A Majorization-Minimization Approach to Design of Power Transmission Networks[1]

Mini-Workshop on Optimization and Control Theory for Smart Grids

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Introduction

Objective design the network, both the graph structure and the line sizes, to achieve best trade-off between efficiency (low power loss) and construction costs.

Our approach was inspired by work of Ghosh, Boyd and Saberi, *Minimizing effective resistance of a graph*, '09.

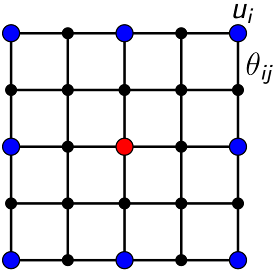
Overview

- resistive network model (DC approximation)
- convex network optimization (line sizing)
- non-convex optimization for *sparse* network design
- *robust* network design

Resistive Network Model

Basic current flow problem (power flow in AC grid):

- graph G
- injected currents b_i
  - sources $b_i > 0$
  - sinks $b_i < 0$
  - junction points $b_i = 0$
- line conductances $\theta_{ij} \geq 0$



Network conductance matrix:

$$K_{i,j}(\theta) = \begin{cases} \sum_{k \neq i} \theta_{ik}, & i = j \\ -\theta_{ij}, & \{i,j\} \in G \end{cases}$$

Solve $K(\theta)u = b$ for node potentials u_i. Line currents given by $b_{i \to j} = \theta_{ij}(u_j - u_i)$.

Resistive Power Loss

For connected G and $\theta > 0$,

$$u = \hat{K}^{-1} b \triangleq (K + 11^T)^{-1} b.$$

Power loss due to resistive heating of the lines:

$$L(\theta) = \sum_{ij} \theta_{ij} (u_i - u_j)^2 = u^T K(\theta) u = b^T \hat{K}(\theta)^{-1} b$$

For random b, we obtain the expected power loss:

$$L(\theta) = \langle b^T \hat{K}(\theta)^{-1} b \rangle = \mathrm{tr}(\hat{K}(\theta)^{-1} \cdot \langle bb^T \rangle) \triangleq \mathrm{tr}(\hat{K}(\theta)^{-1} B)$$

Importantly, this power loss is a *convex* function of θ.

Convex Network Optimization

We may size the lines (conductances) to minimize power loss subject to linear budget on available conductance:

$$\begin{aligned} \text{minimize} \quad & L(\theta) \\ \text{subject to} \quad & \theta \geq 0 \\ & \alpha^T \theta \leq C \end{aligned}$$

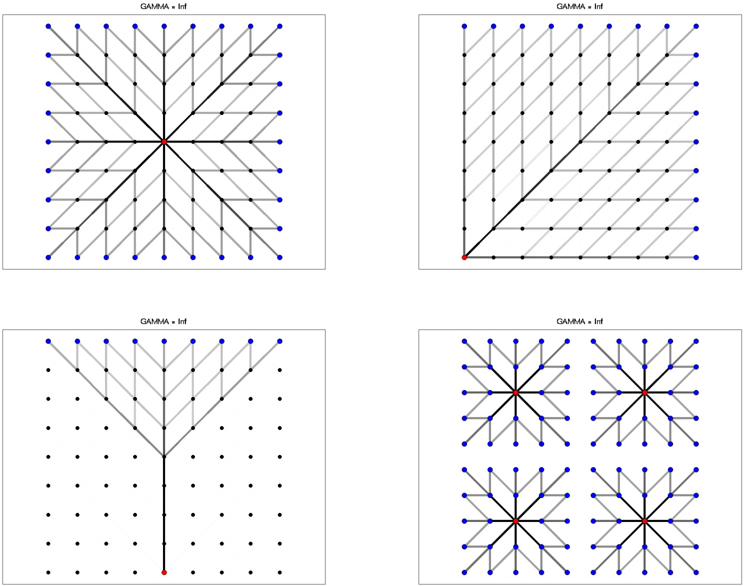
α_ℓ is cost of conductance on line ℓ.

Alternatively, one may find the most cost-effective network:

$$\min_{\theta \geq 0} \{ L(\theta) + \lambda \alpha^T \theta \}$$

where λ^{-1} represents the cost of power loss (accrued over lifetime of network).

Four Examples – Optimal Network Design



Imposing Sparsity (Non-convex continuation)

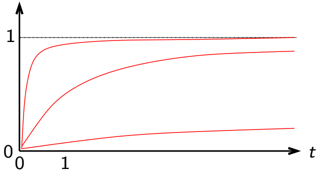
We now add a zero-conductance cost on lines:

$$\min_{\theta \geq 0} \{ L(\theta) + \alpha^T \theta + \beta^T \phi(\theta) \}$$

where $\phi(t)$ is (element-wise) unit step function.

We smooth this combinatorial problem to a continuous one, replacing the discrete step ϕ by a smooth one:

$$\phi_\gamma(t) = \frac{t}{t + \gamma}$$



The convex optimization is recovered as $\gamma \to \infty$ and the combinatorial one as $\gamma \to 0$.

Majorization-Minimization Algorithm

Approach inspired by Candes and Boyd, *Enhancing sparsity by reweighted L_1 optimization*, 2009.

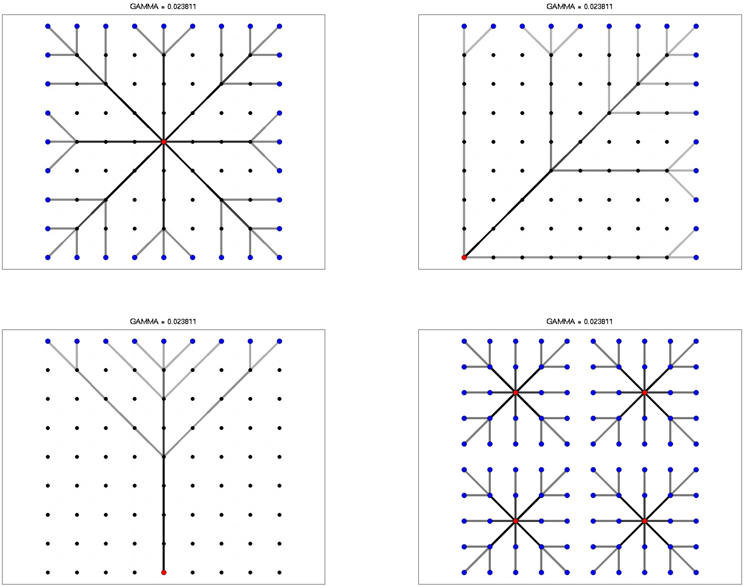
Uses *majorization-minimization* heuristic for non-convex optimization. Iteratively linearize ϕ_γ about previous solution:

$$\theta^{(t+1)} = \arg\min_{\theta \geq 0} \{ L(\theta) + [\alpha + \beta \circ \nabla \phi_\gamma(\theta^{(t)})]^T \theta \}$$

results in a sequence of convex network optimization problems with reweighted α.

Monotonically decreases the objective and (almost always) converges to a local minimum.

Four Examples – Sparse Network Design



Adding Robustness

To obtain solutions that are robust of failure of k lines, we replace $L(\theta)$ by the *worst-case* power dissipation after removing k lines:

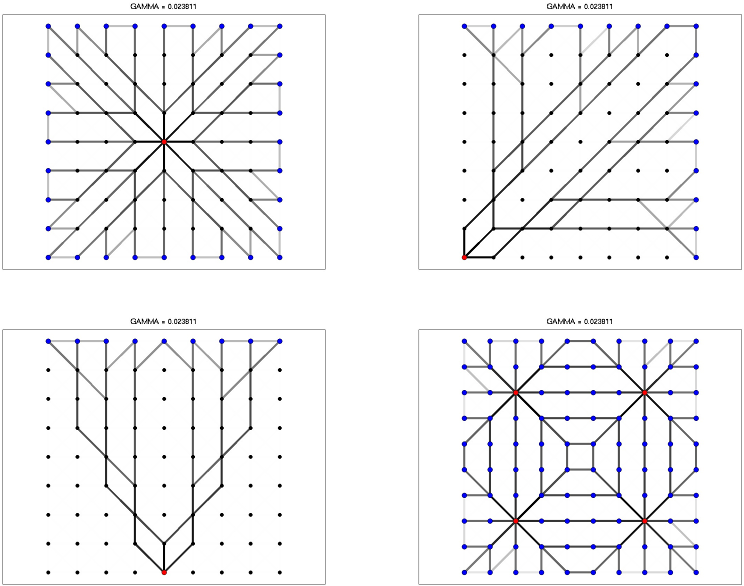
$$L^{\setminus k}(\theta) = \max_{z \in \{0,1\}^m | 1^T z = k} L((1 - z) \circ \theta)$$

This is still a convex function. It is tractable to compute only for small values of k.

Method can be extended to also treat failures of generators.

Solved using essentially the same methods as before...

Four Examples – Robust Network Design



Conclusion

A promising heuristic approach to design of power transmission networks. However, cannot guarantee global optimality.

Future Work:

- Bounding optimality gap?
- Use non-convex continuation approach to place generators
- possibly useful for graph partitioning problems
- adding further constraints (e.g. don't overload lines)
- better approximations to AC power flow?

Thanks!