Multi-Commodity Flow Models for Dynamic Energy Management

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Energy Grid Modeling

- optimize generation, transmission, consumption of multiple energy types
- handle conversion between energy types
- preserve optimization tractability
Simple Power Grid

- Natural Gas Source
- Electric Generator
- Cogen
- Boiler
- Turbine
- Electric Load
- Heat Load
Multi-Commodity Flow Model: Components

- energy grid: a set of systems (nodes) and edges

- system: a set of terminals, a constitutive set, and a real-valued objective function

- terminal: an orientation (IN or OUT), a type (drawn from a finite set), and a real-valued, nonnegative power

- constitutive set: a subset of \( \mathbb{R}_+^r \), where \( r \) is the number of terminals belonging to the constitutive set’s system

- edge: a real-valued, nonnegative flow, and a pair of terminals of the same type, with opposite orientations
Multi-Commodity Flow Model: Semantics

- at every terminal power is conserved (edges are lossless)
- the vector of terminal powers of a system must lie in that system’s constitutive set
- overall objective is the sum of system objectives
- optimal operation:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{m} \phi_i(p_i) \\
\text{subject to} & \quad p_i \in C_i \quad i = 1, \ldots, m \\
& \quad \text{flow conservation}
\end{align*}
\]
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Example System: Generator

- one OUT terminal of type ELEC, with power flow $p$
- maximum output power $P_{\max}$
- constitutive set $C = [0, P_{\max}]$
- objective can include fuel cost, mechanical degradation
Example System: Cogen

• one IN terminal of type GAS and two OUT terminals of type ELEC and HEAT with powers $p_{gas}$, $p_{elec}$, $p_{heat}$

• maximum electrical output power $P_{max}$

• natural gas conversion efficiencies to electricity and heat of 40% and 30%, respectively

• constitutive set

$$C = \{(p_{gas}, p_{elec}, p_{heat}) \mid p_{elec} = 0.4p_{gas}, p_{heat} = 0.3p_{gas}, p_{elec} \leq P_{max}\}$$

• objective measures (say) profit and CO₂ emissions
Example System: Lossy Electrical Line

- one IN and one ONE terminal, both of type ELEC, with power flows $p_{in}$, $p_{out}$
- maximum input power $P_{max}$
- flow dependent line-loss $\ell(p_{in})$
- constitutive set

\[ C = \{(p_{in}, p_{out}) \mid p_{in} = p_{out} + \ell(p_{in}), \ 0 \leq p_{in} \leq P_{max}\} \]

- zero objective
Convexity

- optimal operation problem is convex when constitutive sets and objectives are convex
  - example: linear conversion efficiencies within given limits

- in many other cases, problem can be solved via convex optimization via relaxation
  - example: power line with convex, nonlinear loss
Relaxations

- system objective is **value seeking** if it is nondecreasing in its input flows and nonincreasing in its output flows
  - flows considered valuable: prefer to get more out for less in

- for a constitutive set $C$, its relaxation, $\text{rel}(C)$, is the hypo-epigraph of the output and input powers in $C$
  \[
  \text{rel}(C) = \{(\tilde{p}_{\text{in}}, \tilde{p}_{\text{out}}) \mid \tilde{p}_{\text{in}} \geq p_{\text{in}}, \tilde{p}_{\text{out}} \leq p_{\text{out}}, (p_{\text{in}}, p_{\text{out}}) \in C\}
  \]
  - equivalent to allowing a system to throw away power
Relaxations

- unrelaxed and nonconvex
Relaxations

- relaxed and convex
Relaxed Optimization Problem

- relaxed optimal operation:

\[
\text{minimize} \quad \sum_{i=1}^{m} \phi_i(p_i) \\
\text{subject to} \quad p_i \in \text{rel}(C_i) \quad i = 1, \ldots, m \\
\text{flow conservation}
\]

- if \( \phi_i \) and \( \text{rel}(C_i) \) are convex, relaxed problem is convex

- if in addition \( \phi_i \) are value seeking, then the minimizer of the relaxed optimal operation problem, \( \{p_i^*\}_{i=1,\ldots,m} \), will also satisfy \( p_i^* \in C_i \), making it an optimal solution to the unrelaxed problem
Simple Power Grid

Natural Gas Source

Electric Generator

Turbine

Cogen

Boiler

Electric Load

Heat Load
Simple Power Grid

- electric load and heat load must be met by combination of turbine, cogen, generator, and boiler
- all have nonlinearly varying efficiencies, capacities
- fixed gas price
- goal: minimize operating cost
• results plausible, but non-obvious even for a simple grid
Implementation

- object-oriented extension to CVXOPT
- form power grid simply by declaring systems and edges
- system highly scalable from ISOs down to individual homes
- allows the use of convex optimization techniques for the energy grid without requiring highly specialized knowledge of convex optimization solvers
Code

• declare systems and grid hierarchy

```python
grid = Network()
grid.subSystems = [GasSource(), ElectricGenerator(),
Turbine(), Cogen(), Boiler(),
ElectricLoad(), HeatLoad()]
```

• form nine connections of the form

```python
grid.connect(gasSource.termGAS, turbine.termGAS)
```

• gather all variables and constraints to solve with CVXOPT

```python
grid.solve()
```
Extensions

- dynamic storage, dynamic constraints (convex in many cases)
- distributed optimization
  - groups of subsystems are optimized separately
  - messages passed to coordinate global solution
- grid design and system sizing
- real-time operation
Conclusion

- a theoretically simple, yet highly extensible model with very general constraints and objectives
- general energy systems, not just large electricity grids
- preserves convexity, which allows for tractability
- object oriented nature allows rapid prototyping and scalability