Distributed Optimization, Control and Dynamic Game Algorithms

for Transmission Network and Self-Organizing Distribution Networks in Smart Grids

Zhihua Qu

Professor and Interim Chair of ECE, University of Central Florida
at
Los Alamos NL

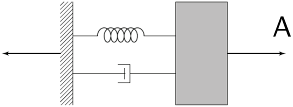
July 19, 2011

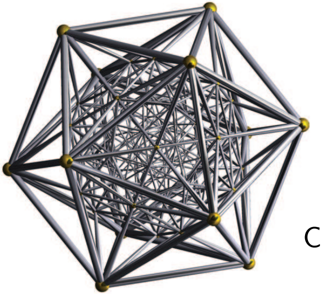
Acknowledgements

- Thanks to Dr. Misha Chertkov for invitation/arrangement and to Ms. Lysa Intrator for assistance.
- Current fundings from NSF, DoE, Army, L-3, FHTC, Harris, etc.
- Current postdocs and graduate students: Ed Daughtery, Hendra Harno, Chaoyong Li, Wei Lin, Ali Maknouninejad, and Mark Snyder.
- Past postdocs and graduate students: Huanhai Xin, Jing Wang, and John Seuss.
- Collaborators: Marwan Simaan, Mary Ann Ingram, Frank Lewis

Outlines

A Simple Networked System (Cyber-Physical System)





Physical: Spring-damper systems

A cyber-physical system if

- Spring-damper forces are replaced by artificial forces
- Physical connections are replaced by local sensing/communication network
- Time-varying topologies, latencies, etc
- Heterogeneous dynamics

Cooperative control:

- distributed
- stability and robustness
- only cumulative information flow

Power System as a Cyber-Physical System

Power systems as a cyber-physical system

- Physical entities of controllable dynamics (generation units, DGs, storage devices, etc)
- Nonlinear algebraic constraints (load flow equations)
- Wide-area monitoring versus local communication: varying topologies and latencies
- Variable operational conditions (loads, DGs, disturbances, etc)
- Diverse economic interests

Core problems:

- Control with partial information
- Robustness under variations of topology, generation and loads.
- Make aggregated DG generation dispatchable.
- Optimize the system operation under different interests

Optimal Control with a Specific Constraint of Information Structure?

Controllable system:

$$\dot{x} = Ax + Bu$$

Desired performance index:

$$J^* = \frac{1}{2}\int_0^\infty (x^T Q^* x + u^T R^* u)\mathrm{d}t.$$

Algebraic Riccati equation:

$$K^* A + A^T K^* + Q^* - K^* B (R^*)^{-1} B^T K^* = 0$$

Optimal control:

$$u = -G^* x = -(R^*)^{-1} B^T K^* x.$$

What happens if

$$u = -G_s x,$$

where G_s has certain structure (i.e., certain elements must be zero)?

Special case: $G_s = FC$, where $y = Cx$.

Constraint on Information Structure

Consider the system with $x(0) = [x_1(0)\ x_2(0)]^T$,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = I_{2\times 2}, \quad Q^* = 2I_{2\times 2}, \quad R^* = I_{2\times 2}.$$

Suppose that the feedback information topology requires

$$G_s = K_s = \text{diag}\{k_1, k_2\}.$$

Standard (unstructured) optimal solution:

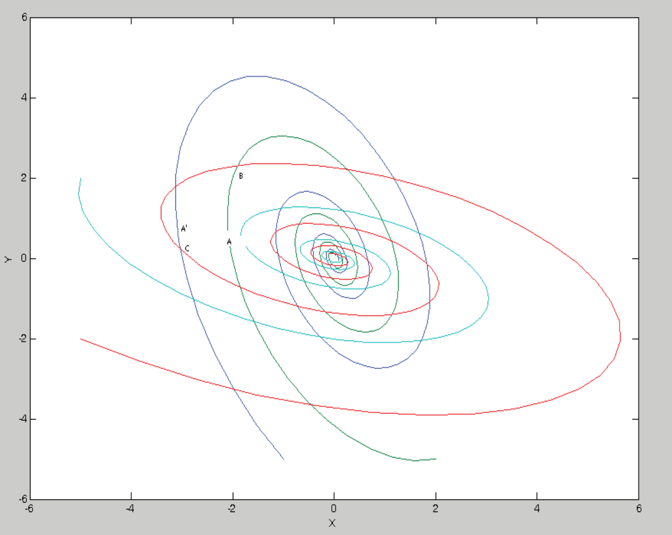
$$G^* = (R^*)^{-1} B^T K^* = \begin{bmatrix} 1.3409 & 0.4495 \\ 0.4495 & 1.6422 \end{bmatrix}$$

Structured optimization:

$$J_p^* = \frac{1}{2} x^T(0) \begin{bmatrix} \frac{2+k_1^2}{2k_1} & \frac{2+k_1^2}{2k_1(k_1+k_2)} \\ \frac{2+k_1^2}{2k_1(k_1+k_2)} & \frac{2+k_1^2}{2k_1 k_2 (k_1+k_2)} + \frac{2+k_2^2}{2k_2} \end{bmatrix} x(0).$$

In general, the problem is NP-hard.

Stability and Robustness under Switching



Cooperative Control Design

Networked Dynamical Systems

Networked dynamical systems: for $j = 1, \cdots, q$,

$$\dot{z}_j = f_j(z_j, u_j) + \Delta f_j(z_j, u_j), \quad y_j = h_j(z_j).$$

Network uncertainties: binary connectivity matrix

$$S(t) = \begin{bmatrix} 1 & s_{12}(t) & \cdots & s_{1q}(t) \\ s_{21}(t) & \ddots & \ddots & s_{2q}(t) \\ \vdots & \ddots & \ddots & \vdots \\ s_{q1}(t) & \cdots & s_{q(q-1)}(t) & 1 \end{bmatrix},$$

and latency matrix

$$S_\tau(t) = \begin{bmatrix} 0 & \tau_{12}(t) & \cdots & \tau_{1q}(t) \\ \tau_{21}(t) & \ddots & \ddots & \tau_{2q}(t) \\ \vdots & \ddots & \ddots & \vdots \\ \tau_{q1}(t) & \cdots & \tau_{q(q-1)}(t) & 0 \end{bmatrix}.$$

Cooperative Control Problem

Cooperative control:

$$u_j(t) = U_j(z_j(t), s_{j1}(t)y_1(t-\tau_{j1}), \cdots, s_{jq}(t)y_q(t-\tau_{jq})).$$

Key features: self and local feedback, pliable to network changes, ...

Closed-loop overall dynamics: for $d_{il}(t) \geq 0$,

$$\begin{aligned} \dot{x}_i(t) &= \mathcal{F}_i(d_{i1}(t)x_1(t-\tau_{i1}), d_{i2}(t)x_2(t-\tau_{i2}), \cdots, x_i(t), \cdots \\ &\qquad d_{in}(t)x_n(t-\tau_{in})), \quad \tau_{ij} \in [0, r], \end{aligned}$$

Information flow: unpredictable connectivity, unknown latencies, etc.

Cooperative stability: $\lim_{t\to\infty} y_j = c$ for all j.

Cooperative control theory: methods and tools to ensure performance in terms of *cumulative* information flow!

Linear Networked Systems and Their Cooperative Control

Linear Systems: $y_i = C_i' z_i$ for $i = 1, \cdots, q$,

$$z_i(k+1) = A_i' z_i(k) + B_i' v_i(k), \quad or \quad \dot{z}_i = A_i' z_i + B_i' v_i.$$

Cooperative control:

$$v_j(t) \;=\; -K_{jj} z_j(t) + \sum_{l \neq j} s_{jl}(t) K_{jl} [y_l(t - \tau_{jl}) - y_j(t)]$$

or its variations.

Cooperative stability: $\lim_{t \to \infty} y_j = c$ for all j.

Goal: Linear methods and design tools in terms of *cumulative* information flow!

Linear Design Procedure: Cooperative Control Canonical Form (with stable internal dynamics)

$$\dot{x}_i = A_i x_i + B_i u_i, \quad y_i = C_i x_i, \quad \dot{\varphi}_i = g_i(\varphi_i, x_i),$$

$$A_i = (J_{l_i} - I_{l_i \times l_i}) \otimes I_{m\times m}, \quad B_i = \begin{bmatrix} 0 \\ I_{m\times m} \end{bmatrix}, \quad C_i = \begin{bmatrix} I_{m\times m} & 0 \end{bmatrix},$$

and J_k is the kth order Jordan block with eigenvalue 0.
Cooperative Control is:

$$u_i(t) = \sum_{j=0}^{q} G_{ij}(t)[s_{ij}(t) y_j] \stackrel{\triangle}{=} G_i(t) y, \quad i = 1, \cdots, q,$$

where $G_{ij}(t) = G_{ij}(t_k^s)$ for $t \in [t_k^s, t_{k+1}^s)$,

$$G_{ij}(t_k^s) = \frac{s_{ij}(t_k^s)}{\sum_{\eta=1}^{q} s_{i\eta}(t_k^s)} K_c, \quad j = 1, \cdots, q, \; ; \; K_c \in \Re^{m\times m} \geq 0, \quad K_c \mathbf{1}_m = \mathbf{1}_m.$$

The Overall Networked System

$$\dot{x} = [A + BG(t)C]x = [-I_{N\times N} + D(t)]x,$$

$$x = [x_1^T, \ \cdots, \ x_q^T]^T, \ \ N = m\sum_{i=1}^{q} l_i,$$

$$A = \text{diag}\{A_1, \ \cdots, \ A_q\}, C = \text{diag}\{C_1, \ \cdots, \ C_q\}, B = \text{diag}\{B_1, \ \cdots, \ B_q\},$$

$$D(t) = \begin{bmatrix} \overline{G}_{11}(t) & \cdots & \overline{G}_{1q}(t) \\ \vdots & \vdots & \vdots \\ \overline{G}_{q1}(t) & \cdots & \overline{G}_{qq}(t) \end{bmatrix},$$

$$\overline{G}_{ii} = \begin{bmatrix} 0 & I_{(l_1-1)\times(l_1-1)} \otimes I_{m\times m} \\ G_{ii} & 0 \end{bmatrix} \in \Re^{l_i m\times l_i m}, \ \ i = 1, \cdots, q,$$

$$\overline{G}_{ij} = \begin{bmatrix} 0 & 0 \\ G_{ij} & 0 \end{bmatrix} \in \Re^{l_i m\times l_j m}, \ \ i = 1, \cdots, q, \ j = 0, 1, \cdots, q, \ \ i \neq j.$$

Underlining Mathematics Problem: Solved

Closed-loop solution:

$$x(t^s_{k+1}) = e^{[-I+D(t^s_k)](t^s_{k+1}-t^s_k)} x(t^s_k),$$

or

$$x(k+1) = P(k)x(k),$$

where $P(k)$ is a Metzler matrix. Choose K_{ij} so that $P(k)$ is row stochastic.

Fundamental question: Is the multiplicative sequence convergent ?

$$\lim_{k\to\infty} P(k)P(k-1)\cdots P(2)P(1) = \mathbf{1}c^T$$

for some $c \in \Re^n$.

Matrix theoretical approach: convergence in terms of cumulative information flow over an infinite sequence of finite intervals.

Necessary and Sufficient Condition on Cooperative Stability

Definition: Communication/sensing sequence $\{S(k) : k \in \aleph^+\}$ is *sequentially complete* if an infinite multiplicative subsequence extracted from $\bigwedge_{k=1}^{\infty} S(k)$ is lower-triangularly complete.

Theorem: Sequence $\{P(k) : k \in \aleph^+\}$ is convergent as

$$\lim_{k\to\infty} \prod_{\eta=1}^{k} P(\eta) = \mathbf{1}c,$$

if and only if $\{S(k) : k \in \aleph^+\}$ is sequentially complete.

Implications:

cooperative controllability

cooperative stability

designs of various behaviors.

Application: 3Φ Inverter Modeling & Cooperative Control Design

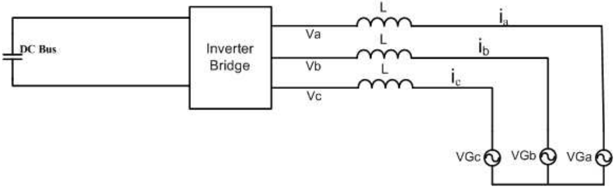


Figure: A typical 3-phase inverter

Dynamic equations:

$$V_{G_{abc}} = L\frac{di_{abc}}{dt} + V_{abc}$$
$$V_{abc} = K * V_{c_{abc}}$$

where K — inverter PWM gain, and $V_{c_{abc}}$ — control input to the inverter.

DQ-Model of Inverters

Applying the park transformation yields:

$$\frac{di}{dt} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} i + \frac{1}{L}(KV_C - V_G)$$

where i — output current, V_c — input command, V_G — the voltage at inverter terminals,

$$i = [i_d \quad i_q]^T, \quad V_c = [V_{cd} \quad V_{cq}]^T, \quad V_G = [V_{Gd} \quad V_{Gq}]^T$$

State space representation:

$$\frac{di}{dt} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} i + B' u'$$

$$u' = [V_{cd} \quad V_{cq} \quad V_{Gd} \quad V_{Gq}]$$

$$B' = \begin{bmatrix} K & 0 & -1 & 0 \\ 0 & K & 0 & -1 \end{bmatrix}$$

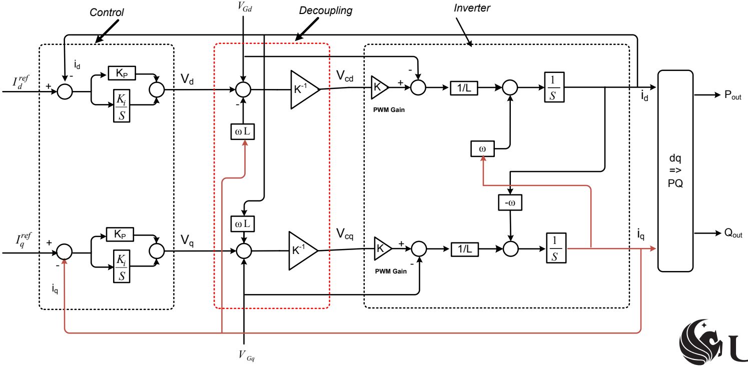
DQ-Model Decoupling & Standard Inverter Block Diagram

Let

$$V = K * V_c - V_G + \omega L[i_q \quad - i_q]^T,$$

where $V = [V_d \quad V_q]^T$. Then,

$$\frac{dI}{dt} = \frac{1}{L} V$$



Cooperative Control Design of 3Φ Inverters

By feedback linearization, we have that, for $k_c > 0$ and letting $d_{ij}(t) = s_{ij}(t)/[\sum_l s_{il}(t)]$,

$$\begin{aligned} \dot{y}_i &= \dot{C}_i x_i + C_i \dot{x}_i = \dot{C}_i x_i + C_i(Ax_i + Bu_i) \\ &\triangleq -k_c y_i + k_c \sum_j d_{ij}(t) y_j. \end{aligned}$$

Solution of u_i:

$$\begin{aligned} u_i &= (C_i B)^{-1}[-k_c y_i + k_c \sum_j d_{ij}(t) y_j - \dot{C}_i x_i - C_i A x_i] \\ &= \begin{bmatrix} \frac{L\overline{P}_i}{k_p V_{G_i}} & 0 \\ 0 & -\frac{L\overline{Q}_i}{k_p V_{G_i}} \end{bmatrix} [-k_c y_i + k_c \sum_j d_{ij}(t) y_j] \\ &\quad - \begin{bmatrix} \left(-\frac{L}{k_p V_{G_i}} \dot{V}_{G_i} + \frac{L}{k_p V_{G_i} \overline{P}_i} \dot{\overline{P}}_i\right) i_{d_i} - \frac{k_i}{k_p} \int (u_{i_1} - i_{d_i}) d\tau + i_{d_i} \\ \left(-\frac{L}{k_p V_{G_i}} \dot{V}_{G_i} + \frac{L}{k_p V_{G_i} \overline{Q}_i} \dot{\overline{Q}}_i\right) i_{q_i} - \frac{k_i}{k_p} \int (u_{i_2} - i_{q_i}) d\tau + i_{q_i} \end{bmatrix} \end{aligned}$$

Simplified Inverter Model: Cooperative Control of DGs

A simple model of the renewables is: $i = 1, \cdots, N_{DG}$,

$$P_{DG_i} \le \overline{P}_{DG_i}, \quad P_{DG_i} = V_{DG_i}(t) I_{d_i}, \quad \dot{I}_{d_i} = v_{i1},$$
$$Q_{DG_i} \le \overline{Q}_{DG_i}, \quad Q_{DG_i} = -V_{DG_i}(t) I_{q_i}, \quad \dot{I}_{q_i} = v_{i2}.$$

Control objectives: fair utilization profiles,

$$y_{P_i} \triangleq \frac{P_{DG_i}}{\overline{P}_{DG_i}} \to \alpha_p, \quad y_{Q_i} \triangleq \frac{Q_{DG_i}}{\overline{Q}_{DG_i}} \to \alpha_q.$$

Cooperative control design: $y_{P_0} = \alpha_p$ being the virtual leader and $k_c > 0$ being a cooperative control gain,

$$v_{i1} = \frac{\overline{P}_{DG_i}}{V_{DG_i}} \left[-\frac{\dot{V}_{DG_i} I_{d_i}}{\overline{P}_{DG_i}} + \frac{P_{DG_i} \dot{\overline{P}}_{DG_i}}{\overline{P}^2_{DG_i}} + k_c \sum_{j=0}^{N_{DG}} d_{ij} y_{P_j} - k_c y_{P_i} \right],$$

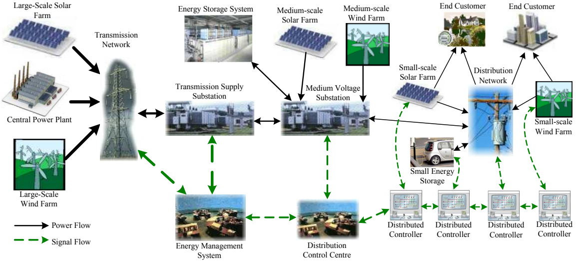
under which

$$\dot{y}_{P_i} = k_c \left[-y_{P_i} + \sum_{j=0}^{N_{DG}} d_{ij} y_{P_j} \right].$$

Smart Grids:

Self-Organizing Cooperative Control
Multi-Level Game-Based Optimization

Problems Addressed



Issues:

- Difficult to dispatch and control DGs due to intermittent and small output
- expensive to have information flow
- difficult negotiation between distribution and transmission part, etc.

Solutions:

- Self-organizing cooperative control of DGs for real power aggregation, storage and injection
- Self-organizing cooperative control for reactive power compensation and voltage stability
- Multi-level multi-entity optimization

UCF

Generation and Transmission

Conventional generation: $i = 1.\cdots, N_g$,

$$\dot{\theta}_i = w_i, \quad M_i \dot{w}_i = P_{m_i} - P_{g_i},$$

$$P_{G_i} = \sum_{i=1}^{N_g} V_i V_j [G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}], \quad \delta_{ij} = \theta_i - \theta_j.$$

Renewables (distributed generation): $i = 1, \cdots, N_{DG}$,

$$P_{DG_i} = V_{DG_i}(t) I_{d_i}, \quad \dot{I}_{d_i} = v_{i1}, \quad Q_{DG_i} = -V_{DG_i}(t) I_{q_i}, \quad \dot{I}_{q_i} = v_{i2}.$$

Power flow equations of transmission network:

$$P^a_{G_i} - P^a_{D_i} = \sum_{j=1}^{N^t_b} V_i V_j \left[G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}\right],$$

$$Q^a_{G_i} - Q^a_{D_i} = \sum_{j=1}^{N^t_b} V_i V_j \left[G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}\right],$$

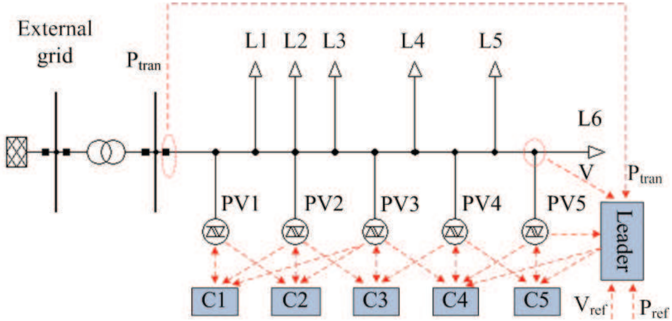
Overall System Model

Distributed generation/storage and Var devices: $i = 1, \cdots, N_{DG}$,

$$\begin{cases} P_{DG_i} = V_{DG_i}(t) I_{d_i} \\ \dot{I}_{d_i} = v_{i1} \end{cases} \qquad \begin{cases} Q_{DG_i} = -V_{DG_i}(t) I_{q_i} \\ \dot{I}_{q_i} = v_{i2} \end{cases}$$

Power flow equations:

$$\begin{cases} g_p(P_1, \cdots, P_{N_{DG}}, X_p) = 0 \\ g_q(Q_1, \cdots, Q_{N_{DG}}, X_q) = 0 \end{cases}$$



Self-Organizing Distributed Control

Cooperative control objective: fair utilization profiles as

$$y_{P_i} \triangleq \frac{P_{DG_i}}{\overline{P}_{DG_i}} \to \alpha_p, \quad y_{Q_i} \triangleq \frac{Q_{DG_i}}{\overline{Q}_{DG_i}} \to \alpha_q,$$

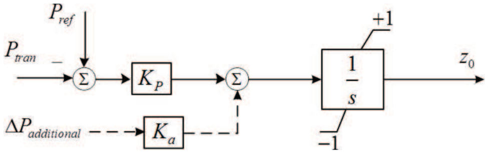
Cooperative control: $y_{P_0} = \alpha_p$ being the virtual leader and $k_c > 0$ being a cooperative control gain,

$$v_{i1} = \frac{\overline{P}_{DG_i}}{V_{DG_i}} \left[-\frac{\dot{V}_{DG_i} I_{d_i}}{\overline{P}_{DG_i}} + \frac{P_{DG_i} \dot{\overline{P}}_{DG_i}}{\overline{P}^2_{DG_i}} + k_c \sum_{j=0}^{N_{DG}} d_{ij} y_{P_j} - k_c y_{P_i} \right].$$

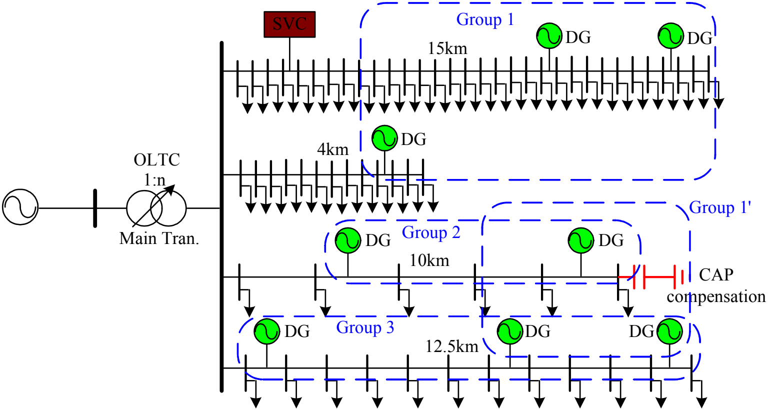
Control objective for self-organizing microgrids: for each virtual leader,

$$\dot{y}_{P_0} = k'_p [P^{ref}_{tran} - P_{tran}], \quad \dot{y}_{Q_0} = k'_q [V^{ref}_c - V_c],$$

where P_{tran} is power flow (downstream or upsteam), and V_c is the critical bus voltage. *Low-level distributed optimization algorithm*



Self-Organizing Microgrids



Power System with Self-Organizing Distributed Control

Closed-loop differential-algebraic system is:

$$
\begin{aligned}
\dot{z}_0 &= k_p[P_{tran}^{ref} - P_{tran}(z_1, \cdots, z_{N_{DG}}, X_p)] \\
\dot{z}_i &= k_c\left[-z_i + d_{i0}z_0 + \sum_{j=1}^{N_{DG}} d_{ij}z_j\right] \\
0 &= g_p(P_1, \cdots, P_{N_{DG}}, X_p),
\end{aligned}
$$

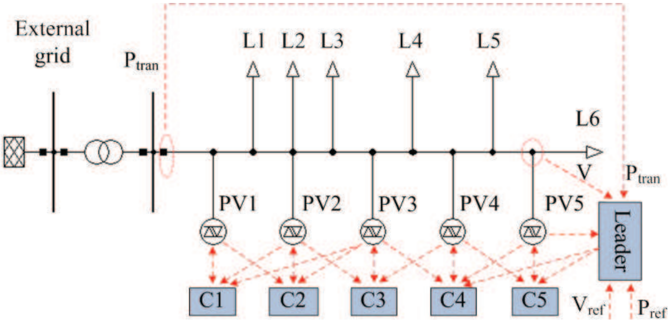
and

$$
\begin{aligned}
\dot{z}_0' &= k_q[V_c^{ref} - V_c(z_1', \cdots, z_{N_{DG}}', X_q)] \\
\dot{z}_i' &= k_c\left[-z_i' + d_{i0}z_0' + \sum_{j=1}^{N_{DG}} d_{ij}z_j'\right] \\
0 &= g_q(Q_1, \cdots, Q_{N_{DG}}, X_q).
\end{aligned}
$$

where $z_0 = \alpha_p$, $z_i = P_{DG_i}/\overline{P}_{DG_i}$, $z_0' = \alpha_q$, $z_i' = Q_{DG_i}/\overline{Q}_{DG_i}$.

Basic Facts on Power System Operations

Fact 1: P_{tran} is an increasing function of P_{DG_i} (and hence of α_p)



Fact 2: Phase angles at the both sides of a transmission line of our concern are relatively close, that is

$$|\sin(\delta_i - \delta_j)| << |\cos(\delta_i - \delta_j)|.$$

Asymptotic Stability under Self-Organizing Distributed Control

Theorem: Consider the system:

$$\begin{aligned}
\dot{z}_0 &= k_p[P_{tran}^{ref} - P_{tran}(z_1, \cdots, z_{N_{DG}}, X_p)] \\
\dot{z}_i &= k_c\left[-z_i + d_{i0}z_0 + \sum_{j=1}^{N_{DG}} d_{ij}z_j\right] \\
0 &= g_p(P_1, \cdots, P_{N_{DG}}, X_p).
\end{aligned}$$

If

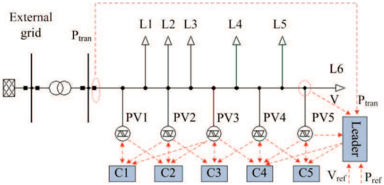
- Gains are chosen such that k_p/k_c is small,
- Facts 1 and 2 hold,
- Communication among the DGs are cumulatively connected (sequentially complete),

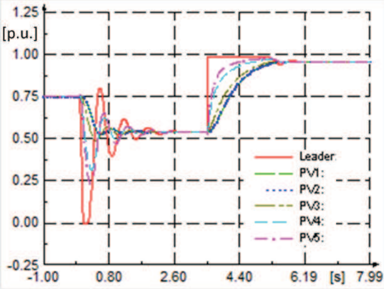
then, the system is asymptotically stable in the sense that $z_i \to z_0 \to \alpha_p^*$ and $P_{tran} \to P_{tran}^{ref}$.

Case Study 1: Radial Distribution Network

Case Study 1: Load Variations in a Radial Distribution Network

All loads experience 10% decrease at $t = 0$ and then a 20% increase at $t = 3.5$s, while active power and reactive power generations of DGs are kept the same. Communication is fixed as shown.



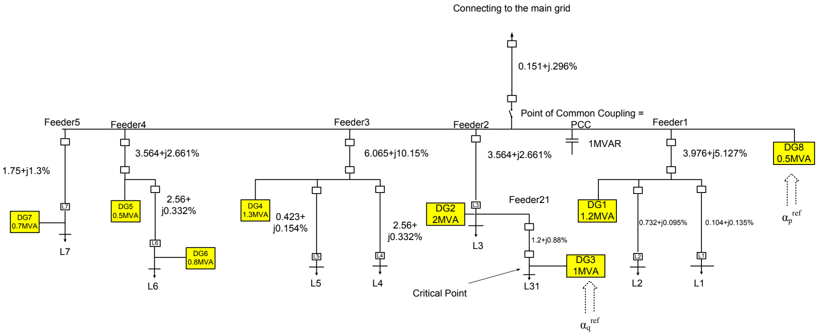


Active power outputs of DGs are adaptively adjusted while converging to the fair utilization profile.

Case Study 2: A Microgrid

Case Study 2: A Microgrid

A modified version in IEEE 399-1997. 8 DGs are distributed along 5 feeders .



Communication Topologies

$$S(t) = 0 \quad t \in ((k-1)T_c + 0^+,\ kT_c], \quad T_c = \frac{1}{f_c}$$

and for $t \in [(k-1)T_c,\ (k-1)T_c + 0^+)$:

$$S_{GlobalConnectivity}(t) = [1],$$

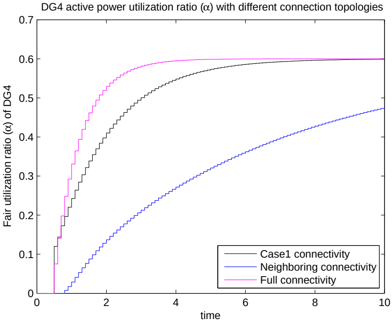
or

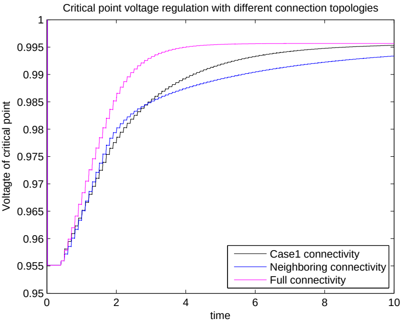
$$S_{case1}(t) = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

or

$$S_{neighboringConnectivity}(t) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

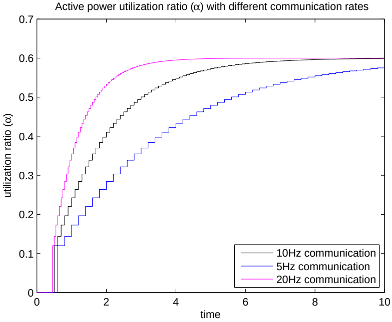
Performance of Cooperative Control versus Communication Topology

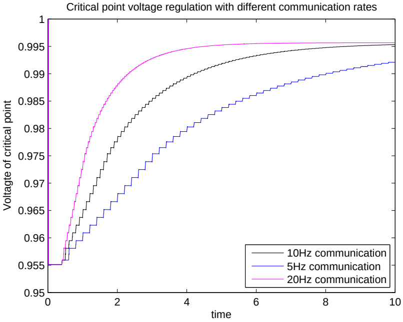




Performance of Cooperative Control versus Communication Frequency

Response of DG4, given $S_{GlobalConnectivity}(t)$ and $\alpha_p^{ref} = 0.6$:

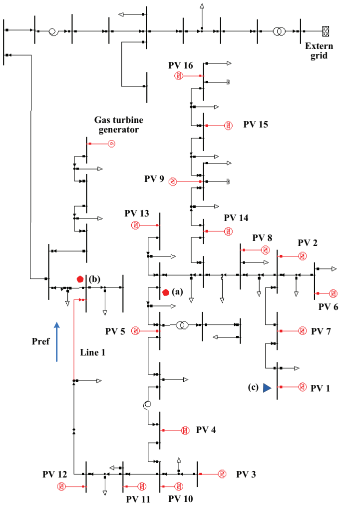




Case Study 3: IEEE 34-bus Distribution Network

Case Study 3: IEEE 34-bus Distribution Network

16 PVs are added: P_{tran} — line 1, and the critical bus voltage (PV1),



Voltage Fluctuations without Cooperative Control

When only PV #1 is active and a reactive compensator is added at the location of gas turbine, voltage fluctuations with respect to the DG penetration level are:

Table 1 Voltage Drop of Central Bus with Different Compensating Capacitors

RCSCC / VDCB / PLPVG	0MVar	0.6MVar	1.2MVar
20%	12.3%	6.6%	5.2%
40%	14.5%	10.0%	8.4%

Table 2 Voltage Drop of Central Bus with Different Synchronous Compensators

RCSC / VDCB / PLPVG	0MVar	0.6MVar	1.2MVar
20%	12.3%	4.2%	3.3%
40%	14.5%	7.3%	5.7%

RCSCC= Rated Capacity of Static Compensating Capacitor

RCSC=Rated Capacity of Synchronous Compensator

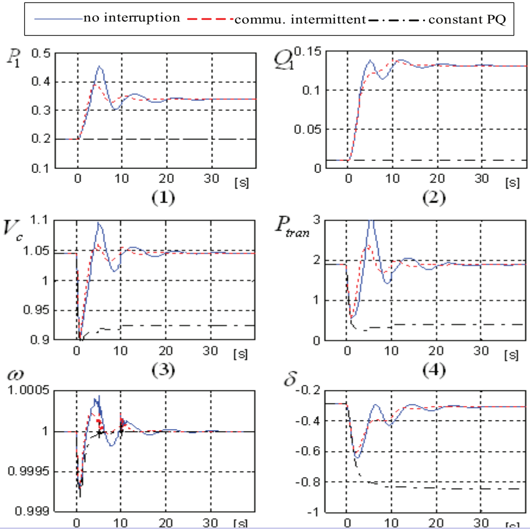
VDCB=Voltage Drop of Central Bus

Voltage Stability under Cooperative Control

For the PV penetration level changing from 0 to over 200%:

PV	0%	50%	100%	220%
1	0.912	0.947	0.978	1.046
2	0.912	0.947	0.978	1.046
3	0.950	0.969	0.986	1.023
4	0.924	0.955	0.983	1.043
5	0.919	0.950	0.979	1.039
6	0.912	0.947	0.978	1.046
7	0.912	0.947	0.978	1.046
8	0.912	0.946	0.978	1.045
9	0.912	0.946	0.977	1.044
10	0.950	0.969	0.966	1.021
11	0.951	0.969	0.986	1.021
12	0.966	0.976	0.985	1.002
13	0.916	0.949	0.978	1.042
14	0.913	0.946	0.978	1.044
15	0.912	0.946	0.978	1.044
16	0.912	0.946	0.978	1.044

Robustness Against Line Fault and Communication Interruptions



Energy Management System: Optimal Power Flow (OPF)

$$\min \sum_{i=1}^{N_b^t} [a_{3i}(P_{G_i})^2 + a_{2i}P_{G_i} + a_{1i} + a_{0i}P_{DG_i}^a],$$

subject to power flow equations and steady-state constraints:

$$\begin{cases} \underline{V}_i \leq V_i(t) \leq \overline{V}_i, \\ \underline{P}_{G_i}(t) \leq P_{G_i}(t) \leq \overline{P}_{G_i}(t), \quad \underline{Q}_{G_i}(t) \leq Q_{G_i}(t) \leq \overline{Q}_{G_i}(t), \\ \underline{P}_{DG_i}^a(t) \leq P_{DG_i}^a(t) \leq \overline{P}_{DG_i}^a(t), \quad \underline{Q}_{DG_i}^a(t) \leq Q_{DG_i}^a(t) \leq \overline{Q}_{DG_i}^a(t). \end{cases}$$

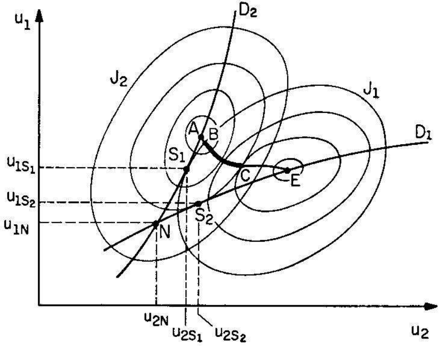
Thermal constraints: $i = 1, \cdots, N_l$,

$$-\underline{T}_i \leq T_i \leq \overline{T}_i.$$

Dynamic security constraints: $k = 1, \cdots, N_c$,

$$|\theta_i^k(t) - \theta_j^k(t)| \leq \overline{\delta}.$$

Multi-Player Optimization: Nash v.s. Stackelberg



N—Nash solution, S_1—Stackelberg solution with P1 as the leader, S_2—Stackelberg solution with P2 as the leader.

Example of Matrix Game

Consider

$$\min_{u_1, u_2} \{J_1, J_2\},$$

where $J_1 = J_1(u_1, u_2)$ and $J_2 = J_2(u_1, u_2)$:

u_1 \ u_2	0.6	0.8	1	1.2	1.4
0.6	{4,5}	{4,3}	{2,3}	{3,1}	{5,9}
0.8	{5,10}	{7,4}	{3,3}	{8,12}	{22,24}
1	{7,8}	{5,6}	{2,2}	{4,4}	{10,11}
1.2	{5,9}	{4,6}	{8,5}	{5,8}	{1,2}
1.4	{1,18}	{10,9}	{5,4}	{6,7}	{10,15}

Nash: $(u_1, u_2) = (1, 1) \to \{2, 2\}$, $(0.6, 1.2) \to \{3, 1\}$, $(1.2, 1.4) \to \{1, 2\}$.

$$\text{Stackelberg: } \begin{cases} (u_1, u_2) = (1.2, 1.4) & \text{if } u_1 \text{ is the leader} \\ (u_1, u_2) = (0.6, 1.2) & \text{if } u_2 \text{ is the leader} \end{cases}$$

Optimization at Transmission Level

$$J_t(\beta_i(k), P_{M_i}(k)) = \min_{\beta_i,\, P_{M_i}} \sum_{i=1}^{N_b^t} \sum_{l=k}^{N} [a_i(l) P_{G_i}(l) + \beta_i(l) P_{M_i}(l)],$$

where N_b^t — bus number, k — index (up to N), and $a_i(l) = a(P_{G_i}(l))$ — cost function.

- "DC" power flow of transmission network: at the ith bus ($i = 1, \ldots, N_b^t$)

$$P^a_{G_i}(k) - P_{D_i}(k) = \sum_{j=1}^{N_b^t} B_{ij} \delta_{ij}(k),$$

  where $P^a_{G_i}(k)$ — aggregated generation (0, or $P_{G_i}(k)$, or $P_{M_i}(k)$, or $P_{G_i}(k) + P_{M_i}(k)$), $P_{D_i}(k)$ — load, and $\delta_{jj} = 0$.

- Steady-state constraints:

$$\underline{P}_{G_i}(t) \leq P_{G_i}(t) \leq \overline{P}_{G_i}(t), \quad \underline{P}_{M_i}(t) \leq P_{M_i}(t) \leq \overline{P}_{M_i}(t).$$

- Thermal constraints: $i = 1, \cdots, N_l$,

$$-\underline{T}_i \leq T_i(k) \leq \overline{T}_i.$$

"Optimal" costs of $J_t(\cdot)$ are found for $\beta_i(k)$ and $P_{M_i}(k)$.

Optimization at Microgrid G_i

$$J_m(\beta_i(k), P_{M_i}(k)) = \max_{P_{M_i}} \sum_{l=k}^{N} \beta_i(l) P_{M_i}(l),$$

for given price $\beta_i(k)$ in the time intervals $t \in [t_0 + kT, t_0 + (k+1)T)$ and subject to

- Power injection into the main grid: $P^a_{DG_i}(k)$ is the aggregated DG/storage power output,

$$P_{M_i}(k) = P^a_{DG_i}(k) - P^{G_i}_{total.load}(k) - P^{G_i}_{total.loss}(k), \quad P^a_{DG_i}(k) = \begin{cases} > 0 & \text{sending power} \\ < 0 & \text{receiving power} \\ = 0 & \text{balanced} \end{cases},$$

  where

$$P^a_{DG_i}(k) = \sum_j [P_{DG_{i,j}}(k) + \Delta E^s_{DG_{i,j}}(k)/T], \quad E^s_{DG_{i,j}}(k) = E^s_{DG_{i,j}}(0) + \sum_{l=0}^{k-1} \Delta E^s_{DG_{i,j}}(l),$$

  where $E^s_{DG_{i,j}}(k)$ is the energy stored in the microgrid at the end of the kth interval.

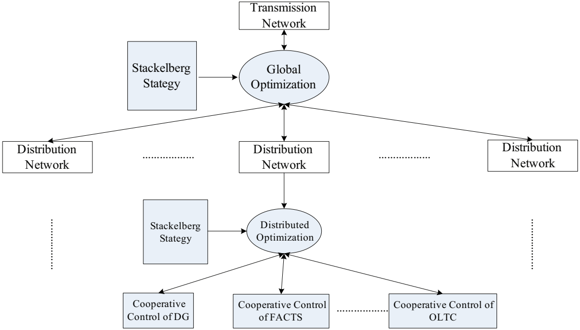
- $\alpha_i(k) \in (-\infty, 1]$ is the fair utilization ratio at stage k as, unless $P_{DG_{i,j}}(k) + E^s_{i,j}(k-1)/T = 0$,

$$\alpha_i(k) = \frac{P^a_{DG_i}(k)}{\sum_j [P_{DG_{i,j}}(k) + E^s_{i,j}(k-1)/T]}.$$

- Constraints:

$$0 \le \Delta E^s_{DG_{i,j}}(l)/T \le \overline{P}^s_{DG_{i,j}}, \quad E^s_{DG_{i,j}}(k) < \overline{E}^s_{DG_{i,j}}.$$

Distributed Optimization and Self-Organizing Control

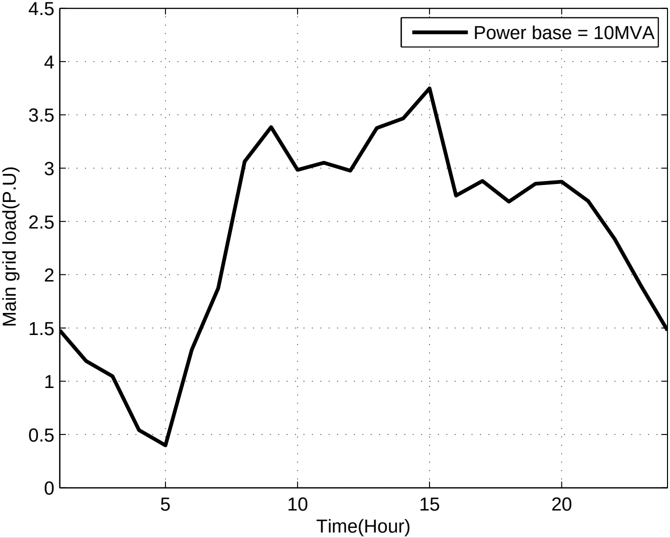


Techniques involved: cooperative control, distributed optimization, scalable game algorithms.

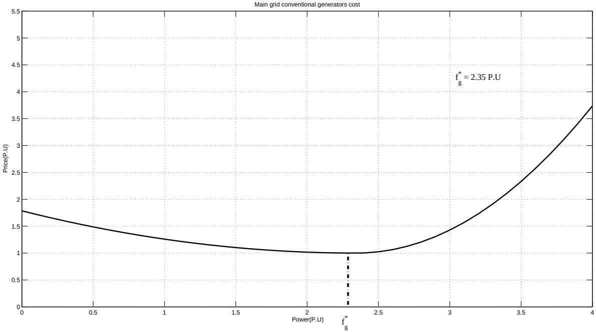
Case Study 4: Application of Stackelberg Algorithm

Simulation Setting: Stackelberg Game for Main Grid versus One Microgrid

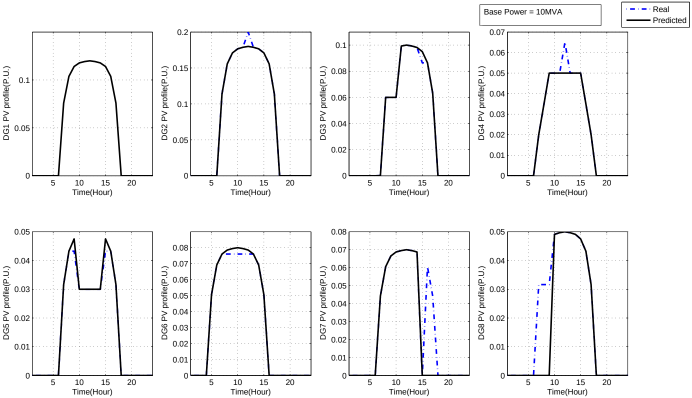
Main Grid Load Profile



Conventional Generation Cost (P.U)



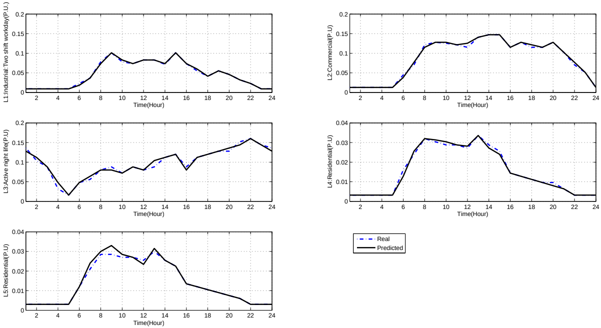
PV Generation Profiles



Microgrid Load Profiles

5 different load profiles considered:

- Loads on feeder 1: Industrial two shift workday
- Loads on feeder 2: Commercial area
- Loads on feeder 3: Active night life area
- Loads on feeder 4 & 5: Small residential areas



Setting: $\beta(l) = 16[1 + \beta_1(P_G - P_G^*)/P_G^*]$, $P_G^\star = 2.35$, $|\Delta E(k)| \leq 0.25$, and $0 \leq E(k) \leq 1$.

The Stackelberg and Nash solutions (with decision variables of β_1 vs. ΔE):

	No Game: $\beta_1 = 1$ and $E(k) = 0.5$	Game: $\beta_1 \in \Omega_{\beta_1}$ and $\Delta E(k) \in \Omega_{\Delta E}$	
		Stackelberg	Nash
$J_t^{(1-24)}$	84.0155	81.0872	81.0872
$J_m^{(1-24)}$	6.4682	9.9812	9.9812

where $E(l) = E(0) + \sum_{k=0}^{l-1} \Delta E(k)$

$$\Omega_{\beta_1} = \{0.5,\ 0.75,\ 1.0,\ 1.25,\ 1.5\},$$

$$\Omega_{\Delta E} = \{\Delta E(k) \in \{-0.25,\ -0.125,\ 0,\ 0.125,\ 0.25\}$$
$$0 \leq E(l) \leq 1,\ \text{and}\ E(24) = E(0).\}$$

Case 2: Increased Reserve Capacity

Setting: $P_G^\star = 2.35$, $|\Delta E(k)| \leq 0.25$, and $0 \leq E(k) \leq 1.5$.

The Stackelberg and Nash solutions:

	No Game: $\beta_1 = 1$ and $E(k) = 0.5$	Game: $\beta_1 \in \Omega_{\beta_1}$ and $\Delta E(k) \in \Omega_{\Delta E}$	
		Stackelberg	Nash
$J_t^{(1-24)}$	84.0155	78.5641	78.5641
$J_m^{(1-24)}$	6.4682	7.9906	7.9906

where $E(l) = E(0) + \sum_{k=0}^{l-1} \Delta E(k)$

$$\Omega_{\beta_1} = \{0.5,\ 0.75,\ 1.0,\ 1.25,\ 1.5\},$$

$$\Omega_{\Delta E} = \{\Delta E(k) \in \{-0.25,\ -0.125,\ 0,\ 0.125,\ 0.25\},$$
$$0 \leq E(l) \leq 1.5, \text{ and } E(24) = E(0).\}$$

Conclusions

Robust and Efficient Operation of Power Systems with DGs:

- Cooeprative controls yield self-organizing microgrids (by utilizing available communication and information flow)
- The aggregated real power can be dispatched real-time: Cooperative behaviors within microgrids by adaptively adjusting local storages and real power outputs from the renewables.
- Voltage stability is ensured: Cooperative behaviors within microgrids by adaptively adjusting reactive power generation.
- Robustness against line/network faults, communication intermittency and latency is ensured.
- Microgrids can be represented by virtual entities which are capable of taking appropriate decisions.
- The main grid and the microgrids can jointly and autonomously optimize their operations by applying game-theoretical algorithms.

Thanks! Questions?