Multi-Commodity Flow Models for Dynamic Energy Management

Matt Kraning and Stephen Boyd

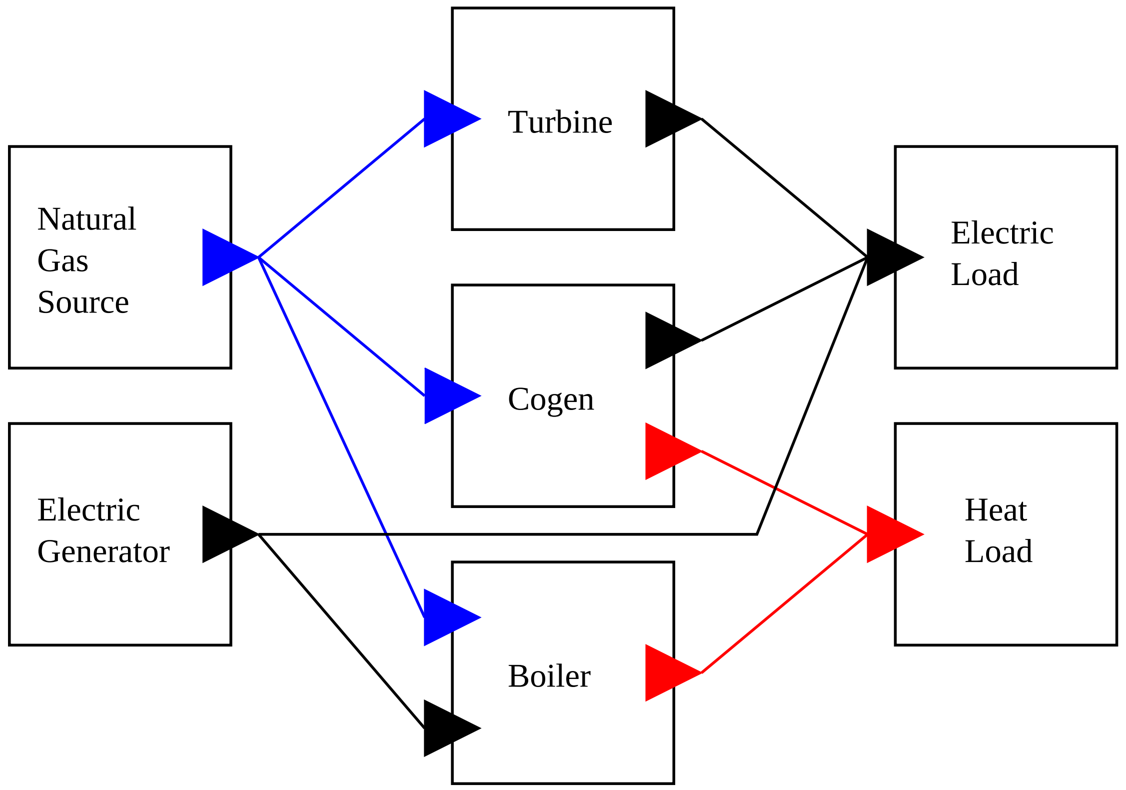
Electrical Engineering Department
Stanford University

Los Alamos CNLS, 8/10/10

Energy Grid Modeling

- optimize generation, transmission, consumption of multiple energy types
- handle conversion between energy types
- preserve optimization tractability

Simple Power Grid



Multi-Commodity Flow Model: Components

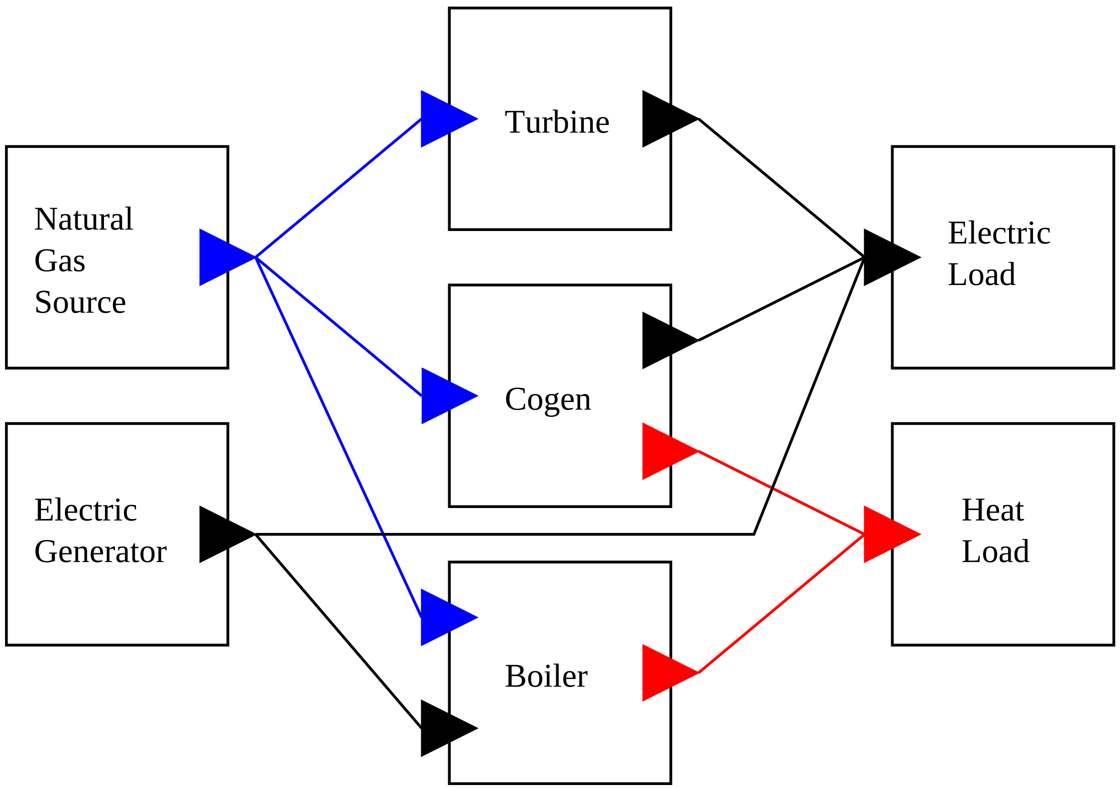
- energy grid: a set of **systems** (nodes) and **edges**
- system: a set of **terminals**, a **constitutive set**, and a real-valued **objective function**
- terminal: an **orientation** (IN or OUT), a **type** (drawn from a finite set), and a real-valued, nonnegative **power**
- constitutive set: a subset of $\mathbf{R}_+^r$, where r is the number of terminals belonging to the constitutive set's system
- edge: a real-valued, nonnegative **flow**, and a pair of terminals of the same type, with opposite orientations

Mutli-Commodity Flow Model: Semantics

- at every terminal power is conserved (edges are lossless)
- the vector of terminal powers of a system must lie in that system's constitutive set
- overall objective is the sum of system objectives
- optimal operation:

$$
\begin{array}{ll}
\text{minimize} & \sum_{i=1}^{m} \phi_i(p_i) \\
\text{subject to} & p_i \in \mathcal{C}_i \quad i = 1, \ldots, m \\
& \text{flow conservation}
\end{array}
$$

Simple Power Grid



Example System: Generator

- one OUT terminal of type ELEC, with power flow p
- maximum output power $P^{\max}$
- constitutive set $\mathcal{C} = [0, P^{\max}]$
- objective can include fuel cost, mechanical degradation

Example System: Cogen

- one IN terminal of type GAS and two OUT terminals of type ELEC and HEAT with powers p_{gas}, p_{elec}, p_{heat}
- maximum electrical output power $P^{\max}$
- natural gas conversion efficiencies to electricity and heat of 40% and 30%, respectively
- constitutive set

$$\mathcal{C} = \{(p_{\mathrm{gas}}, p_{\mathrm{elec}}, p_{\mathrm{heat}}) \mid p_{\mathrm{elec}} = 0.4 p_{\mathrm{gas}},\ p_{\mathrm{heat}} = 0.3 p_{\mathrm{gas}},\ p_{\mathrm{elec}} \leq P^{\max}\}$$

- objective measures (say) profit and CO_2 emissions

Example System: Lossy Electrical Line

- one IN and one ONE terminal, both of type ELEC, with power flows $p_{\rm in}$, $p_{\rm out}$

- maximum input power $P^{\rm max}$

- flow dependent line-loss $\ell(p_{\rm in})$

- constitutive set

$$\mathcal{C} = \{(p_{\rm in}, p_{\rm out}) \mid p_{\rm in} = p_{\rm out} + \ell(p_{\rm in}),\ 0 \le p_{\rm in} \le P^{\rm max}\}$$

- zero objective

Convexity

- optimal operation problem is convex when constitutive sets and objectives are convex
  - example: linear conversion efficiencies within given limits
- in many other cases, problem can be solved via convex optimization via relaxation
  - example: power line with convex, nonlinear loss

Relaxations

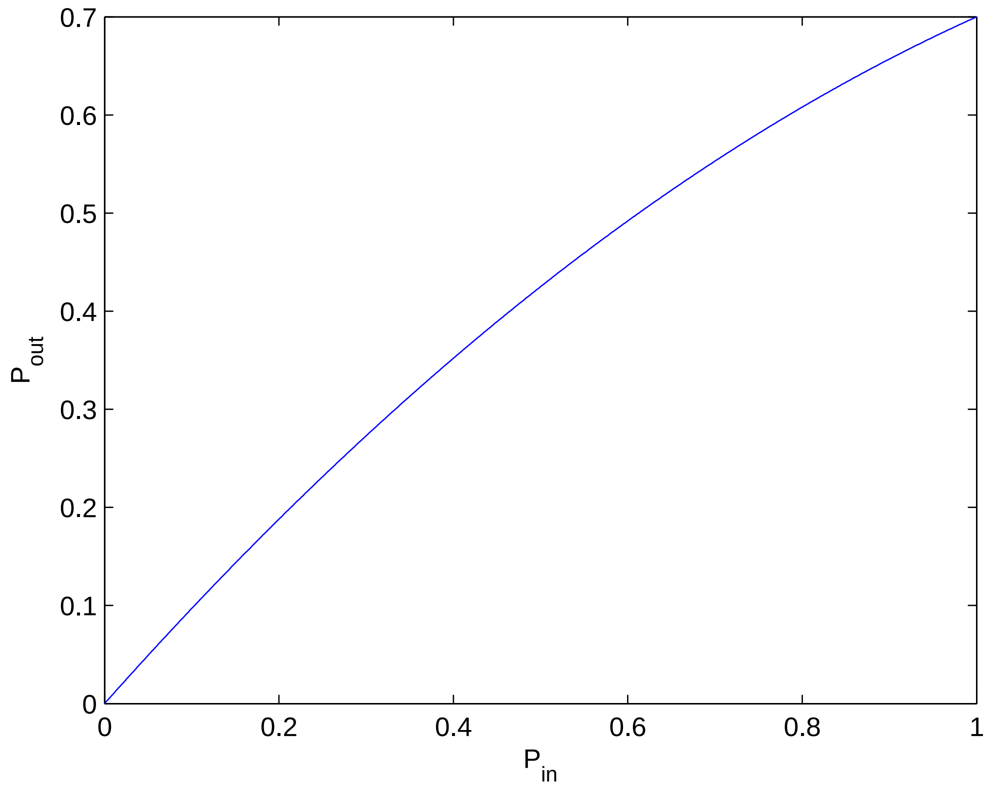
- system objective is **value seeking** if it is nondecreasing in its input flows and nonincreasing in its output flows
  - flows considered valuable: prefer to get more out for less in

- for a constitutive set $\mathcal{C}$, its relaxation, $\mathbf{rel}(\mathcal{C})$, is the hypo-epigraph of the output and input powers in $\mathcal{C}$

$$\mathbf{rel}(\mathcal{C}) = \{(\tilde{p}_{\rm in}, \tilde{p}_{\rm out}) \mid \tilde{p}_{\rm in} \geq p_{\rm in},\ \tilde{p}_{\rm out} \leq p_{\rm out},\ (p_{\rm in}, p_{\rm out}) \in \mathcal{C}\}$$

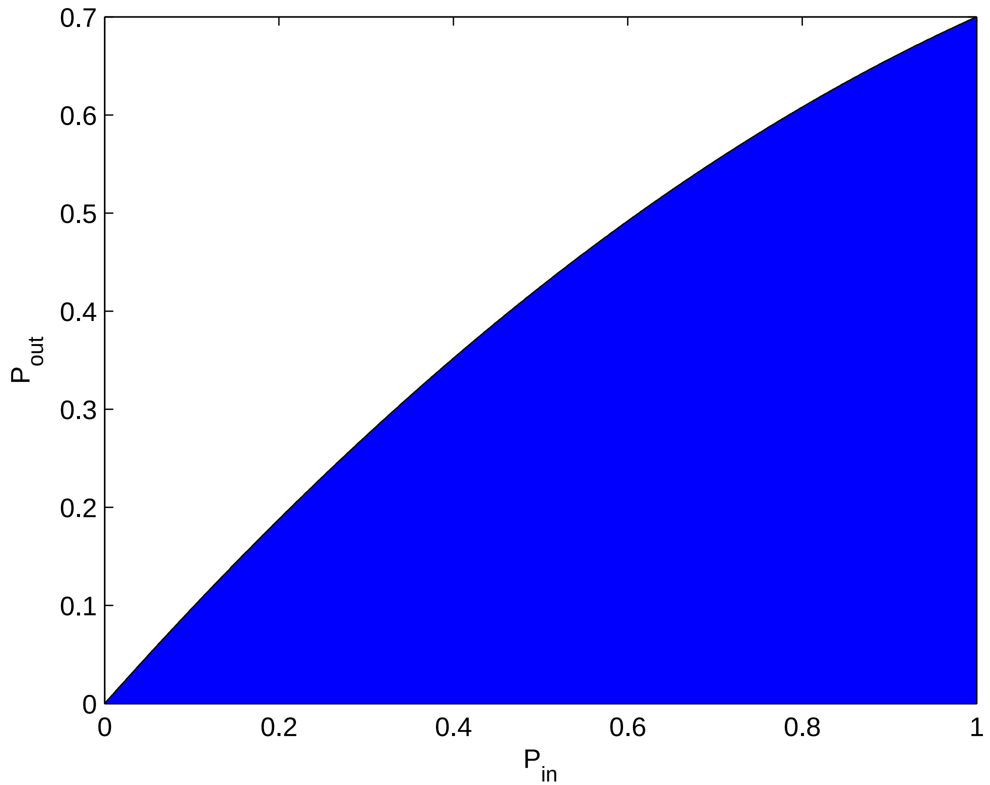
  - equivalent to allowing a system to throw away power

Relaxations



- unrelaxed and nonconvex

Relaxations



- relaxed and convex

Relaxed Optimization Problem

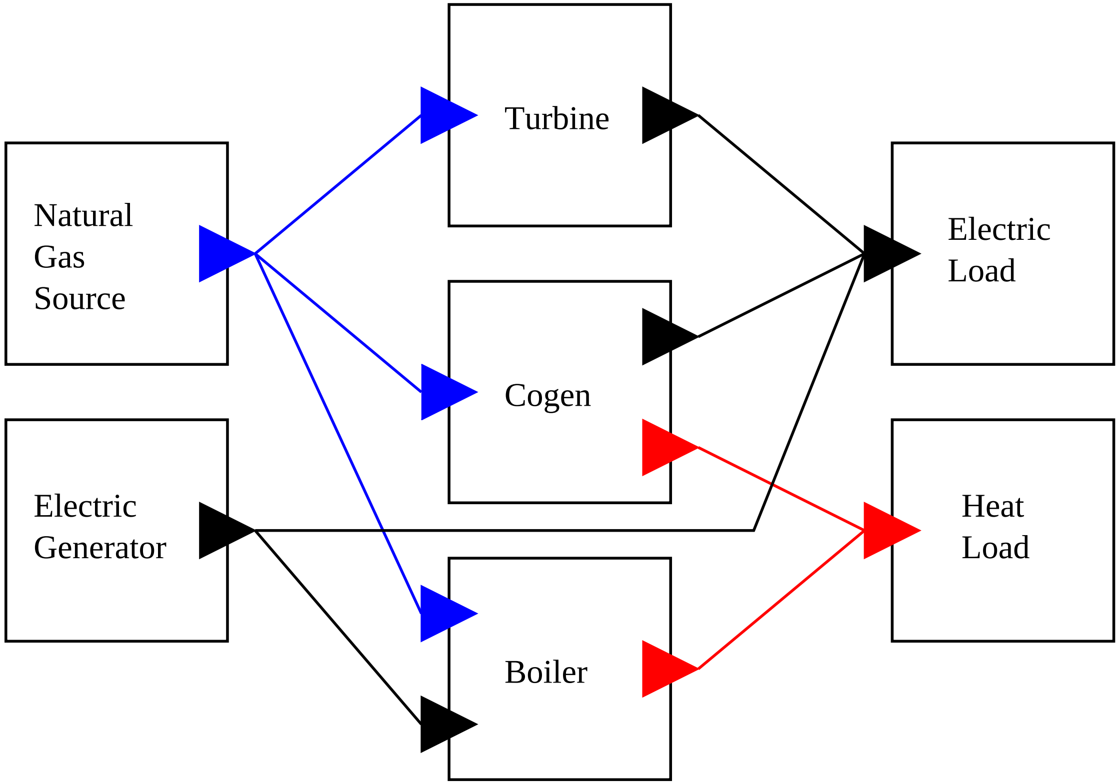
- relaxed optimal operation:

$$
\begin{array}{ll}
\text{minimize} & \sum_{i=1}^{m} \phi_i(p_i) \\
\text{subject to} & p_i \in \mathbf{rel}(\mathcal{C}_i) \quad i = 1, \ldots, m \\
& \text{flow conservation}
\end{array}
$$

- if ϕ_i and $\mathbf{rel}(\mathcal{C}_i)$ are convex, relaxed problem is convex

- if in addition ϕ_i are value seeking, then the minimizer of the relaxed optimal operation problem, $\{p_i^\star\}_{i=1,\ldots,m}$, will also satisfy $p_i^\star \in \mathcal{C}_i$, making it an optimal solution to the unrelaxed problem

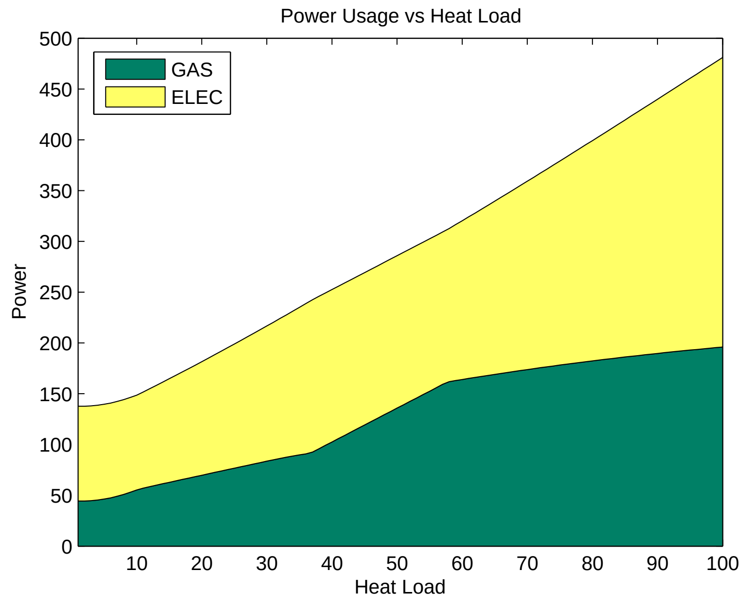
Simple Power Grid

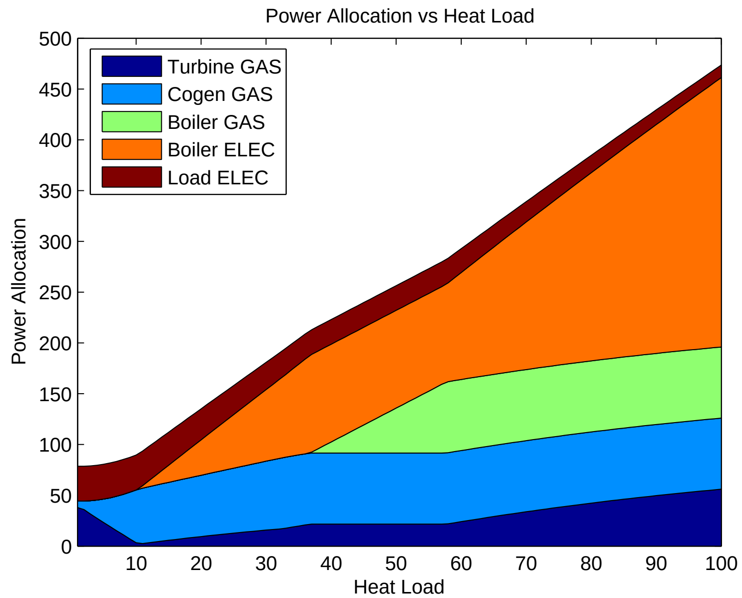


Simple Power Grid

- electric load and heat load must be met by combination of turbine, cogen, generator, and boiler
- all have nonlinearly varying efficiencies, capacities
- fixed gas price
- goal: minimize operating cost

Flow Model Results





- results plausible, but non-obvious even for a simple grid

Implementation

- object-oriented extension to CVXOPT
- form power grid simply by declaring systems and edges
- system highly scalable from ISOs down to individual homes
- allows the use of convex optimization techniques for the energy grid without requiring highly specialized knowledge of convex optimization solvers

Code

- declare systems and grid hierarchy

```
grid = Network()
grid.subSystems = [GasSource(), ElectricGenerator(),
                   Turbine(), Cogen(), Boiler(),
                   ElectricLoad(), HeatLoad()]
```

- form nine connections of the form

```
grid.connect(gasSource.termGAS, turbine.termGAS)
```

- gather all variables and constraints to solve with CVXOPT

```
grid.solve()
```

Extensions

- dynamic storage, dynamic constraints (convex in many cases)
- distributed optimization
  - groups of subsystems are optimized seperately
  - messages passed to coordinate global solution
- grid design and system sizing
- real-time operation

Conclusion

- a theoretically simple, yet highly extensible model with very general constraints and objectives
- general energy systems, not just large electricity grids
- preserves convexity, which allows for tractability
- object oriented nature allows rapid prototyping and scalability