# Cournot oligopoly and PHEV scheduling

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- Cournot Oligopoly
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# Social welfare analysis

1. We establish an efficiency bound, which depends only on one single parameter extracted from the model, for Cournot oligopoly games.
2. We formulate the centralized PHEV scheduling as a dynamic programming problem, and then introduce several approximate dynamic programming methods to approach the optimal solution.
3. We propose a dynamic game theoretical model to study the decentralized PHEV scheduling problem.

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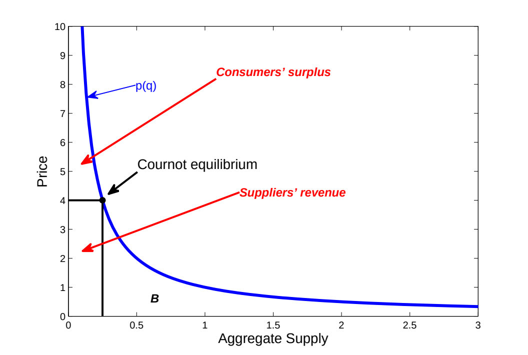
# Cournot Oligopoly

- Consider a market for a single homogeneous good with $N$ suppliers.
- Supplier $n \in \{1, 2, \ldots, N\}$ has a cost function $C_n : [0, \infty) \to [0, \infty)$.
- The inverse demand function $p : [0, \infty) \to [0, \infty)$, which maps the total supply into price.
- Suppose that supplier $n$ produces $x_n$ amount of good, and the price will be $p(X)$, where $X = \sum_{n=1}^{N} x_n$.
- The payoff of supplier $n$ is

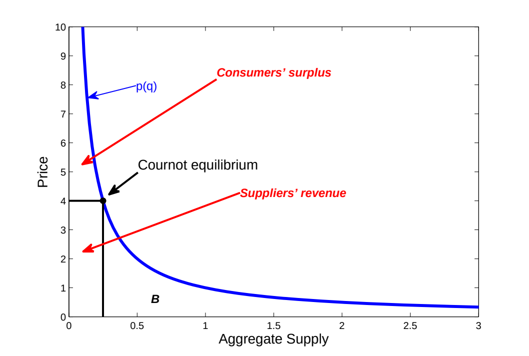
$$\pi_n(x_n, x_{-n}) = x_n p\left(\sum_{n=1}^{N} x_n\right) - C_n(x_n).$$

# Social Welfare in a Cournot oligopoly




- The aggregate utility received by the consumers is given by $U(X) = \int_0^X p(q)dq$.
- The consumer surplus is given by $\int_0^X p(q)dq - p(X)X$.

# Social Welfare in a Cournot oligopoly




- The profit earned by suppliers is $p(X)X - \sum_{n=1}^{N} C_n(x_n)$.
- The social welfare: $\int_0^X p(q)dq - \sum_{n=1}^{N} C_n(x_n)$.

# Price of Anarchy in Cournot Oligopoly

The **efficiency** of a nonnegative vector $\mathbf{x} = (x_1, \ldots, x_N)$ is defined as

$$\gamma(\mathbf{x}) = \frac{\int_0^{\sum_{n=1}^N x_n} p(q)dq - \sum_{n=1}^N C(x_n)}{\int_0^{\sum_{n=1}^N x_n^S} p(q)dq - \sum_{n=1}^N C(x_n^S)}, \tag{1}$$

where $(x_1^S, \ldots, x_N^S)$ is an optimal solution to the following optimization problem,

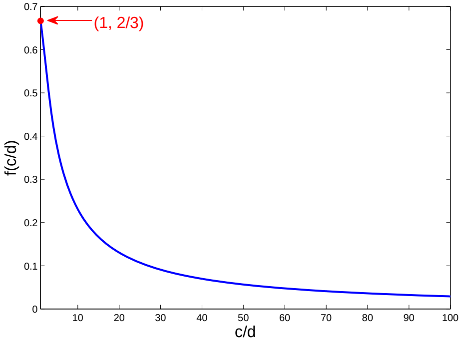
$$\begin{array}{ll} \text{maximize} & \int_0^X p(q)dq - \sum_{n=1}^N C_n(x_n) \\ \text{subject to} & x_n \geq 0, \quad n = 1, 2, \ldots, N. \end{array} \tag{2}$$

# Related work

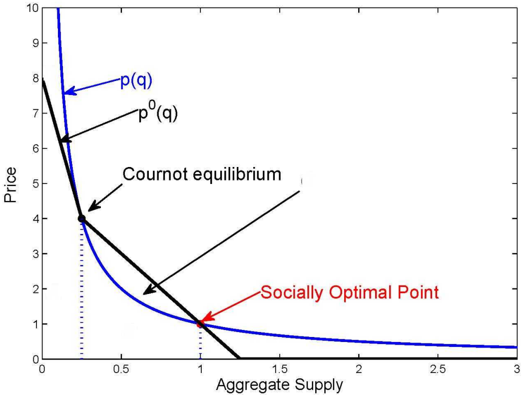
- (Anderson and Renault 2003) quantifies the efficiency loss in Cournot oligopoly models with concave demand functions. However, most of their results are not on the relation between the social welfare achieved at a Cournot equilibrium and a socially optimal competitive equilibrium.
- (Kluberg and Perakis 2008) compares the social welfare under Cournot competition to the corresponding maximum possible, for the case where the price demand relationship is linear.
- (Johari and Tsitsiklis 2005) establishes a $2/3$ lower bound on the efficiency of a Cournot equilibrium, when the inverse demand function is affine.

# Theorem

1. If $p(X) = p(X^S)$, then $\gamma(\mathbf{x}) = 1$.
2. Otherwise, let $c = p'(X)$, $d = (p(X^S) - p(X))/(X^S - X)$ and $\overline{c} = c/d$. Then, $\gamma(\mathbf{x}) \geq f(\overline{c}), \quad \overline{c} \geq 1$.



# Piece-wise Linear Inverse Demand Functions



# Application

## A lucky case

- Consider a group of convex inverse demand functions (eq.6, Bulow and Pfleiderer 1983))

$$p(q) = \alpha - \beta \log q, \quad \alpha,\ \beta > 0, \quad 0 < q < \exp(\alpha/\beta).$$

- There exists at least one Cournot equilibrium, and the efficiency of a Cournot equilibrium is no less than 0.5237.

- Consider a group of constant elasticity demand curves (eq.4, Bulow and Pfleiderer 1983))

$$p(q) = \alpha q^{-\beta}, \quad 0 < \alpha,\ 0 < \beta < 1,\ 0 \leq q.$$

- The efficiency of a Cournot equilibrium is no less than $f\left((1-\beta)^{\frac{-\beta-1}{\beta}}\right)$.

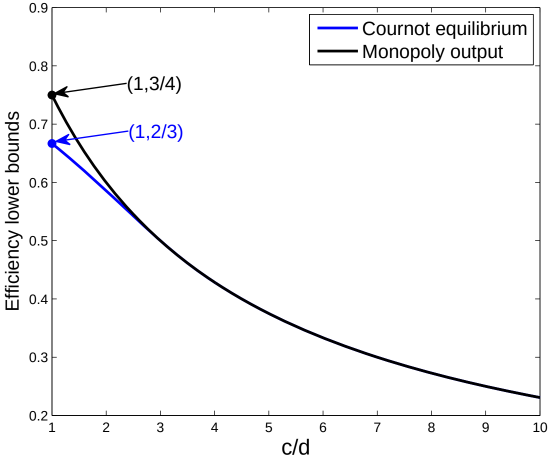
# Summary

- Social optimum (SO), where the social welfare is maximized.
- Monopoly Output (MO): where the aggregate profit of suppliers is maximized.

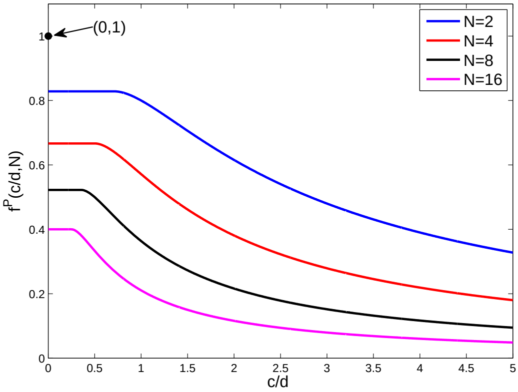
| | CE vs SO | CE vs MO | MO vs SO |
|---|---|---|---|
| Social welfare | $\gamma(\mathbf{x}) \leq 1$ efficiency lower bound[1] | $\leq 1$ or $> 1$ | $\gamma(\mathbf{x}^P) \leq 1$ efficiency lower bound[1] |
| Consumer surplus | $\leq 1$ | $\geq 1$ | $\leq 1$ |
| Profit | $\leq 1$ or $> 1$ | $\eta(\mathbf{x}) \leq 1$ profit lower bound[2] | $\eta(\mathbf{x}^S) \leq 1$ |

1. Results derived for convex inverse demand functions.
2. Results hold when the function $qp(q)$ is concave over the interval where $p(q)$ is positive.

# Efficiency lower bounds for CE and MO



# A lower bound on the profit ratio of a Cournot equilibrium.




The profit ratio bound decreases with the number of suppliers.

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# Plug-in (hybrid) electric vehicles

- Plug-in (hybrid) electric vehicles (PHEVs) may help to reduce greenhouse gas emissions.
- Large penetration of PHEVs will put considerable additional loads onto existing power grids.
- Proper scheduling of PHEV charging may help to balance the load.

## Objective

Appropriately schedule the charging of electric vehicles to maximize the social welfare.

# Dynamic Programming Approach

## Elements

1. Consider a **discrete time** model, where time periods are indexed by $t = 0, 1, \ldots, T$.
2. At each stage $t$, let $\mathcal{I}_t$ be the set of electric vehicles which has arrived at stage $t$, with a battery which is not fully charged at stage $t$.
3. For each vehicle $i \in \mathcal{I}_t$, let $\alpha_i$ and $\beta_i$ be its arrival and departure time, respectively. It can be charged from stage $\alpha_i + 1$ through $\beta_i$. We assume that each vehicle can stay at the station for at most $B$ time units, where $B$ is a positive integer.
4. At stage $t$, let $\gamma_{i,t}$ be the number of time units needed to fully charge vehicle $i$'s battery.
5. Let $a_{i,t} = 1$ if vehicle $i$ is charged at stage $t$; otherwise let $a_{i,t} = 0$.
6. A feasible **action** at stage $t$ is to charge $A_t$ arrived electric vehicles, where $A_t = \sum_{i \in \mathcal{I}_t} a_{i,t}$.

# Dynamic Programming Approach II

## Utility received by a vehicle

1. At stage $t$, let $x_{i,t} = (\beta_i - t, \gamma_{i,t})$ denote the state of vehicle $i$.
2. For example, a vehicle arrives at stage $\alpha_i = 0$, will leave at stage $\beta_i = 8$, and requires 5 time units of charging. At stage 1, it is charged for one time unit. Its state at stage 2 is $(7, 4)$.
3. For each vehicle $i$ in $\mathcal{I}_t$, it receives a utility $U(a_{i,t}, x_{i,t}) = a_{i,t} \cdot V(x_{i,t})$, where $V(\cdot)$ is a function of the current state of the vehicle.
4. For example, a vehicle with a state $(7, 4)$ receives a higher utility than a vehicle with a state $(7, 2)$, i.e., $V(7, 4) > V(7, 2)$.

# Dynamic Programming Approach III

## State of the grid

1. At each stage $t$, let $s_t$ be the state of the grid. The cost caused by battery charging of electric vehicles is $C(s_t, A_t)$.
2. The state of the grid, $s_t$, may present the the electric capacity available for electric vehicles. The cost caused by battery charging of electric vehicles, $C(s_t, A_t)$, depends on the residual capacity and the capacity used by electric vehicles.

## Elements-Continued

1. The grid state, $s_t$, evolves as a controlled Markov chain, where the transition probability depends on the current grid state and the current action, $A_t$.
2. The probability distribution on the number and state of the vehicles which will come in future stages is calculated according to the state of all vehicles in $\mathcal{I}_t$.

# Dynamic Programming Approach III

## Dynamic Programming Model

1. At each stage $t$, the **system state**, $x_t$, consists of the state of all vehicles in $\mathcal{I}_t$, and the grid state $s_t$.
2. For $t = 1, \ldots, T$, the **stage cost function**, $g_t(x_t, u_t)$, is $C(s_t, A_t) - \sum_{i \in \mathcal{I}_t} a_{i,t} \cdot V(x_{i,t})$.
3. A feasible **action** at stage $t$, $u_t$, is to charge $A_t$ electric vehicles in the set $\mathcal{I}_t$.
4. The system state, $x_t$, evolves as a controlled Markov chain, where the transition probability depends on the current system state and the current action taken by the centralized operator, $u_t$.

# Approximate Dynamic Programming

## Dynamic Programming Model-Continued

The system state evolves as a controlled Markov chain, where the transition probability, $P_t(x_{t+1} \mid x_t, u_t)$, depends on the current system state and the action taken by the operator, $u_t$.

## Challenges and the proposed approach

1. Since the system state includes the state of all arrived vehicles, the state space grows exponentially with the number of arrived vehicles.
2. To address this issue, we plan to first reduce the number of states by combining many of them into **aggregate states**.
3. Solve the DP problem with aggregate state space.
4. The cost-to-go function of the aggregate problem will then be used as heuristics for the limited look ahead policy for the original problem.

# State Aggregation

1. We choose a positive integer $b$ which is no more than $B$.
2. Given the set of arrived vehicles $\mathcal{I}_t$, $\widetilde{\mathbf{m}}_t$ is a $b$-dimensional vector such that its $n$th component reflects the number of battery units which has to be charged before time $t + n - 1$.
3. We use $\widetilde{\mathbf{m}}_t$ as the aggregate state for the arrived vehicles.

## Example

At stage 1, there are 3 vehicles which arrive during stage 0. Consider two possibilities:

1. Two vehicles will leave at stage 3, which require to be charged for 4 time units in total. The other one leaves at stage 4 and requires to be charged for 3 time units.
2. All three vehicles leave at stage 3, which require to be charged for 4 time units in total.

What if $b = 3$? How about $b = 4$?

# State Aggregation II

## Example

At stage 1, there are 3 vehicles that arrive during stage 0. Consider two possibilities:

1. Two vehicles will leave at stage 3, which require to be charged for 4 time units in total. The other one leaves at stage 4 and requires to be charged for 3 time units.
2. All three vehicles leave at stage 3, which require to be charged for 4 time units in total.

If $b = 3$, for both cases we have $\widetilde{\mathbf{m}}_1 = (0, 0, 4)$.
If $b = 4$, for the first case we have $\widetilde{\mathbf{m}}_1 = (0, 0, 4, 3)$, and for the second case we have $\widetilde{\mathbf{m}}_1 = (0, 0, 4, 0)$.

## Aggregate State

At each time $t$, the **aggregate system state**, $\widetilde{x}_t$, consists of the vector $\widetilde{\mathbf{m}}_t$, and the state of the grid $s_t$.

# State Aggregation III

## Comparison on the state space

Suppose that there are $N$ vehicles, and the battery capacity of each vehicle is $C$.

1. The state space of the original model is larger than $C^N$.
2. The state space of the aggregate model is in the order of $(CN)^b$.

# Cost function in the aggregate model

## Action

In the aggregate model, given the current system state $\widetilde{x}_t$, an feasible action, $\widetilde{u}_t$, is a $b$-dimensional vector such that its $n$-th component denotes the number of vehicles which will leave at stage $t+n-1$ and are charged at stage $t$.

## Cost function

In the aggregate model, given the current system state $\widetilde{x}_t$ and the current action $\widetilde{u}_t$, the stage cost function is given by

$$\widetilde{g}_t(\widetilde{x}_t, \widetilde{u}_t) = -C(s_t, A_t) + \sum_{y=1}^{b} \widetilde{u}_t(y) \cdot \left( \sum_{z=1}^{Z} w_z V(y,z) \right) / Z$$

where $A_t = \sum_{y=1}^{b} \widetilde{u}_t(y)$, $Z$ is the maximum capacity of a vehicle's battery, and $w_z$ are weighted factors such that $\sum_{z=1}^{Z} w_z = 1$.

# Transition probability in the aggregate model

1. The randomness of the original dynamic programming problem come from two aspects: the future grid status, as well as the number and the states of vehicles that will arrive in the next stage.
2. In the original model, given the current system state $x_t$, for an event $e$ on the vehicles that arrive at the next stage $t+1$, let $P_t(e \mid x_t)$ denote the probability that event $e$ occurs.
3. In the aggregate model, given the current system state $\widetilde{x}_t$, for an event $e$ on the vehicles that arrive at the next stage $t+1$, the probability that event $e$ occurs, $P_t(e \mid \widetilde{x}_t)$, is calculated by

$$P_t(e \mid \widetilde{x}_t) = \sum_{x \in \mathcal{S}(\widetilde{x}_t)} P_t(e \mid x)/\mathcal{N}(\widetilde{x}_t),$$

where $\mathcal{S}(\widetilde{x}_t)$ is the set of states in the original model that are presented as $\widetilde{x}_t$ in the aggregate model, and $\mathcal{N}(\widetilde{x}_t)$ is the number of elements in the set $\mathcal{S}(\widetilde{x}_t)$.

# Transition probability in the aggregate model II

## Transition probability

In the aggregate model, given the current system state, $\widetilde{x}_t = (\widetilde{m}_t, s_t)$, and the current action $\widetilde{u}_t$, the probability that the next system state is $\widetilde{x}_{t+1}$, $P_t(\widetilde{x}_{t+1} \mid \widetilde{x}_t, \widetilde{u}_t)$, is given by

$$P_t(\widetilde{x}_{t+1} \mid \widetilde{x}_t, \widetilde{u}_t) = P_t(s_{t+1} \mid s_t) \sum_{x \in \mathcal{S}(\widetilde{x}_t)} \sum_{e \in \mathcal{T}(\widetilde{m}_{t+1} - (\widetilde{m}_t - \widetilde{u}_t))} P_t(e \mid x)/\mathcal{N}(\widetilde{x}_t),$$

where $\widetilde{m}_t - \widetilde{u}_t$ is the aggregate state (a $b$-dimensional vector) at the end of stage $t$, $\mathcal{T}(\widetilde{m}_{t+1} - (\widetilde{m}_t - \widetilde{u}_t))$ is the set of all possible events (on vehicles which will arrive at $t+1$) which lead to the state $\widetilde{m}_{t+1}$, and $P_t(s_{t+1} \mid s_t)$ is the transition probability of the grid state.

# Solving the aggregate dynamic programming problem

## A $T$-stage dynamic programming problem

1. For the last stage, the cost-to-go function can be obtained by

$$\widetilde{J}_T(\widetilde{x}_T) = \min_{\widetilde{u}_T} \{\widetilde{g}_T(\widetilde{x}_T, \widetilde{u}_T)\}, \quad \forall \widetilde{x}_T.$$

2. For $t = 1, \ldots, T-1$, the cost-to-go function can be calculated through:

$$\widetilde{J}_t(\widetilde{x}_t) = \min_{\widetilde{u}_t} \left\{ \widetilde{g}_t(\widetilde{x}_t, \widetilde{u}_t) + \sum\nolimits_{\widetilde{x}_{t+1}} P_t(\widetilde{x}_{t+1} \mid \widetilde{x}_t, \widetilde{u}_t) \cdot \widetilde{J}_{t+1}(\widetilde{x}_{t+1}) \right\}, \quad \forall \widetilde{x}_t.$$

# One-step look ahead policy

## One-step look ahead policy

1. First calculate the probability $P_t(\widetilde{x}_{t+1} \mid x_t, u_t)$ by

$$P_t(\widetilde{x}_{t+1} \mid x_t, u_t) = \sum_{x \in \mathcal{S}(\widetilde{x}_{t+1})} P_t(x \mid x_t, u_t).$$

2. At stage $t$, under state $x_t$, one-step look ahead policy chooses an action $\overline{u}_t$ such that

$$\overline{u}_t \in \arg\min_{u_t} \left\{ g_t(x_t, u_t) + \sum_{\widetilde{x}_{t+1}} P_t(\widetilde{x}_{t+1} \mid x_t, u_t) \cdot \widetilde{J}_{t+1}(\widetilde{x}_{t+1}) \right\}.$$

# Performance bound for one-step look ahead policy

It is known that one-step look ahead policy achieves better performance than the heuristics, i.e.,

$$\min_{u_t} \left\{ g_t(x_t, u_t) + \sum_{\widetilde{x}_{t+1}} P_t(\widetilde{x}_{t+1} \mid x_t, u_t) \cdot \widetilde{J}_{t+1}(\widetilde{x}_{t+1}) \right\} \leq \widetilde{J}_t(\widetilde{x}_t),$$

where $x_t$ is included in the aggregate state $\widetilde{x}_t$.

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# Decentralized Scheduling

## Introduction

1. Suppose that the decision that a vehicle is charged or not is made by the owner of the vehicle.
2. The charging decision is made by each individual who aim to maximize her own payoff.

## Objective

To design a proper pricing mechanism for electric vehicles to benefit the social welfare when each individual makes her own charing plan to maximize her payoff.

# Dynamic game model I

1. The game is played in **discrete time**. We index time periods $t = 0, 1, \dots, T$.
2. At each stage $t$, let $s_t \in \mathcal{S}$ be the state of the grid, which evolves as a Markov chain and its transition is independent of each vehicle's decision.
3. At each stage $t$, for each vehicle $i \in \mathcal{I}_t$, the **state of vehicle** $i$ is denoted by $x_{i,t} \in \mathcal{X}$, which is a two dimensional vector such that the first component indicates the number of time units it will stay at the station, and the second component reflects the number of time units it needs to be charged before departure.
4. For example, a vehicle arrives at stage $\alpha_i = 0$, will leave at stage $\beta_i = 8$, and requires 5 time units of charging. At stage 1, it is charged for one time unit. Its state at stage 2 is $(7, 4)$.

# Dynamic game model II

1. At stage $t$, for each vehicle $i \in \mathcal{I}_t$, it chooses whether or not to charge the vehicle, $a_{i,t}$. $a_{i,t} = 1$ indicates it is charged at stage $t$ and $a_{i,t} = 0$ means it is not.
2. At stage $t$, for each vehicle $i \in \mathcal{I}_t$, it receives a utility $a_{i,t} \cdot V(x_{i,t})$.
3. At stage $t$, for each vehicle $i \in \mathcal{I}_t$, it pays $a_{i,t} \cdot p_t$, where $p_t$ is the current price set for electric vehicles. Its payoff function is given by $a_{i,t} \cdot (V(x_{i,t}) - p_t)$.

# Dynamic game model III

## State transition

1. For those unarrived vehicles, we say they are in a state "NULL". For a vehicle which has been fully charged or left the station, it is in a state "Completed".
2. At the first stage 0, the initial states of all vehicles are drawn from a given distribution.
3. For an arrived vehicle, the first component of a vehicle's state is easily calculated. The second component is reduced by 1 at stage $t + 1$ if the vehicle is charged at stage $t$.
4. For a vehicle at the NULL state, its state transits to other (arrived vehicle's) states under a given state transition probability vector.
5. For a vehicle at the Completed state, its state transits to the state of NULL with a given probability.

# Dynamic game model IV

## Cost function and social welfare

1. Under current grid state $s_t$, let $C(s_t, A_t)$ denote the cost caused by battery charging of electric vehicles, where $A_t = \sum_i a_{i,t}$ is the number of electric vehicles charged at stage $t$.
2. The social welfare realized at stage $t = 1, \ldots, T$ is given by

$$\sum_{i \in \mathcal{I}-t} a_{i,t} \cdot V(x_{i,t}) - C(s_t, A_t).$$

## Problem

How to design a pricing mechanism, i.e., a sequence of prices $\{p_t\}_{t=1}^{T}$, to maximize the social welfare when each vehicle aims to maximize its own benefit.

# Equilibrium

- Let $f_t$ denote the **distribution** on vehicles' state.
- Let $h_t = (s_0, s_1, \ldots, s_t)$ denote the grid history up to stage $t$.
- A **strategy** $\nu$, which maps a grid history, $h_t$, as well as the current state of the vehicle, $x_{i,t}$ into an action $a_{i,t}$.

- Suppose all vehicles use a strategy $\nu$. It leads to a history-dependent distribution on vehicles' state, i.e.,

$$\mathcal{D}_\nu : h_t \to (f_0, f_1, ..., f_t), \quad t = 0, ..., T.$$

- A strategy $\nu$ is said to be an equilibrium if it maximizes each vehicle's long-term expected payoff with respect to the distribution it induces, i.e., $\nu(h_t, x)$ maximizes the following expected payoff

$$a \cdot (V(x) - p_t) + E\left[\sum_{\tau=t+1}^{T} \nu(h_\tau, x_{i,\tau})(V(x_{i,\tau}) - p_\tau)\right].$$

# Equilibrium II

Suppose that the current price, $p_t$, is a function of the current grid state, $s_t$, and the aggregate demand $A_t$. If all vehicles use a strategy $\nu$, then the expected payoff

$$a \cdot (V(x) - p_t) + E\left[\sum_{\tau=t+1}^{T} \nu(h_\tau, x_{i,\tau})(V(x_{i,\tau}) - p_\tau)\right],$$

is a function of the distribution on vehicles' state $(f_t, \ldots, f_T)$, where the expectation is over future grid states $(s_{t+1}, \ldots, s_T)$.

# Equilibrium III

## An example

- Suppose that the grid state remains unchanged.
- Suppose that all vehicles arrive with a state $(2, 1)$. There are totally four states, i.e., $(2, 1), (1, 1)$, NULL and Completed.
- Suppose that the strategy to charge only at state $(1, 1)$ leads to a stationary distribution on vehicles' state, and results in a stable price $p > 0$.
- The utility functions are given by: $V(2, 1) = p/2$ and $V(1, 1) = 1.5p$.
- The strategy is an equilibrium.

# The proposed pricing mechanism

- Suppose that the cost function $C(s_t, A_t)$ is increasing in $A_t$, and $C(s_t, A) - C(s_t, A-1) \geq C(s_t, B) - C(s_t, B-1)$ for any pair of positive integers such that $A > B$.
- Let $\overline{C}(s_t, A)$ be an increasing, convex and continuously differentiable function in $A$ such that $\overline{C}(s_t, A) = C(s_t, A)$ for any positive integer $A$.
- We let $p_t = \overline{C}'(s_t, A_t)$.

# Conjecture

Suppose that all vehicles use a strategy $\mu$. Given a history $h_t$ and a distribution on vehicles' state, $f_t$, the social welfare achieved at stage $t$ is given by

$$W_t(h_t, f_t, \mu) = -C(s_t, A_t) + \sum_x f_t(x) \cdot \mu(h_t, x) V(x),$$

where $A_t = \sum_x f_t(x) \cdot \mu(h_t, x)$.

## Conjecture

The pricing mechanism maximizes the social welfare at an equilibrium, i.e., if $\nu$ is an equilibrium, then it maximizes the expected social welfare,

$$W_t(h_t, f_t, \mu) + E\left[\sum_{\tau=t+1}^{T} W_\tau(h_\tau, f_\tau, \mu)\right],$$

among all possible strategies.