Solution Methodologies for Integrated Network Design and Scheduling Problems

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Los Alamos National Laboratory
Student Smart Grid Seminar

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What are Integrated Network Design and Scheduling (INDS) Problems?

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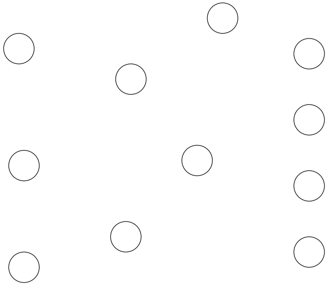

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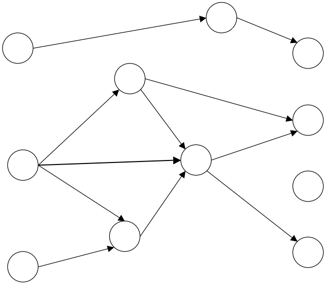

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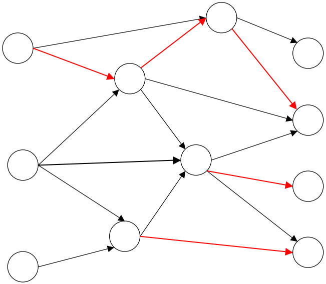

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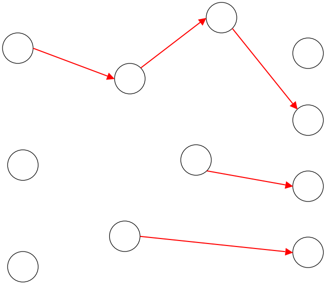

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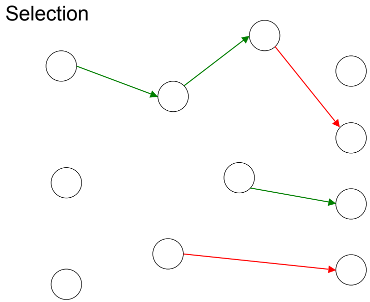

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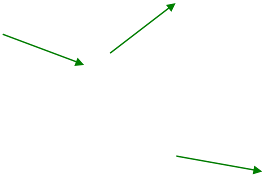

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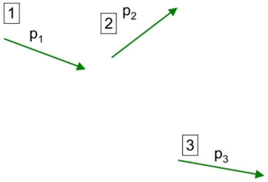

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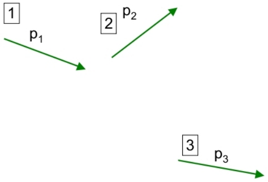

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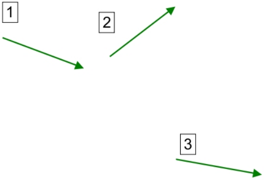

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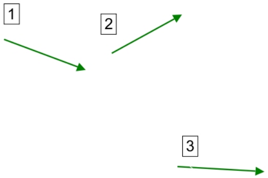

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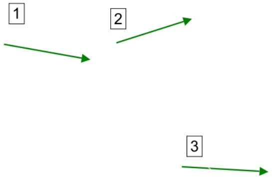

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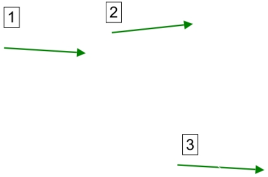

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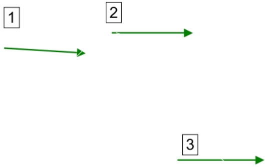

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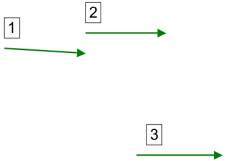

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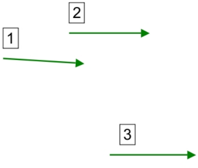

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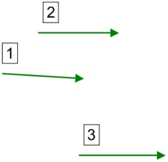

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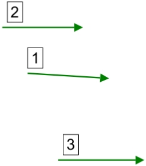

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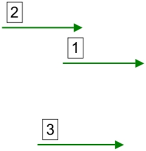

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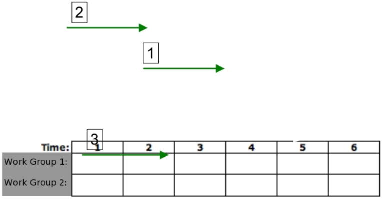

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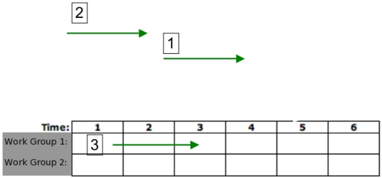

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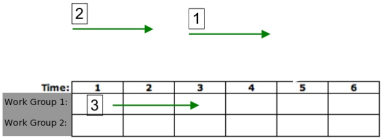

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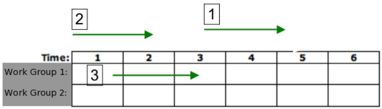

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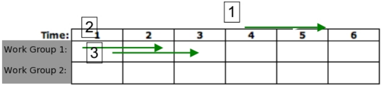

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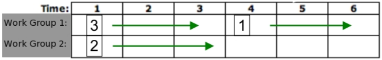

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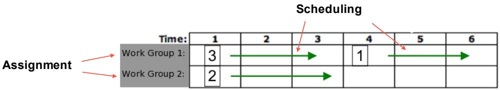


Decisions for an Integrated Network Design and Scheduling Problem

- Three main decisions occurring *simultaneously*:
  - Selection: select arcs to install into the network
  - Assignment: assign arcs to a work group
  - Scheduling: schedule arcs assigned to a work group

Motivation and Applications for Network-based Scheduling Problems

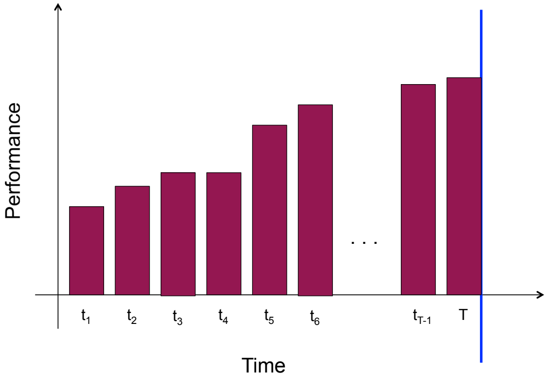
- Infrastructure Restoration after extreme events
  - Install components (either arcs or nodes) into the networks.
  - Power, telecommunications, water, waste water, and transportation networks.
- Humanitarian Logistics
  - Most often install nodes but also can install arcs into the network.
  - Location set-up for distribution of essential products such as food and water.
- PHEV Exchange Station Operations
  - Schedule the charging and discharging of batteries within an exchange station

Metrics for Network Performance Evaluation

- Maximum Flow
- Minimum Cost Flow
- Shortest Path
- Minimum Spanning Tree

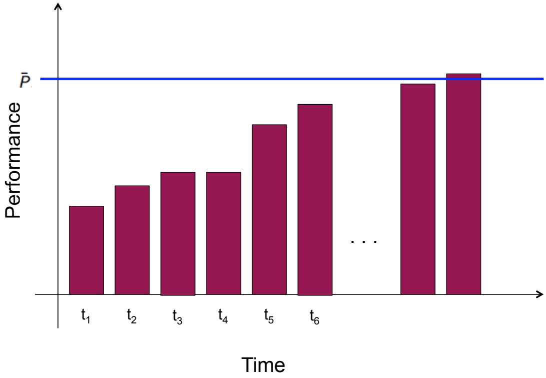
Objective Functions

- Cumulative objective
  - Set time horizon T.
  - Sum performance of the network for $t = 1, \ldots, T$, $\sum_{t=1}^{T} w_t P(G(t))$.



Objective Functions

- Makespan objective
  - Set performance value, $\bar{P}$.
  - Minimize the amount of time needed to reach or exceed $\bar{P}$.



Complexity Results Overview

Problem	Objective Function	NP-Hard	Strongly NP-Hard	Approximation ln(n)
Minimum Cost Flow	Cumulative		x	
	Makespan			x
Maximum Flow	Cumulative		x	
	Makespan			x
Shortest Path - Multiple Destinations	Cumulative		x	
	Makespan			x
Shortest Path - Multiple, Max Path	Cumulative		x	
	Makespan			x
Shortest Path - Single Destination	Cumulative			
	Makespan	x		
Minimum Spanning Tree	Cumulative			
	Makespan	x		

Weighted Shortest Processing Time (WSPT) Dispatching Rule

- Have a set of jobs J that need to be completed on M work groups.
- Each job $j \in J$ has an associated weight w_j that represents the added value to the objective or priority level.
- Each job $j \in J$ has an associated processing time p_j.
- How do we decide which job should be scheduled next?

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- How do we decide which job should be scheduled next?
- WSPT Rule selects the job to be processed next that maximizes:

$$\frac{w_j}{p_j} \text{ for } j \in J$$

Adapt the WSPT Rule to solve INDS Problems

- What are the jobs?
- What are the weights of jobs, w_j?
- What the processing times of jobs, p_j?
- How do you find the job that maximizes $\frac{w_j}{p_j}$?
- We need to make sure to answer the 3 main decisions: selection, assignment, and scheduling.

Maximum Flow NBSP Dispatching Rule

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- Observation:
  - If we know the optimal numerator, w_j (min residual capacity).
  - Alter network to only look at arcs with residual capacity greater than or equal to this value.
  - Shortest path in this altered network maximizes $\frac{w_j}{p_j}$.

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- Solve for shortest path for every possible numerator value.
- $O(m)$ shortest path problems to find job that maximizes $\frac{w_j}{p_j}$.

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- $O(m)$ shortest path problems to find job that maximizes $\frac{w_j}{p_j}$.
- Three decision answers:
  - Selection: According to this algorithm, select a path
  - Assignment and Scheduling: Order arcs in path according to longest processing time first, and assign to the first machine or work group that becomes available.

Minimum Cost Flow INDS Dispatching Rule

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- Solve the min cost to time ratio problem for every possible residual capacity.
- $O(m)$ min cost to time ratio problems to find job that minimizes $\frac{w_j}{p_j}$.

Shortest Path INDS Dispatching Rule

- We can solve the shortest path problem by solving a minimum cost flow problem.
- Source node supply = number of destination nodes.
- Each destination node has a demand of 1.
- The flow on arc (i, j) signifies the number of destination nodes that use arc (i, j) in their shortest path.
- Only O(number of destination nodes) possible residual capacity values.
- Solve O(number of destination nodes) min cost to time ratio problems to find the cycle that minimizes $\frac{w_j}{p_j}$.

Minimum Spanning Tree INDS Dispatching Rule

- Optimality Conditions: A tree is the minimum spanning tree if and only if for every non-tree arc (i, j), its cost is greater than or equal to the cost of every arc on the unique path from i to j in the tree.

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- What the processing times of jobs, p_j?
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- How do you find the job that minimizes $\frac{w_j}{p_j}$?
  - $O(m)$ searches to find paths.

Case Study: Lower Manhattan Power Network

- Data:
  - Represents the realistic power network of Lower Manhattan
  - 1603 Nodes
  - 2621 Arcs
- Tests:
  - Randomly ‘knock out’ 25%, 50%, and 75% of the arcs.
  - Randomly assign a processing time of 1, 2, or 3 to ‘knocked out’ arcs.
  - Implemented the Maximum Flow cumulative and makespan dispatching rules.
  - Compared to the Integer Programming formulation using CPLEX 12.0.
  - Constant weights $w_1 = w_2 = \ldots = w_T = 1$.
  - Tested for 1, 2, and 3 work groups.
  - Tested for two different makespan performance goals $\bar{P} = 75\%$ of damage and $\bar{P} = 100\% =$ max flow.
  - Results presented are the average over 5 random instances.

Case Study: Cumulative Results

		Dispatching Rule		CPLEX 12.0	
Work Groups	Percentage	Time (s)	Performance (%)	Time (s)	Performance(%)
1	25%	28.75	0.72%	12335	0.34%
	50%	31.97	0.07%	5178	0.03%
	75%	19.33	0.40%	14400	0.40%
2	25%	32.13	0.85%	14400	0.66%
	50%	68.10	0.15%	7363	0.03%
	75%	43.22	0.39%	12100	0.27%
3	25%	29.33	0.61%	14400	0.51%
	50%	139.59	0.67%	14400	0.46%
	75%	57.45	0.51%	14400	0.35%

Case Study: Makespan 75% Results

		Dispatching Rule		CPLEX 12.0	
Work Groups	Percentage	Time (s)	Performance (%)	Time (s)	Performance(%)
1	25%	18.90	2.34%	1.37	0.00%
	50%	61.35	1.32%	3.79	0.00%
	75%	87.02	0.36%	8.29	0.00%
2	25%	16.63	3.27%	2881.07	0.91%
	50%	57.87	1.82%	2888.42	0.27%
	75%	81.84	0.70%	3916.89	0.22%
3	25%	18.77	6.36%	2882.46	1.33%
	50%	82.56	1.56%	98.60	0.00%
	75%	98.73	0.88%	11521.32	0.71%

Case Study: Makespan 100% Results

		Dispatching Rule		CPLEX 12.0	
Work Groups	Percentage	Time (s)	Performance (%)	Time (s)	Performance(%)
1	25%	21.03	4.88%	1.61	0.00%
	50%	112.35	6.99%	1347.62	0.00%
	75%	130.07	7.11%	13759.31*	1.33%
2	25%	18.93	6.70%	6155.98	0.98%
	50%	83.55	10.81%	14400	3.44%
	75%	136.73	11.05%	14400	3.17%
3	25%	16.68	10.01%	8677.14	2.34%
	50%	75.89	11.46%	14400	3.50%
	75%	121.09	12.04%	14400	3.15%

* For two instances CPLEX ran into a memory error before the 4 hour time limit.

Conclusions

- Looked at the new class of problems integrated Network Design and Scheduling problems
- Created Novel Dispatching Rules based on Optimality Conditions of the problems
- Applied Dispatching Rule to real life data set and provided near-optimal solutions quickly

Basic PHEV Exchange Station Model

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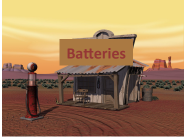

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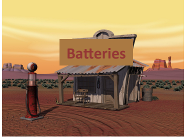

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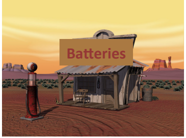

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Basic PHEV Exchange Station Model

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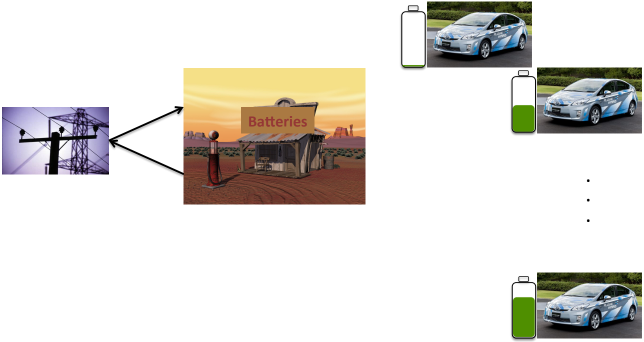

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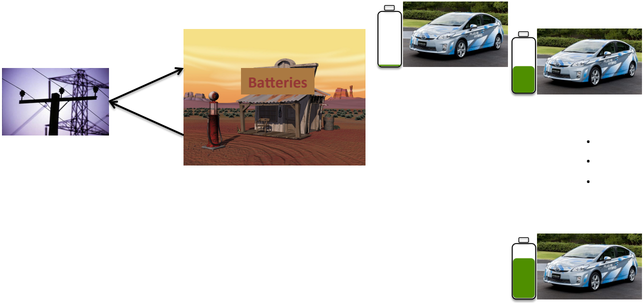

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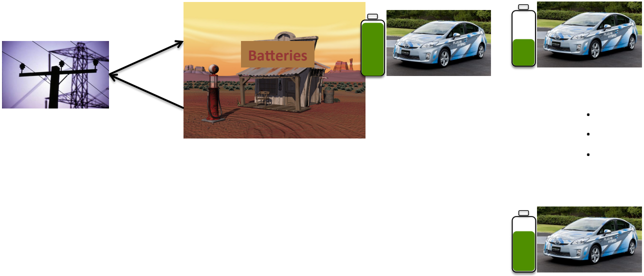

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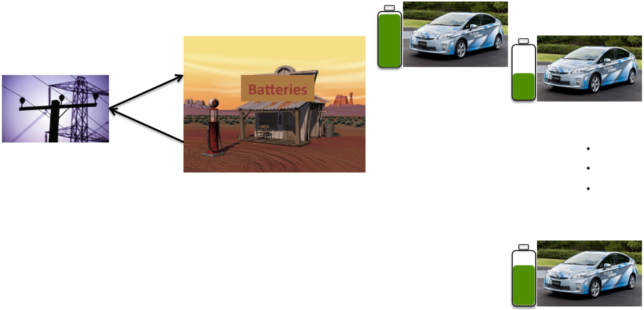

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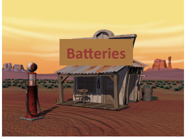

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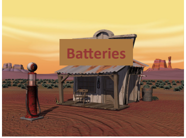

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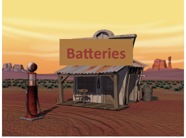

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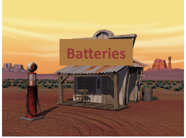

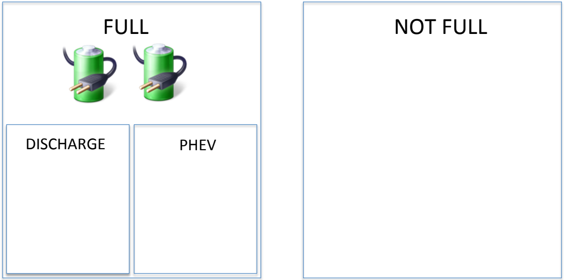

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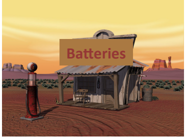

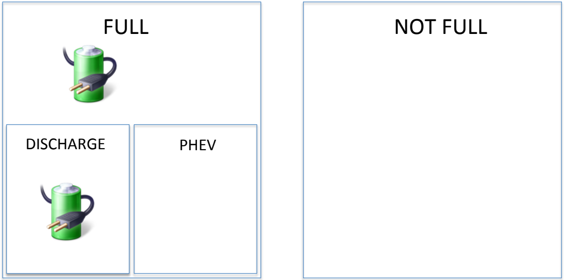

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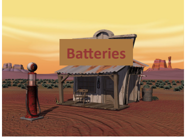

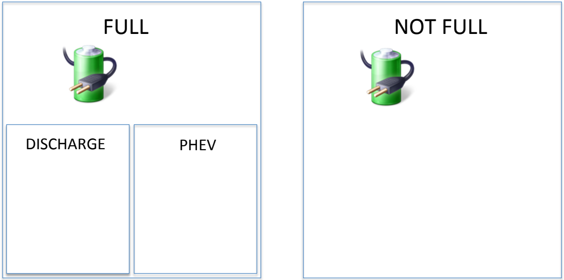

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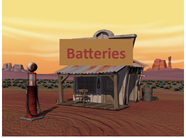

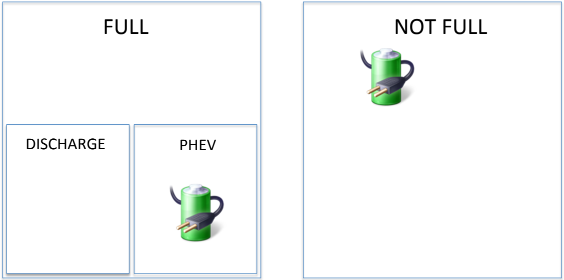

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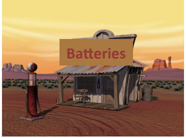


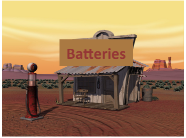
FULL

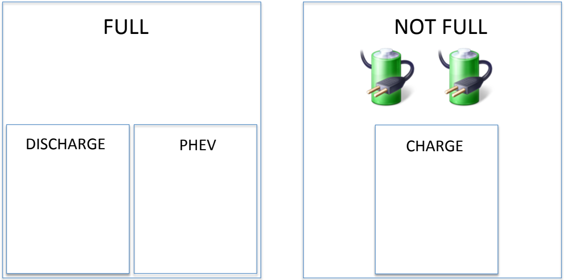
DISCHARGE

PHEV

NOT FULL

Basic PHEV Exchange Station Model





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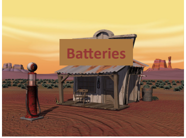

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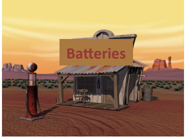

FULL

DISCHARGE

PHEV

NOT FULL

CHARGE

Basic PHEV Exchange Station Model

- Model the Exchange Station operations
- Determine the optimal number of batteries to charge, discharge, and exchange at each time
- Stochastic model with variable battery demand and power network flow
- Consider three different models that factor in different objective functions
  - One solution factoring in all different scenarios for the stochastic input
  - A solution for each scenario
  - A solution for each node of the tree that incorporates all scenarios

Reduction of Set Cover Problem to Maximum Flow INDS

- Show a reduction of known Strongly NP-Complete problem
- Set Cover Problem
  - Set of elements $X = \{x_1, x_2, \ldots, x_n\}$.
  - Set of sets $S = \{S_1, S_2, \ldots, S_m\}$.
  - Each $S_i \subseteq X$, for example $S_1 = \{x_1, x_2, x_4\}$.
  - Want to select the least amount of sets S_i such that each x_i is selected, i.e. select a set $\bar{S} \subseteq S$ such that $\cup_{S_i \in \bar{S}} S_i = X$.

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- Use this reduction idea for:
  - Max Flow Cumulative and Makespan
  - Min Cost Flow Cumulative and Makespan
  - Shortest Path Multiple Destinations Cumulative and Makespan
  - Shortest Path Multiple, Max Path Cumulative and Makespan

Formulation of Set Cover as a Max Flow INDS

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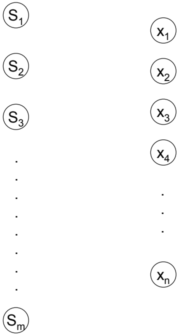

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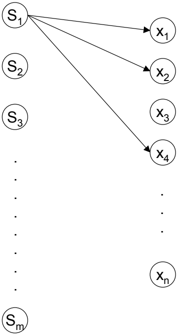

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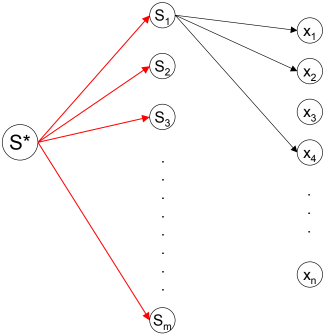

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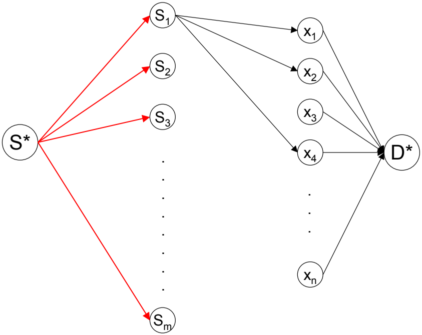

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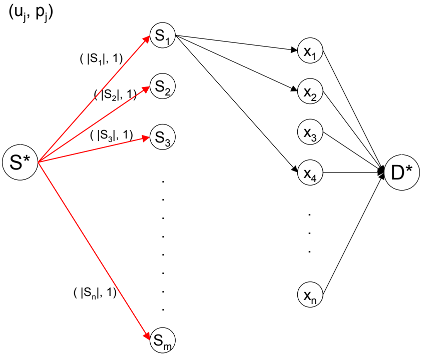

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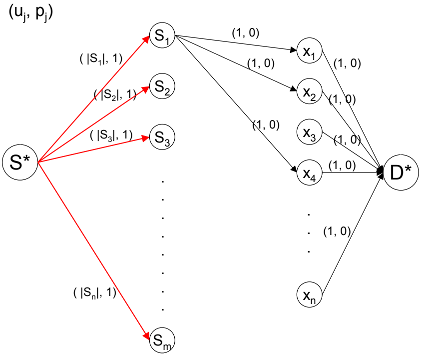

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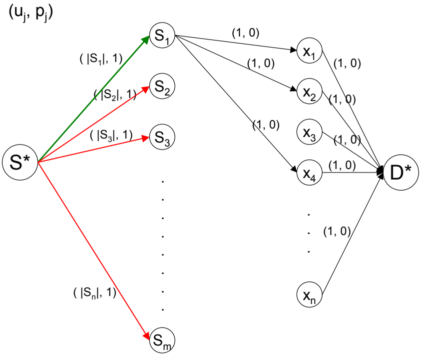

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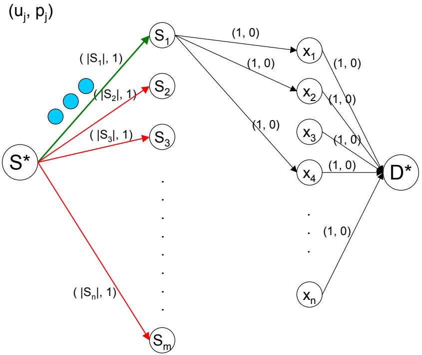

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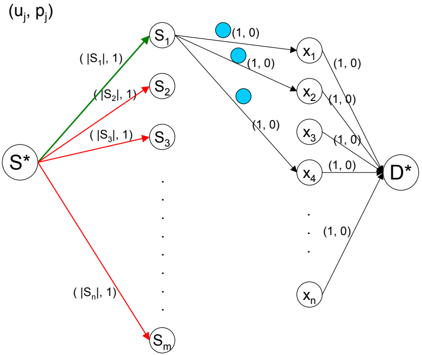

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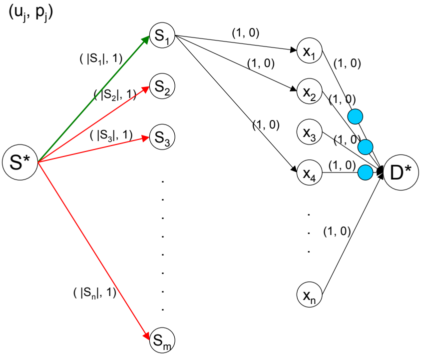
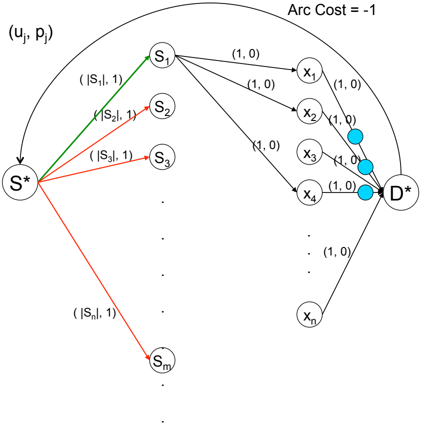
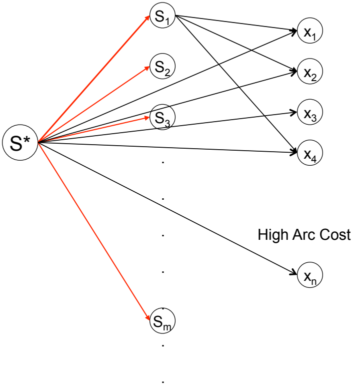


Formulation of Set Cover as a Min Cost Flow INDS



Formulation of Set Cover as a Shortest Path INDS



Theorem

The Integrated Network Design and Scheduling Problem (INDS) with maximum flow cumulative performance objective is Strongly NP-Complete with one work group.

Theorem

The Integrated Network Design and Scheduling Problem (INDS) with maximum flow cumulative performance objective is Strongly NP-Complete with one work group.

Proof: Reduction of Set Cover to Maximum Flow INDS

- Solution to this formulation of INDS $\rightarrow$ Decision for Set Cover problem
- Set $w_1 = w_2 = \ldots = w_{T-1} = 0$, and $w_T = 1$
- Cumulative objective = max flow at time T
- If the max flow at time T equals $|X| \rightarrow$ A set cover is possible with $\leq T$ sets
- Conclusion: Maximum Flow Cumulative INDS is Strongly NP-Complete

Theorem

The Integrated Network Design and Scheduling Problem (INDS) with maximum flow makespan performance objective is Strongly NP-Complete with one work group.

Theorem

The Integrated Network Design and Scheduling Problem (INDS) with maximum flow makespan performance objective is Strongly NP-Complete with one work group.

Proof:

- Use same reduction
- Set performance value $\bar{P} = |X|$
- Minimum makespan value T' for INDS $\rightarrow$ minimum number of sets needed to cover all elements in X
- Conclusion: Maximum Flow Makespan INDS is Strongly NP-Complete

Theorem

The INDS with minimum cost flow cumulative and makespan performance objectives is Strongly NP-Complete with one work group.

Proof: Can formulate a Maximum Flow problem as a Minimum Cost Flow problem

Theorem

The INDS with multiple destination shortest path cumulative and makespan performance objectives is Strongly NP-Complete with one work group.

Proof:

- If cumulative shortest path equals n, a set cover is possible in T time.
- Makespan objective equals the minimum number of sets needed to cover all elements.

Reduce Partition Problem to Shortest Path Single Destination Makespan INDS

- Show a reduction of known NP-Complete problem to INDS.
- Partition Problem
  - Set of elements $S = \{a_1, a_2, \ldots, a_n\}$.
  - Is there $\bar{S} \subset S$ such that $\sum_{a_i \in \bar{S}} a_i = \sum_{a_j \notin \bar{S}} a_j = \sum_{k=1}^{n} \frac{a_k}{2}$.

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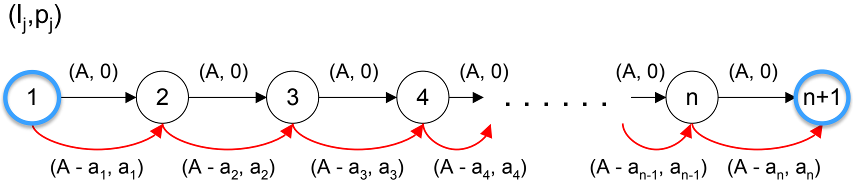
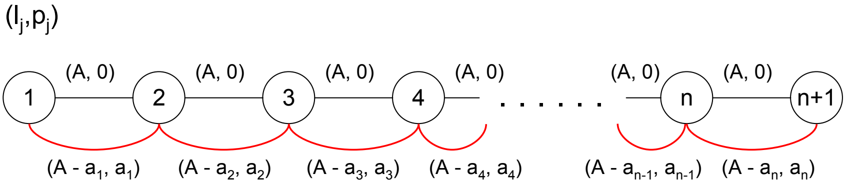
- Use this reduction idea for:
  - Shortest Path Single Destination Makespan objective
  - Minimum Spanning Tree Makespan objective

Formulation of Partition as a Shortest Path INDS



Formulation of Partition as a MST INDS



Theorem

The INDS with shortest path single destination makespan performance objective is NP-Complete with one work group.

Proof: Reduction of Partition to Shortest Path INDS

- Set Makespan performance value to be $nA - \sum_{i=1^n} \frac{a_i}{2}$.
- If makespan objective equals $\sum_{i=1^n} \frac{a_i}{2}$ then we know a partition is possible.

Theorem

The INDS with minimum spanning tree makespan performance objective is NP-Complete with one work group.

Proof: Use same idea with reduction from partition