Message passing for integrating and assessing renewable generation in a redundant power grid

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Assessing renewable generation

- Intermittent renewable-sources-based generation destabilizes the grid.
  How to improve grid control schemes?

- If renewable sources produce power x, how much can be saved on the level of the firm generation?

Improvement trough redundancy

- Build additional power lines and introduce switches (on / off = power line connected to / disconnected from the network)

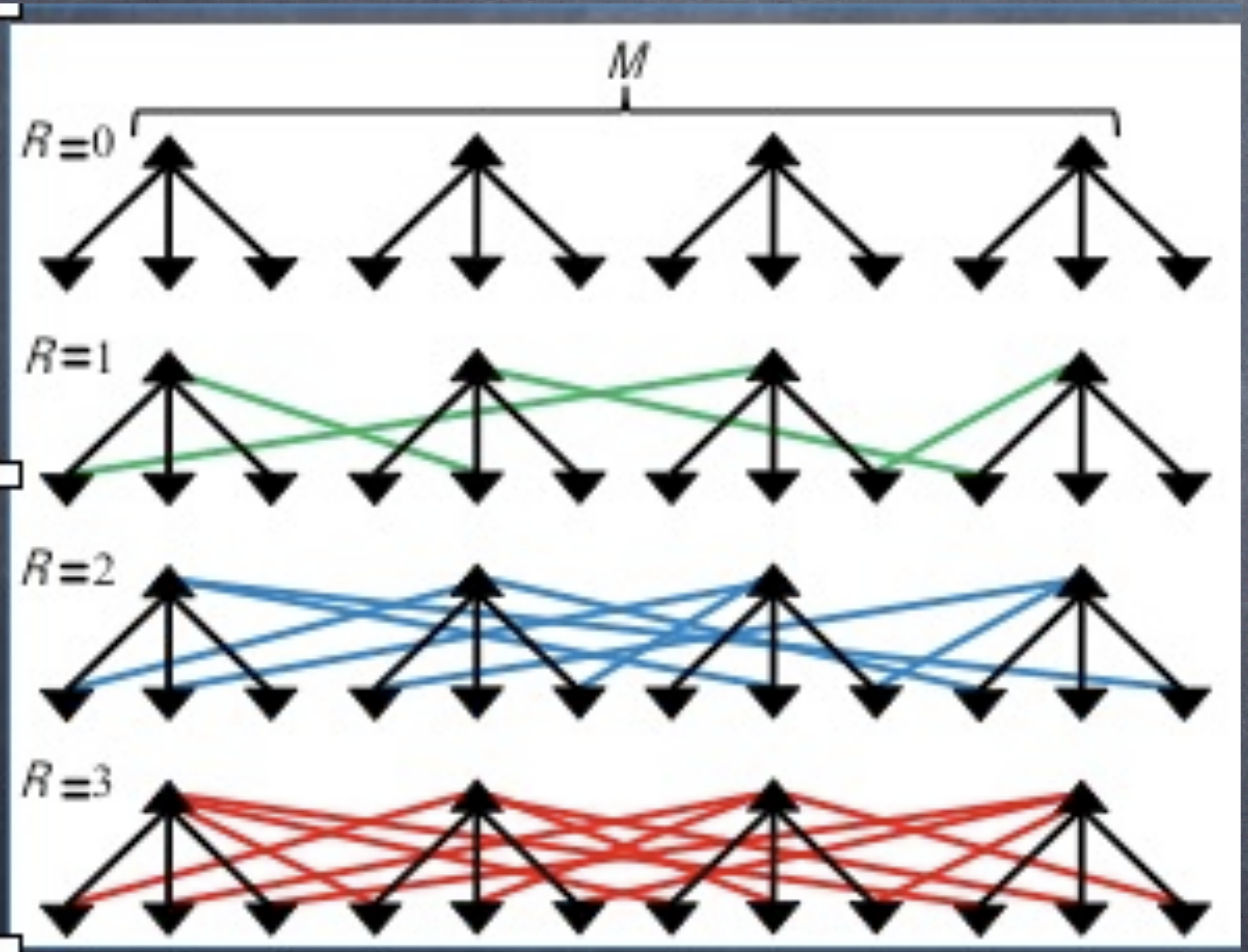
- Redundancy must help to optimize both stability and efficiency - larger space to optimize over. But how much does redundancy help?

Methodology:

- Approach A: Take a realistic power grid model and several computers and run simulations. Do again when details change ...
- Approach B (probabilistic + physicist way): Study behavior of simple abstract models that facilitate the analysis, and look for universal properties, dependencies and behavior.
  Model choice criteria (in physics): The simpler and richer the better.

Our power grid model

- M producers, N=DM consumers
- Out of every D consumers R have auxiliary lines



Consumer "i" consumes x_i

produces z_i

Producer "a" capability y_a

M=4, N=12, D=3

Setting

Switch variables for power lines:

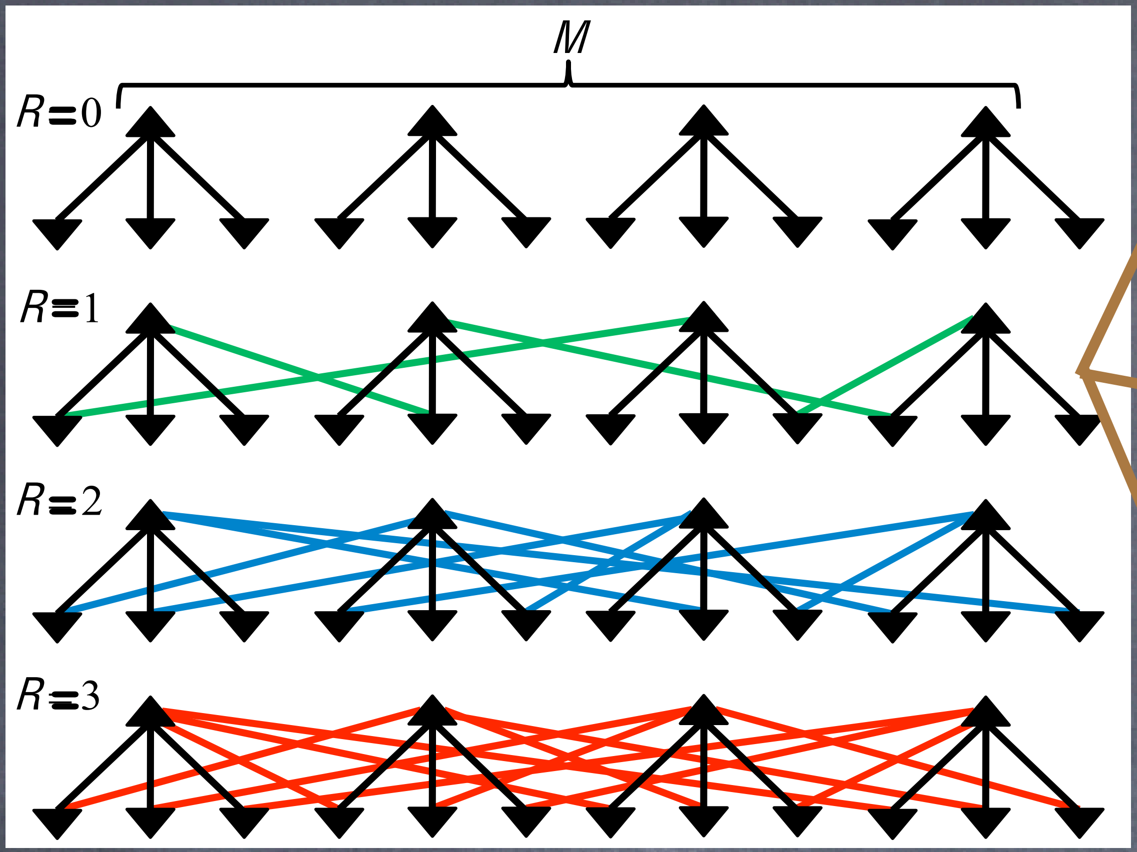
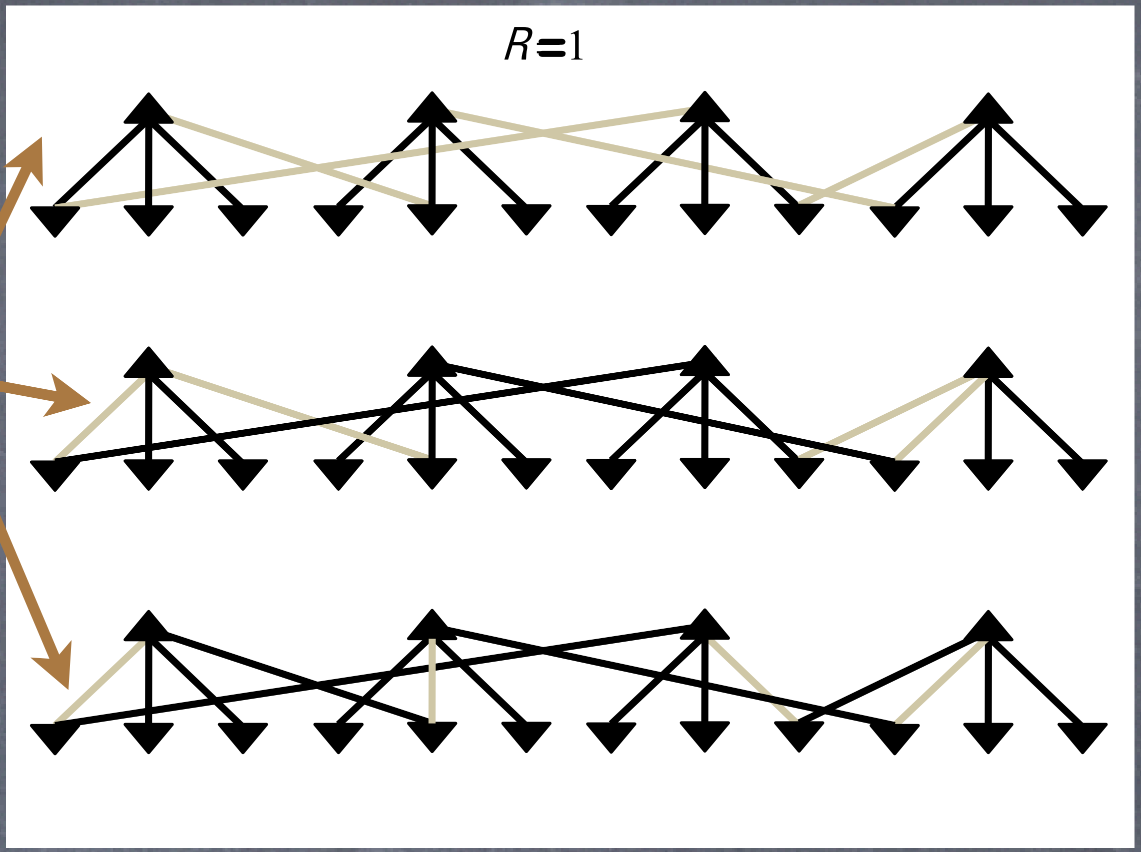
$$\sigma_{ia} = 0 / \sigma_{ia} = 1$$

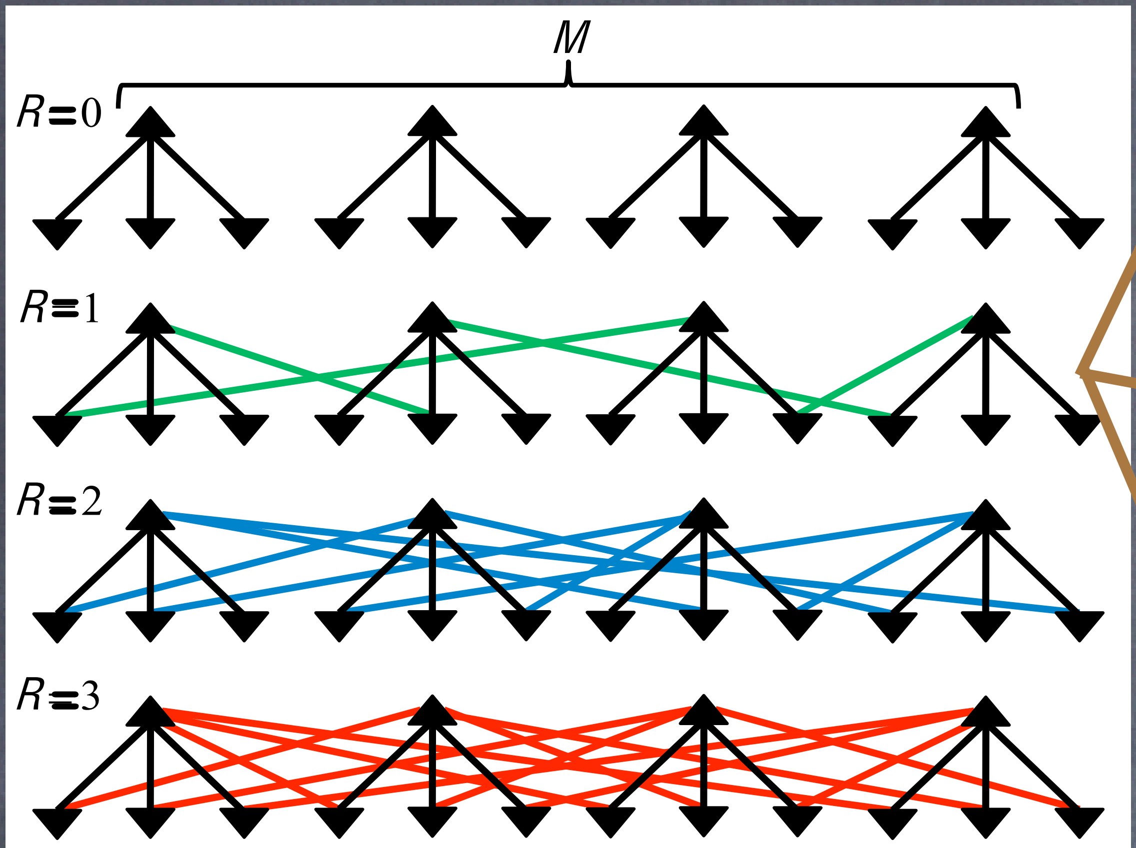
Each consumer has exactly one line on.

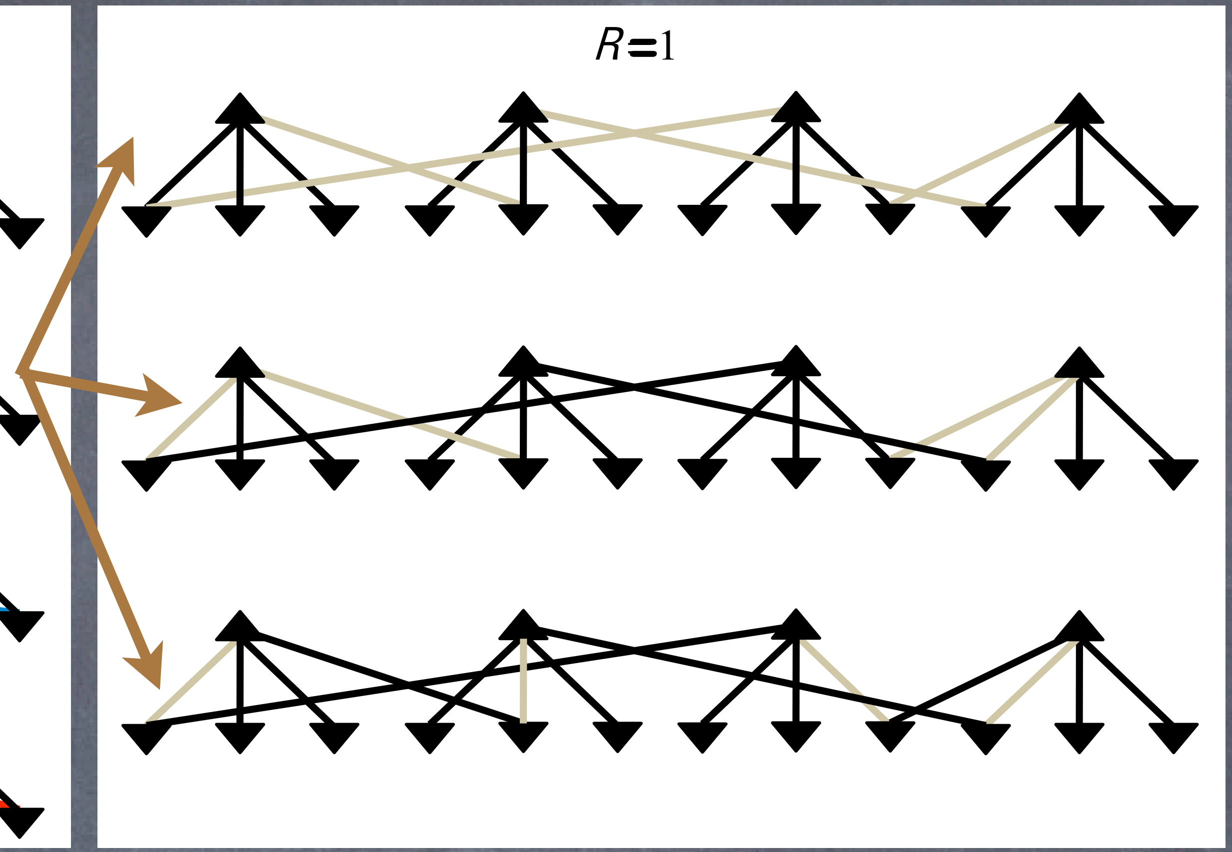
Constraints

$$\sum_{a \in \partial i} \sigma_{ia} = 1$$ Every consumer one connection

$$\sum_{i \in \partial a} \sigma_{ia}(x_i - z_i) \leq y_a$$ Producers not overloaded


M
R=0
R=1
R=2
R=3

R=1





Note that the final topology is a tree, hence the Kirchhoff's laws satisfied.

However, general power flow optimum cannot be worse than the tree case!

Questions

Given $\{x_i\}, \{z_i\}, \{y_a\}$ can all the constraints be simultaneously satisfied? (Nobody overloaded.)

If yes, then how many satisfying configurations of the switches are there? Is it easy to find one?

Answer: via Belief Propagation

How does BP work?

Prob. that line “ia” is in state σ_{ia} conditioned

$\psi_{\sigma_{ia}}^{a\rightarrow i}$ constraint on “i” is missing

$\chi_{\sigma_{ia}}^{i\rightarrow a}$ constraint on “a” is missing

$$\Rightarrow$$

Iterative “message passing” scheme

Belief Propagation Equations

$$
\begin{aligned}
\chi_1^{i\to a} &= \frac{1}{Z^{i\to a}} \prod_{b\in\partial i\setminus a} \psi_0^{b\to i} \\
\chi_0^{i\to a} &= \frac{1}{Z^{i\to a}} \sum_{b\in\partial i\setminus a} \psi_1^{b\to i} \prod_{c\in\partial i\setminus a,b} \psi_0^{c\to i} \\
\psi_1^{a\to i} &= \frac{1}{Z^{a\to i}} \sum_{\sigma_{\partial a\setminus ia}} \theta(y_a - w_i - \sum_{j\in\partial a\setminus i} \sigma_{ja} w_j) \prod_{j\in\partial a\setminus i} \chi_{\sigma_{ja}}^{j\to a} \\
\psi_0^{a\to i} &= \frac{1}{Z^{a\to i}} \sum_{\sigma_{\partial a\setminus ia}} \theta(y_a - \sum_{j\in\partial a\setminus i} \sigma_{ja} w_j) \prod_{j\in\partial a\setminus i} \chi_{\sigma_{ja}}^{j\to a}
\end{aligned}
$$

Belief Propagation

- Distributed approximative way of:
- (a) computing the probability that a given switch is on or off.
- (b) estimating number of valid (not overloading) configurations.
- For large number of customers and producers (thermodynamic limit) - average analysis solvable.

Switching model without renewable generation

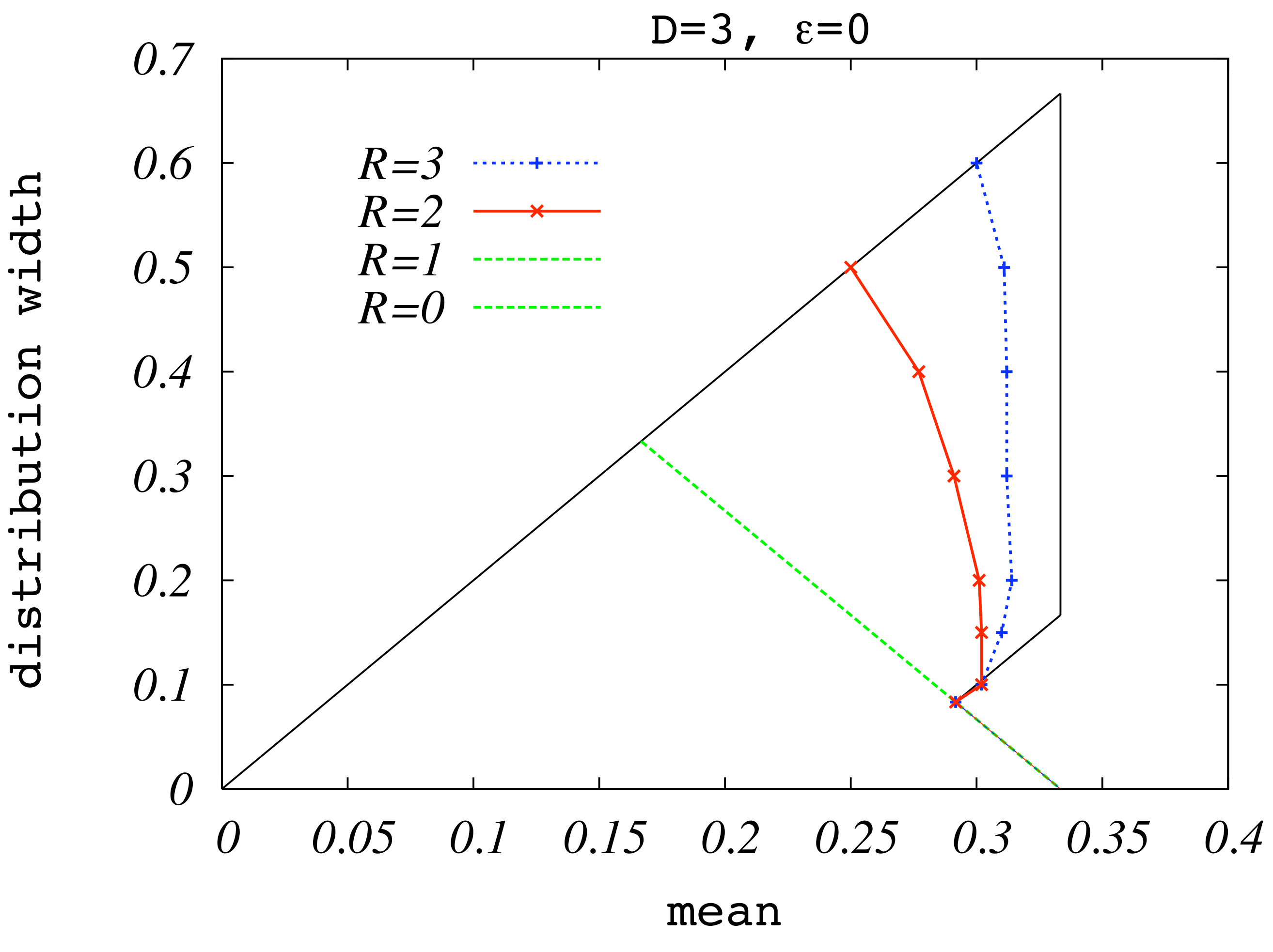
Average case analysis

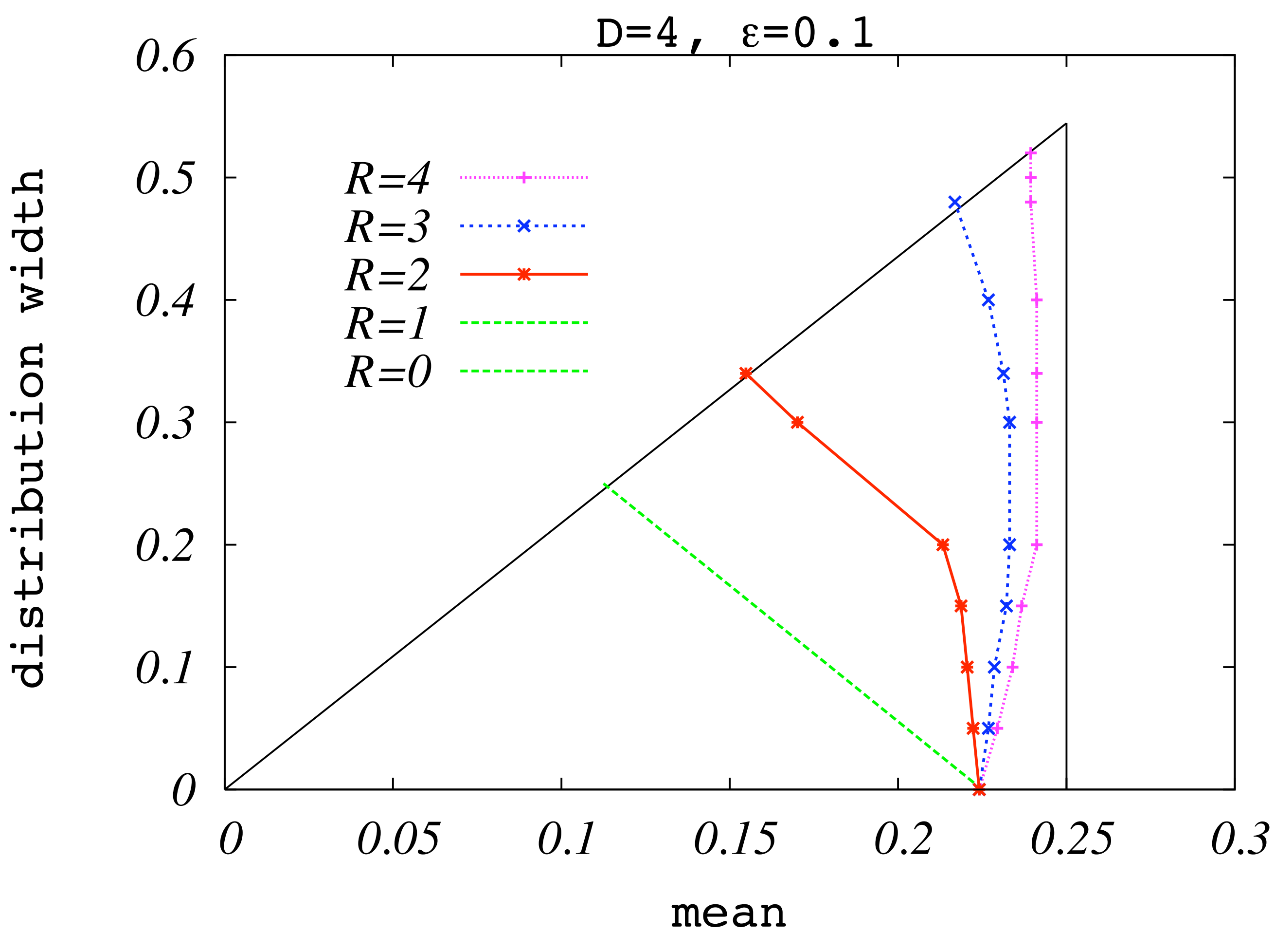
Consumption random number between $(\overline{x} - \Delta/2)$ and $(\overline{x} + \Delta/2)$

$\mathcal{E}$ fraction of consumers with no demand.

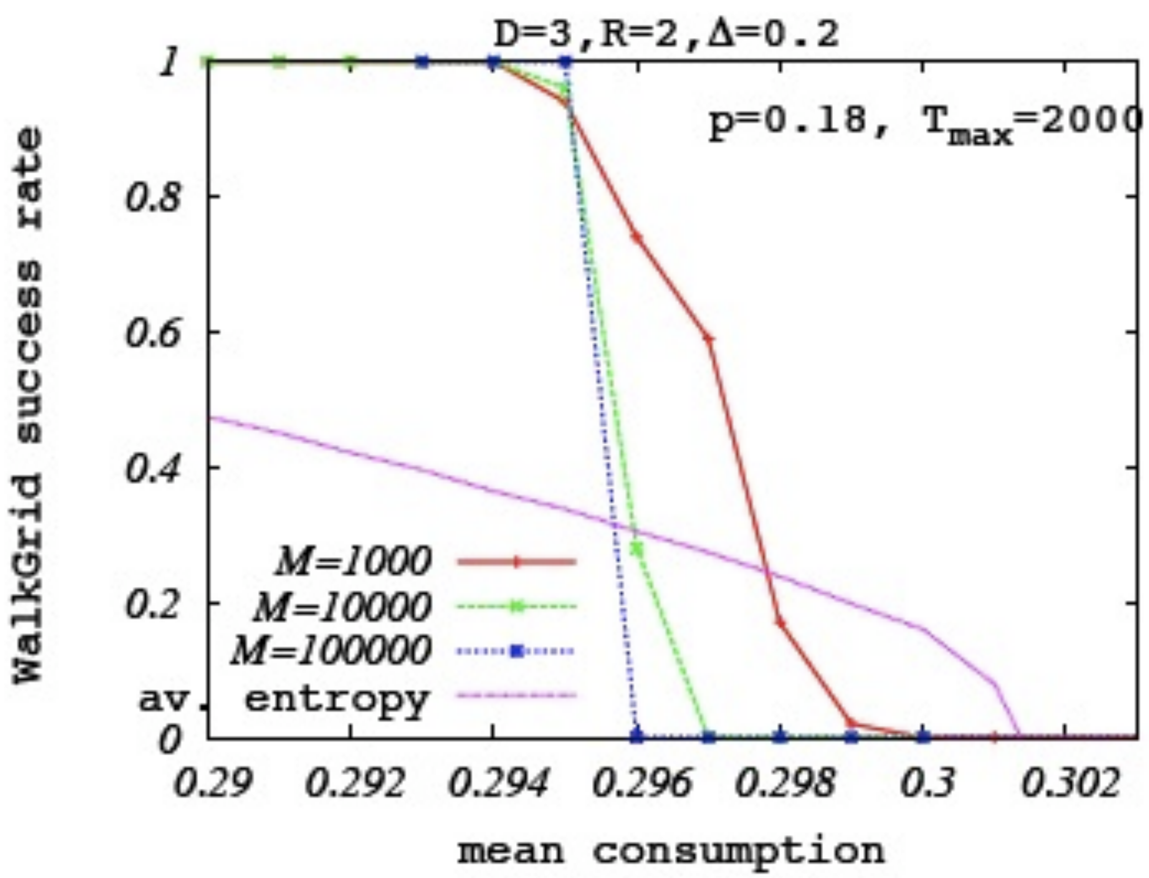
$y_a = 1 \qquad \forall a \qquad$ flat production

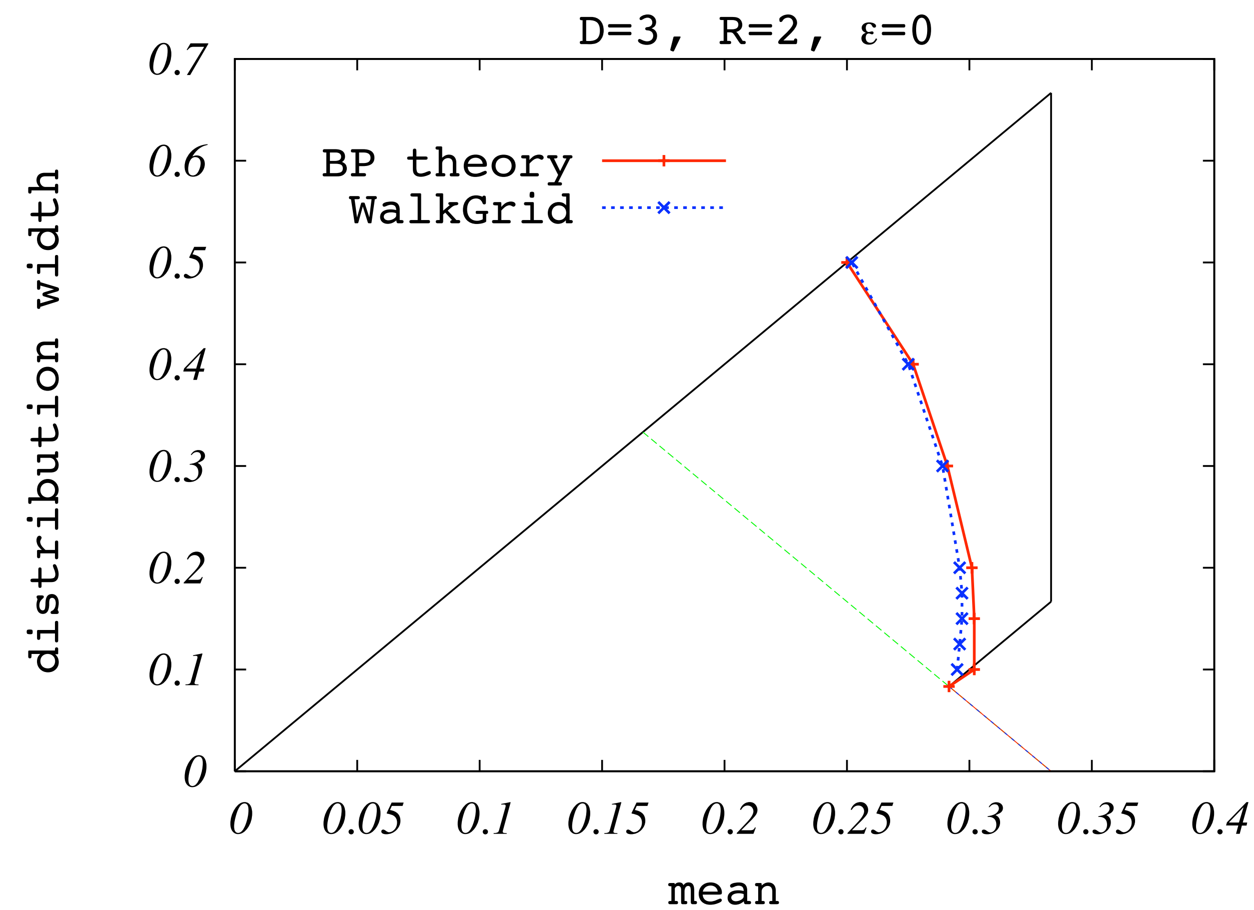
$z_i = 0 \qquad \forall i \qquad$ no renewable resources





WalkGrid efficient algorithm for switching





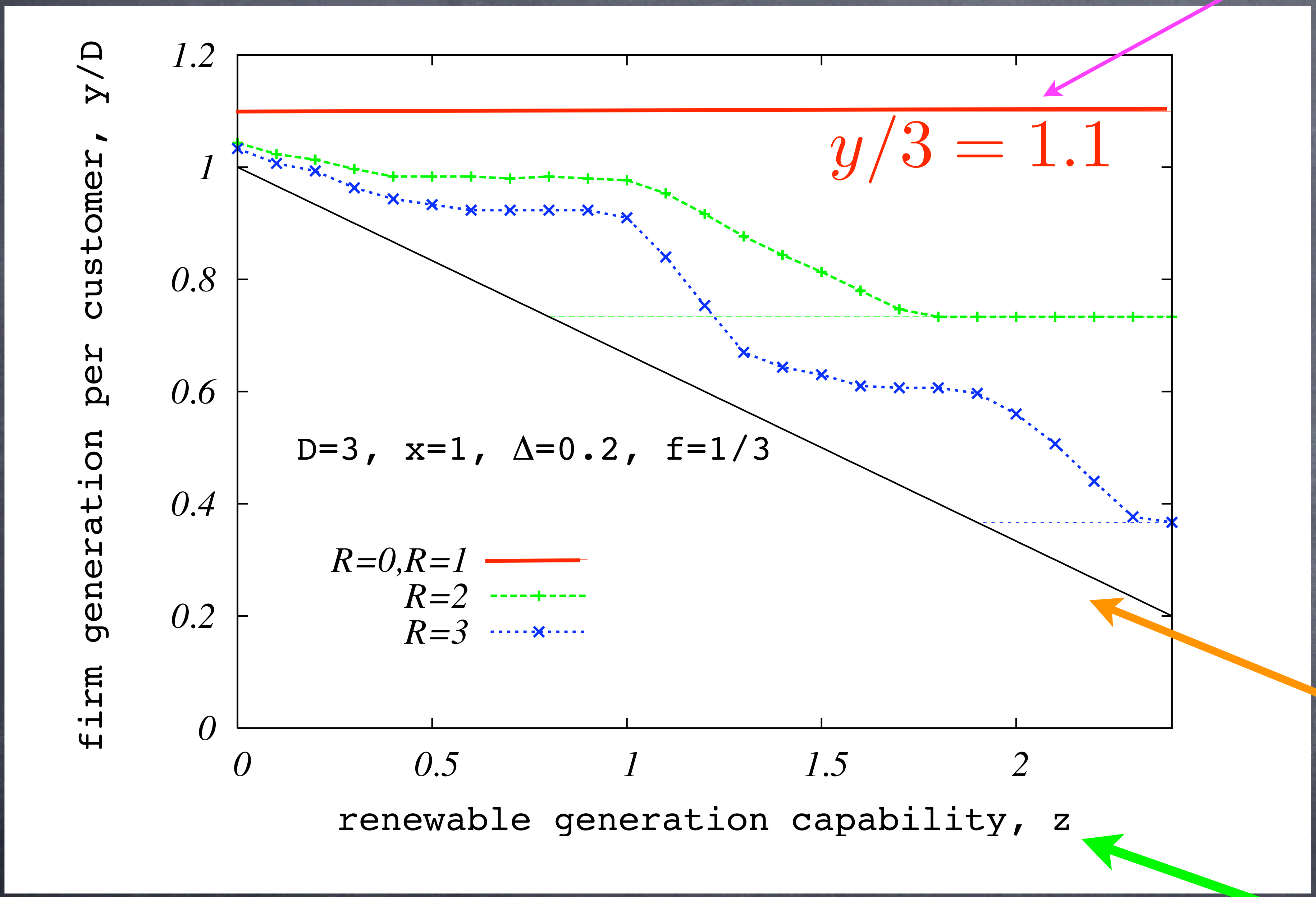
WALKGRID

Assign each value of σ 0 or 1 randomly (but such that $\forall i \in G : \sum_{\alpha \in \partial i} \sigma_{i\alpha} = 1$);
repeat Pick a random power generator α which shows an overload, and denote the value of the overload, δ;
Choose a random consumer i connected to the generator α, i.e. $\sigma_{i\alpha} = 1$;
Pick an arbitrary other generator which is not overloaded and consider switching connection from $(i\alpha)$ to $(i\beta)$.
if (in the result of this switch α is relieved from being overloaded
and β either remains under the allowed load or it is overloaded but by the amount less than δ)
Accept the move, i.e. disconnect i from α and connect it to β thus setting $\sigma_{i\beta} = 1$, $\sigma_{i\alpha} = 0$.
else With probability p connect consumer i to β instead of α;
until Solution found or number of iterations exceeds $M T_{\max}$.

With renewable generation

Example n. 1

somebody must serve D-R+1 fully demanding consumers



\# producers

$$M \to \infty$$

\# consumers

$$N = 3M$$

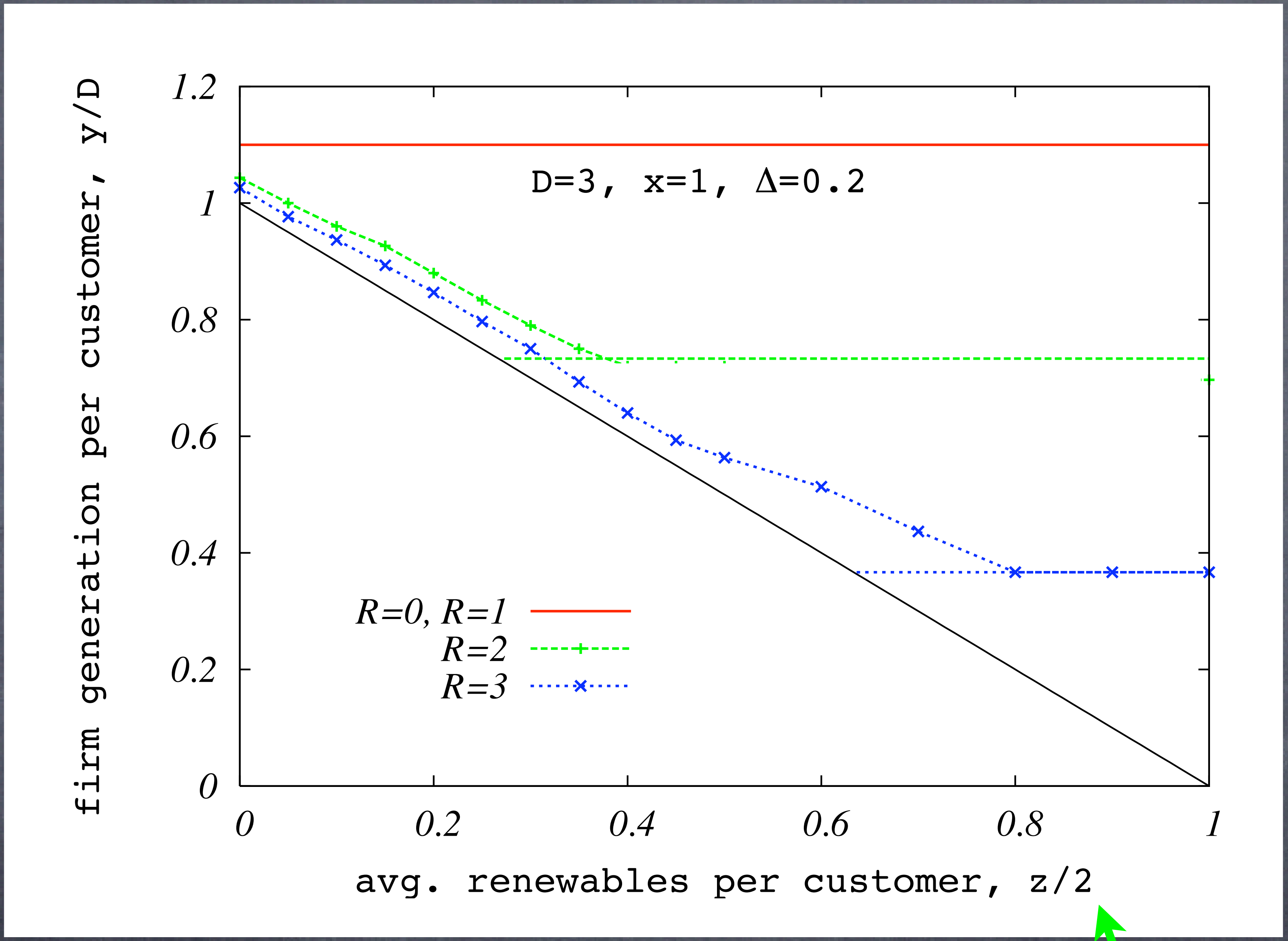
generation > consumption

$$y/3 = 1 - z/3$$

Fraction 1/3 of consumers produce amount z

Every consumer consumes random number in (0.9,1.1)

Example n. 2

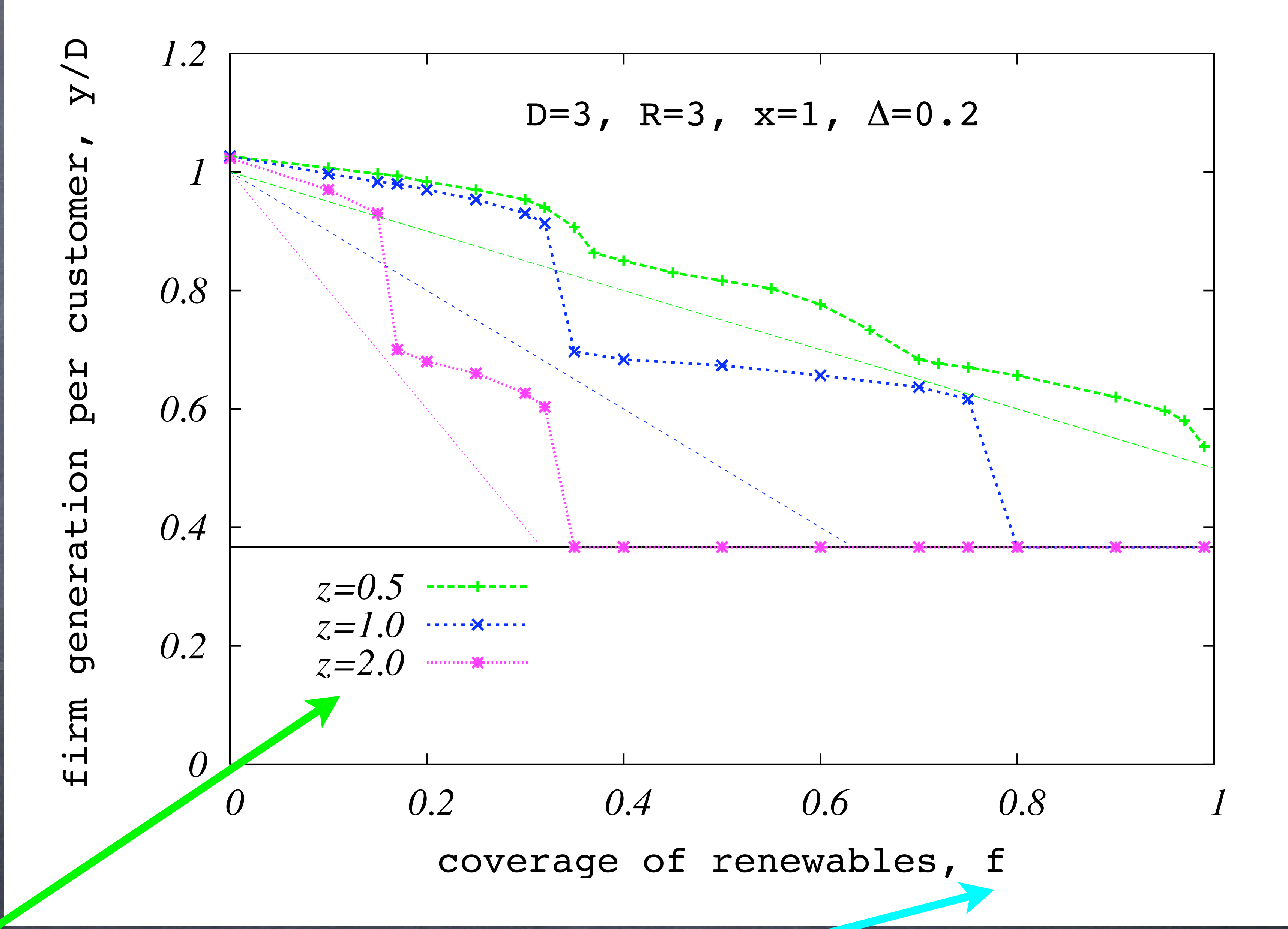


Every consumer produced a random number between (0,z)

Example n. 3

produced > consumed

$$y/3 > 1 - fz$$



amount z is produced by fraction f of consumers

Conclusions and Perspectives

- Existence of SAT/UNSAT phase transition and regimes where higher penetration useful or futile.

- Redundancy + switches help renewable integrations. Belief propagation a tool of analysis but also distributed control algorithm.

- In physics: Study of toy models (and phase transitions) leads to qualitative understanding. Is that true also for the Smart Grid?

- Combine belief propagation with DC or AC power flow rules on a non-tree topology.

References

- L. Zdeborová, A. Decelle, M. Chertkov; Phys. Rev. E 90, 046112 (2009).
- L. Zdeborová, S. Backhaus, M. Chertkov; in HICSS 43.