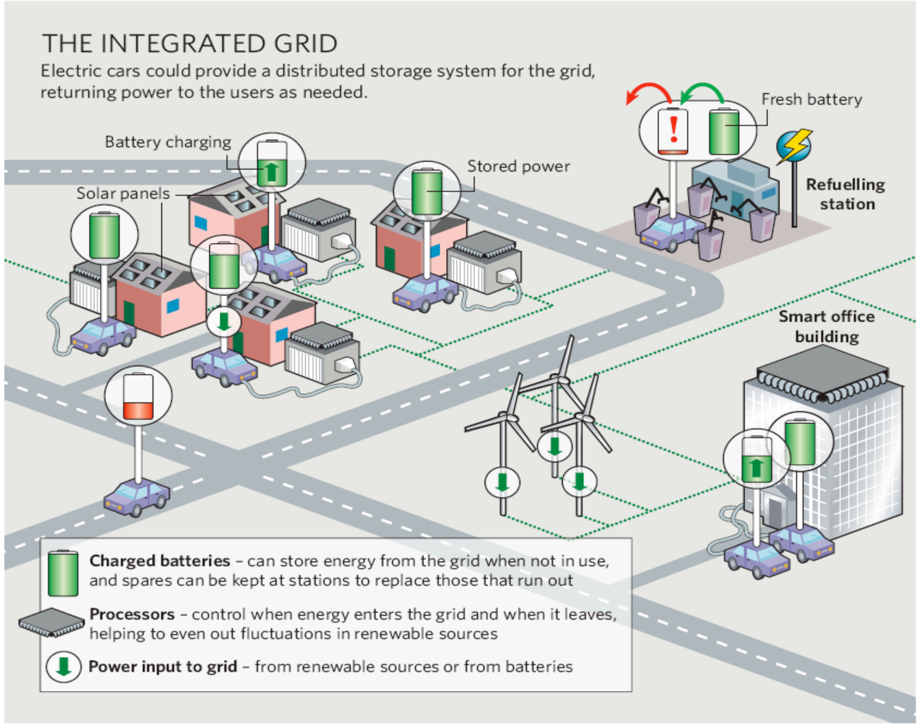
Smart Grid and Analysis of Large-Scale Interconnected Dynamical Systems

Igor Mezić

Department of Mechanical Engineering, University of California, Santa Barbara

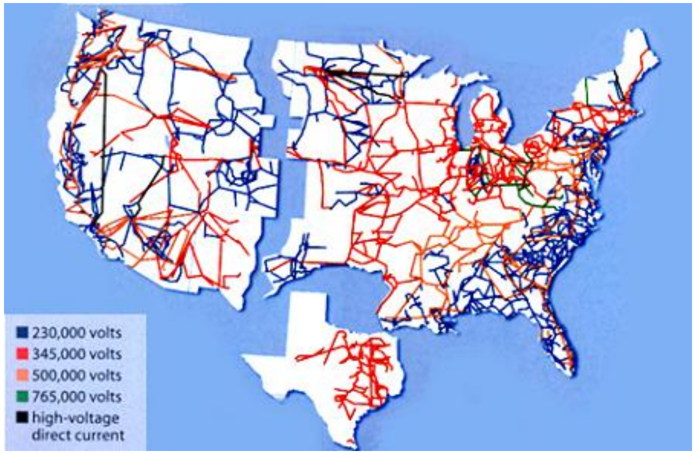
Introduction



NATURE Vol. 456, 27 November 2008

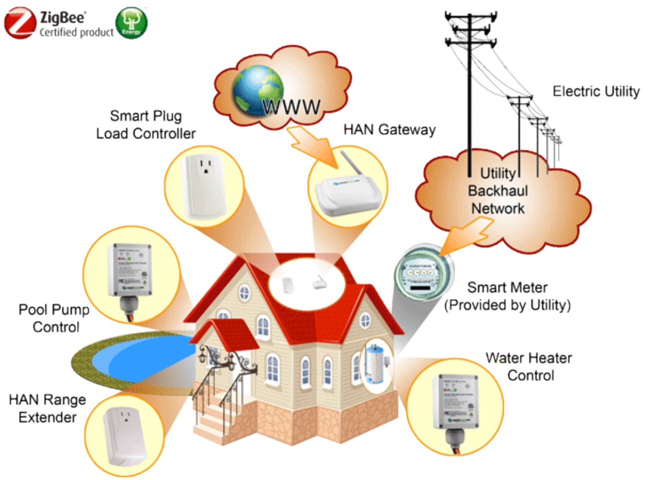
"The current U.S. system for transmitting and distributing electricity is in critical need of an upgrade…Grid-related power outages and problems with power quality reportedly cost the nation **$80 billion to $188 billion per year**…"

MIT Technology Review, February 2009



Power grid

Introduction



-(Nonlinear) Dynamics at every node.

-Extremely large number of degrees of freedom.

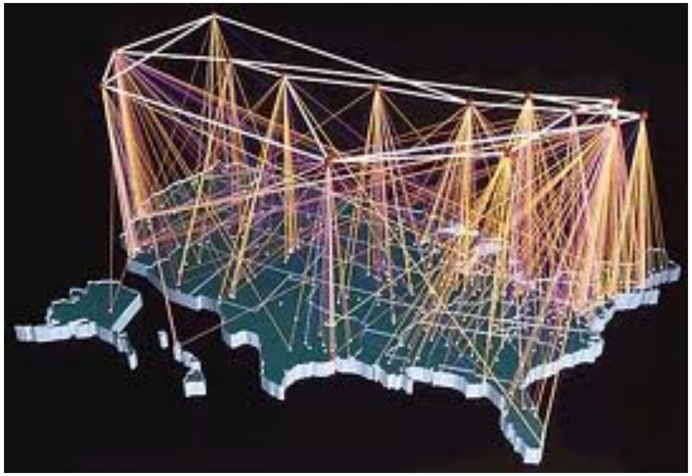
-Multiscale in space and time.

-Uncertainty in parameters describing dynamics.

-Stochastic effects.

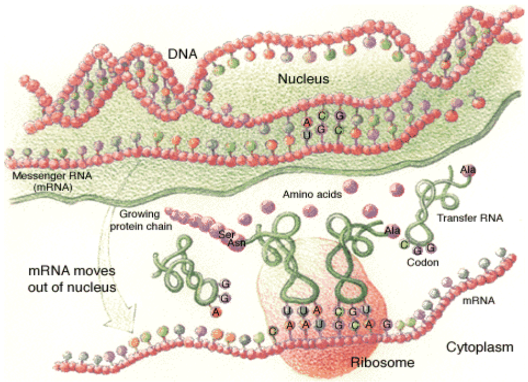
-Mixture of discrete and continuous dynamics.

-Lack of analysis tools.



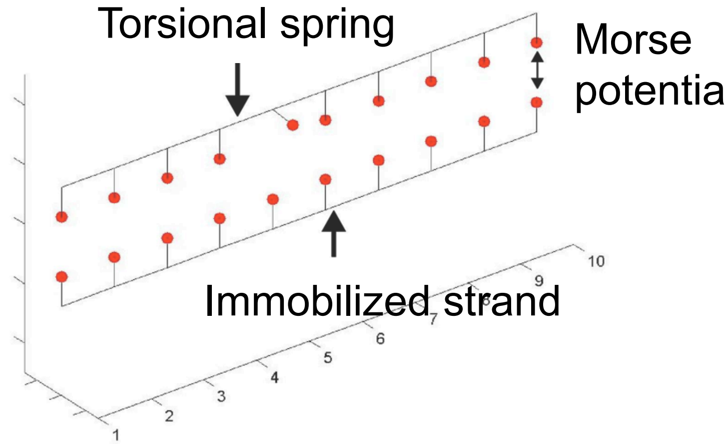
UC SANTA BARBARA

A coupled oscillator system

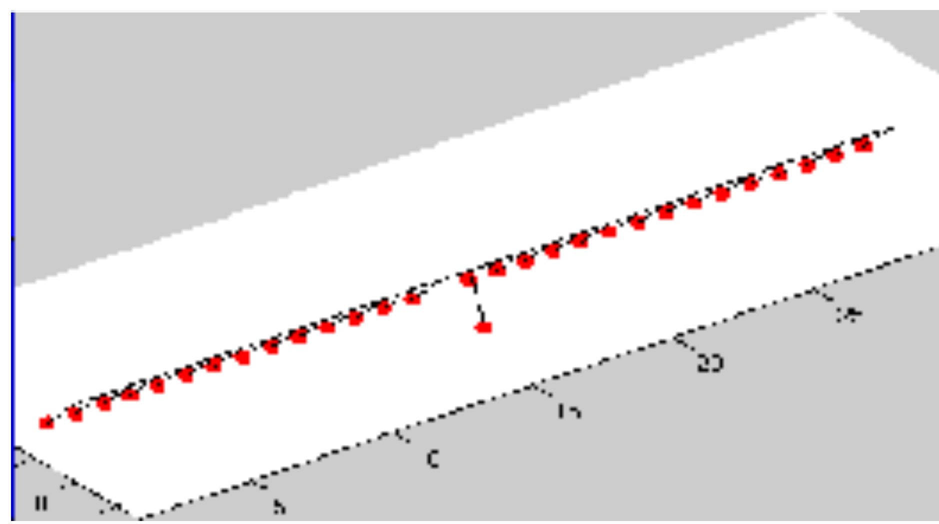


Englander et al (1980)
Peyrard, Bishop and collaborators.

I.M. PNAS (2006)



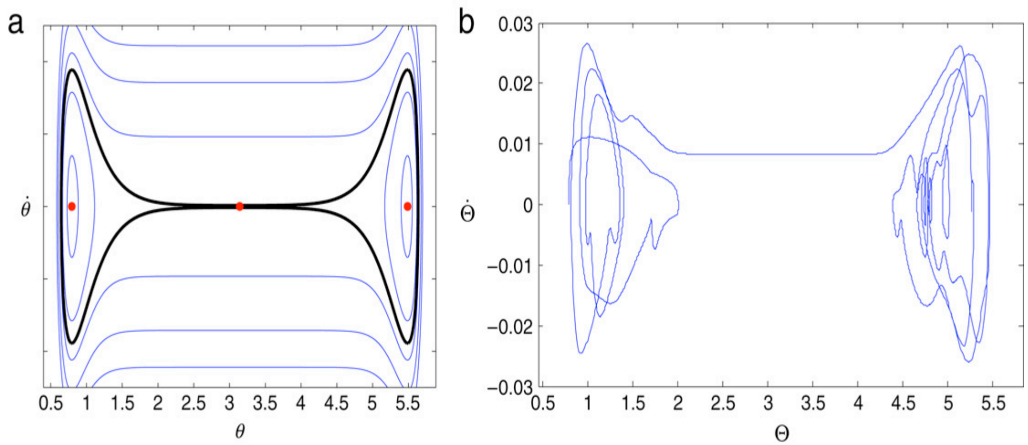
G. Gilmore, UCSB (2009)



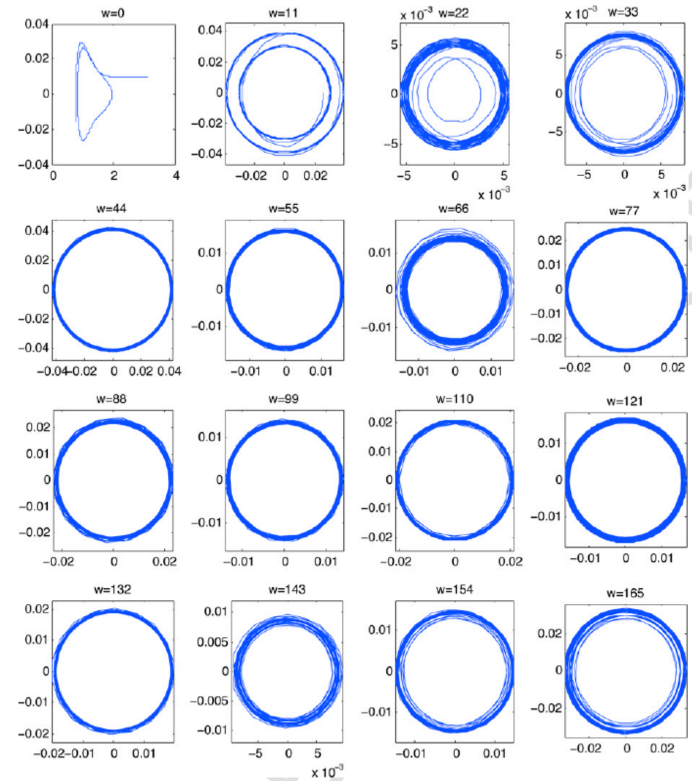
Inverse cascade: small scale $\longrightarrow$ large scale

No scale separation…

$$
\begin{aligned}
\dot{\theta}_k =& v_k,\\
\dot{v}_k =& \frac{1}{L^2}(\exp(-a_d(h(1-cos(\theta_k))-x_0))-1)\\
&\cdot \exp(-a_d(h(1-\cos(\theta_k))-x_0))\sin(\theta_k)\\
&+(\theta_{k+1}-2\theta_k+\theta_{k-1}).
\end{aligned}
$$



200 DOF



Cf. Goedde et al. PRL (1992)

$$
\begin{aligned}
\ddot{\theta}_k &= \theta_{k+1} - 2\theta_k + \theta_{k-1} + \epsilon F_k(\theta, t) \qquad k = 1, \ldots, N\\
\theta_k(0) &= a_k \qquad \dot{\theta}_k(0) = b_k
\end{aligned}
$$

P. DuToit, I.M., J. Marsden Physica D (2009)

Harmonic field approximation

Let $\hat{\theta}_0 := \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \theta_k$

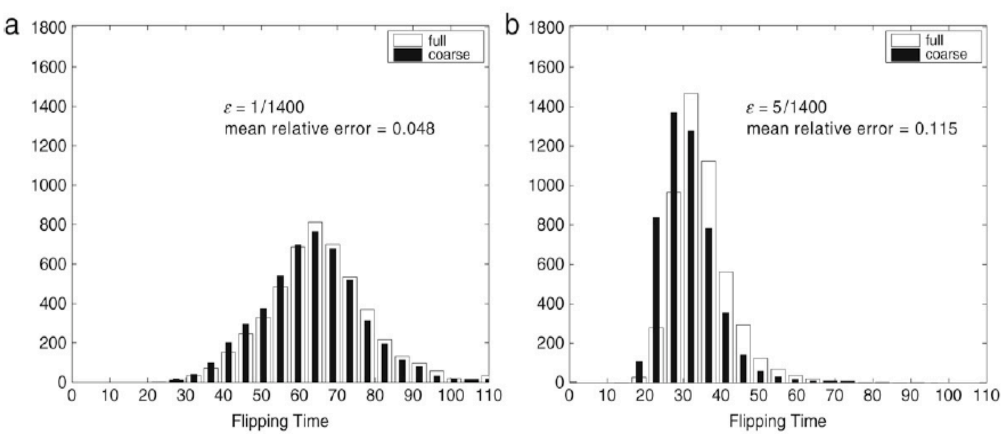
In normal mode coordinates: $\hat{\theta}_w(t) = \hat{a}_w \cos \alpha_w t + \frac{\hat{b}_w}{\alpha_w} \sin \alpha_w t \quad w = 1, \ldots, N-1.$

Define $\bar{\theta} = (\bar{\theta}_0, \hat{\theta}_1, \ldots, \hat{\theta}_{N-1})$

$$\ddot{\bar{\theta}}_0 = \frac{\epsilon}{\sqrt{N}} \sum_{k=1}^{N} F_k(P\bar{\theta})$$

Harmonic field approximation

Cf. mean field approximation



There is
no separation of scales.
Yet, there is reduced order representation!

P. DuToit, I.M., J. Marsden Physica D (2009)

Operator theory: history and setup

Observable

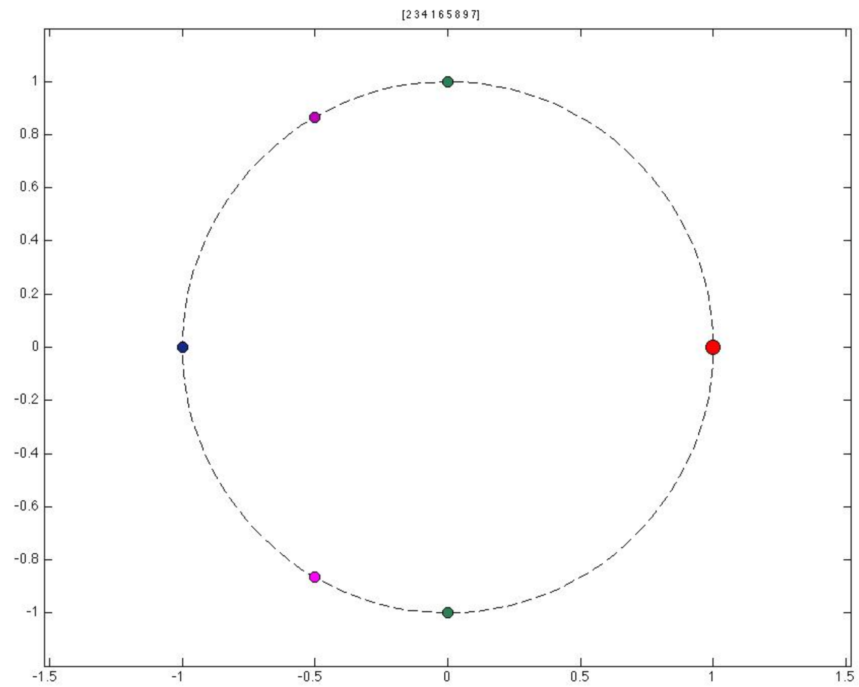
Koopman op

ery $t \in \mathbb{R}$

B.O. Koopn

AS (1931)

Vector field



On the attractor, Koopman operator is unitary and thus admits a spectral decomposition:

$$U = U_p + U_c = \sum_{j=1}^{n} e^{i2\pi\omega_j} P_T^{\omega_j} + U_c$$

where $P_T^{\omega_j}$ is orthogonal projection on the algebraic eigenspace associated with $e^{i2\pi\omega_j}$.

Operator theory: history and setup

C'. The system possesses no invariant subset of Ω of positive finite measure, and in the case where $\mu\Omega$ is finite, no angle variables.

We will show that under this hypothesis all the initially observed properties of the system are obliterated by the lapse of time: the method of elementary mechanics of computing the final from the initial state must be replaced by the methods of the theory of probability. Contrary to the case of classical statistical mechanics, this situation is not dependent upon the system's having an enormous number of degrees of freedom: this number may perfectly well reduce to two.

THEOREM I. Under the hypothesis C, there exists a zero t-set I such that, for any two P-sets M and N of finite μ-measure, as $t \longrightarrow +\infty$ (or $t \longrightarrow -\infty$) through values not on I,

$$\mu(M_t \cdot N) \longrightarrow \frac{\mu M \cdot \mu N}{\mu\Omega}, \qquad (1)$$

B.O. Koopman and J. von Neumann "Dynamical Systems of Continuous Spectra", PNAS (1932)

<u>Methods based on analysis of the **Perron-Frobenius operator**:</u>

Lasota and Mackey, "Chaos, fractals, and noise: stochastic aspects of dynamics",
David Ruelle, Lai-Sang Young, , Vivian Baladi,
Michael Dellnitz, Oliver Junge, Erik Bollt, Gary Froyland…

Koopman and Von Neumann on chaos

Theorem I may also be expressed by saying that the states of motion corresponding to any set M of Ω become more and more spread out into an amorphous everywhere dense chaos. Periodic orbits, and such like, appear only as very special possibilities of negligible probability.

We have already called attention to the essential difference between this situation, which may exist in very simple systems, and that envisaged in the kinetic theory of gases, in which the confused character of the motion is an intuitively evident consequence of the large number of degrees of freedom of the system.

B.O. Koopman and J. von Neumann "Dynamical Systems of Continuous Spectra", PNAS (1932)

UC SANTA BARBARA

Operator theory and harmonic analysis

We call the function f^* the time average of a function f under T if

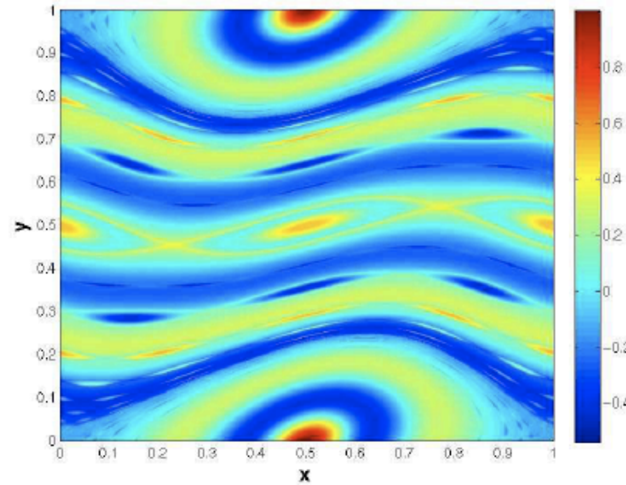
$$f^*(x) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} f(T^i x$$

Of importance in study of design of search algoritms

(c.f. G. Mathew work, Mon AM)

And characterizing ergodicity in ocean flows

(c.f. S. Scott talk, Monday)



f^* is constant on orbits i.e.

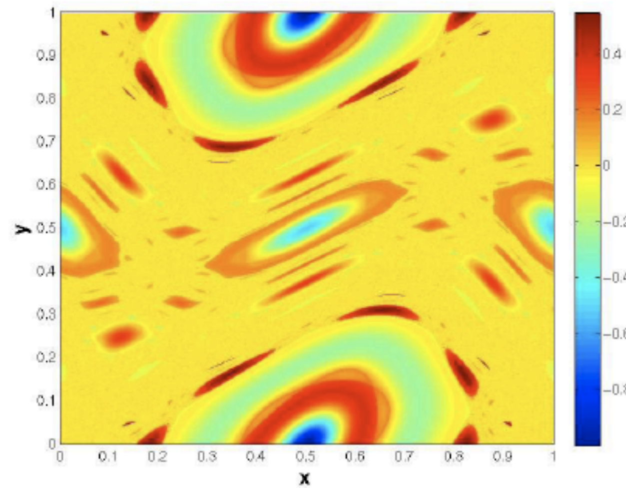
$$Uf^*(x) = f^*(x).$$

The operator

$$P_T(f) = f^*$$

can be considered as a member

operators P_T^ω,



$$P_T^\omega(f) = f_\omega^* = \lim_{n\to\infty} \frac{1}{n} \sum_{j=0}^{n-1} e^{i2\pi j}$$

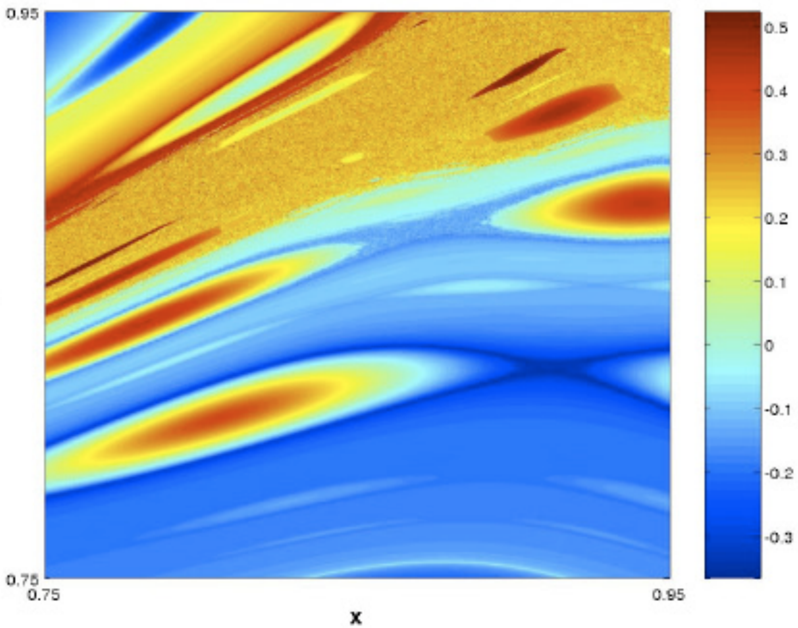
Cf. M. Dellnitz, O. Junge,, SIAM J. Numer. Anal.) (1999).

Ergodic partition

Rokhlin(1940;s), Oxtoby, Ulam, Yosida, Mane,

Lemma 1 *Let M be a compact metric space and $T : M \to M$ a $C^r, r \geqslant 1$, diffeomorphism . Assume there exist a complete system of functions $\{f_i\}, f_i \in C(M)$, i.e. finite linear combinations of f_i are dense in $C(M)$. The ergodic partition of a $C^r, r \geqslant 1$ diffeomorphism $T : M \to M$ on M is*

$$\zeta_e = \bigvee_{i \in \mathbb{N}} \zeta_{f_i}.$$



I.M. and A. Banaszuk, Physica D (2004)

Statistical Takens Theorem:

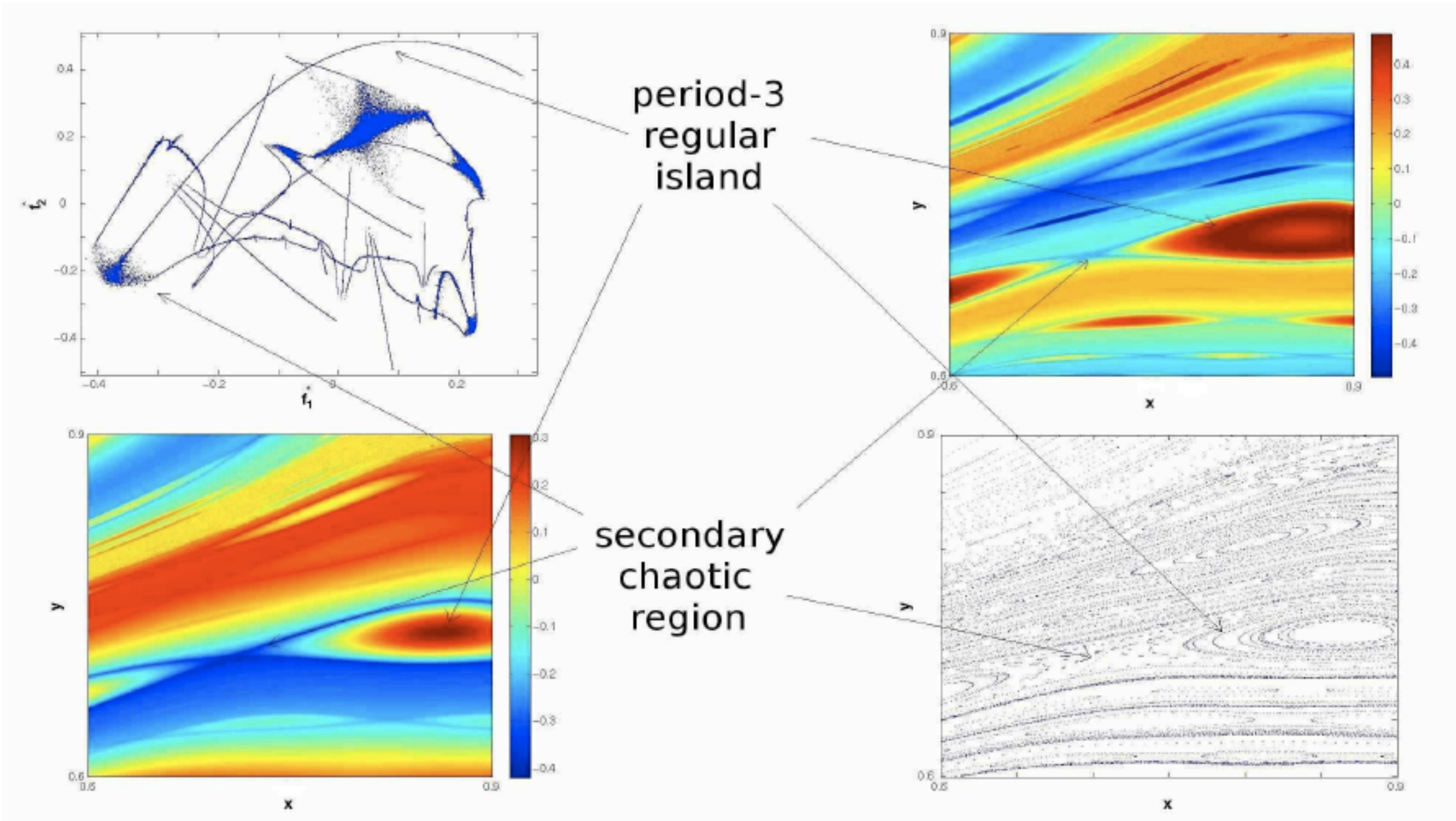
Theorem 1 *For $C^r, r \geqslant 1$ pairs (f, T) it is a generic property that the ergodic partition of a dynamical system T on M is*

$$\zeta_e = \bigvee_{i_1,\ldots,i_{2m+1}} \zeta_{f_{i_1,\ldots,i_{2m+1}}}.$$

Invariant sets by Koopman eigenfunctions

Quotient space embedding, $\mathbf{R}^2$

$$f_1 = \sin(3\pi x + 8\pi y)$$



$$f_2 = \sin(7\pi x + 9\pi y)$$

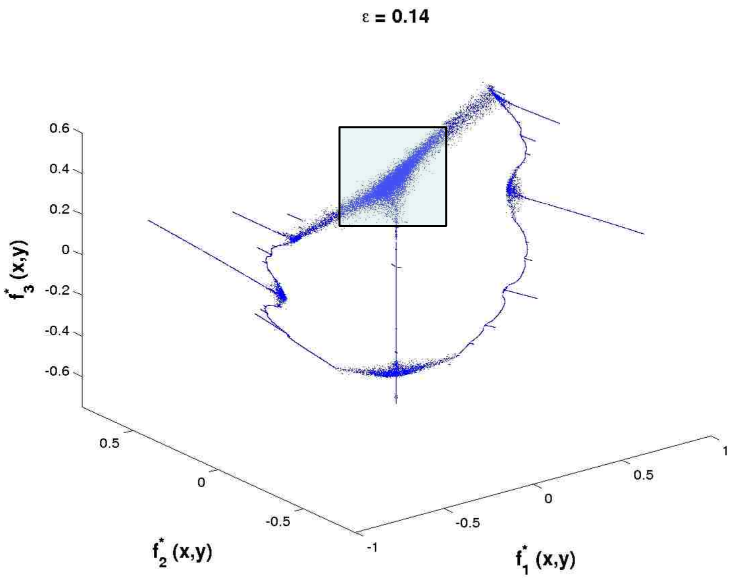
Trajectories of the Standard Map.

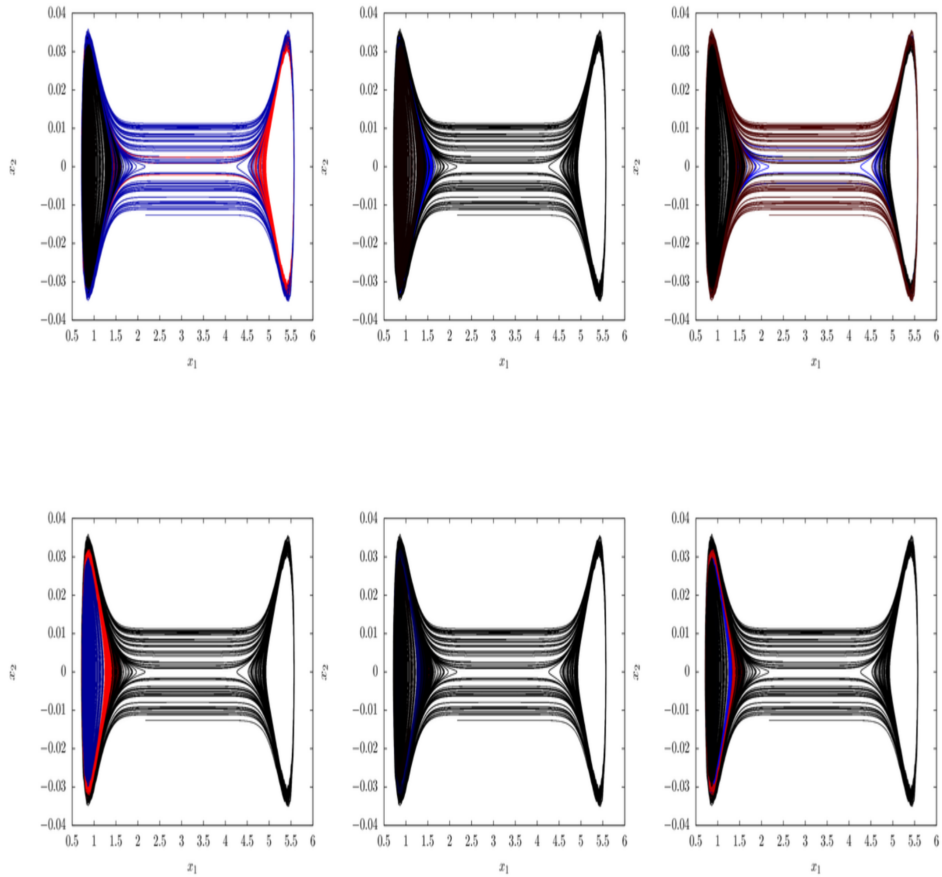
$$\begin{aligned} x' &= x + y + \varepsilon \sin(2\pi x) \quad [mod\ 1] \\ y' &= y + \varepsilon \sin(2\pi x) \quad [mod\ 1] \end{aligned}$$

Z. Levnajic and I.M., ArXiv (2009)

Invariant sets by Koopman eigenfunctions

Quotient space embedding, **R^3**





-Use spectral technique of Belkin, Lafon, Coifman and collaborators,

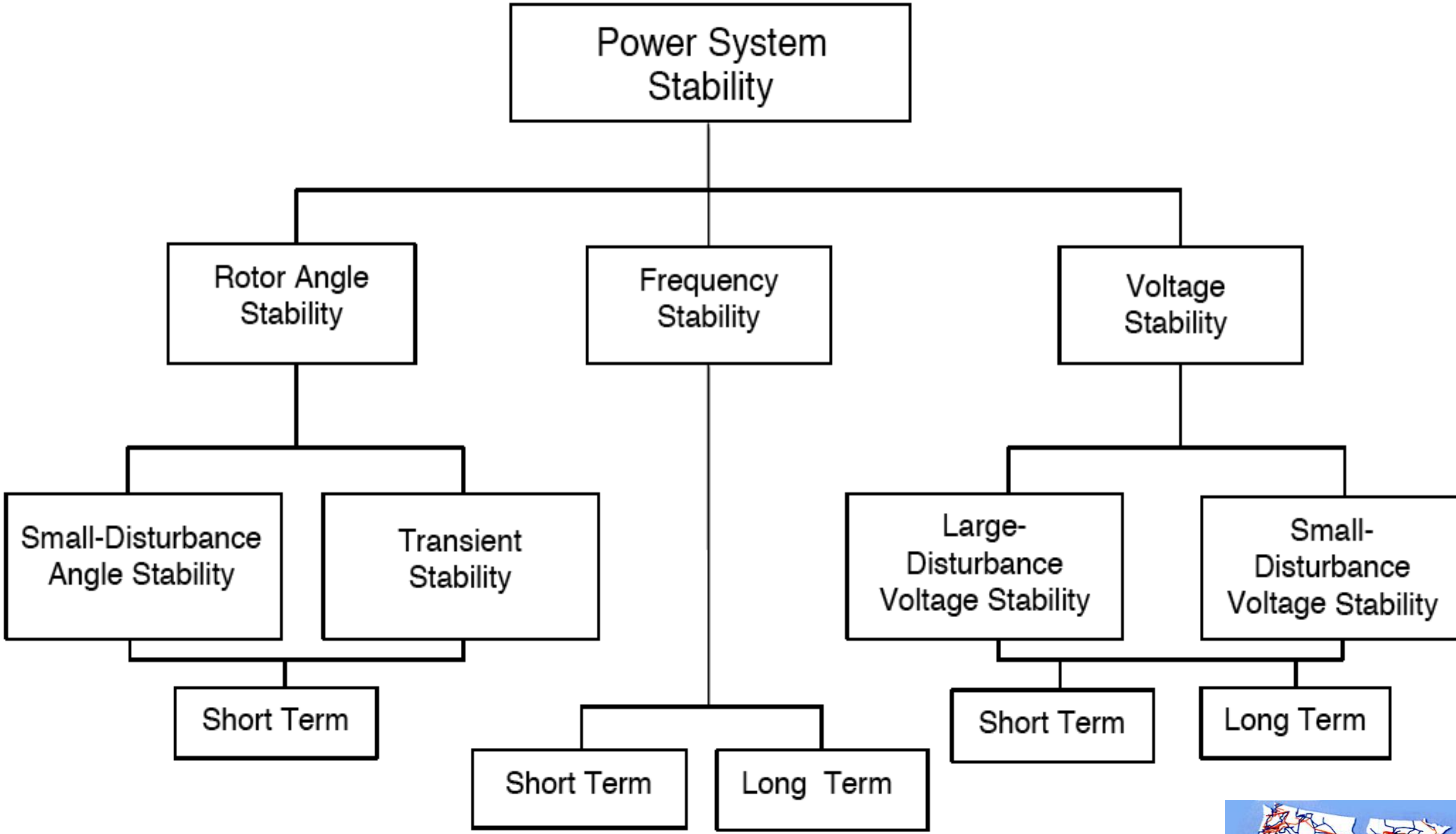
-Replace Euclidean distance (L^2 norm) with a negative Sobolev space-type modification:

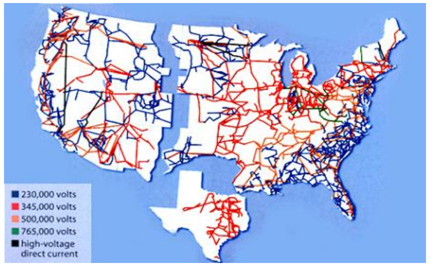
$$d(\mathrm{f}^*(\mathrm{x}) - \mathrm{f}^*(\mathrm{y})) = \sum_k \frac{||f_k^*(\mathrm{x}) - f_k^*(\mathrm{y})||}{1+k^2}$$

Cf. M. Budisic talk, CP31 Thu 3-4

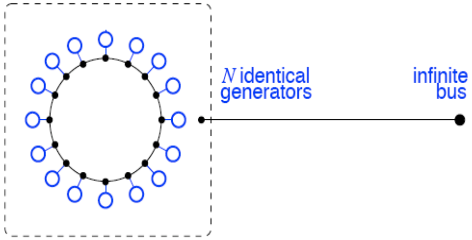
UC SANTA BARBARA

Power Grid Instabilities



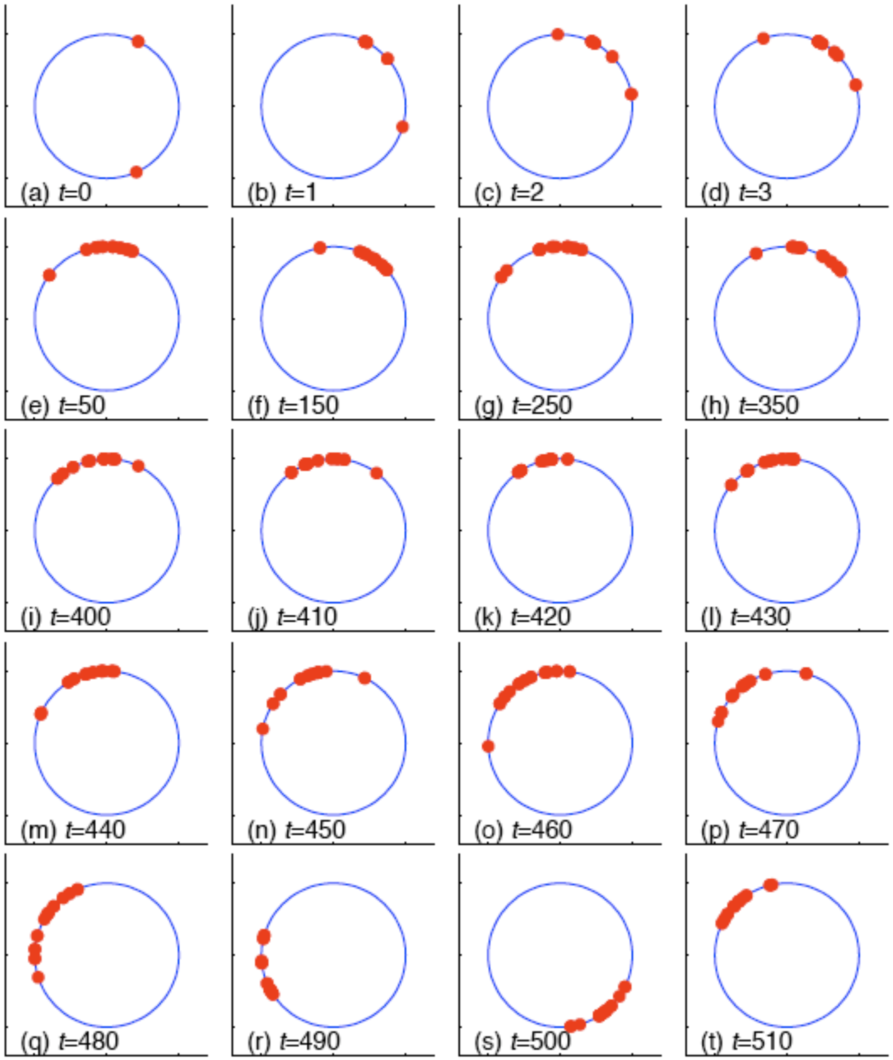


A Power Grid Model



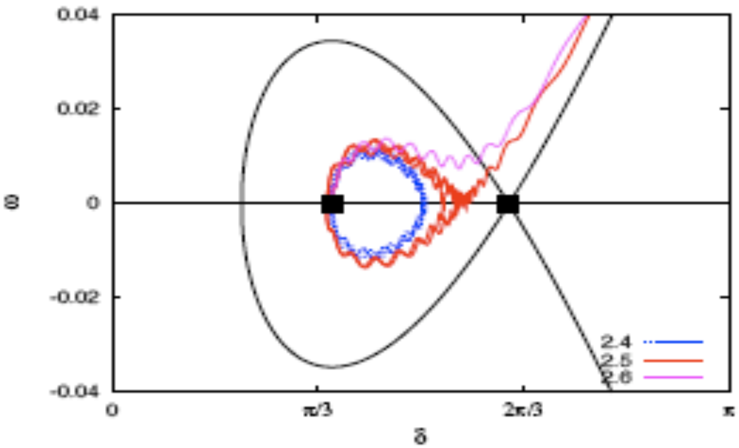
$$\frac{d\delta_i}{dt} = \omega_i,$$

$$\frac{d\omega_i}{dt} = p_m - b\sin\delta_i + b_{int}\{\sin(\delta_{i-1}-\delta_i) - \sin(\delta_i - \delta_{i+1})\},$$



Y. Susuki, T. Hikihara (Kyoto) and I.M.
(Journal of Nonlinear Science, 2011)

A Power Grid Model



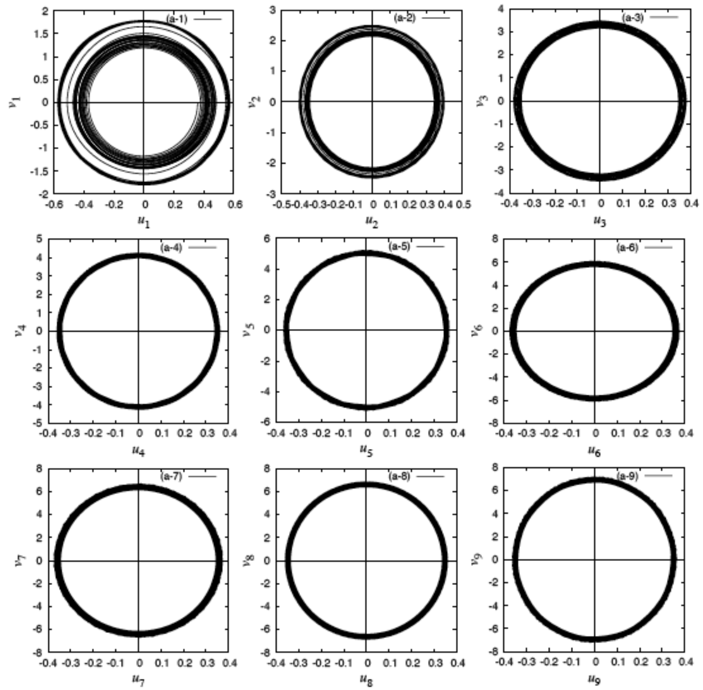
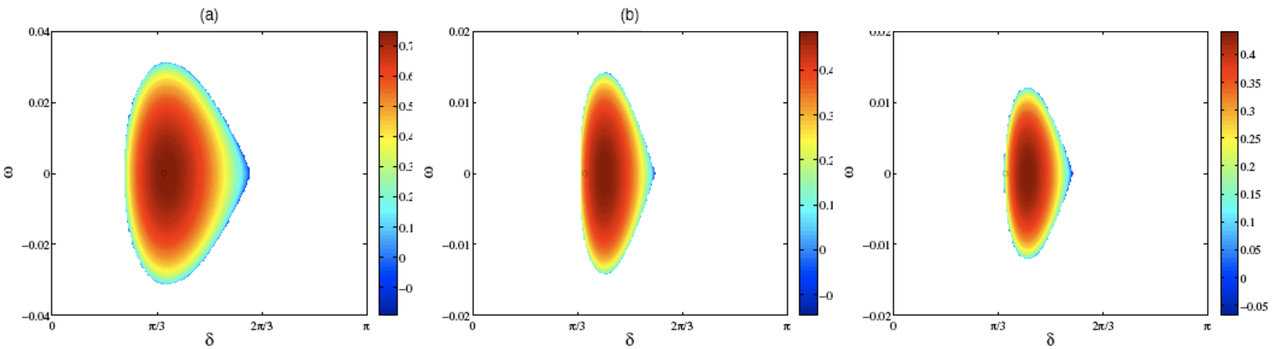


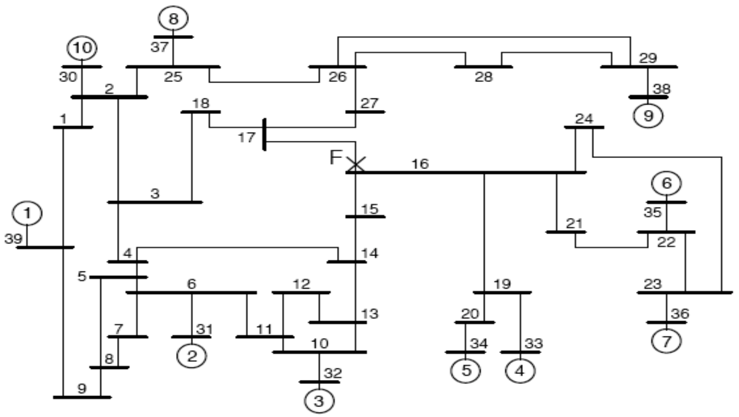
Figure 6: Dynamics of the oscillatory modes under coherent swing instability. We show trajectories of the swing equations (3) projected onto the modal variable (u_j, v_j) planes: (a) local disturbance at $\delta_{N/2}(0) = -0.352$. For (a) the other trajectories of oscillatory modes are omitted.

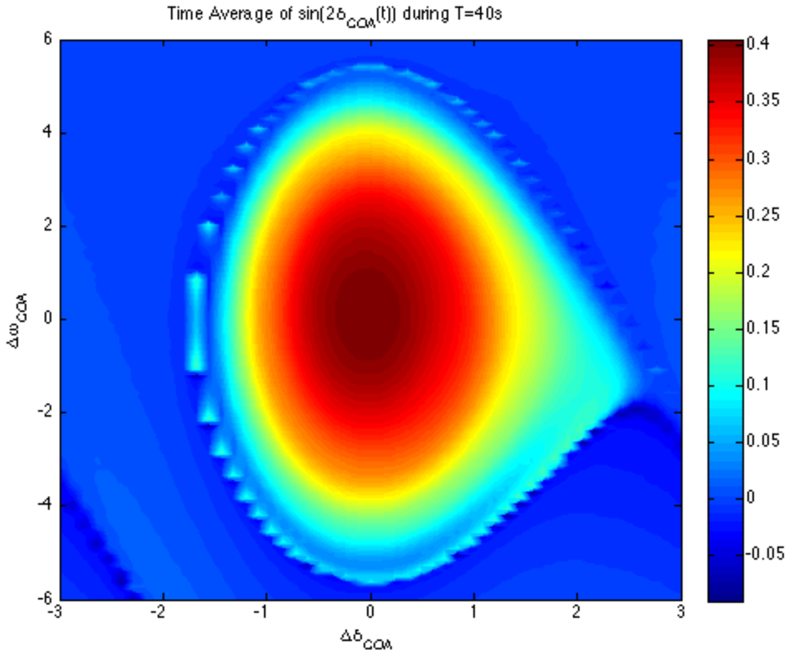
Harmonic field approximation



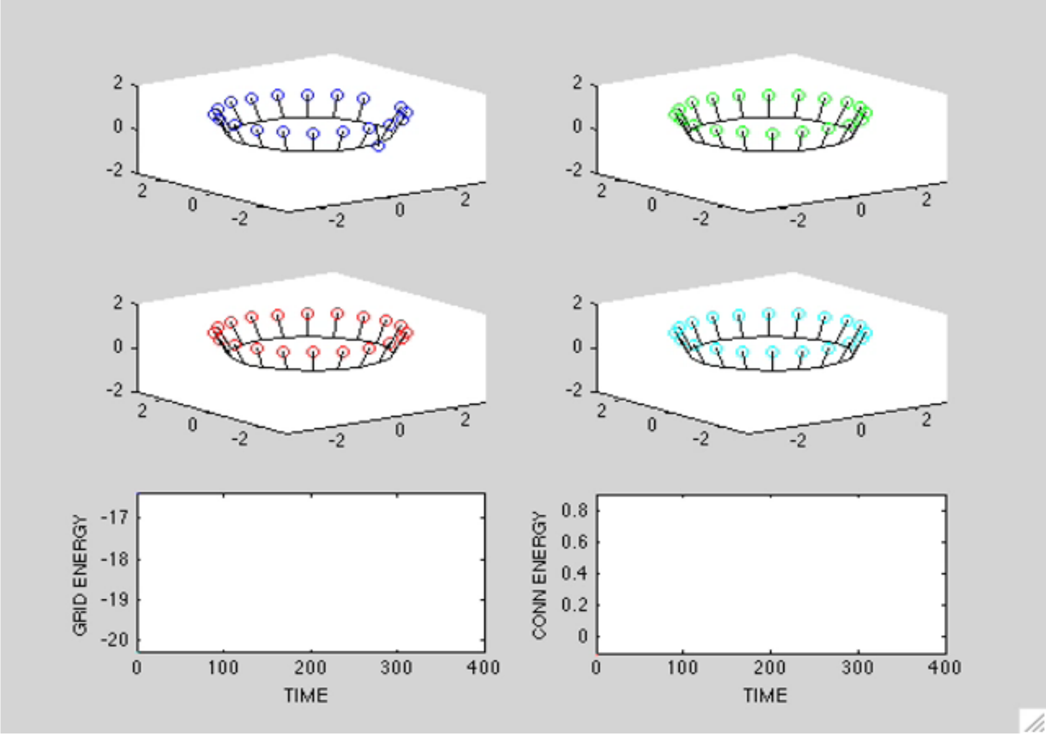
A Realistic Power Grid Model

NE Power grid model: 10 generators





Y. Susuki, T. Hikihara (Kyoto)
And I.M. (2009)



A Realistic Power Grid Model

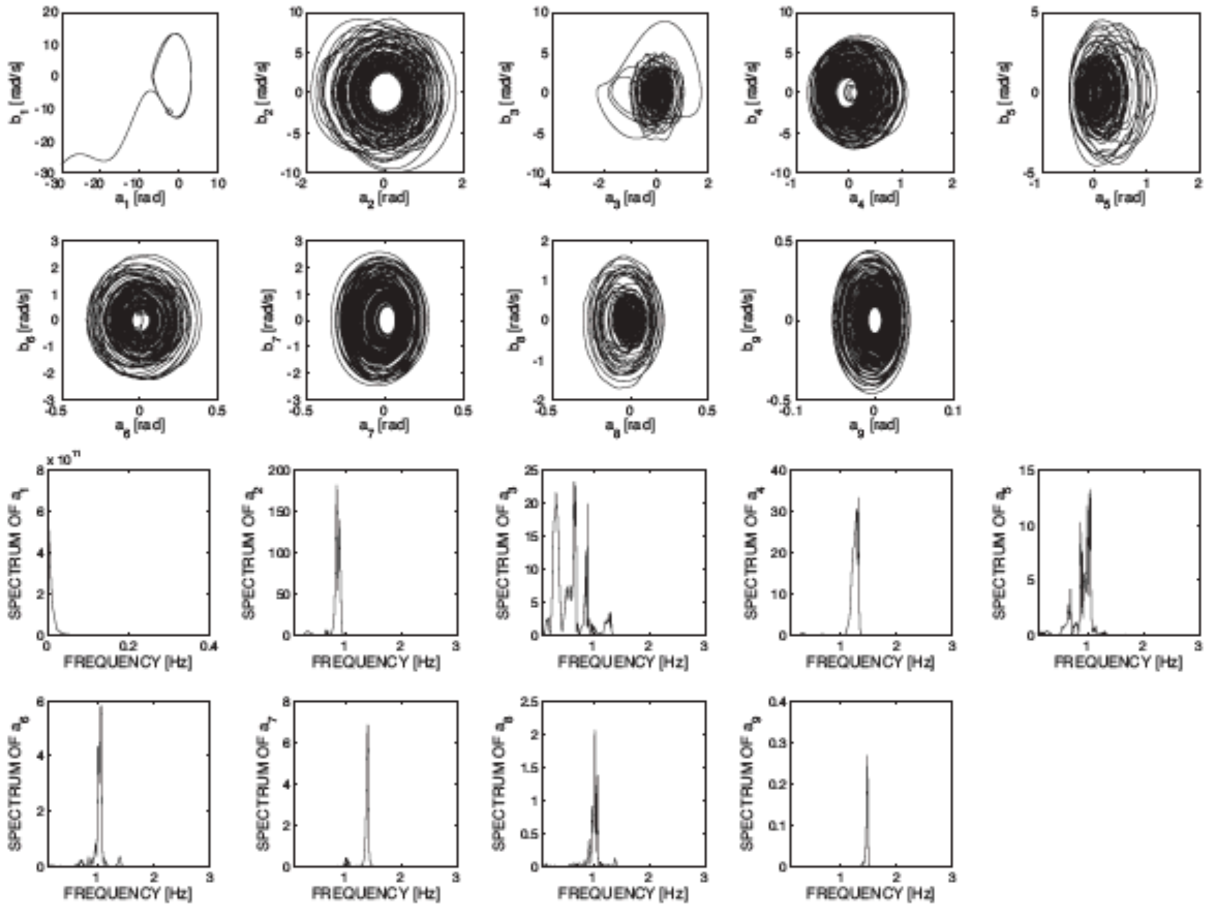


Figure 11: Projected trajectories onto Proper Orthonormal Mode (POM) planes and power spectra of time series of coefficients a_j during [1 s, 90 s] with sampling frequency 60 Hz. Zero-frequency components for the 2nd to 9th POM are not shown here.

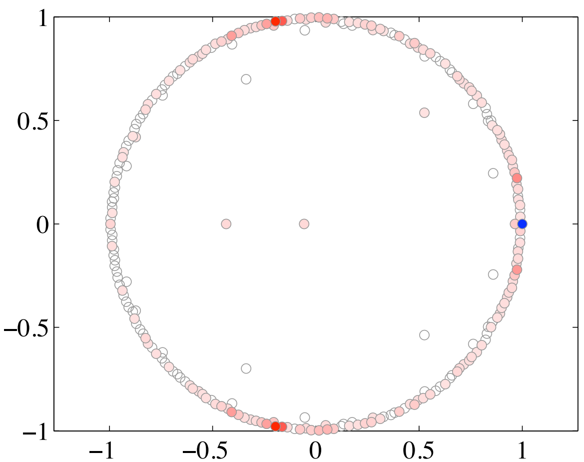
Koopman Modes

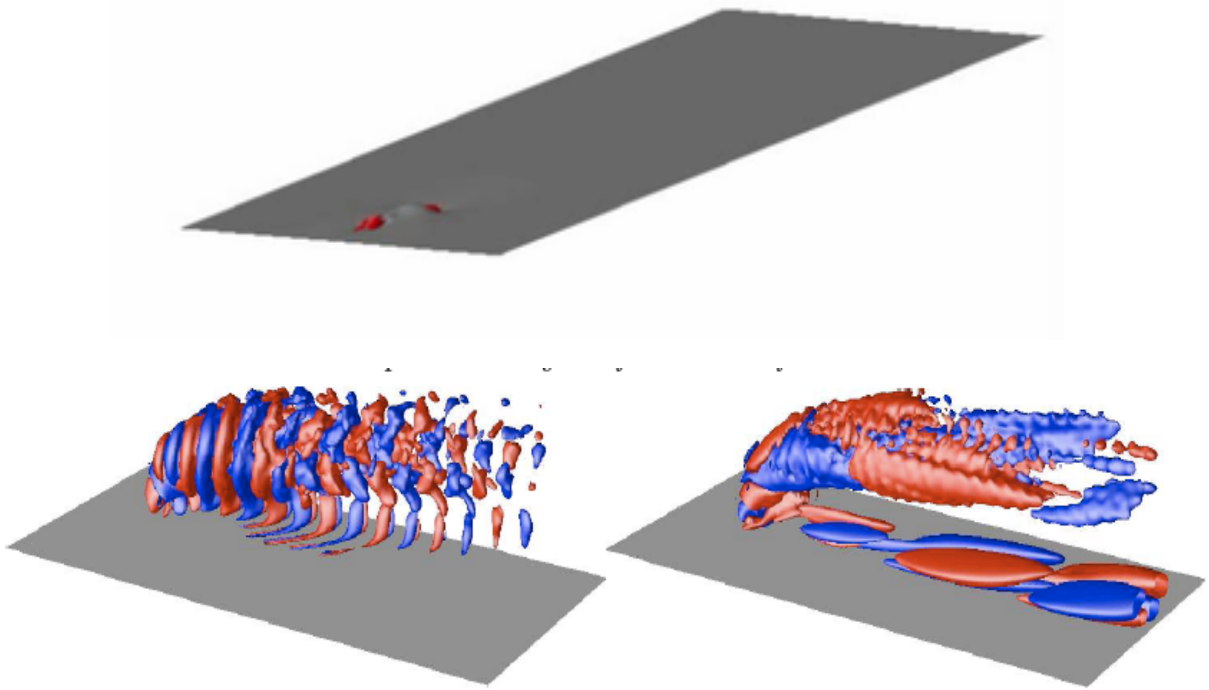
$$v^{n+1}(x) = N(v^n(x), p)$$

I.M., Nonl.Dyn (2005)

$$v_x^n(m) = U_s^n v_x(m) = v^*(x) + \sum_{j=1}^{k} \lambda_j^n f_j(m) s_j(x) + \int_0^1 \exp(i2\pi\alpha)\, dE(\alpha) v(x,m)$$

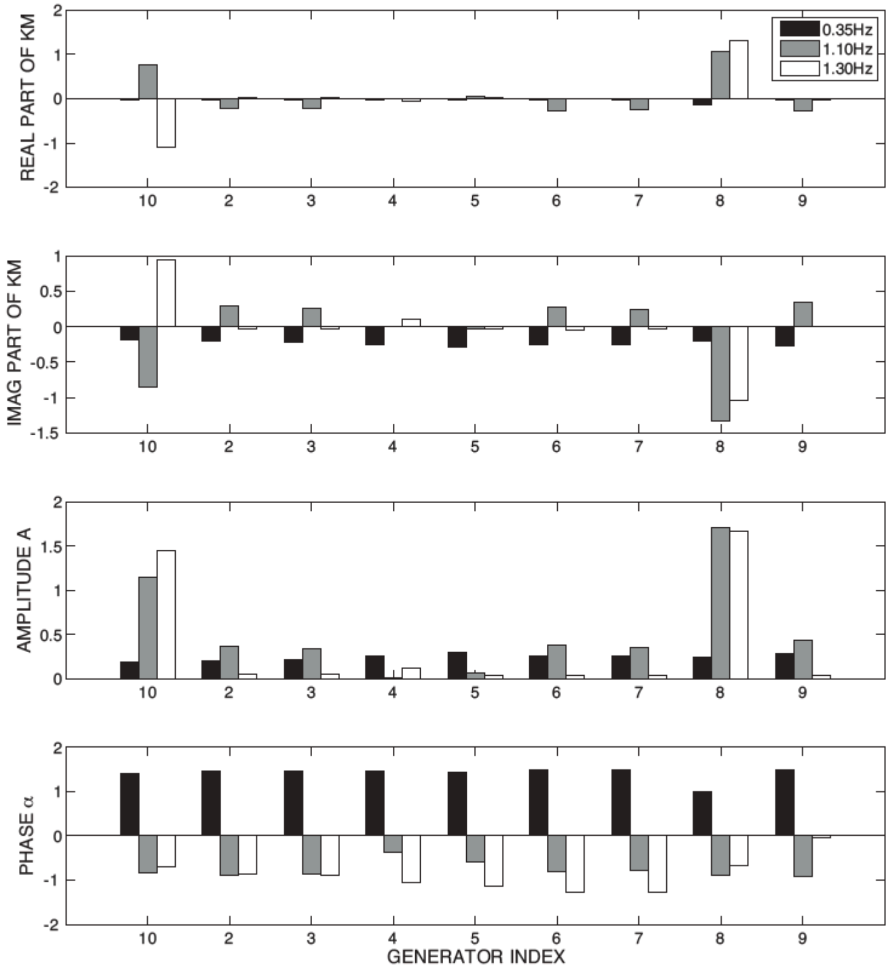
Koopman modes

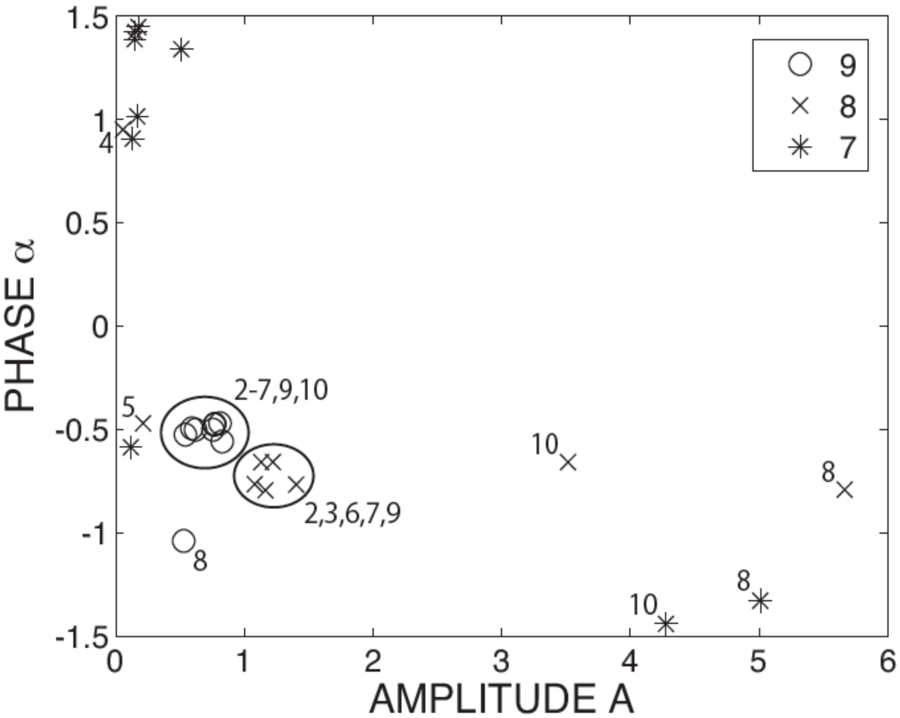




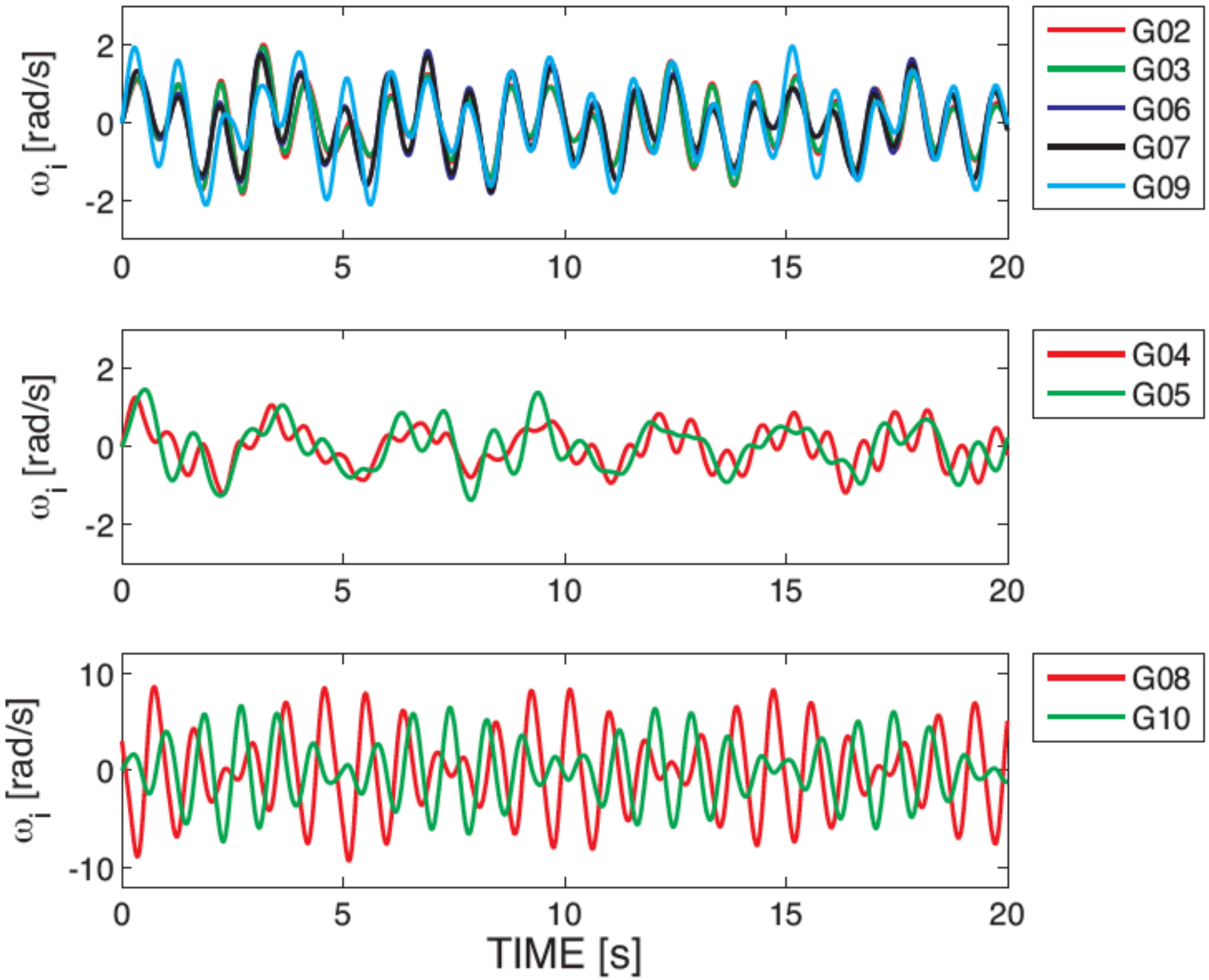
C.W. Rowley, I. Mezic, S. Bagheri, P. Schlatter, and D.S. Henningson (Journal Fluid Mech (2010))

Koopman Modes and Coherency



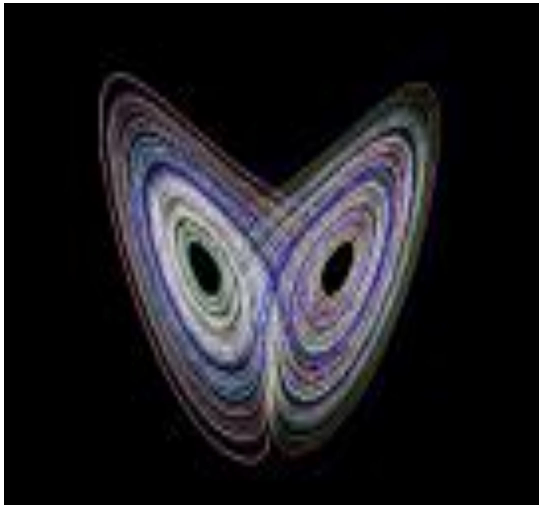


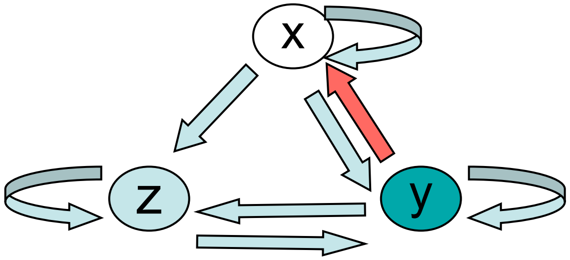
Koopman Modes and Coherency

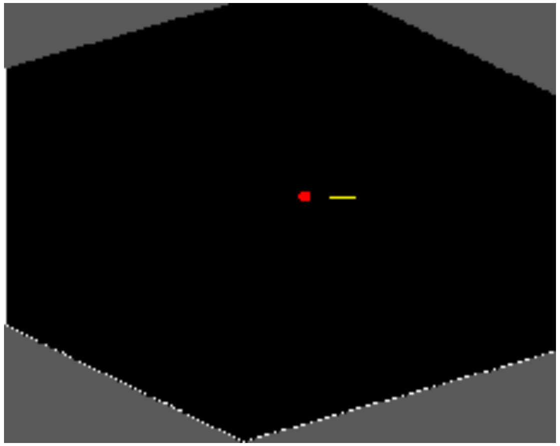


Intro to graph-theoretic techniques

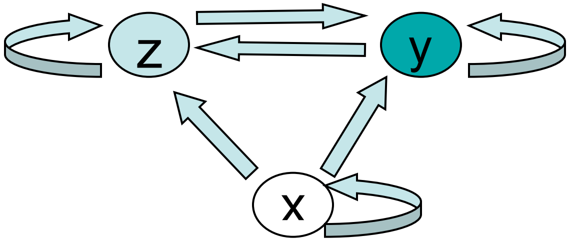
$b = 8/3, r = 28, s_1 = 10, s_2 = 10$

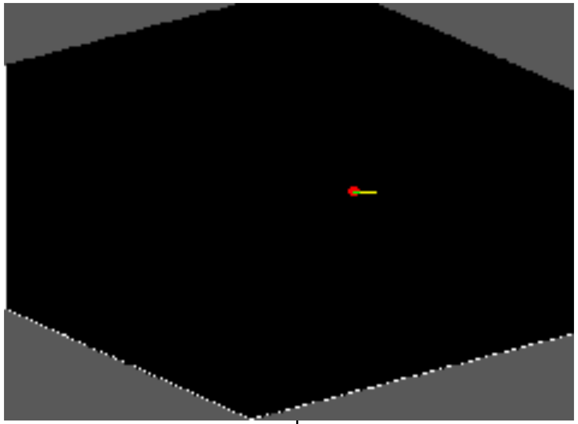






$$\begin{aligned} \dot{x} &= s_1 y - s_2 x, \\ \dot{y} &= -xz + rx - y, \\ \dot{z} &= xy - bz. \end{aligned}$$





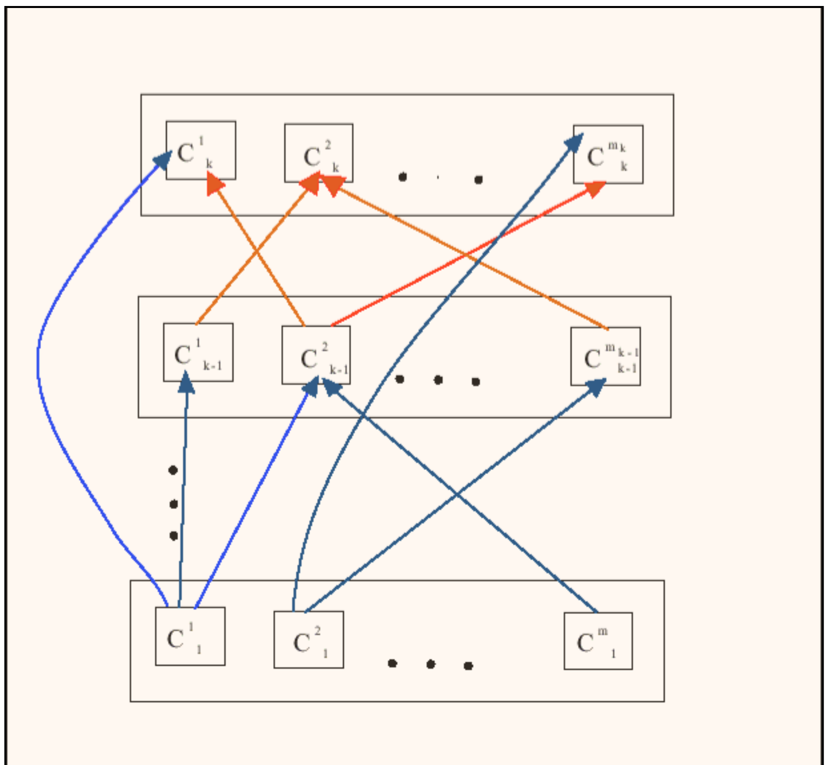
$b = 8/3, r = 28, s_1 = 0, s_2 = 10$

Graph indicates no chaos

Horizontal-Vertical Decomposition

I.M., Proc. CDC (2004)

Theorem (Horizontal-vertical decomposition)
A dynamical system with a stochastic matrix M can be decomposed into k vertical levels, such that each higher level is driven by the dynamics of levels below. Every horizontal level $i \subset \{1, \ldots, k\}$ can be decomposed into m_i sets, which have dynamics independent of each other.



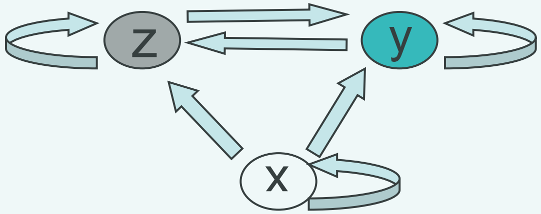
Skew-product structure

$$\dot{x}_{j_n} = f(x_{j_n}, \mathbf{x}_{\leq j})$$

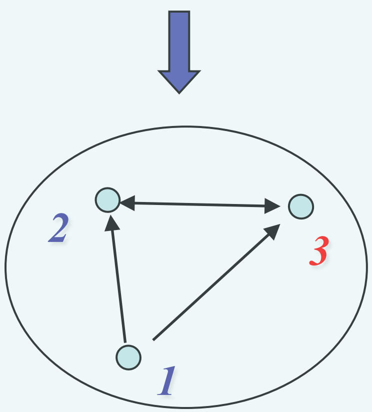
Cf. E. Shea-Brown an L.-S. Young
on reliability in neural networks (ArXiv2007)

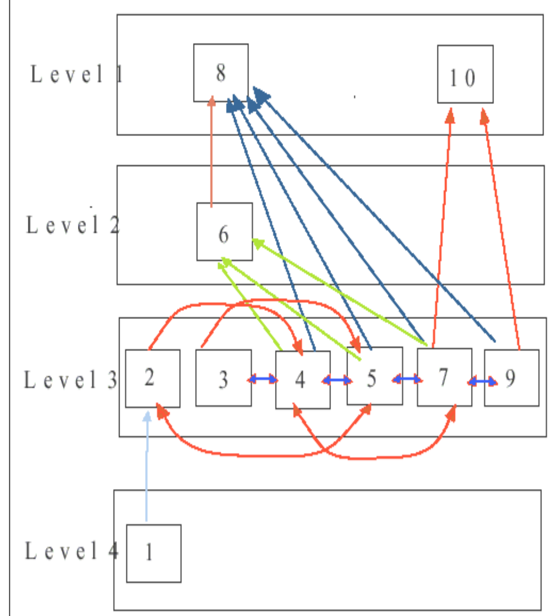
Cf. Alice Hubenko talk Wed 5:15 MS 104

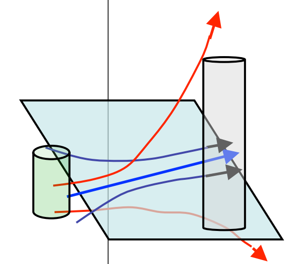
Propagation of uncertainty



$$M = \begin{pmatrix} 1/3 & 0 & 0 \\ 1/3 & 1/2 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{pmatrix}$$







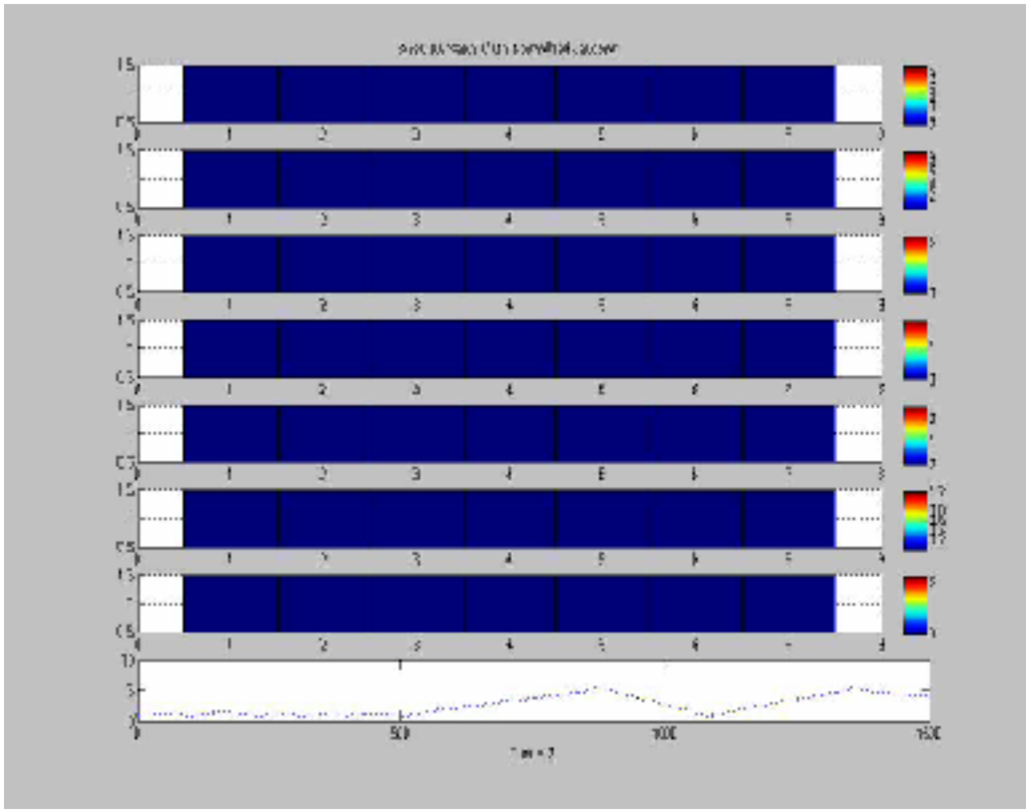
$$\int_{-\infty}^{\infty} p(\mathbf{x}, t, t_0) d\mathbf{x}_{>i} = p_0^{\leq i}(S_{<i}^{to-t}(\mathbf{x}_{\leq i}, t)) \exp\left[-\int_{t_0}^{t} \nabla_{\leq i} \cdot \mathbf{v}_{\leq i}(S_{\leq i}^{t_0-\bar{t}}(\mathbf{x}_{\leq i}, t)) d\bar{t}\right]$$

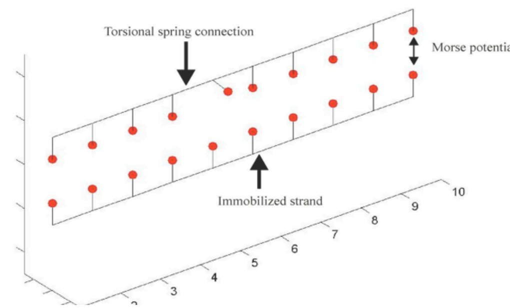
SODE’s:Feynman-Kac

Asymptotically: Lyapunov exponents

Dynamical graph decomposition

B. Eisenhower and I.M. (2009)





$$
\begin{bmatrix} \dot{\bar\theta} \\ \dot\phi_1 \\ \dot\phi_2 \\ \vdots \\ \dot\phi_n \\ \dot{\bar\omega} \\ \dot J_1 \\ \dot J_2 \\ \vdots \\ \dot J_n \end{bmatrix}
=
\begin{bmatrix}
\frac{\partial\dot{\bar\theta}}{\partial\bar\theta} & \frac{\partial\dot{\bar\theta}}{\partial\phi_1} & \frac{\partial\dot{\bar\theta}}{\partial\phi_2} & \cdots & \frac{\partial\dot{\bar\theta}}{\partial\phi_n} & \frac{\partial\dot{\bar\theta}}{\partial\bar\omega} & \frac{\partial\dot{\bar\theta}}{\partial J_1} & \frac{\partial\dot{\bar\theta}}{\partial J_2} & \cdots & \frac{\partial\dot{\bar\theta}}{\partial J_n} \\
\frac{\partial\dot\phi_1}{\partial\bar\theta} & \frac{\partial\dot\phi_1}{\partial\phi_1} & \frac{\partial\dot\phi_1}{\partial\phi_2} & \cdots & \frac{\partial\dot\phi_1}{\partial\phi_n} & \frac{\partial\dot\phi_1}{\partial\bar\omega} & \frac{\partial\dot\phi_1}{\partial J_1} & \frac{\partial\dot\phi_1}{\partial J_2} & \cdots & \frac{\partial\dot\phi_1}{\partial J_n} \\
\frac{\partial\dot\phi_2}{\partial\bar\theta} & \frac{\partial\dot\phi_2}{\partial\phi_1} & \frac{\partial\dot\phi_2}{\partial\phi_2} & & \frac{\partial\dot\phi_2}{\partial\phi_n} & \frac{\partial\dot\phi_2}{\partial\bar\omega} & \frac{\partial\dot\phi_2}{\partial J_1} & \frac{\partial\dot\phi_2}{\partial J_2} & & \frac{\partial\dot\phi_2}{\partial J_n} \\
 & & \vdots & & & & & \vdots & & \\
\frac{\partial\dot\phi_n}{\partial\bar\theta} & \frac{\partial\dot\phi_n}{\partial\phi_1} & \frac{\partial\dot\phi_n}{\partial\phi_2} & & \frac{\partial\dot\phi_n}{\partial\phi_n} & \frac{\partial\dot\phi_n}{\partial\bar\omega} & \frac{\partial\dot\phi_n}{\partial J_1} & \frac{\partial\dot\phi_n}{\partial J_2} & & \frac{\partial\dot\phi_n}{\partial J_n} \\
\frac{\partial\dot{\bar\omega}}{\partial\bar\theta} & \frac{\partial\dot{\bar\omega}}{\partial\phi_1} & \frac{\partial\dot{\bar\omega}}{\partial\phi_2} & & \frac{\partial\dot{\bar\omega}}{\partial\phi_n} & \color{red}{\frac{\partial\dot{\bar\omega}}{\partial\bar\omega}} & \color{red}{\frac{\partial\dot{\bar\omega}}{\partial J_1}} & \color{red}{\frac{\partial\dot{\bar\omega}}{\partial J_2}} & & \color{red}{\frac{\partial\dot{\bar\omega}}{\partial J_n}} \\
\frac{\partial\dot J_1}{\partial\bar\theta} & \frac{\partial\dot J_1}{\partial\phi_1} & \frac{\partial\dot J_1}{\partial\phi_2} & & \frac{\partial\dot J_1}{\partial\phi_n} & \color{red}{\frac{\partial\dot J_1}{\partial\bar\omega}} & \color{red}{\frac{\partial\dot J_1}{\partial J_1}} & \color{red}{\frac{\partial\dot J_1}{\partial J_2}} & & \color{red}{\frac{\partial\dot J_1}{\partial J_n}} \\
\frac{\partial\dot J_2}{\partial\bar\theta} & \frac{\partial\dot J_2}{\partial\phi_1} & \frac{\partial\dot J_2}{\partial\phi_2} & \cdots & \frac{\partial\dot J_2}{\partial\phi_n} & \color{red}{\frac{\partial\dot J_2}{\partial\bar\omega}} & \color{red}{\frac{\partial\dot J_2}{\partial J_1}} & \color{red}{\frac{\partial\dot J_2}{\partial J_2}} & \cdots & \color{red}{\frac{\partial\dot J_2}{\partial J_n}} \\
 & & \vdots & & & & & \vdots & & \\
\frac{\partial\dot J_n}{\partial\bar\theta} & \frac{\partial\dot J_n}{\partial\phi_1} & \frac{\partial\dot J_n}{\partial\phi_2} & & \frac{\partial\dot J_n}{\partial\phi_n} & \color{red}{\frac{\partial\dot J_n}{\partial\bar\omega}} & \color{red}{\frac{\partial\dot J_n}{\partial J_1}} & \color{red}{\frac{\partial\dot J_n}{\partial J_2}} & & \color{red}{\frac{\partial\dot J_n}{\partial J_n}}
\end{bmatrix}
\begin{bmatrix} \bar\theta \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \\ \bar\omega \\ J_1 \\ J_2 \\ \vdots \\ J_n \end{bmatrix}
$$

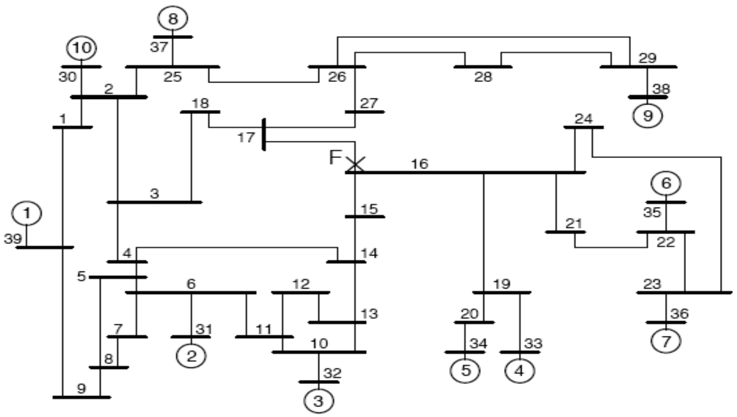
Jacobian: **H-V decomposition!!!**

Collective coordinates: actions

Cf B. Eisenhower talk, Tue 5:15 CP 13.

A Realistic Power Grid Model

NE Power grid model: 10 generators



$$(\delta_i(0), \omega_i(0)) = \begin{cases} (\delta_i^* + 1.000\,\text{rad}, 3\ \text{rad/s}) & i = 8, \\ (\delta_i^*, 0\,\text{rad/s}) & \text{else}, \end{cases} \quad (15)$$

and

$$(\delta_i(0), \omega_i(0)) = \begin{cases} (\delta_i^* + 1.575\,\text{rad}, 3\ \text{rad/s}) & i = 8, \\ (\delta_i^*, 0\,\text{rad/s}) & \text{else}. \end{cases} \quad (16)$$

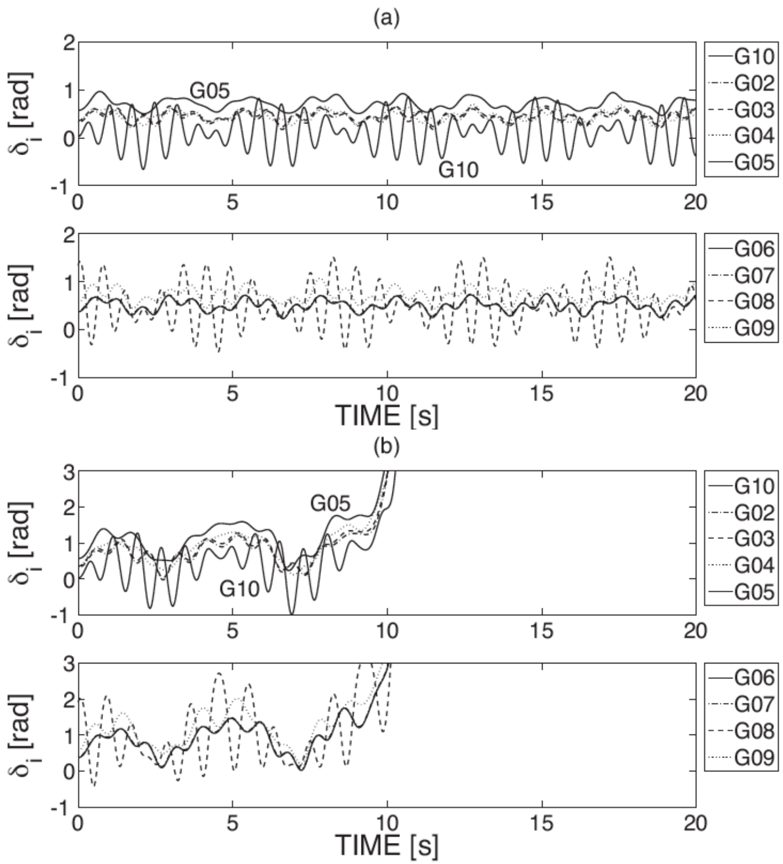
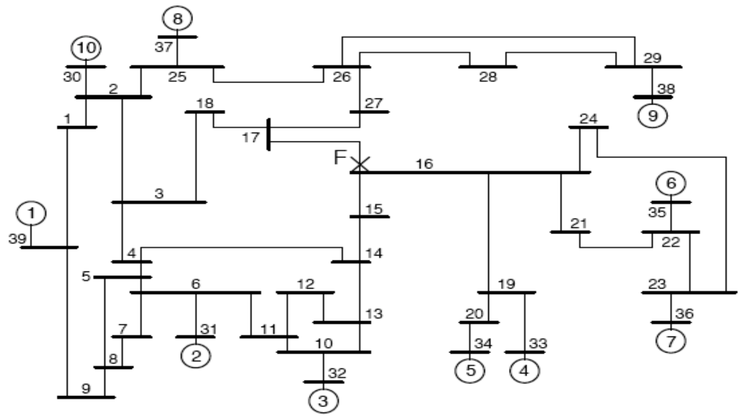


Fig. 2. Coupled swings and instability observed in in the New England 39-bus test system ($D_i = 0$): (a) the initial condition (15) and (b) the initial condition (16).

UC SANTA BARBARA

A Realistic Power Grid Model

NE Power grid model: 10 generators



$$(\delta_i(0), \omega_i(0)) = \begin{cases} (\delta_i^* + 1.000\,\mathrm{rad}, 3\ \mathrm{rad/s}) & i = 8, \\ (\delta_i^*, 0\,\mathrm{rad/s}) & \text{else}, \end{cases} \quad (15)$$

and

$$(\delta_i(0), \omega_i(0)) = \begin{cases} (\delta_i^* + 1.575\,\mathrm{rad}, 3\ \mathrm{rad/s}) & i = 8, \\ (\delta_i^*, 0\,\mathrm{rad/s}) & \text{else}. \end{cases} \quad (16)$$

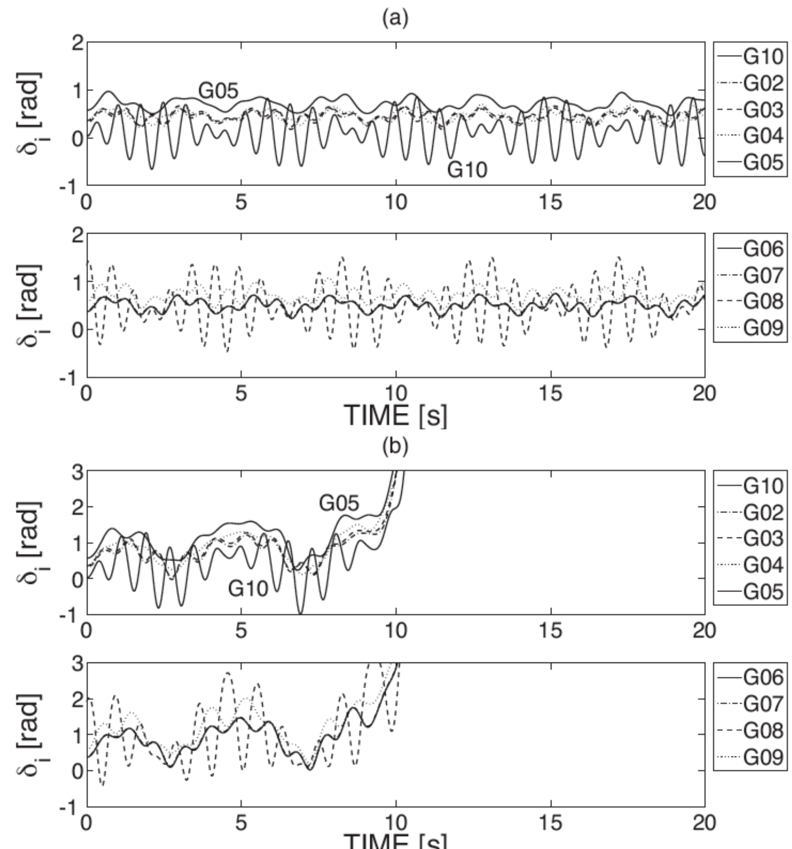


Fig. 2. Coupled swings and instability observed in in the New England 39-bus test system ($D_i = 0$): (a) the initial condition (15) and (b) the initial condition (16).

A Realistic Power Grid Model

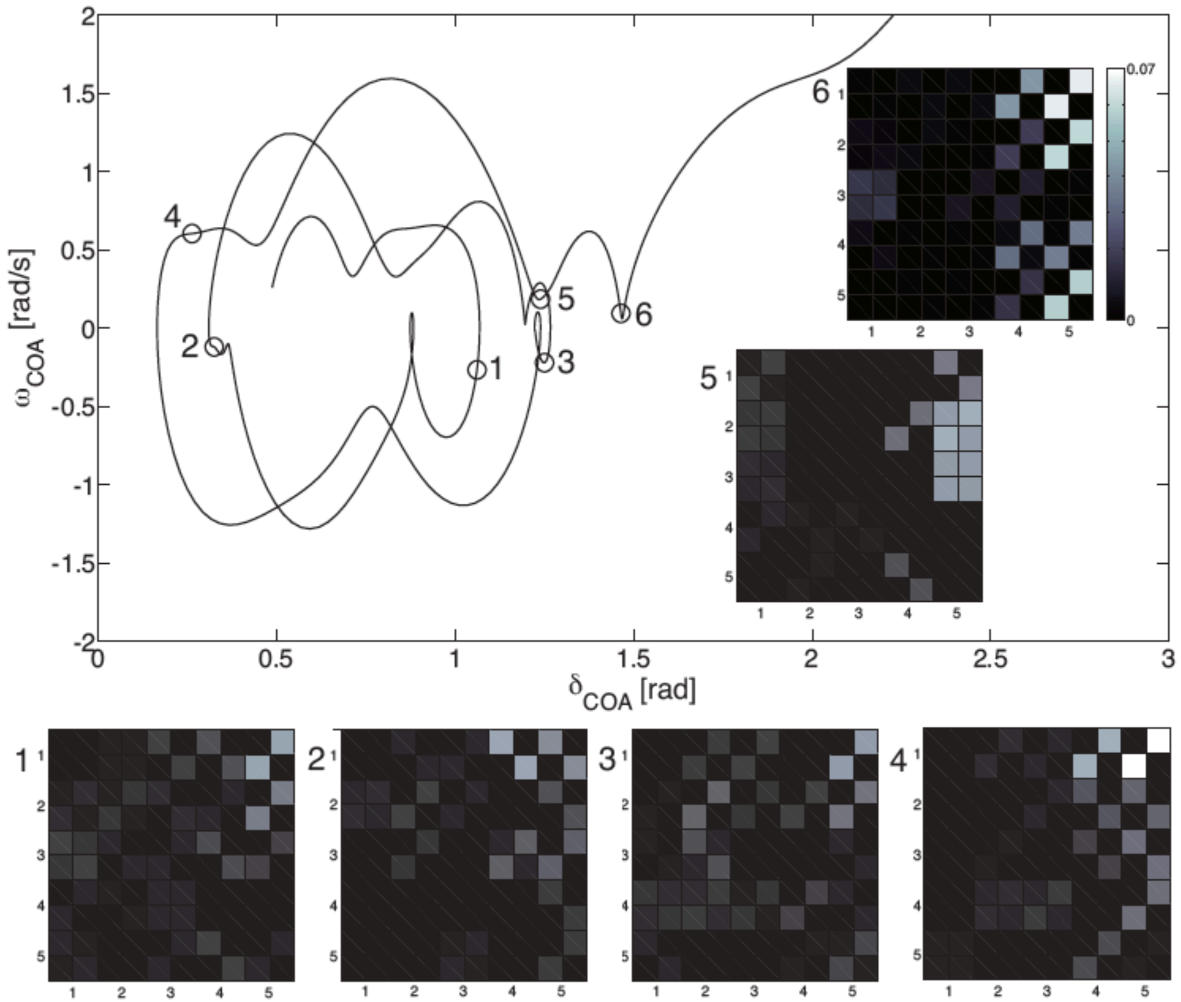


Fig. 5. Collective dynamics of the CSI phenomenon shown in Fig. 3(b) and snapshots of the Jacobi matrix J. The six snapshots are at time (1) 1.2 s, (2) 2.8 s, (3) 5.0 s, (4) 7.2 s, (5) 8.4 s, and (6) 9.0 s.

Conclusions

- Structure of inertial network equations with weak local and strong coupling terms lead to switching between global equilibria.
- Both simple (circle) and realistic grid topologies lead to an instability that we named Coherent Swing Instability (CSI)
- Koopman operator formalism enables study of invariant partitions (fixed, periodic, quasiperiodic) despite the large interconnected and nonsmooth nature of the systems.
- The same (spectral formalism enables extraction of quasiperiodic, stable and unstable modes for large systems. This is a dynamically consistent (as opposed to energy-based, POD) decomposition.
- This decomposition enables extraction of coherent (single frequency) groups of generators
- Graph theoretic methods for decomposition and uncertainty propagation are coupled to operator formalism.
- Utilization of global coordinates and graph theory leads to a precursor to CSI.

UC SANTA BARBARA

Acknowledgments

Students:

Marko Budisic
Bryan Eisenhower
George Gilmore
Ryan Mohr
Blane Rhoads
Gunjan Thakur

Postdocs:

Alice Hubenko
Symeon Grivopoulos
Sophie Loire
Maud-Alix Mader
George Mathew

Visiting Professors:

Yoshihiko Susuki (Kyoto)
Yueheng Lan (Tsinghua)

Collaborators:

S. Bagheri (KTH)
Andrzej Banaszuk (UTRC)
Takashi Hikihara (Kyoto)
D.S. Henningson (KTH)
Jerry Marsden (Caltech)
Clancy Rowley (Princeton)
P. Schlatter (KTH)
Phillip du Toit (Caltech)

Sponsors: