results from entanglement between particle and field states.

The decoherence induced by radiation pressure coupling with vacuum fluctuations is a very slow effect, when compared with the the time scale of the free evolution. Yet, it is remarkable, from a conceptual point-of-view, that classical behavior of a macroscopic system emerges from the formalism of Quantum Mechanics itself, even though in very long time scale, provided that the quantum vacuum radiation field is taken into account.

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