Optimization and Control Theory for Smart Grids

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April 21, 2010, MIT/LIDS

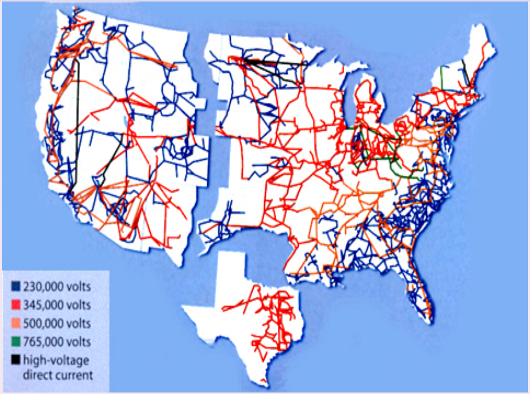
Outline

Optimization and Control Theory for Smart Grids

- So what? Impact.
  - savings: (a) 30b$ annually is the cost of power losses,
    10% efficiency improvement=> 3b$ savings,
    (b) cost of 2003 blackout is 7-10b$, 80b$ is the total cost of blackouts annually in US
  - further challenges (more vulnerable, cost of not doing planning, control, mitigation)
- Grid is being redesigned [stimulus]

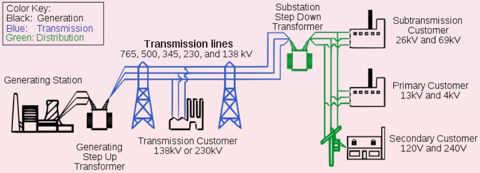
The research is timely.

-2T$ in 20 years (at least)



US power grid

The greatest Engineering Achievement of the 20th century



will require **smart revolution** in the 21st century

Preliminary Remarks

The power grid operates according to the laws of electrodynamics

- Transmission Grid (high voltage) vs Distribution Grid (low voltage)
- Alternating Current (AC) flows ... but DC flow is often a valid approximation
- No waiting period $\Rightarrow$ power constraints should be satisfied immediately. Adiabaticity.
- Loads and Generators are players of two types (distributed renewable will change the paradigm)
- At least some generators are adjustable - to guarantee that at each moment of time the total generation meets the total load
- The grid is a graph ... but constraints are (graph-) global

Our (LANL) *Road Map* for Smart Grids

New Systems

Renewables

PHEV & Storage

Metering

Grid Stability

- distance to failure
- dynamic stability
- outages/rare events
- cascading
- signature detection

Grid Control

- load balancing
- queuing and scheduling
- optimal power flows
- feeder lines control
- distribution and switching with redundancy

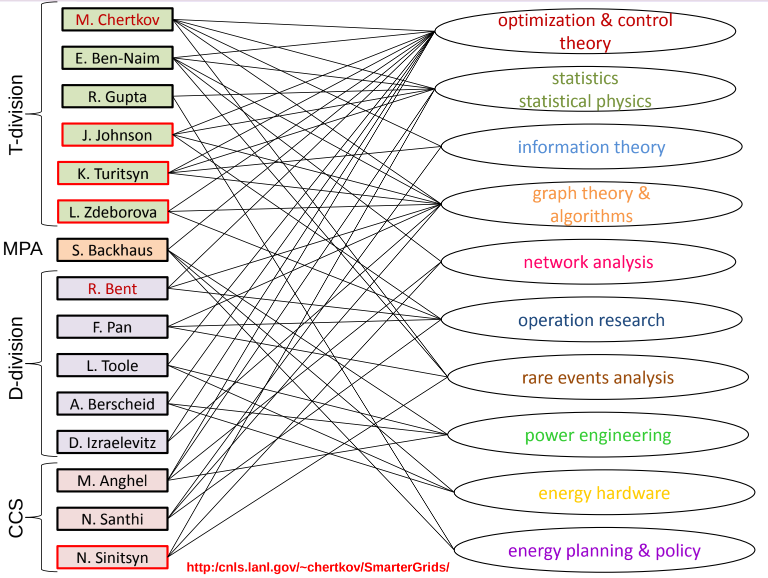
Grid Design

- placement of generations and storage
- intermittent wind as an integrated capacity
- accounting for outages and intermittency in planning

New Challenges

All of the above also requires scientific advances in

- Analysis & Control
- Stability/Reliability Metrics
- State Estimation
- Data Aggregation & Assimilation
- Middleware for the Grid
- Modeling Consumer Response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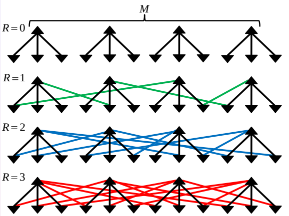
graph theory & algorithms
network analysis
operation research
rare events analysis
power engineering
energy hardware
energy planning & policy
http:/cnls.lanl.gov/~chertkov/SmarterGrids/

Outline

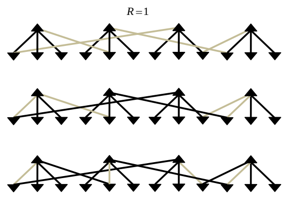
Methodology/Disclaimer

- Approach A: Take a realistic power grid model and run simulations. Time Consuming ... and need a new setting when details change
- Approach B (probabilistic + physicist/applied.math way): Study behavior of simple abstract models that facilitate the analysis, and look for universal features. Model choice criteria (in physics): The simpler but richer the better.

Optimization & Control of Power Grid [L. Zdeborova, A. Decelle, MC '09]



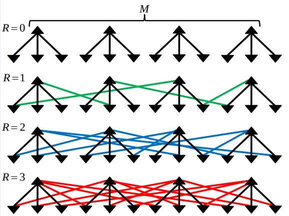
A: $R = 0; 1; 2; 3$. Graph samples. Ancillary connections to foreign generators/consumers are shown in color.



B: $R = 1$. Three valid (SAT) configurations (shown in black, the rest is in gray) for a sample graph shown in Fig. A.

- Can the ancillary lines (redundancy) help?
- Design and efficient switching algorithm for finding SAT solution.

Combinatorial Model of Switching



- M producers, $N = DM$ consumers
- Of D consumers R has an auxiliary line
- Consumer i consumes x_i
- In [Zdeborova, Backhaus, MC '09] we also model renewables = allow consumer to generate z_i

- Capacity of producer α is y_α
- Setting: switch variables $\sigma_{i\alpha} = 0, 1$ for power lines
- Constraint (1): each consumer has exactly one line - $\sum_{\alpha\sim i} \sigma_{i\alpha} = 1$
- Constraint (2): Producers are not overloaded - $\sum_{i\sim\alpha} \sigma_{i\alpha}(x_i - z_i) \le y_\alpha$
- Assume certain statistics of consumption/production for consumers

Highlights of our Approach

- To analyze the SAT-UNSAT transition we solved Cavity Equations with Population Dynamics Algorithm. The approach allows
  (a) to compute the probability that the given switch is off/on
  (b) to estimate number of valid (not overloading) configurations.
- We also developed Belief Propagation and WalkGrid (greedy stochastic search, younger brother of WalkSum for K-SAT) algorithms which find SAT-switching efficiently.

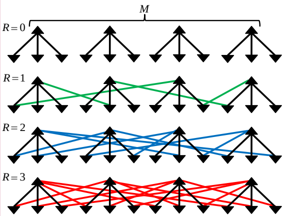
Simple (no renewables, $z = 0$) Model of Switching

Statistics of load fluctuations

$$\mathcal{P}(\mathbf{x}) = \prod_i [(1-\epsilon)p(x_i) + \epsilon\delta(x_i)]$$

$$p(x) = \begin{cases} 1/\Delta, & |x-\bar{x}| < \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$

- $\bar{x}, \Delta$ and ϵ are parameters.

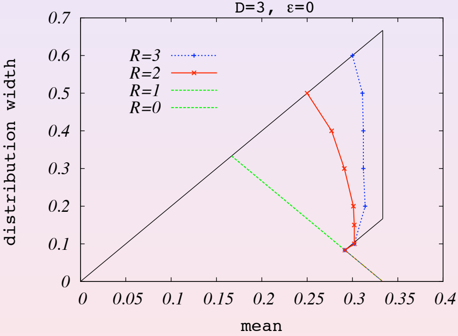


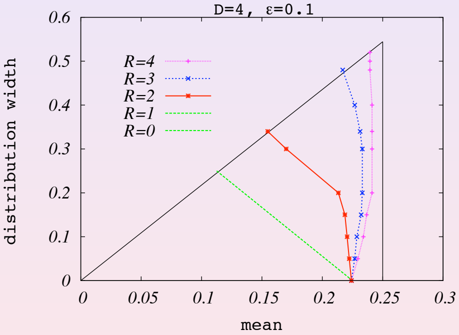
Some simple conditions

- $2\bar{x} \geq \Delta$ - individual consumption is non-negative
- $(1-\epsilon)\bar{x} \leq \frac{M}{N} = \frac{1}{D}$ - typical case is SAT
- $(D-R+1)\left(\bar{x} + \frac{\Delta}{2}\right) \leq 1$ - SAT condition (for a generator) with redundancy
- $\epsilon = 0: \quad (D+1)(\bar{x} - \frac{\Delta}{2}) \leq 1$ - $(D+1)$ consumers connected to a generator and drawing minimum amount do not overload the generator

Average case (cavity/population dynamics) analysis

Less trivial conditions (in color)

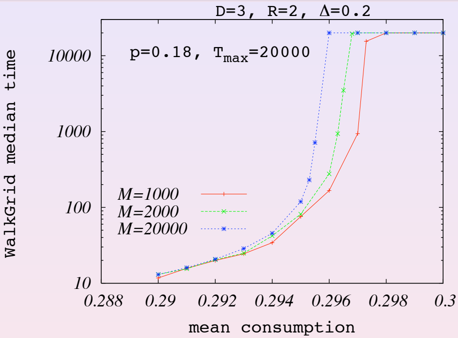


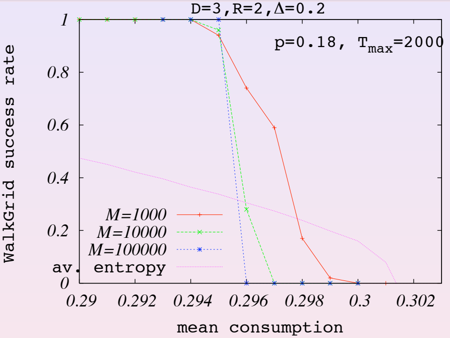


- $(1-\epsilon)\bar{x}$ and Δ are the distribution mean and width
- SAT/UNSAT = bottom-left/up-right

▸ The Story of Renewables

Walkgrid is an efficient algoritm for switching





WalkGrid

Assign each value of σ 0 or 1 randomly (but such that $\forall i \in G : \sum_{\alpha \in \partial i} \sigma_{i\alpha} = 1$);
repeat Pick a random power generator α which shows an overload, and denote the value of the overload, δ;
Choose a random consumer i connected to the generator α, i.e. $\sigma_{i\alpha} = 1$;
Pick an arbitrary other generator which is not overloaded and consider switching connection from $(i\alpha)$ to $(i\beta)$.
if (in the result of this switch α is relieved from being overloaded
and β either remains under the allowed load or it is overloaded but by the amount less than δ)
Accept the move, i.e. disconnect i from α and connect it to β thus setting $\sigma_{i\beta} = 1$, $\sigma_{i\alpha} = 0$.
else With probability p connect consumer i to β instead of α;
until Solution found or number of iterations exceeds $MT_{\max}$.

We are working on

- Application of the approach to more realistic distribution grids
- Extending the story beyond "the commodity flow" approach towards accounting for AC/DC specifics of the power flows
- Switching vs Contingency. Off-line games. Control Algorithms.

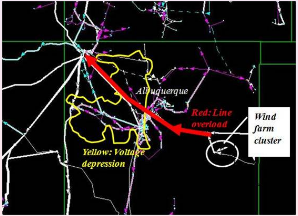
Bibliography

- L. Zdeborova, A. Decelle, M. Chertkov, Phys. Rev. E 90, 046112 (2009).
- L. Zdeborova, S. Backhaus, M. Chertkov, in proceedings of HICSS 43 (Jan 2010).

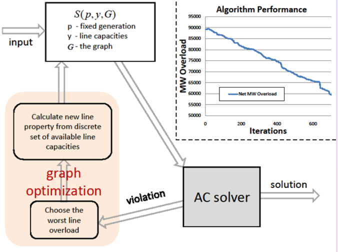
Outline

Grid Design: Motivational Example

- Cost dispatch only (transportation,economics)
- Power flows highly approximate
- Unstable solutions
- Intermittency in Renewables not accounted



An unstable grid example



Hybrid Optimization - is current "engineering" solution developed at LANL: Toole,Fair,Berscheid,Bent 09 extending and built on NREL "20% by 2030 report for DOE

Network Optimization ⇒

- Design of the Grid as a tractable global optimization

The Kirchhoff Laws. Loss Function. Power flows.

The Kirchhoff Laws

$$\forall a \in \mathcal{G}_0: \quad \sum_{b\sim a} J_{ab} = J_a \text{ for currents}$$
$$\forall (a,b) \in \mathcal{G}_1: \quad J_{ab} z_{ab} = U_a - U_b \text{ for potentials}$$

Loss Function

$$Q(\hat{G}) = \sum_{\{a,b\}\in\mathcal{G}_1} \Re\left(\frac{1}{z_{ab}}\right)|U_a - U_b|^2 = \mathbf{J}^+(\hat{G}')^{-1}\mathbf{J}^* = \mathbf{U}^+\hat{G}\mathbf{U}^*$$
$$\mathbf{U} = (\hat{G}')^{-1}\mathbf{J}, \quad \hat{G}' = \hat{G} + \mathbf{1}\mathbf{1}^+, \quad \hat{G} = (G_{ab}|a,b \in \mathcal{G}_0)$$
$$G_{ab} = \begin{cases} 0, & a \neq b, \quad a \nsim b \\ -g_{ab}, & a \neq b, \quad a \sim b \\ \sum_{c\neq a}^{c\sim a} g_{ac}, & a = b. \end{cases}, \quad \underbrace{(z_{ab})^{-1}}_{admittance} = \underbrace{g_{ab}}_{conductance} + i\underbrace{\beta_{ab}}_{susceptance}$$

Complex Power Flow [balance of power]

$$\forall a: \quad P_a = U_a J_a^* = J_a^* \sum_b (\hat{G}')^{-1}_{ab} J_b = U_a \sum_{b\sim a} J_{ab}^* = U_a \sum_{b\sim a} \frac{U_a^* - U_b^*}{z_{ab}^*}$$
$$\forall a: \quad U_a = u_a \exp(i\varphi_a), \quad P_a = p_a + i q_a$$

DC flow approximation

(0) The amplitude of the complex potentials are all fixed to the same number (unity, after trivial re-scaling): $\forall a: \quad u_a = 1$.

(1) $\forall\{a,b\}: \quad |\varphi_a - \varphi_b| \ll 1$ - phase variation between any two neighbors on the graph is small

(2) $\forall\{a,b\}: \quad r_{ab} \ll x_{ab}$ - resistive (real) part of the impedance is much smaller than its reactive (imaginary) part. Typical values for the r/x is in the $1/27 \div 1/2$ range.

(3) $\forall a: \quad p_a \gg q_a$ - the consumed and generated powers are mainly real, i.e. reactive components of the power are much smaller than their real counterparts

It leads to

- Linear relation between powers and phases (at the nodes): $\hat{B}\boldsymbol{\varphi} = \mathbf{p}$ and $\sum_a p_a = 0$
- Losses (in the leading order): $\mathcal{Q} = \mathbf{p}^+(\hat{B}')^{-1}\hat{G}(\hat{B}')^{-1}\mathbf{p}, \quad \hat{B}' = \hat{B} + \mathbf{1}\mathbf{1}^+$
- If all lines are of the same grade: $\hat{B} = \hat{G}/\alpha$ and $\mathcal{Q} = \alpha^2\mathbf{p}^+(\hat{G}')^{-1}\mathbf{p}$, i.e. the system is equivalent to "resistive network", where $\hat{B}/\alpha^2$, $\mathbf{p}$ and $\boldsymbol{\varphi}$ play the roles of the resistivity matrix, vector of currents and vector of voltages

Network Optimization (for fixed production/consumption $\mathbf{p}$)

$$\underbrace{\min_{\hat{g}} \mathbf{p}^{+}\left(\hat{G}(\hat{g})\right)^{-1}\mathbf{p},}_{\text{convex over } \hat{g}} \qquad \underbrace{G_{ab} = \begin{cases} 0, & a \neq b, \quad a \nsim b \\ -g_{ab}, & a \neq b, \quad a \sim b \\ \sum_{c\neq a}^{c\sim a} g_{ac}, & a = b. \end{cases}}_{\text{Discrete Graph Laplacian of conductance}}$$

Network Optimization (averaged over $\mathbf{p}$)

$$\min_{\hat{g}}\langle \mathbf{p}^{+}\left(\hat{G}(\hat{g})\right)^{-1}\mathbf{p}\rangle = \min_{\hat{g}} \mathrm{tr}\left(\left(\hat{G}(\hat{g})\right)^{-1}\langle \mathbf{p}\mathbf{p}^{+}\rangle\right) =$$

$$\underbrace{\min_{\hat{g}} \mathrm{tr}\left(\left(\hat{G}(\hat{g})\right)^{-1}\hat{P}\right),}_{\text{still convex}} \quad \hat{P} - \text{covariance matrix of load/generation}$$

Boyd,Ghosh,Saberi '06 in the context of resistive networks

also Boyd, Vandenberghe, El Gamal and S. Yun '01 for Integrated Circuits

Network Optimization: Losses+Costs [J. Johnson, MC '10]

Costs need to account for

- "sizing lines" - grows with g_{ab}, linearly or faster (convex in $\hat{g}$)
- "breaking ground" - l_0-norm (non convex in $\hat{g}$) but also imposes desired sparsity

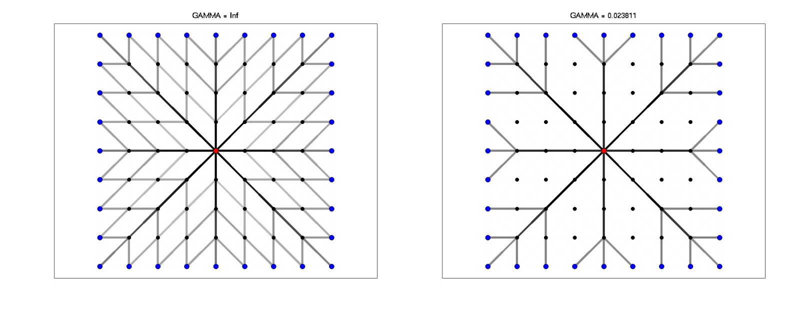
Resulting Optimization is non-convex

$$\min_{\hat{g}>0}\left(\mathrm{tr}\left(\left(\hat{G}(\hat{g})\right)^{-1}\hat{P}\right)+\sum_{\{a,b\}}\left(\alpha_{ab}g_{ab}+\beta_{ab}\phi_\gamma(g_{ab})\right)\right),\quad \phi_\gamma(x)=\frac{x}{x+\gamma}$$

Tricks (for efficient solution of the non-convex problem)

- "annealing": start from large (convex) γ and track to $\gamma\to 0$ (combinatorial)
- Majorization-minimization (from Candes, Boyd '05) for current γ: $\hat{g}^{t+1}=\mathrm{argmin}_{\hat{g}>0}\left(\mathrm{tr}(\mathcal{L})+\hat{\alpha}.*\hat{g}+\hat{\beta}.*\phi'_\gamma(g^t_{ab}).*g_{ab}\right)$

Single-Generator Examples



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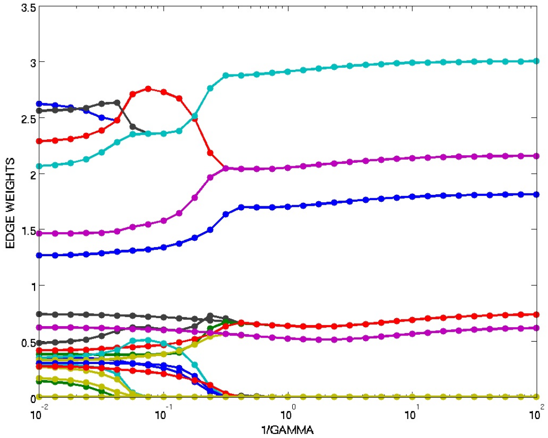

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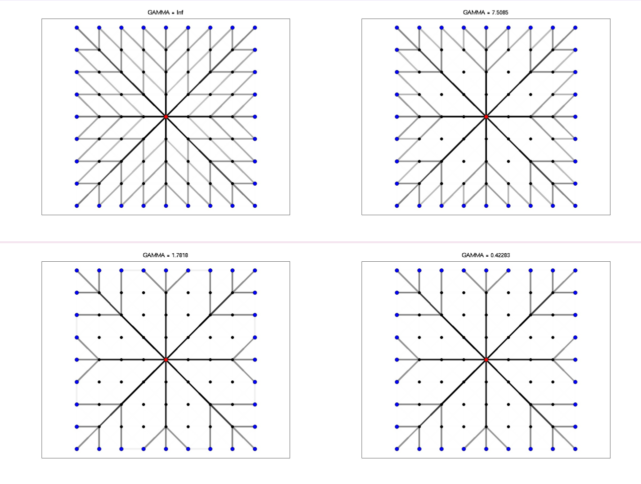

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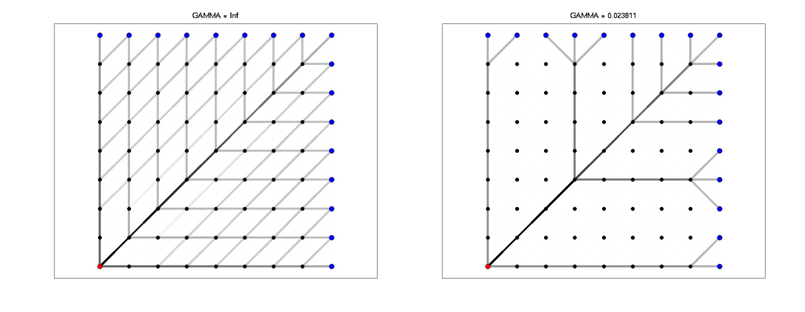

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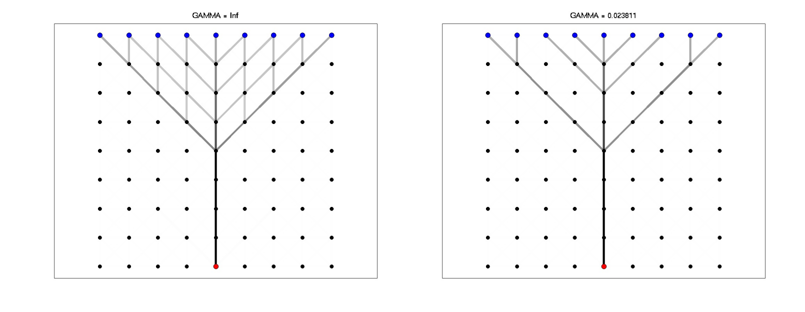

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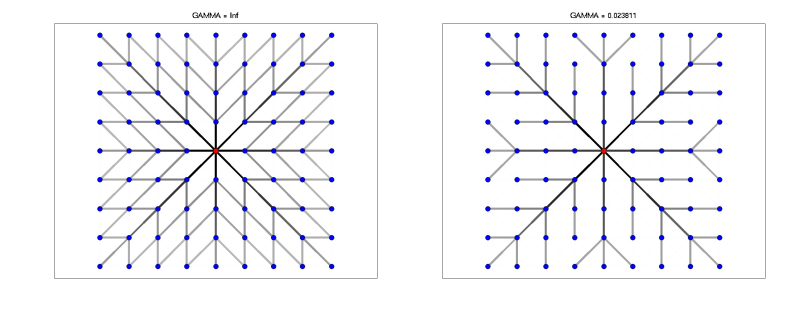

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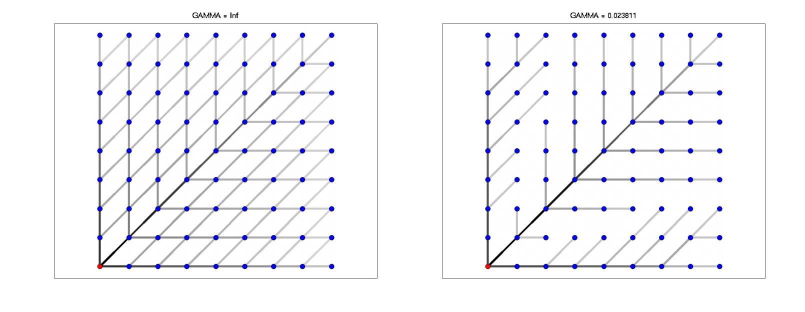
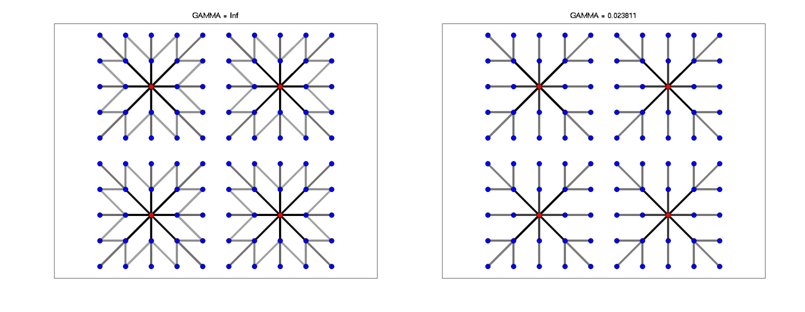


Multi-Generator Example



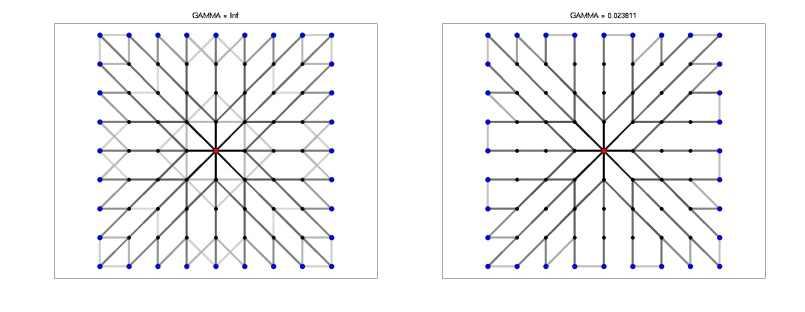
Adding Robustness

To impose the requirement that the network design should be robust to failures of lines or generators, we use the worst-case power dissipation:

$$\mathcal{L}^{\setminus k}(\hat{g}) = \max_{\forall\{a,b\}:z_{ab}\in\{0,1\}|\sum_{\{a,b\}} z_{ab}=N-k} \mathcal{L}(\hat{z}.*\hat{g}))$$

- It is tractable to compute only for small values of k.
- Note, the point-wise maximum over a collection of convex function is convex.
- So the linearized problem is again a convex optimization problem at every step continuation/MM procedure.

Single-Generator Examples



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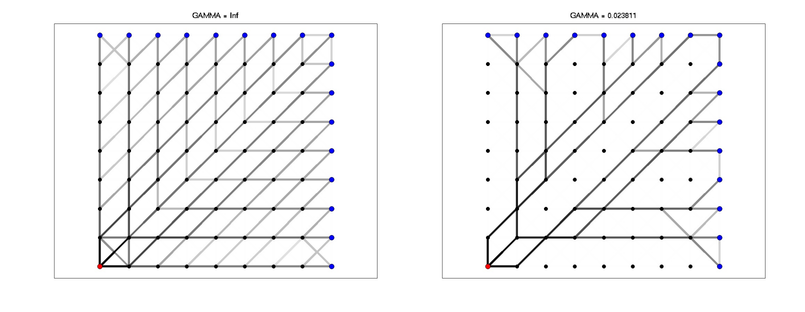

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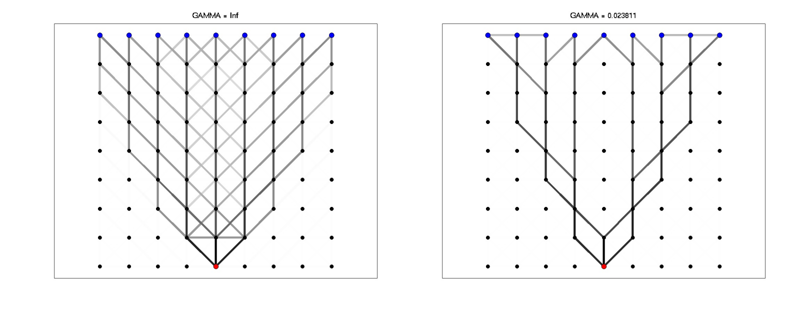

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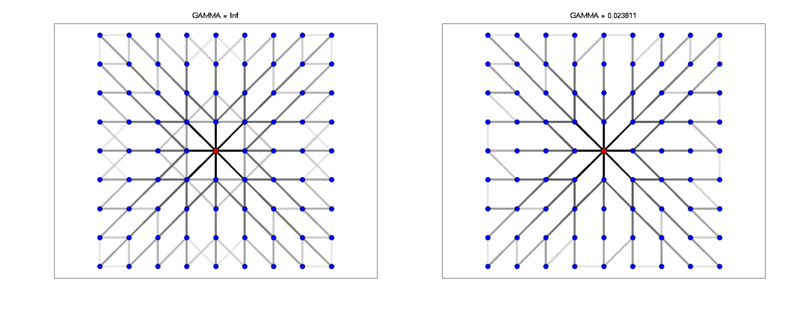

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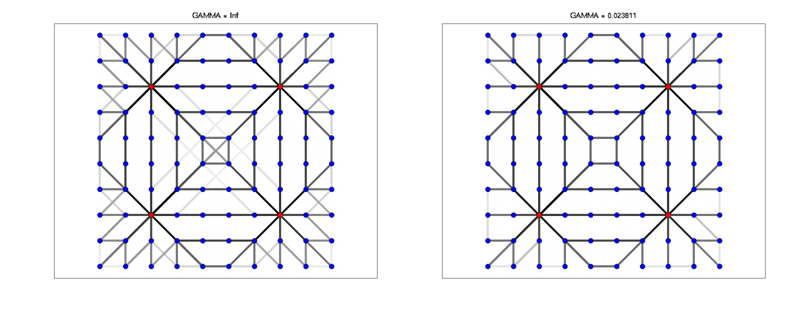


Conclusion (for the Network Optimization part)

A promising heuristic approach to design of power transmission networks. However, cannot guarantee global optimum.

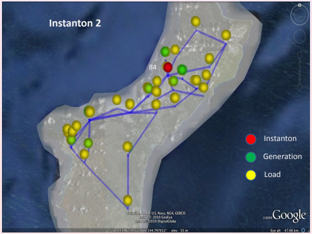
- Submitted to CDC10: `http://arxiv.org/abs/1004.2285`

Future Work:

- Bounding optimality gap?
- Use non-convex continuation approach to place generators
- possibly useful for graph partitioning problems
- adding further constraints (e.g. don't overload lines)
- extension to (exact) AC power flow?

Outline

- Normally the grid is ok (SAT) ... but sometimes failures (UNSAT) happens
- How to estimate a probability of a failure?
- How to predict (anticipate and hopefully) prevent the system from going towards a failure?
- Phase space of possibilities is huge (finding the needle in the haystack)



Model of Load Shedding [MC, F.Pan & M.Stepanov '10]

Minimize Load Shedding = Linear Programming for DC

$$LP_{DC}(\mathbf{d}|\mathcal{G};\mathbf{x};\mathbf{u};\mathbf{P}) = \min_{\mathbf{f},\varphi,\mathbf{p},\mathbf{s}} \left(\sum_{a\in\mathcal{G}_d} s_a\right)_{COND(\mathbf{f},\varphi,\mathbf{p},\mathbf{d},\mathbf{s}|\mathcal{G};\mathbf{x};\mathbf{u};\mathbf{P})}$$

$$COND = COND_{flow} \cup COND_{DC} \cup COND_{edge} \cup COND_{power} \cup COND_{over}$$

$$COND_{flow} = \left(\forall a: \quad \sum_{b\sim a} f_{ab} = \begin{cases} p_a, & a\in\mathcal{G}_p \\ -d_a+s_a, & a\in\mathcal{G}_d \\ 0, & a\in\mathcal{G}_0\setminus(\mathcal{G}_p\cup\mathcal{G}_d)\end{cases}\right)$$

$$COND_{DC} = \left(\forall\{a,b\}: \varphi_a-\varphi_b+x_{ab}f_{ab}=0\right), \quad COND_{edge} = \left(\forall\{a,b\}: -u_{ab}\le f_{ab}\le u_{ab}\right)$$

$$COND_{power} = \left(\forall a: \quad 0\le p_a\le P_a\right), \quad COND_{over} = \left(\forall a: \quad 0\le s_a\le d_a\right)$$

φ -phases; f -power flows through edges; x - inductances of edges

SAT/UNSAT & Error Surface

Statistics of Loads

$$\mathcal{P}(\mathbf{d}|\mathbf{D};c) \propto \exp\left(-\frac{1}{2c}\sum_i \frac{(d_i - D_i)^2}{D_i^2}\right)$$

D is the normal operational position in the space of demands

Instantons (special instances of demands from the error surface)

- Points on the error-surface maximizing $\mathcal{P}(\mathbf{d}|\mathbf{D};c)$ - locally!
- $\arg\max_{\mathbf{d}} \mathcal{P}(\mathbf{d})|_{LP_{DC}(\mathbf{d})>0}$ - most probable instanton
- The maximization is not concave (multiple instantons)

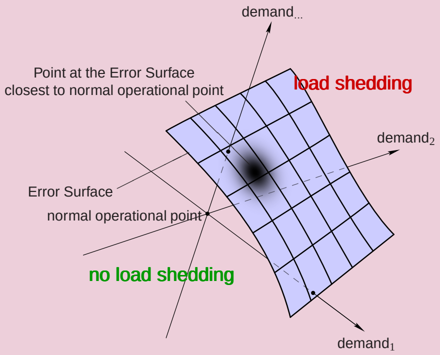
No Shedding (SAT) - Boundary - Shedding (UNSAT) = Error Surface

The task: to find the most probable failure modes [instantons]

Instanton Search Algorithm [Sampling]

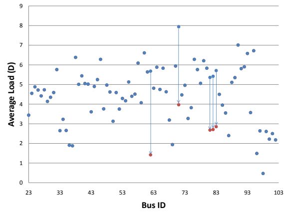
Borrowed (with modifications) from Error-Correction studies: analysis of error-floor [MC, M.Stepanov, et al '04-'10]

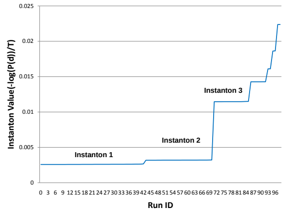
- Construct $\mathcal{Q}(\mathbf{d}) = \begin{cases} \mathcal{P}(\mathbf{d}), & LP_{DC}(\mathbf{d}) > 0 \\ 0 & , LP_{DC}(\mathbf{d}) = 0 \end{cases}$
- Generate a simplex (N+1 points) of UNSAT points
- Use Amoeba-Simplex [Numerical Recepies] to maximize $\mathcal{Q}(\mathbf{d})$
- Repeat multiple times (sampling the space of instantons)

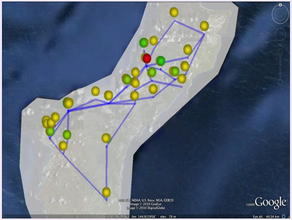


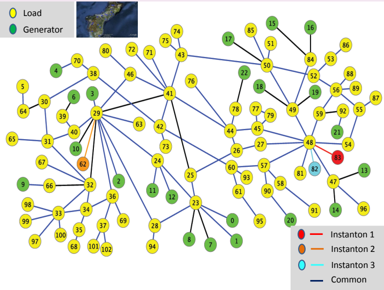
Example of Guam

[MC, F.Pan & M.Stepanov '10]



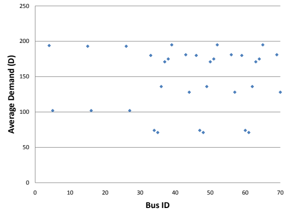


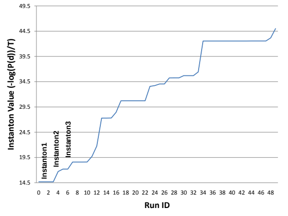


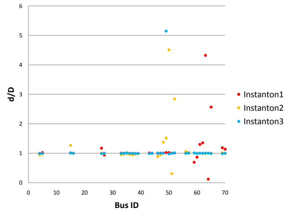


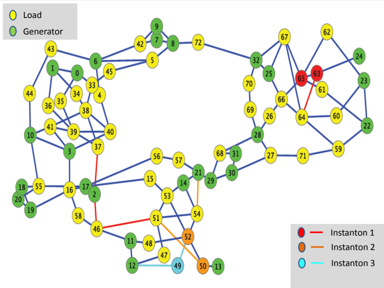
- The instantons are sparse (localized on troubled nodes)
- The troubled nodes are repetitive in multiple-instantons
- Instanton structure is not sensitive to small changes in **D** and statistics of demands

Example of IEEE RTS96 system [MC, F.Pan & M.Stepanov '10]









- The instantons are well localized (but still not sparse)
- The troubled nodes and structures are repetitive in multiple-instantons
- Instanton structure is not sensitive to small changes in **D** and statistics of demands

Concusions and Path Forward (for the distance to failure)

Conclusions

- Formulated Load Shedding (SAT/UNSAT condition) as an LP_{DC}
- Posed the problem of the Error-Surface and Instantons description in the power-grid setting
- Instanton-amoeba algorithm was suggested and tested on examples

Path Forward

- The instanton-amoeba allows upgrade to other (than LP_{DC}) network stability testers, e.g. for AC flows and transients
- Instanton-search can be accelerated, utilizing LP-structure of the tester (in the spirit of [MC,MS '08])
- This is an important first step towards exploration of "next level" problems in power grid, e.g. on interdiction [Bienstock et. al '09], optimal switching [Oren et al '08], cascading outages [Dobson et al '06], and control of the extreme [outages] [Ilic et al '05]

Bottom Line

- A lot of interesting power grid settings for CS/IT, Physics analysis
- The research is timely (blackouts, renewables, stimulus)
- Stay tuned ... the field is growing explosively

Other Problems under investigation by the team

- Control of Reactive Power Flow in a radial circuit [K. Turtisyn, P. Sulc, S. Backhaus, MC '09 = arXiv:0912.3281 + selected for Super Session of IEEE/PES Gen Mtg 2010]
- Efficient PHEV charging via queuing/scheduling with and without communications and delays [S. Backhaus, MC, K. Turitsyn, N. Sinitsyn]

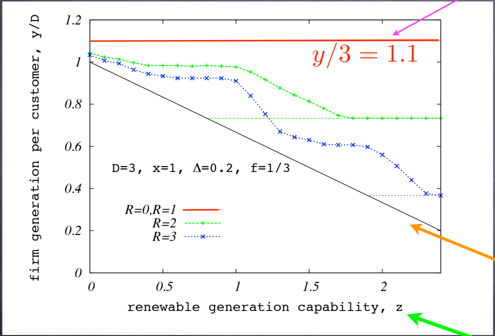
For more info - check:

Thank You!

Switching/control with renewables (I)

Example n. 1

somebody must serve D-R+1 fully demanding consumers



\# producers

$M \to \infty$

\# consumers

$N = 3M$

generation > consumption

$y/3 = 1 - z/3$

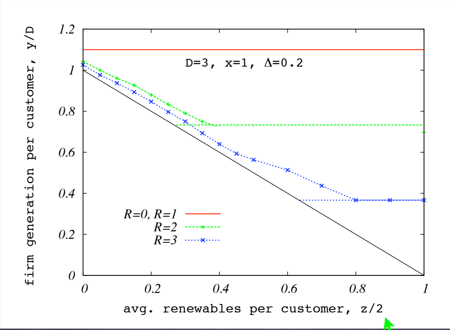
Fraction 1/3 of consumers produce amount z

Every consumer consumes random number in (0.9,1.1)

Main Switching Story

Switching/control with renewables (II)

Example n. 2



Every consumer produced a random number between (0,z)

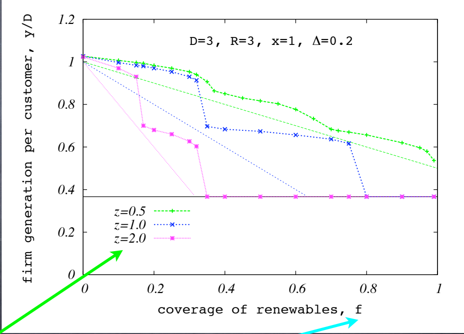
Main Switching Story

Switching/control with renewables (III)

Example n. 3

produced > consumed

$y/3 > 1 - fz$



amount z is produced by fraction f of consumers

Main Switching Story