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Fluid Mechanics and Turbulence in the Wind-Turbine Array Boundary Layer

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Collaboration with

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- **Marc Calaf (JHU & EPFL) - LES**
- **José Lebrón-Torres (RPI) - exp**
- **Dr. Hyung-Suk Kang (JHU) - exp**
- **Mr. Max Gibson (Portland State Univ.) - exp**
- **Prof. Johan Meyers (Univ. Leuven) - LES**

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Simulations: NCAR allocation (NSF)

NSF

The windturbine-array boundary layer (WTABL)

Land-based HAWT

Shell's Rock River windfarm in Carbon County, Wyoming, USA
Source: http://www.the-eic.com/News/Archive/2005/May/Article503.htm

Horns Rev HAWT
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Arrays are getting big:

When *L >> 10 H (H: height of PBL)*,

starts to approach "fully developed"

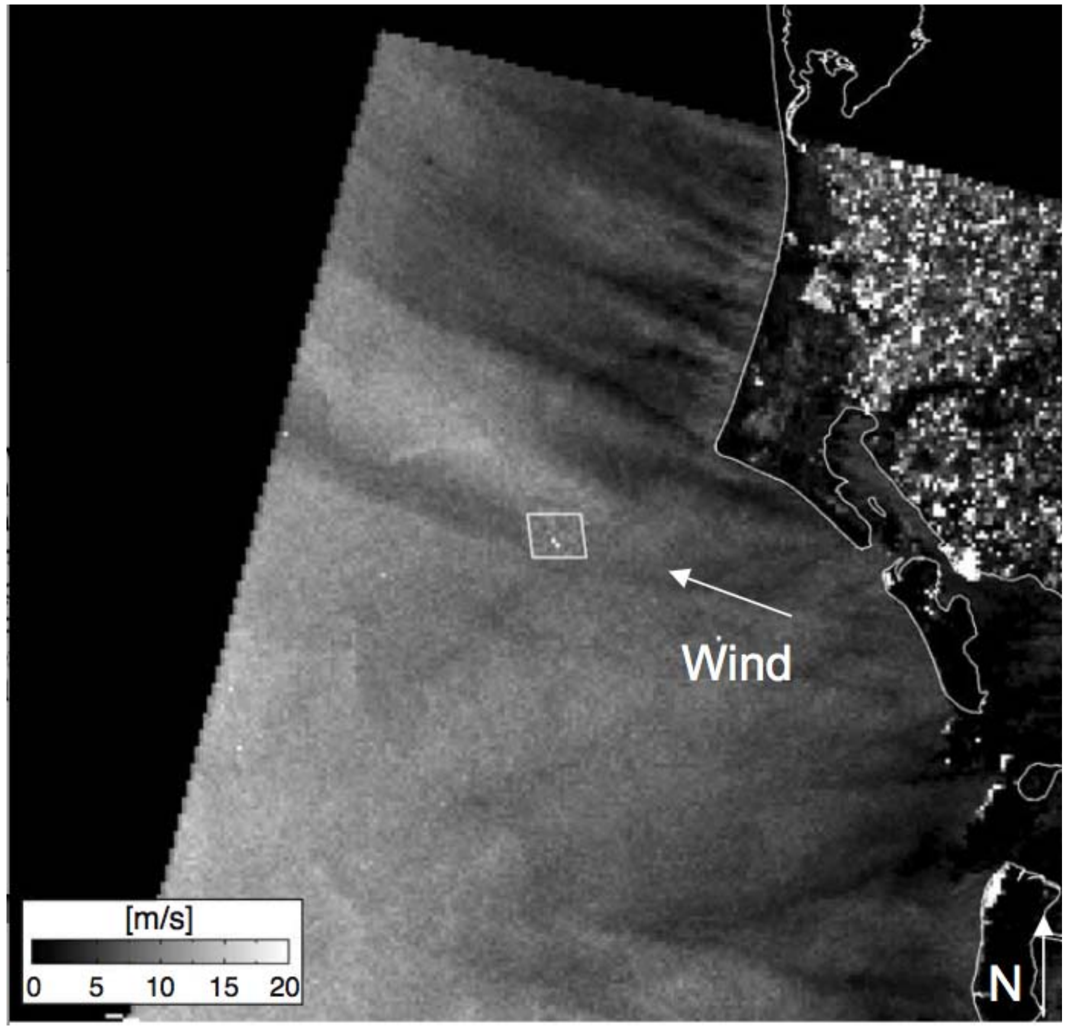
The windturbine-array boundary layer (WTABL)

Photo by Uni-Fly A/S (Wind turbine maintenance company)

Long downstream effects:

Horns Rev wind farm:

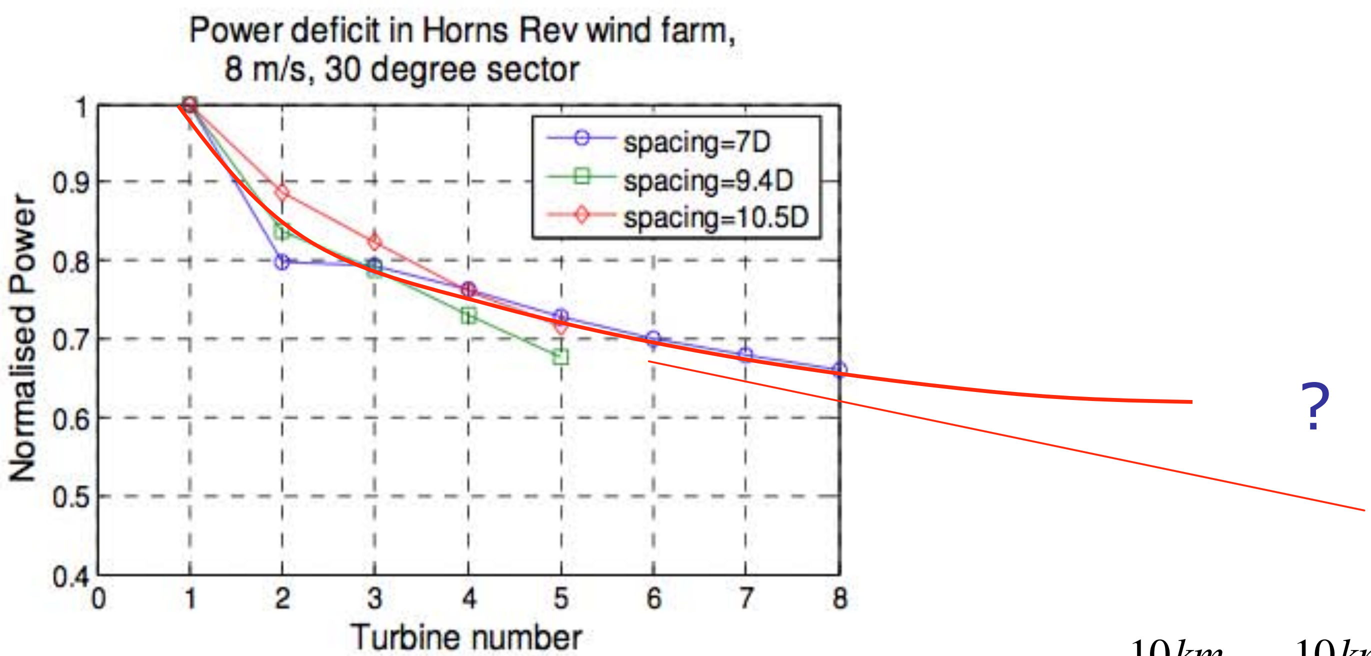
Christiansen & Hasager: “Wake effects of large wind farms identified from satellite synthetic aperture radar (SAR)”, Remote Sensing Env. 98, 251 (2005)



Long downstream effects:

Modelling and measurements of wakes in large wind farms
Barthelemie, Rathmann, Frandsen, Hansen et al…
J. Physics Conf. Series 75 (2007), 012049

Power extraction at Horns Rev wind farm:



$$10H \approx 10km: \quad \frac{10km}{7D} = \frac{10km}{0.56km} = 18$$

Importance of wake fluid dynamics and turbulence

Elkinton, Manwell & McGowan (2006)
Offshore Wind Farm Layout Optimization (OWFLO) Project:
Preliminary Results AIAA paper AIAA-2006-998

Total cost optimization for various layouts.

PARK model for wake velocity reduction (wake loss):

$$U(x) = U_0\left[1 - a\left(\frac{1}{1+\theta(x/D)}\right)^2\right]$$

Entrainment parameters depend on turbulence structure…

Fluid dynamics and details of wake structure very important all the way to the economic analysis

Sensitivity of cost to various components included in their cost analysis:

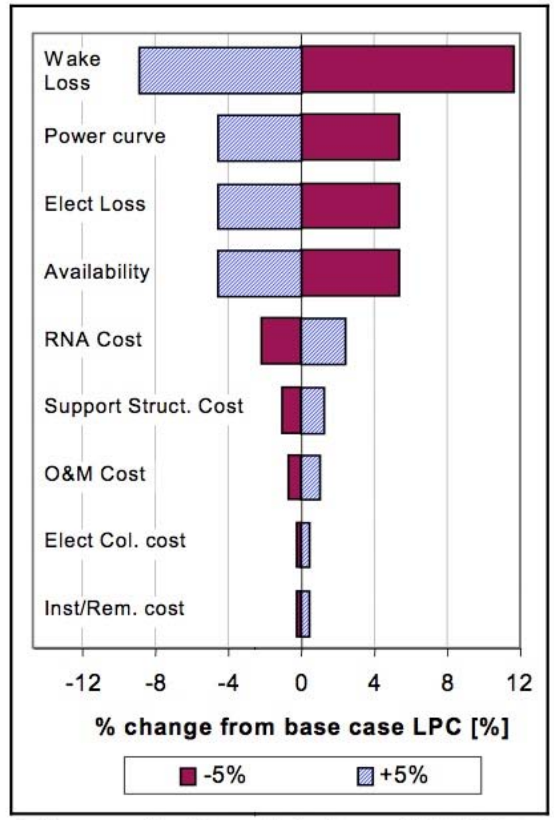


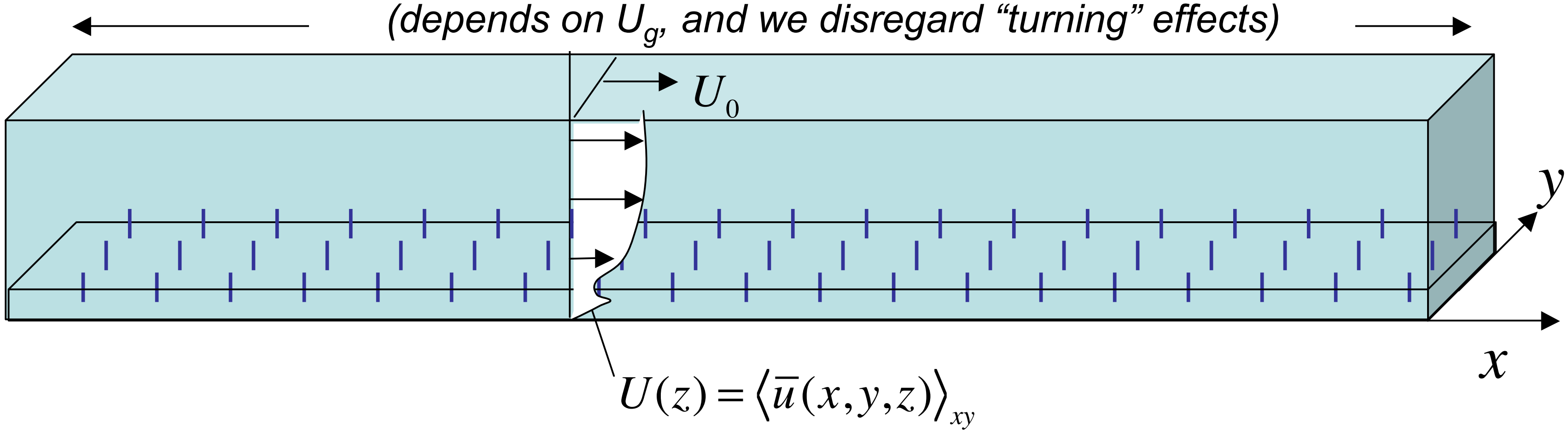
Figure 4. Sensitivity of LPC to component models

Sample of works on modelling and measurements of wind turbine wakes and arrays:

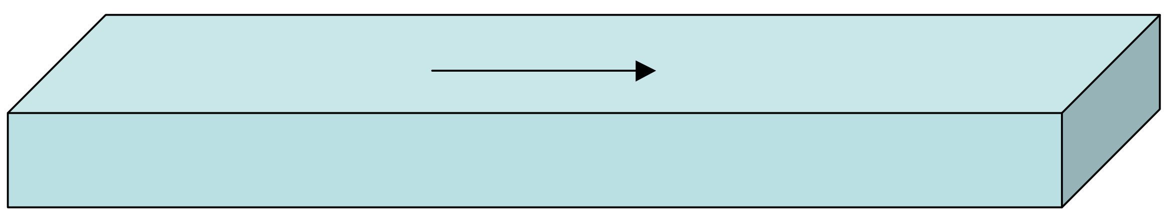
- Lissaman PBS. Energy effectiveness of arbitrary arrays of wind turbines. In: AIAA Paper 79-0114. p. 1-7, 1979
- Voutsinas SG, Rados KG, Zervos A. On the analysis of wake effects in wind parks. Journal of Wind Eng Ind Aerodyn **14**, 204-219, 1990.
- S Frandsen: wind speed reduction… J. Wind Eng & Ind Appl **39**, 1992.
- Crespo, Hernandez & Frandsen: Survey of modeling methods… Wind Energy **2**, 1-24, 1999.
- Magnusson M, Smedman AS. Air flow behind wind turbines. J Wind Eng Ind Aerodyn **80**, 169- 89, 1999.
- Frandsen & Thogersen: Integrated fatigue loading for wind turbines in wind farms by combining ambient turbulence and wakes. J Wind Engin 23, 327-339, 1999.
- Ebert & Wood: Renewable Energy, detailed single wake measurements.. 1999.
- Vermeer, Sorensen & Crespo: Wind turbine wake aerodynamics. Prog Aerospace Sci 39, 467-510, 2003.
- Corten, Schaak & Hegberg: Turbine interaction in large offshore wind farms, Report ECN-C-04-048, 2004.
- Medici & Alfredsson: Measurement on a wind turbine wake: 3D effects and bluff body vortex shedding. Wind Energy 9, 219-236, 2006.
- Jakob Mann: Simulation of Turbulence, Gusts and Wakes for Load Calculations, Wind EnergyProceedings of the Euromech Colloquium, 2007.
- Chamorro & Porté-Agel: A wind-tunnel investigation of wind-turbine wakes: Boundary-layer turbulence effects Wind Energy, 2009 (in press)
- …..

The “fully developed” WTABL:

Assume flow driven by imposed (unaffected) dpdx
(depends on U_g, and we disregard “turning” effects)



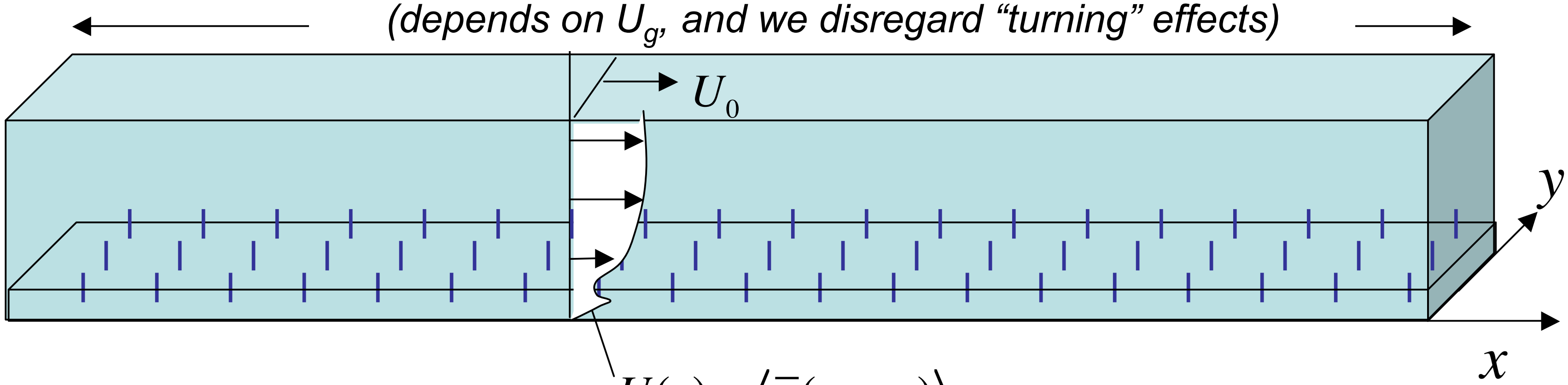
I = V/R



Models for R so that we may evaluate flow (I) for a given driving force (V)

The "fully developed" WTABL:

Assume flow driven by imposed (unaffected) dpdx
(depends on U_g, and we disregard "turning" effects)



$$U(z) = \langle \bar{u}(x,y,z) \rangle_{xy}$$

- Momentum theory (time averaged + "canopy average"):

$$0 = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \frac{d}{dz}\left(-\langle \overline{u'w'} \rangle_{xy} - \langle \bar{u}''\bar{w}'' \rangle_{xy} \right) + \langle f_x \rangle_{xy}$$

thrust force due to WT

$$\bar{u}'' = \bar{u} - \langle \bar{u} \rangle_{xy}$$

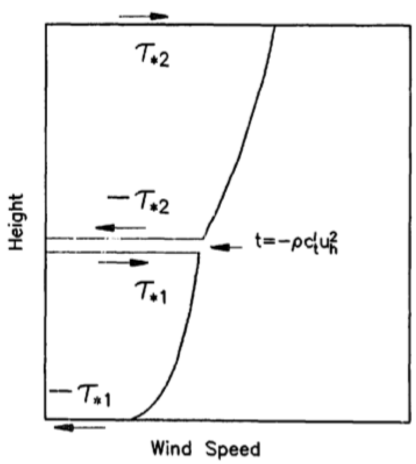
Horizontal average
of turbulent Reynolds stresses

We must include correlations
between mean velocity deviations
from their spatial mean
(Raupach et al. Appl Mech Rev **44**, 1991, Finnigan,
Annu Rev Fluid Mech **32**, 2000)

The fully developed WTABL: momentum theory

Horizontally averaged variables

Sten Frandsen,
J. Wind Eng & Ind
Appl **39**, 1992)*:*



$$0 = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \frac{d}{dz}\left(-\left\langle \overline{u'w'}\right\rangle_{xy} - \left\langle \overline{u}''\overline{w}''\right\rangle_{xy}\right) + \left\langle f_x \right\rangle_{xy}$$

Integrate in z-direction:

If top of WT canopy still

falls in the "surface layer", where $\frac{dp}{dx} z \approx 0$

and if wakes have "diffused" so that $\left\langle \overline{u}''\overline{w}''\right\rangle_{xy} \approx 0$

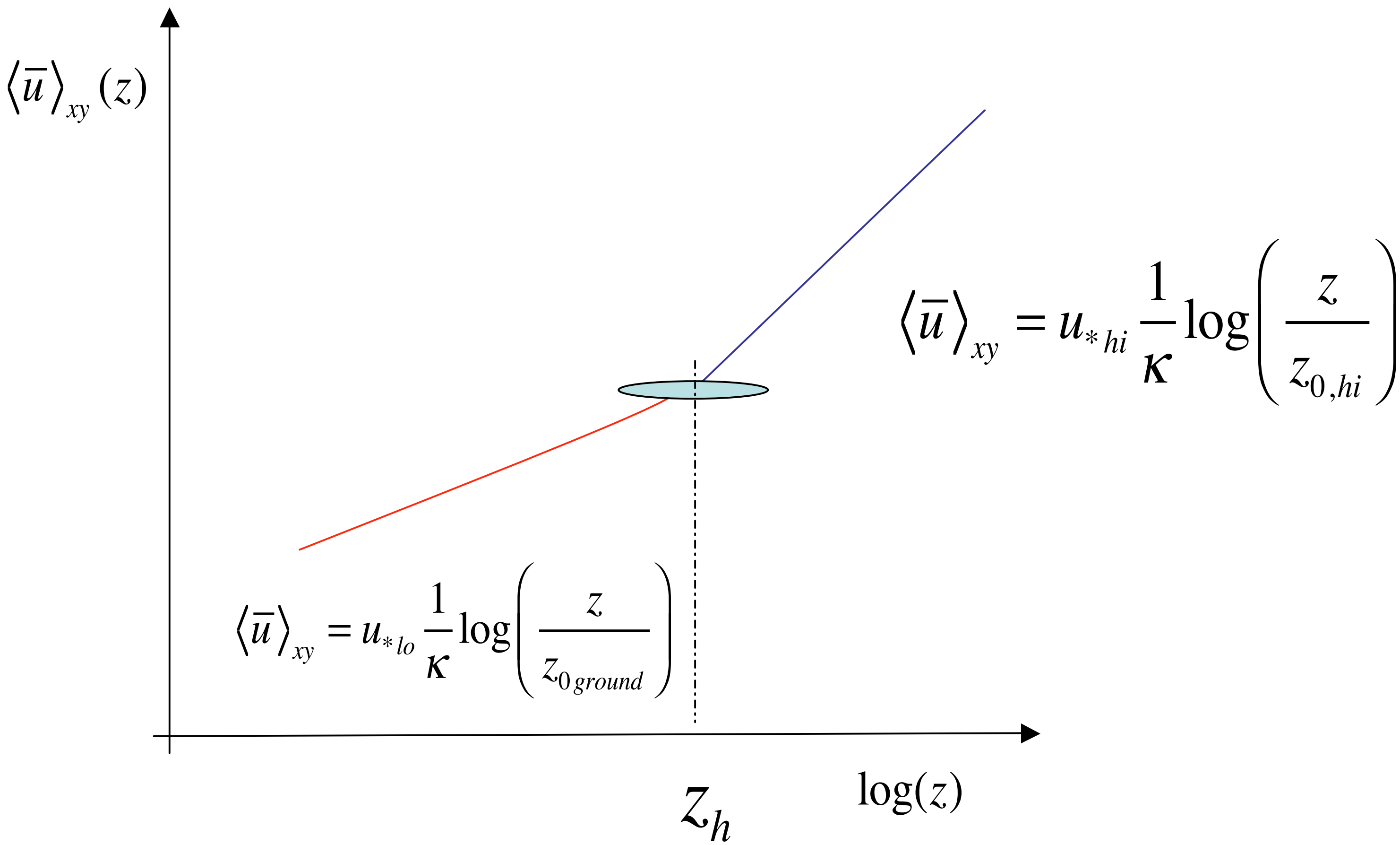
$$-\left\langle \overline{u'w'}\right\rangle_{xy}(z_{top}) \approx -\left\langle \overline{u'w'}\right\rangle_{xy}(z_{bottom}) + \frac{1}{2} C_T \frac{A_{disk}}{A_{xy}} U_R^2$$

$$u_{*\,hi}^2 \approx u_{*\,lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_R^2$$

The fully developed WTABL: momentum theory

$$u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_R^2$$

Frandsen 1992: postulated the existence of 2 log laws with u_{*hi} and u_{*lo}



The zeroth-order question:
Flow resistance as effective roughness length

S. Frandsen, 1992:

$$\langle \overline{u} \rangle_{xy} = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z}{z_{0,hi}} \right)$$

This is of the form **"I = V/R"**

I = velocity (output),

V=u* is applied driving force, and

R=resistance: $R = \left(\frac{1}{\kappa} \log\left(\frac{z}{z_{0,hi}} \right) \right)^{-1}$

If $z_{0,hi}$ is known, we can relate u_* and mean velocity u(z)

Note: for Geostrophic forcing, u* must Further be related to geostrophic wind (straight-forward if $z_{0,hi}$ is known)

The fully developed WTABL concept and effective roughness: sample application

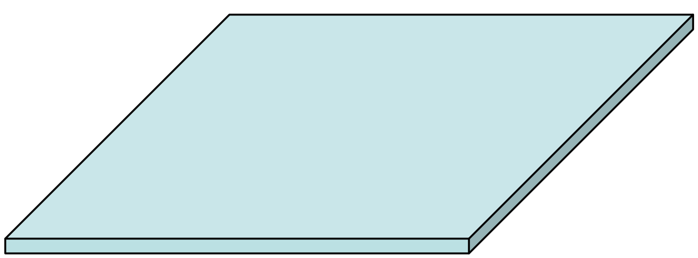
Keith et al. “The influence of large-scale wind power on climate” PNAS (2004)

Barrie & Kirk-Davidoff: “Weather response to management of large Wind turbine array”, Atmos. Chem. Phys. Discuss. **9**, 2917–2931, 2009

(under review for ACP)

Use $z_{0,hi}$ ~ 0.8 m - assumes rough-wall log-law (MO)

Grid-spacings 100’s of km, first vertical point ~ 80m



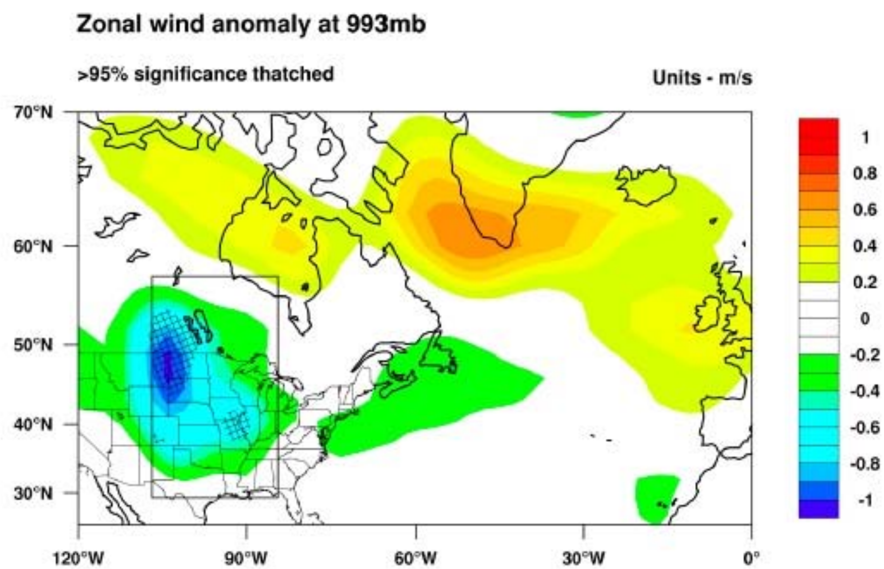


Fig. 1. 993 mbar zonal wind anomaly. The mean difference in the eastward wind in the lowest model level between the control and perturbed model runs highlights regions of atmospheric modification. Regions where significance exceeds 95%, as determined by a Student’s t-test, are thatched. The wind farm is located within the rectangular box over the central United States and central Canada. Areas of the wind farm located over water are masked out during the model runs.

The zeroth-order question:

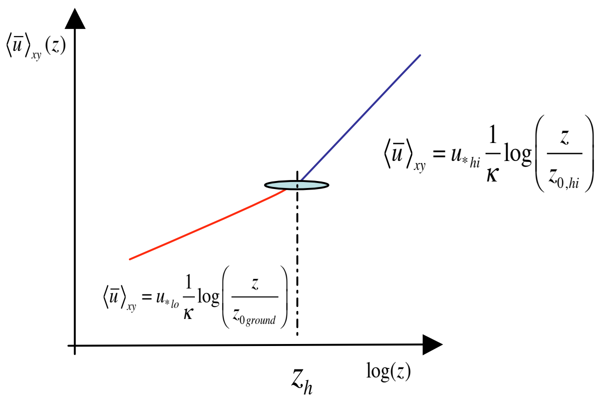
Flow resistance as effective roughness length

S. Frandsen 1992, 2006:

Knowns: $u_{*hi}, z_{0,ground}, C_T, s_x, s_y$

3 unknowns: $z_{0,hi}, U_R, u_{*lo}$

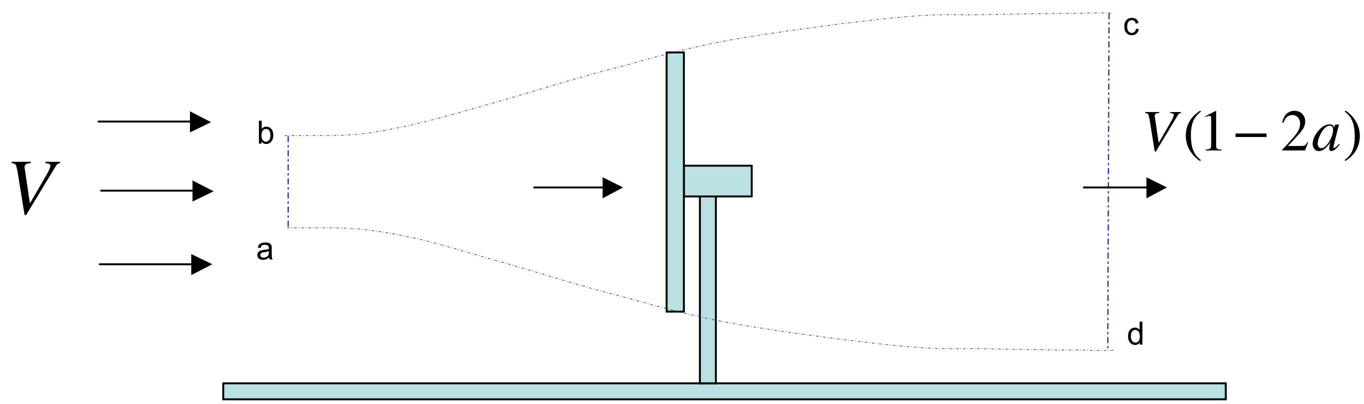
$$\begin{cases} u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_R^2 \\ U_R = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) \\ u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) = u_{*lo} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,ground}}\right) \end{cases}$$



Solve for effective roughness:

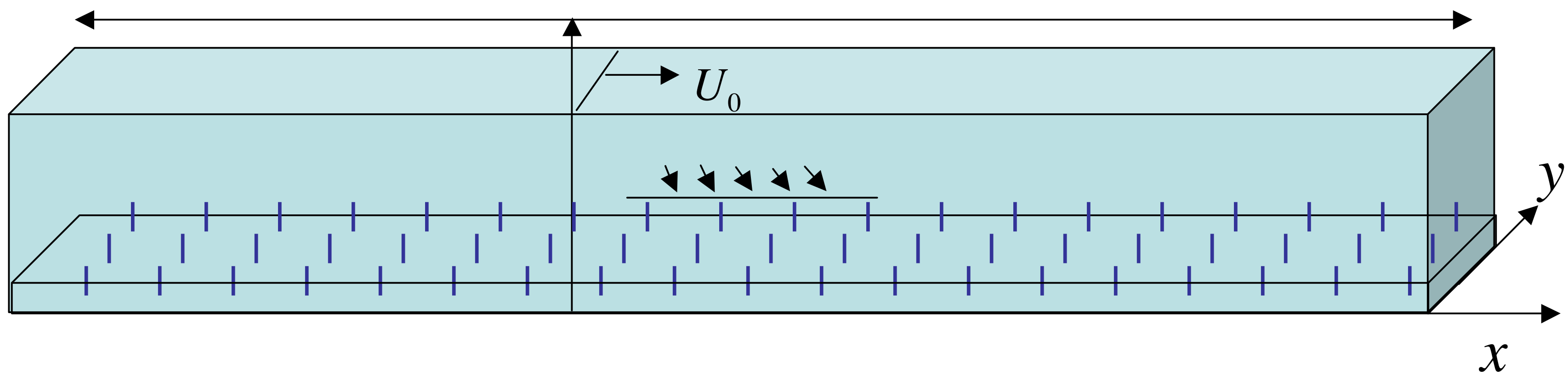
$$z_{0,hi} = z_h \exp\left(-\kappa \left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})} \right) \right]^{-1/2} \right)$$

Another important question: Fluxes of kinetic energy



For **single** wind turbine, extracted power =
difference in front and back fluxes of kinetic energy

Another important question: Fluxes of kinetic energy



For **multiple** (∞) wind turbines in fully developed WTABL, extracted power = must be brought to wind turbine by vertical fluxes of kinetic energy:

$$\frac{d\,\frac{1}{2}\langle u\rangle_{xy}^2}{dt} = -\varepsilon_{turb} - \varepsilon_{canop} - \frac{d}{dz}\left(\langle\overline{u'w'}\rangle_{xy}\langle u\rangle_{xy} + \langle\overline{u}''\overline{w}''\rangle_{xy}\langle u\rangle_{xy}\right) - \langle u\rangle_{xy}\frac{1}{\rho}\frac{dp_\infty}{dx} - P_T(z)$$

Objectives:

- Measure profiles of horizontally averaged momentum fluxes & mean velocity for various wind turbine arrangements and conditions (neutral for now)

- Verify existence of double log-law structure, check Frandsen formula for $z_{0,hi}$, *develop improvements?*

- *Quantify fluxes of kinetic energy: correct magnitude? Turbulence or mean flow?*

- Challenges: requires statistics at entire 3-D volume surrounding a turbine elementary volume, including in transverse direction

- LES: fully developed, but contains many modeling assumptions

- Wind tunnel: more realistic, but not fully developed and exp. uncertainties

Large Eddy Simulation

- LES equations (formally filtered Navier-Stokes eqns. - here: neutral):

$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial}{\partial x_k}\tau_{ik} - \frac{1}{\rho}\frac{\partial \langle \tilde{p} \rangle}{\partial x_i} + f_{Ti}$$

Subgrid scale stress tensor

Imposed PG forcing of flow

Wind-turbine force model

- Closure problems:

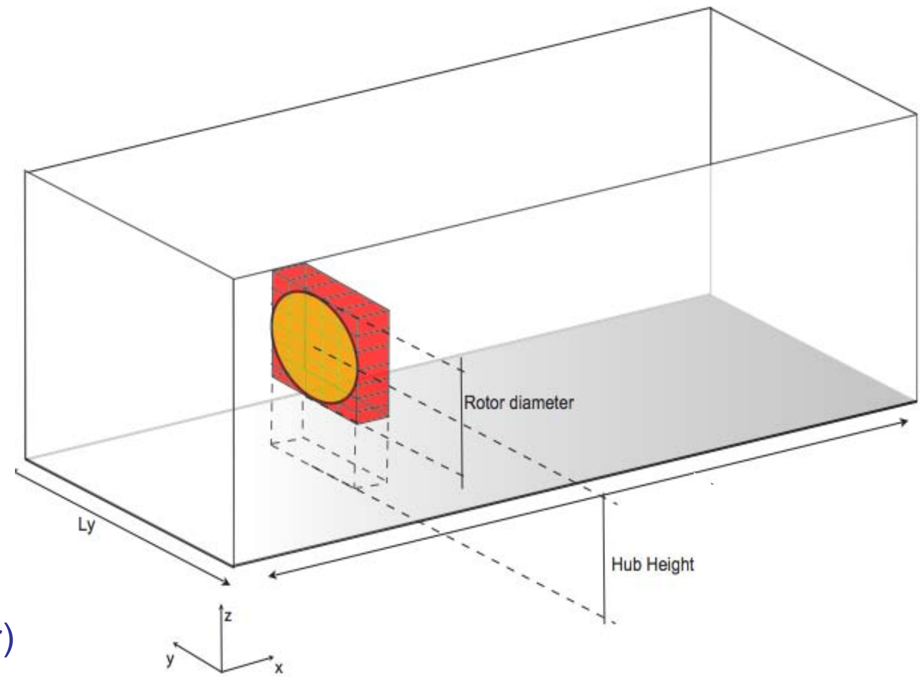
  - need a model for $\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$ (Lagrangian dynamic Smag. model)

  - need a model for wall stress τ_{wi} (log-law wall function)

  - need a model for f_{Ti}

Actuator disk modeling of turbines in LES

Jimenez et al., J. Phys. Conf. Ser. **75** (2007) simulated single turbine in LES using dynamic Smag. model



They used fixed reference (undisturbed) velocity:

$$f_{Tx} = -\frac{1}{2} C_T U_{ref}^2 \frac{\delta A_{yz}}{\delta V}, \quad C_T = 0.75$$

Here we use disk-averaged and time-averaged velocity, but local at the disk (see Meyers & Meneveau 2010, 48th AIAA conf., paper)

$$f_{Tx} = -\frac{1}{2} C_T \left(\frac{1}{1-a} \overline{U} \right)^2 \frac{\delta A_{yz}}{\delta V} = -\frac{1}{2} C_T' \overline{U}^2 \frac{\delta A_{yz}}{\delta V}$$

$$\overline{U}(t) = (1-\varepsilon)\overline{U}(t-dt) + \varepsilon U_{disk}(t)$$

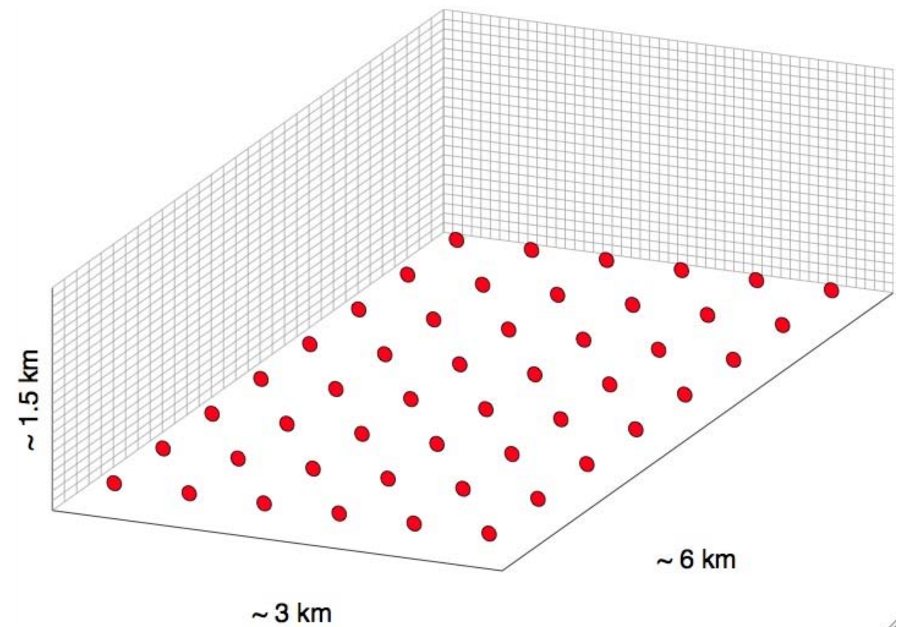
$$C_T = 0.75 \Rightarrow a \approx 0.25 \rightarrow C_T' = 1.33$$

Simulations setup:

• Code: horizontal pseudo-spectral (periodic B.C.), vertical: centered 2nd order FD (Moeng 1984, Albertson & Parlange 1999, Porté-Agel et al. 2000, Bou-Zeid et al. 2005)

$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$

$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128$$



• Top surface: zero stress, zero w

• Bottom surface B.C.: Zero w +
Wall stress: Standard wall function relating wall stress to first grid-point velocity

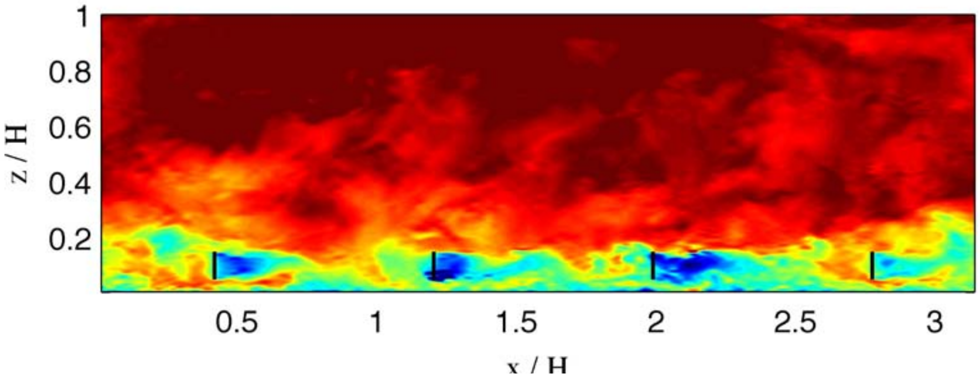
• Scale-dependent dynamic Lagrangian model (*no* adjustable parameters)

• Calaf, Meneveau & Meyers, (Phys Fluids 2010, **22**)

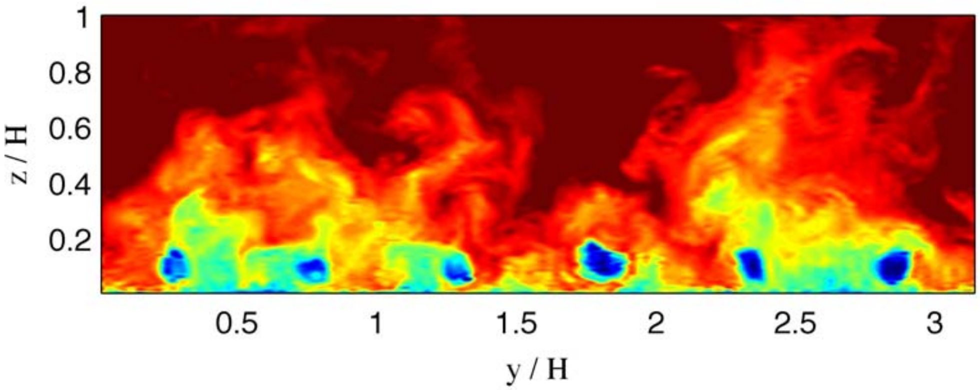
Simulations results:

Instantaneous stream-wise velocity contours:

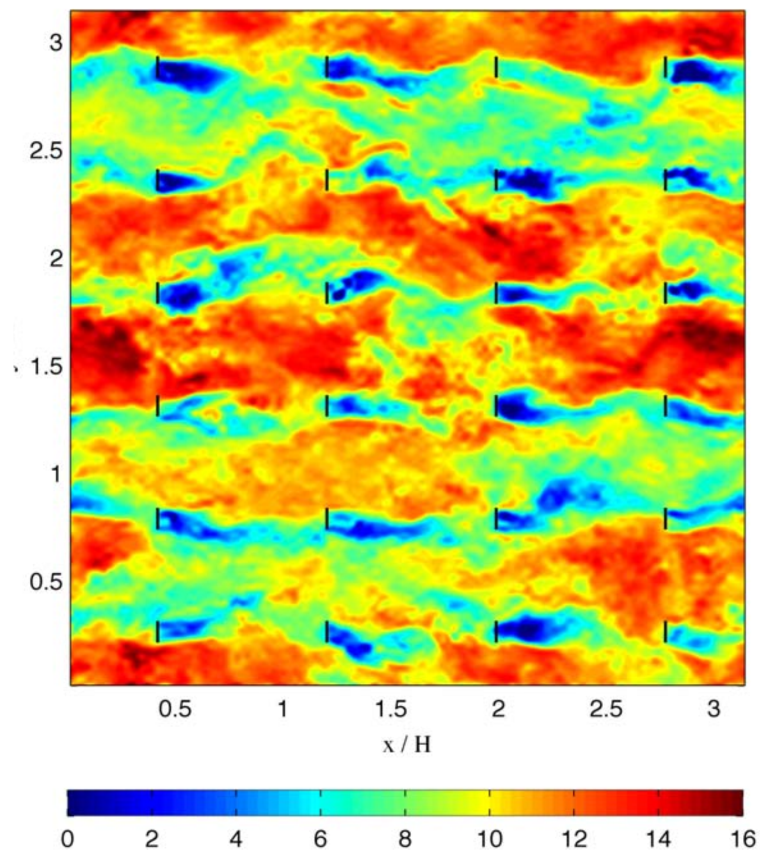
side-view



front-view

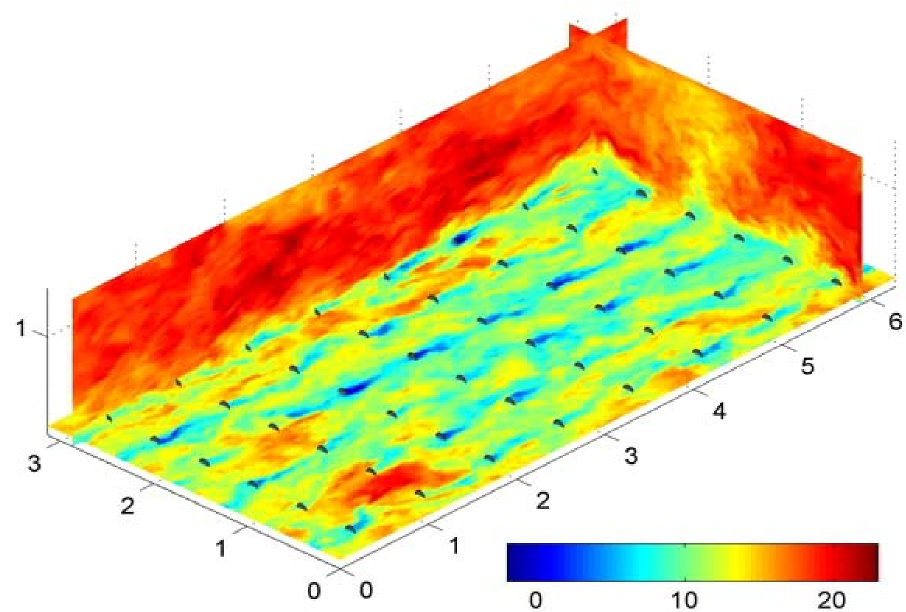


top-view



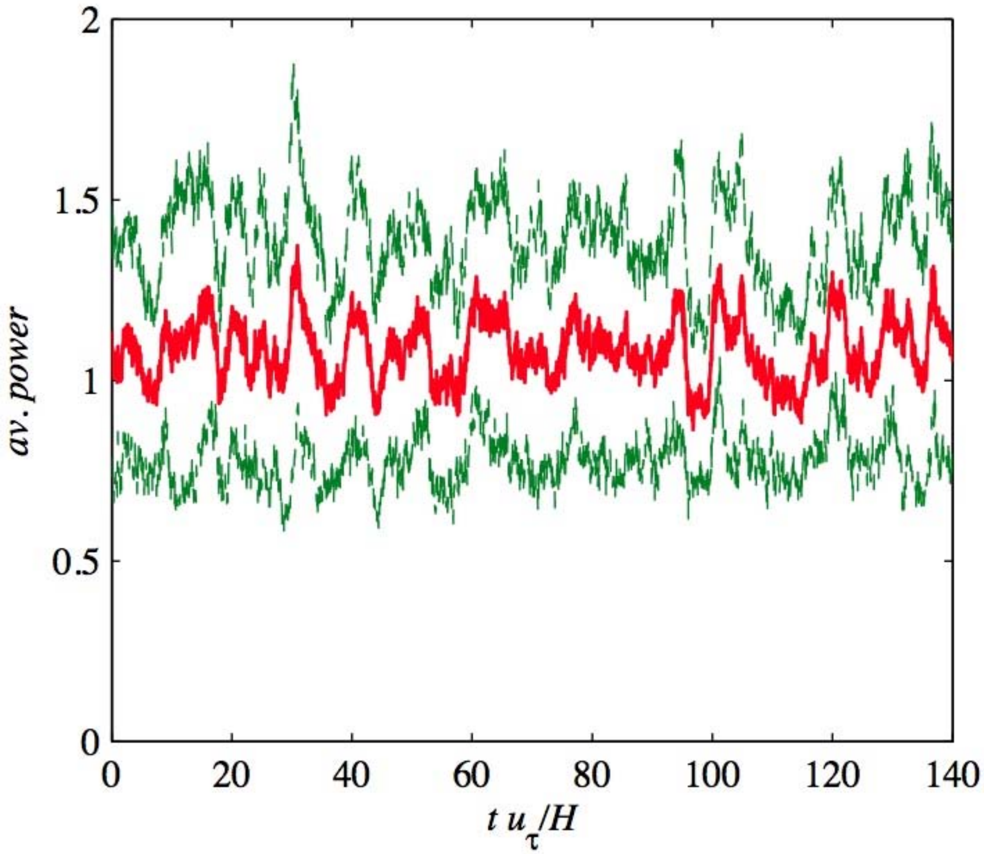
Simulations results:

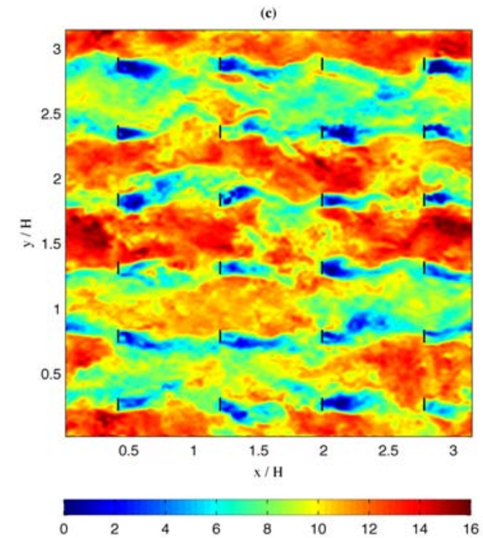
Instantaneous stream-wise velocity contours:



Simulations results:

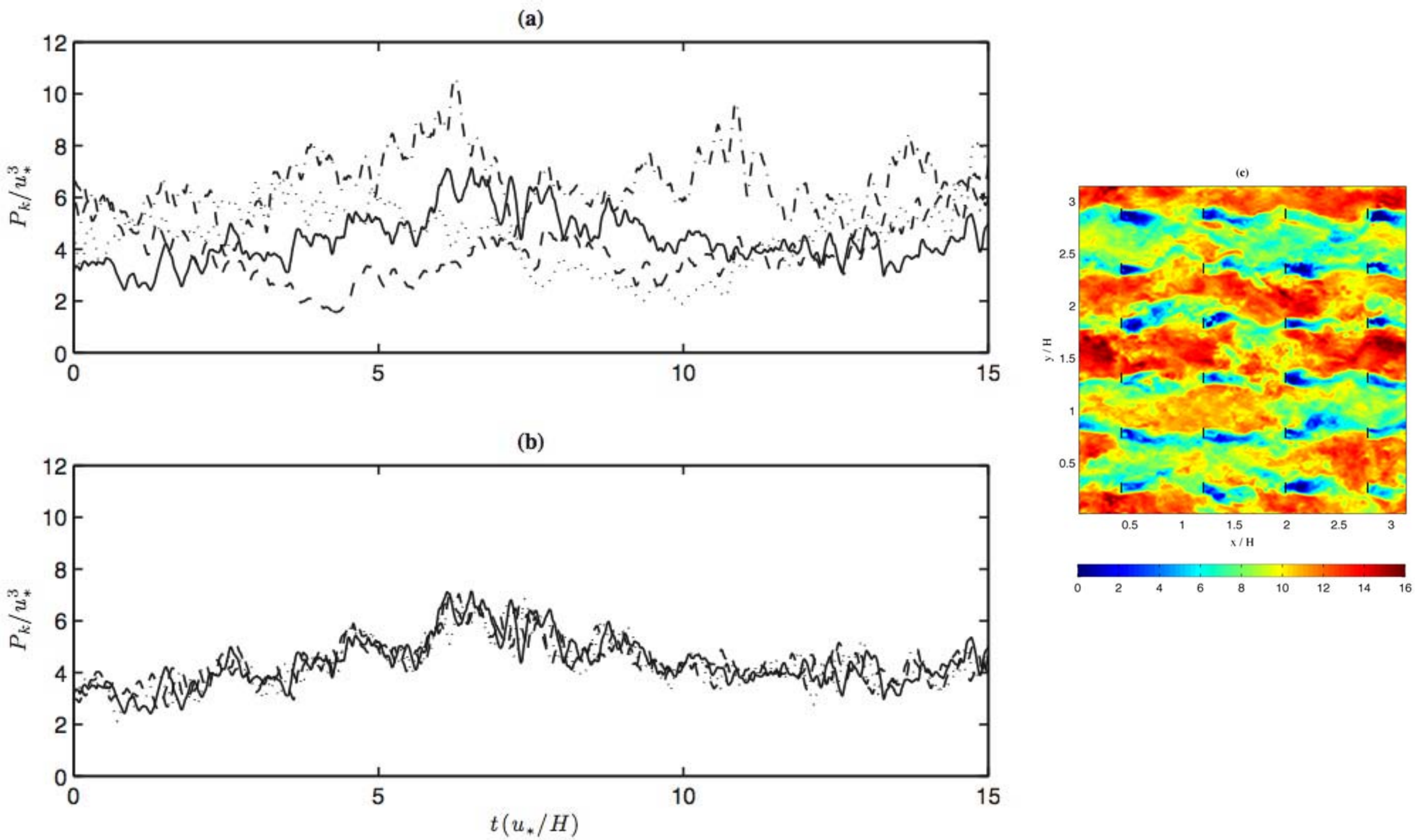
Instantaneous power output (mean over all WT and ±rms):





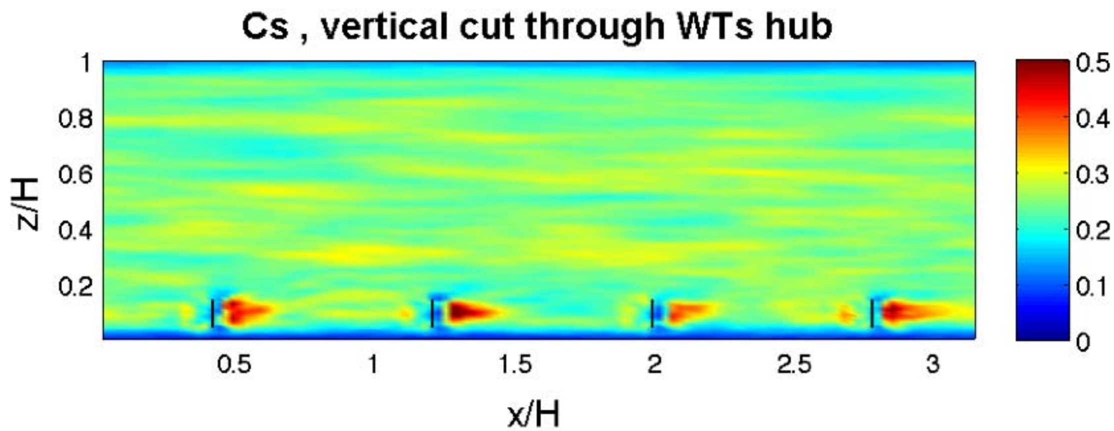
Simulations results:

Instantaneous power output (along rows and columns):



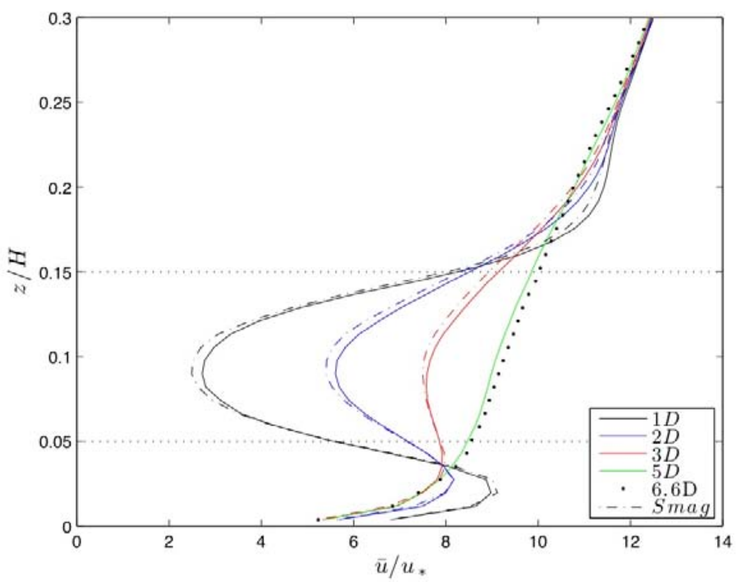
Simulations results:

Dynamic Smagorinsky coefficient: increases in wake region, while decreases near wall



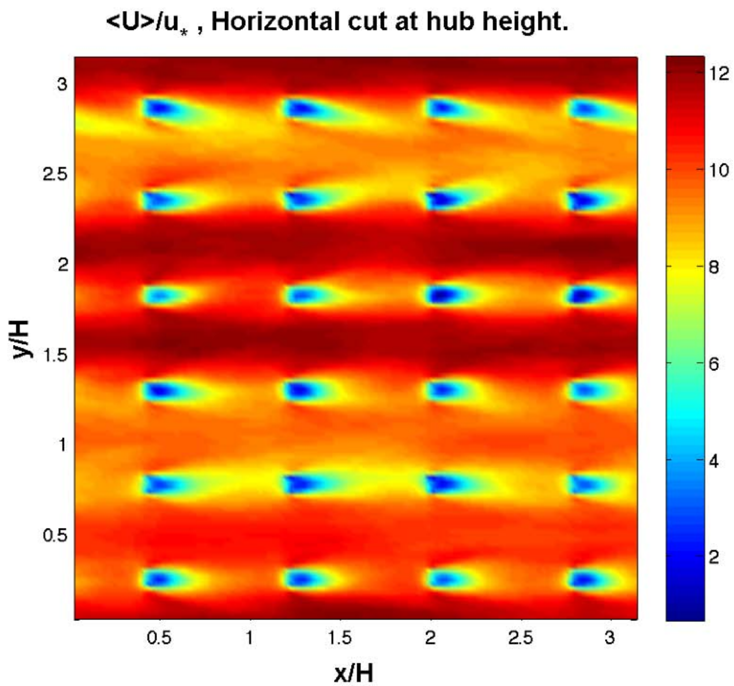
$$\left\langle c_s^2 \right\rangle^{1/2}$$

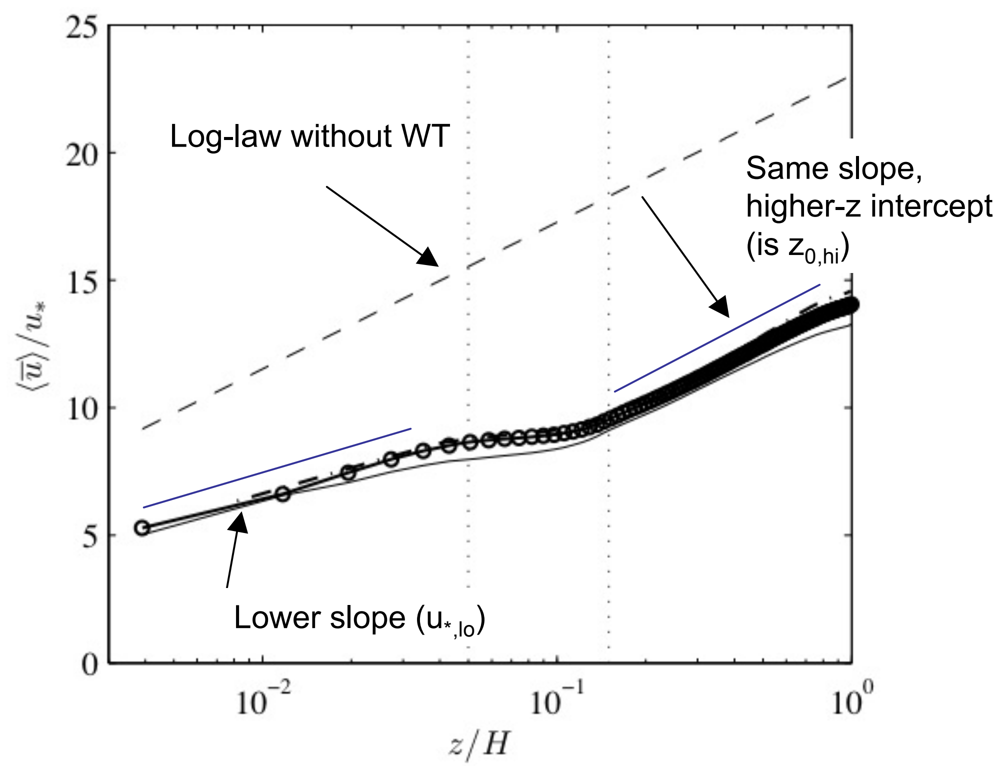
Comparison of wake profiles, regular Smagorinsy (with wall damping) and dynamic model:



Simulations results: horizontally averaged profiles

Mean velocity profile: $\langle \overline{u} \rangle_{xy}$

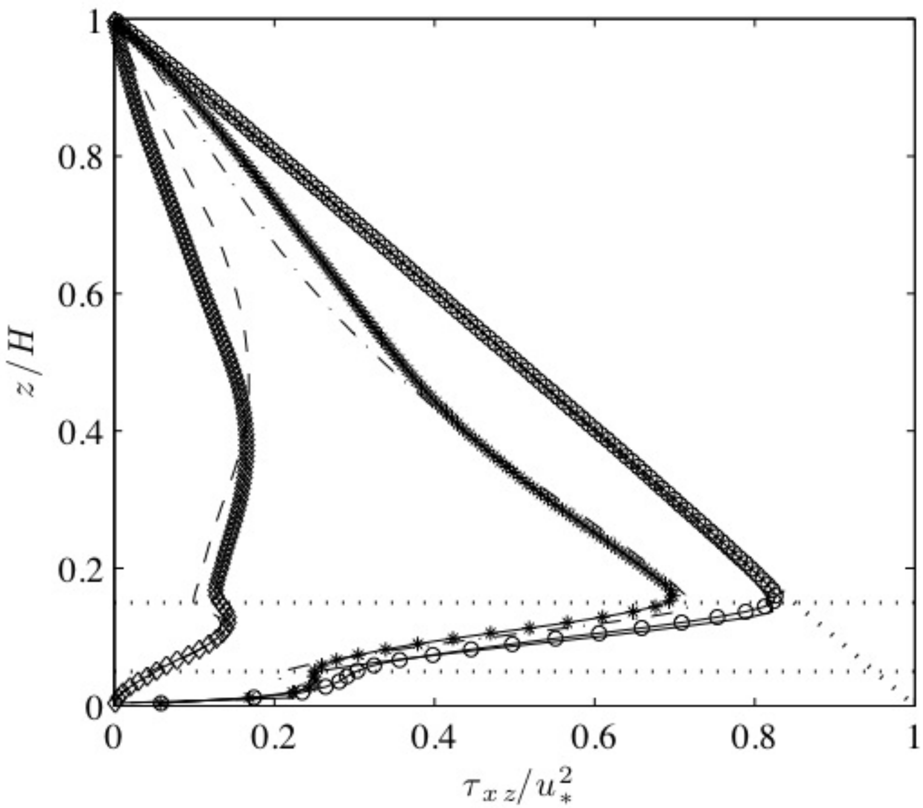




Simulations results: horizontally averaged profiles

Mean Reynolds stress profile: $-\langle u'w' \rangle_{xy}$

Spatial “canopy stress” profiles, $-\langle \overline{u}''\overline{w}'' \rangle_{xy}$



Conclusions: Momentum fluxes carried by cross-stream heterogeneity are significant

Suite of LES cases:

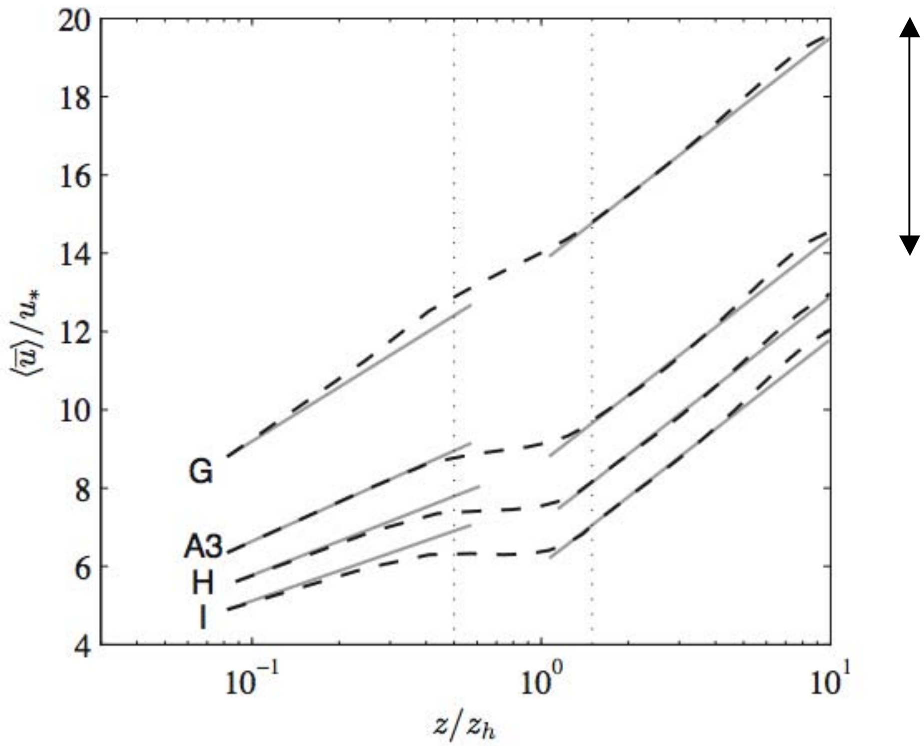
Calaf, Meneveau, and Meyers

Phys. Fluids **22**, 1 (2010)

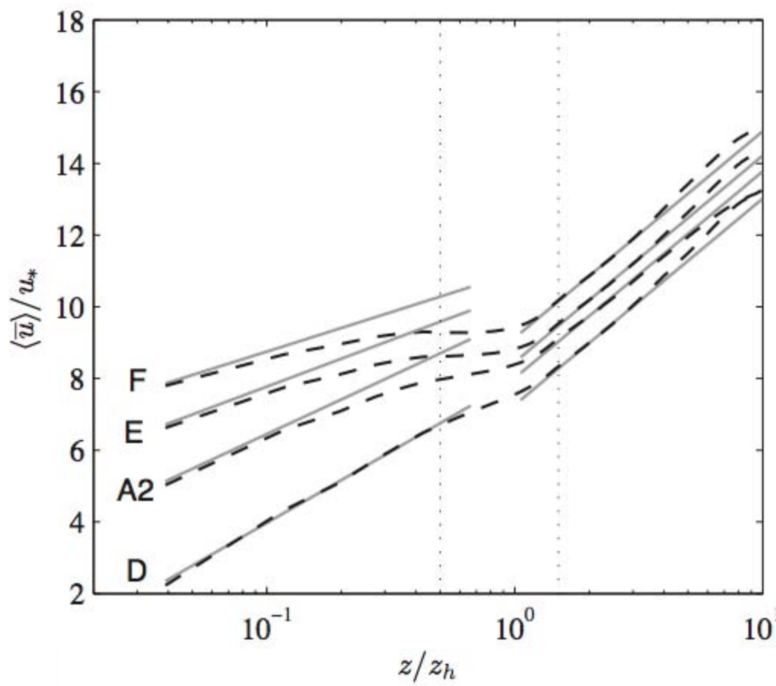
TABLE I. Summarizing parameters of the various LES cases. Between brackets is indicated which code is used: "L" refers to the KULeuven code and "J" refers to the JHU-LES code.

	s_x/s_y	s_x	$4s_xs_y/\pi$	N_t	$L_x\times L_y\times H$	$N_x\times N_y\times N_z$	$z_{0,\text{lo}}$	C_T'	c_{ft}'
A1 (L)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	1.33	0.025
A2 (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	1.33	0.025
A3 (L)	1.5	7.85	52.36	8×6	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.025
A4 (L)	1.5	7.85	52.36	8×6	$2\pi\times\pi\times1.5$	$128\times192\times92$	10^{-4}	1.33	0.025
B (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	2.00	0.038
C (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	0.60	0.012
D (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-3}	1.33	0.025
E (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-5}	1.33	0.025
F (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-6}	1.33	0.025
G (L)	1.5	15.7	209.4	4×3	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.0064
H (L)	1.5	6.28	33.51	10×8	$2\pi\times1.07\pi\times1$	$128\times192\times57$	10^{-4}	1.33	0.040
I (L)	1.5	5.24	23.27	12×9	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.057
J (L)	2	9.07	52.36	7×7	$2.02\pi\times1.01\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.025
K (L)	1	6.41	52.36	10×5	$2.04\pi\times1.02\pi\times1$	$128\times192\times60$	10^{-4}	1.33	0.025

Suite of LES cases:



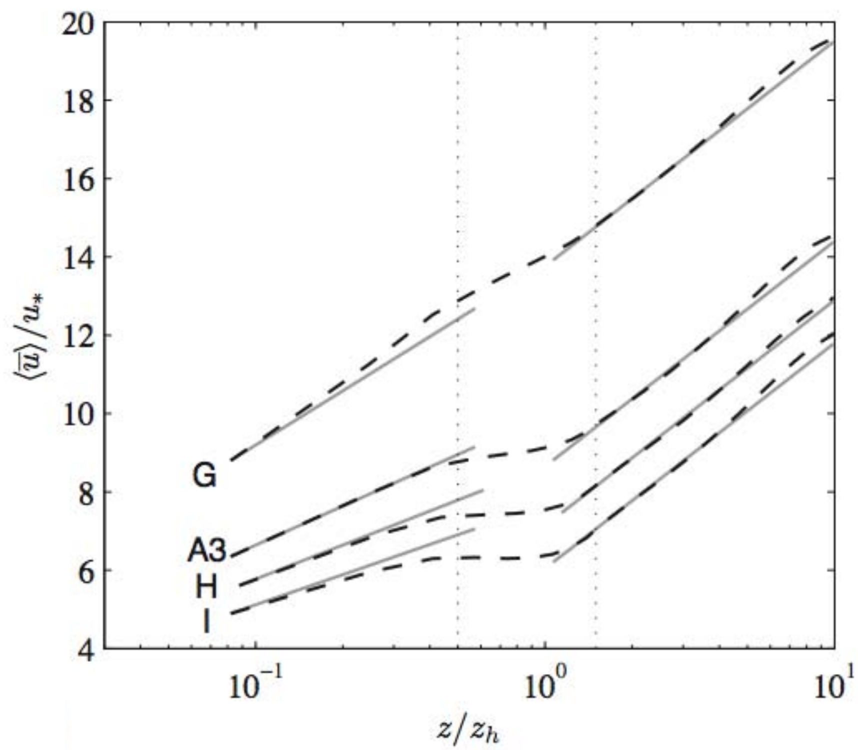
measure $z_{0,hi}$ from intercept



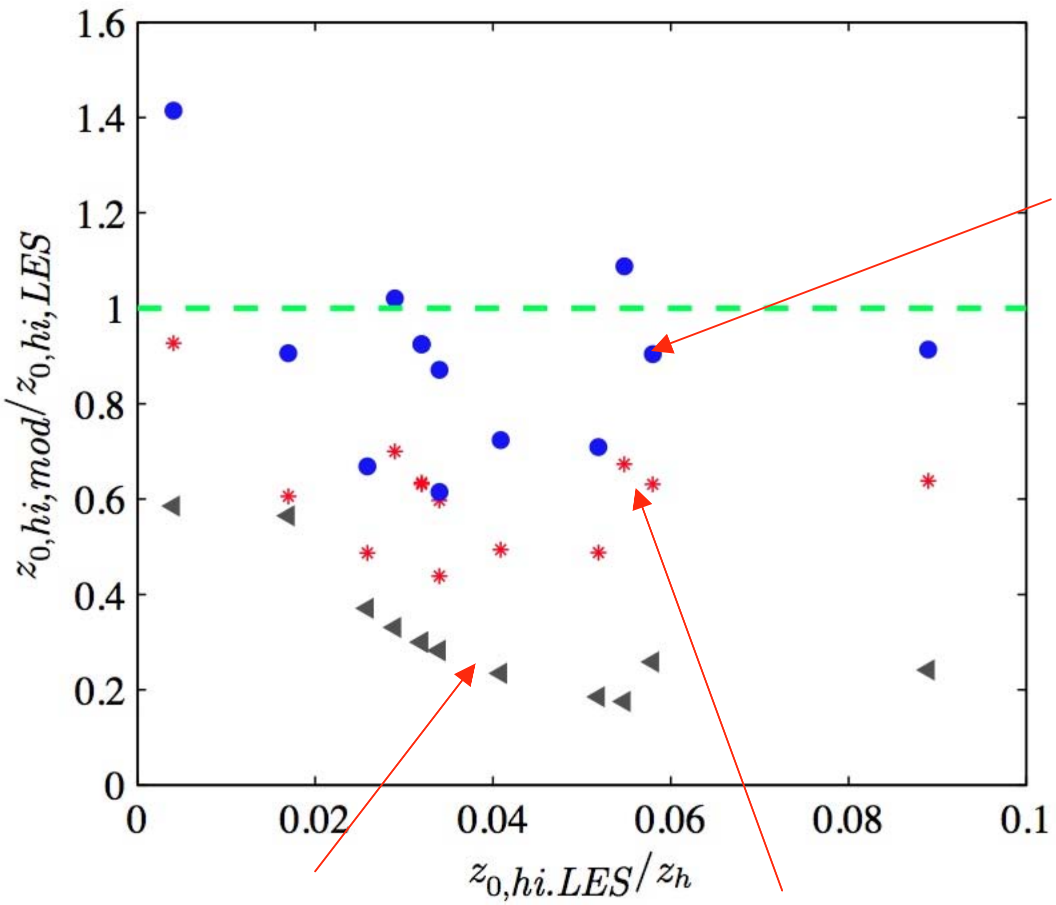
Improvement to Frandsen's model:

$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(- \left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}} \left(1 - \frac{D}{2z_h}\right)^{\beta} \right] \right)^{-2} \right]^{-1/2} \right)$$

where $\beta = \dfrac{\nu_w^*}{1+\nu_w^*}$, and $\nu_w^* = \dfrac{\nu_T}{\kappa u_* z_h}$, eddy viscosity due to wake



Comparison of LES results with models:



Circles: improved Frandsen model
Calaf, Meneveau & Meyers,
(Phys. Fluids 2010, **22**)

$$z_{0,hi} = z_h \left(1+\frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

where $\beta = \frac{\nu_w^*}{1+\nu_w^*}$, and $\nu_w^* = \frac{\nu_T}{\kappa u_* z_h}$, eddy viscosity due to wake

Triangles: Lettau formula

Asterisks: Frandsen (2006) formula

$$z_{0,hi} = z_h \exp\left(-\kappa\left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})}\right)^2\right]^{-1/2}\right)$$

Using the roughness model for array optimization - find s-opt:

What is the optimal averaged turbine spacing s?

Normalized farm power per unit farm area, and geostrophic wind

$$P^{+} = \frac{P}{S\rho G^3/2} = \frac{\frac{1}{2}C'_T \rho U_d^3 A}{S\rho G^3/2} = \frac{\pi C'_T}{4s}\left(\frac{u_{*hi}}{G}\right)^3\left(\frac{U_d}{u_{*hi}}\right)^3$$

Normalized farm power per unit cost?

$$cost\,[\$] = cost_L\,[\$/m^2]\times S + cost_T\,[\$] \;\rightarrow\; \text{Define: } \alpha = \frac{cost_T/A}{cost_L}$$

Then

$$P^*(s, C'_T, \alpha) = P^{+}\frac{cost_L}{cost_T/S + cost_L} = \frac{C'_T}{4s/\pi + \alpha}\left(\frac{u_{*hi}}{G}\right)^3\left(\frac{U_d}{u_{*hi}}\right)^3$$

Using the roughness model for array optimization - find s-opt:

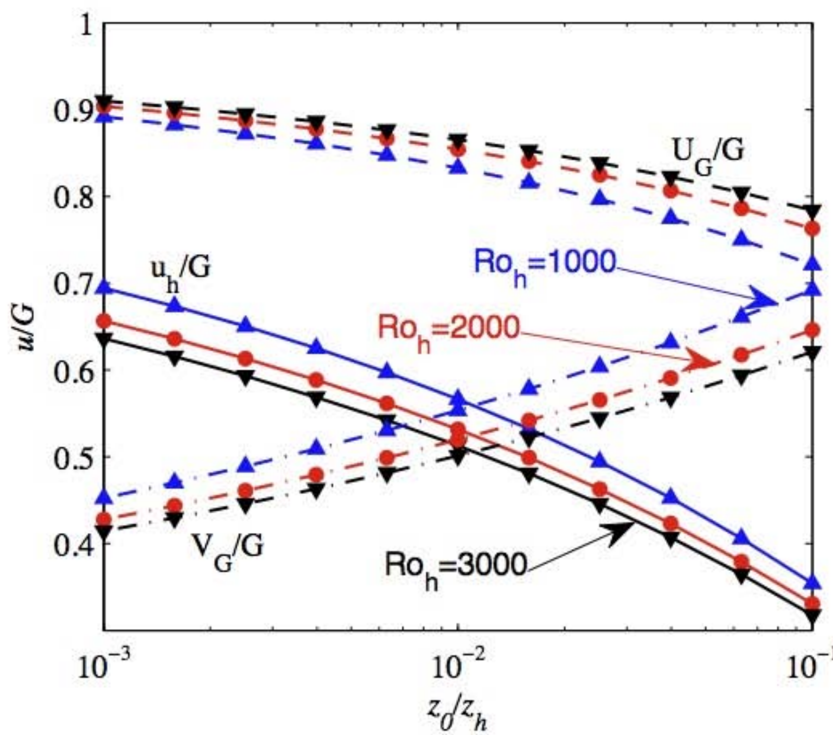
u_*/G ?? → use standard ABL relations

see, e.g., Tennekes & Lumley, 1972

$$\frac{G}{u_*} = \left(A^2 + \left[\frac{1}{\kappa}\ln\left(\frac{u_*}{G}Ro\right) - C\right]^2\right)^{1}$$

with

$$Ro = \frac{G}{fz_h}\frac{z_h}{z_0} = Ro_h\frac{z_h}{z_0}$$

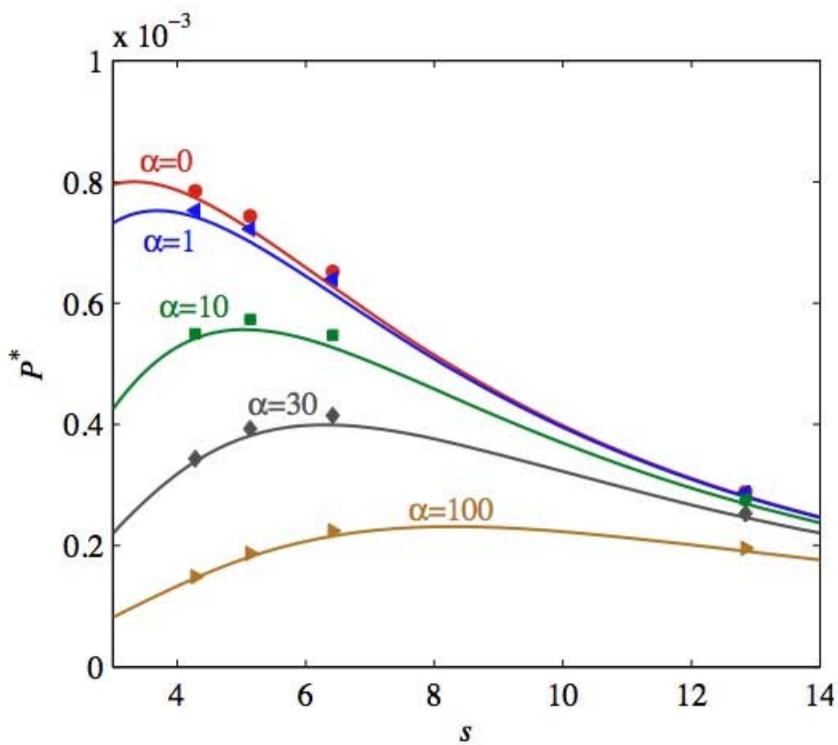


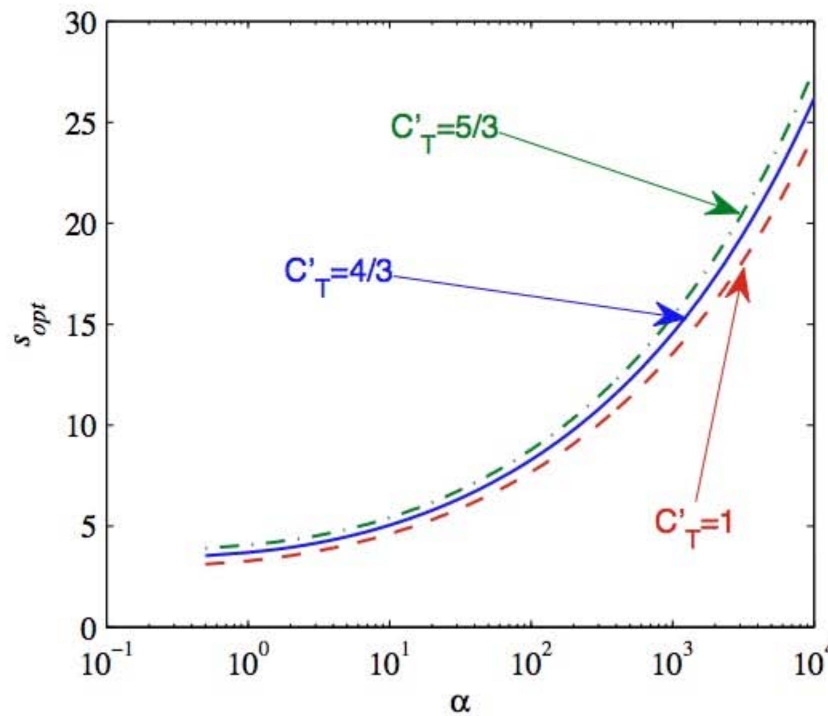
Using the roughness model for array optimization - find s-opt:

Optimization of P^*

Take $Ro_h = 2000$, $z_{0,lo}/z_h = 10^{-3}$

$P^*(s, C'_T, \alpha)$ $\qquad$ $s_{opt} = \max_{s>0} P^*(s, C'_T, \alpha)$



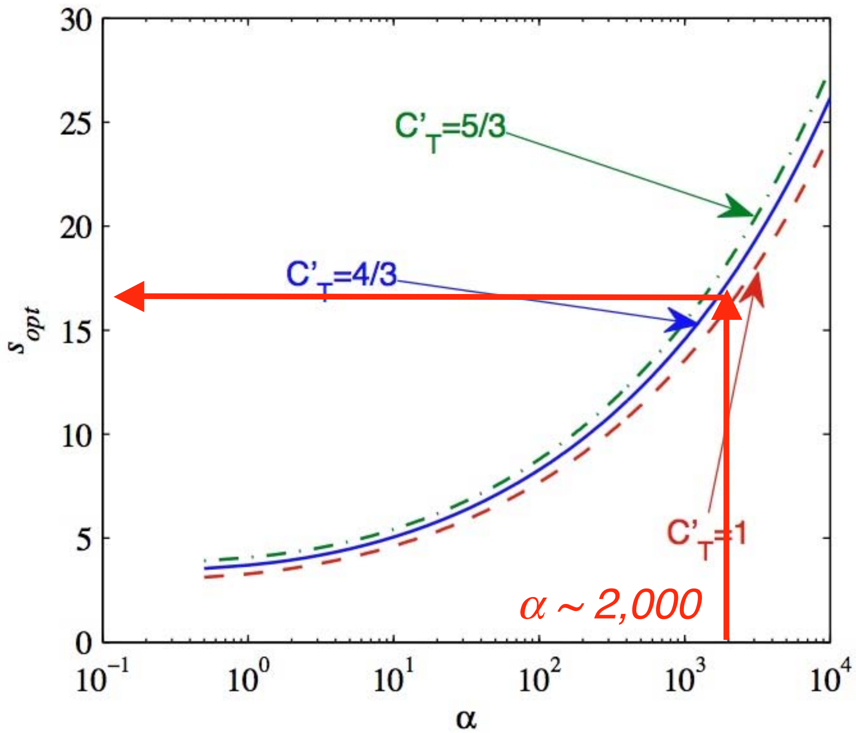


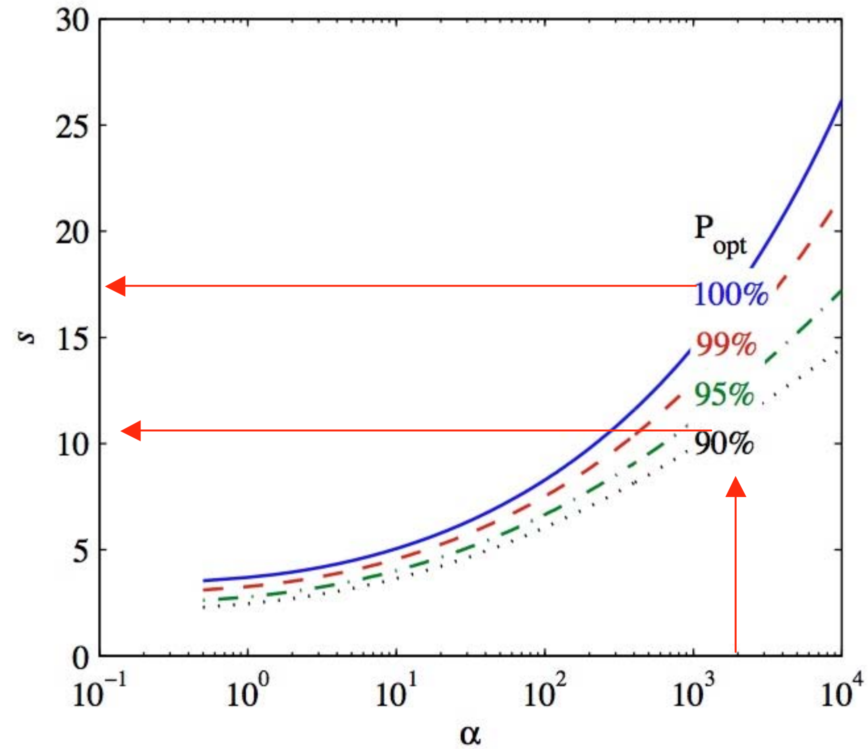
Using the roughness model for array optimization - find s-opt:

- Lease of land, yearly payout per wind turbine is around US$ 5,000 for present typical spacings of 500x500m (e.g. *http://www.windustry.org/how-much-do-farmers-get-paid- to-host-wind-turbines*).
- So over a 20-year lease: $p_L \approx 0.4$ US$/$m^2$.
- Land purchase e.g. in Texas ~ US$ 1,000 per acre (e.g. *http://recenter.tamu.edu/data/agp*), or $cost_L \approx 0.25$ US$/$m^2$
- Representative cost of a wind-turbine US$3.5x$10^6$ for a 2MW-rated wind turbine. With D= 70m: $cost_T/A \approx 700$ US$/$m^2$ (e.g. *http://www.windustry.org/how-much-do-wind-turbines-cost)*.
- The corresponding parameter α is then roughly

$$\alpha = (cost_T/A)/cost_L \sim 1{,}700 \text{ to } 3{,}000$$

Using the roughness model for array optimization - find s-opt:

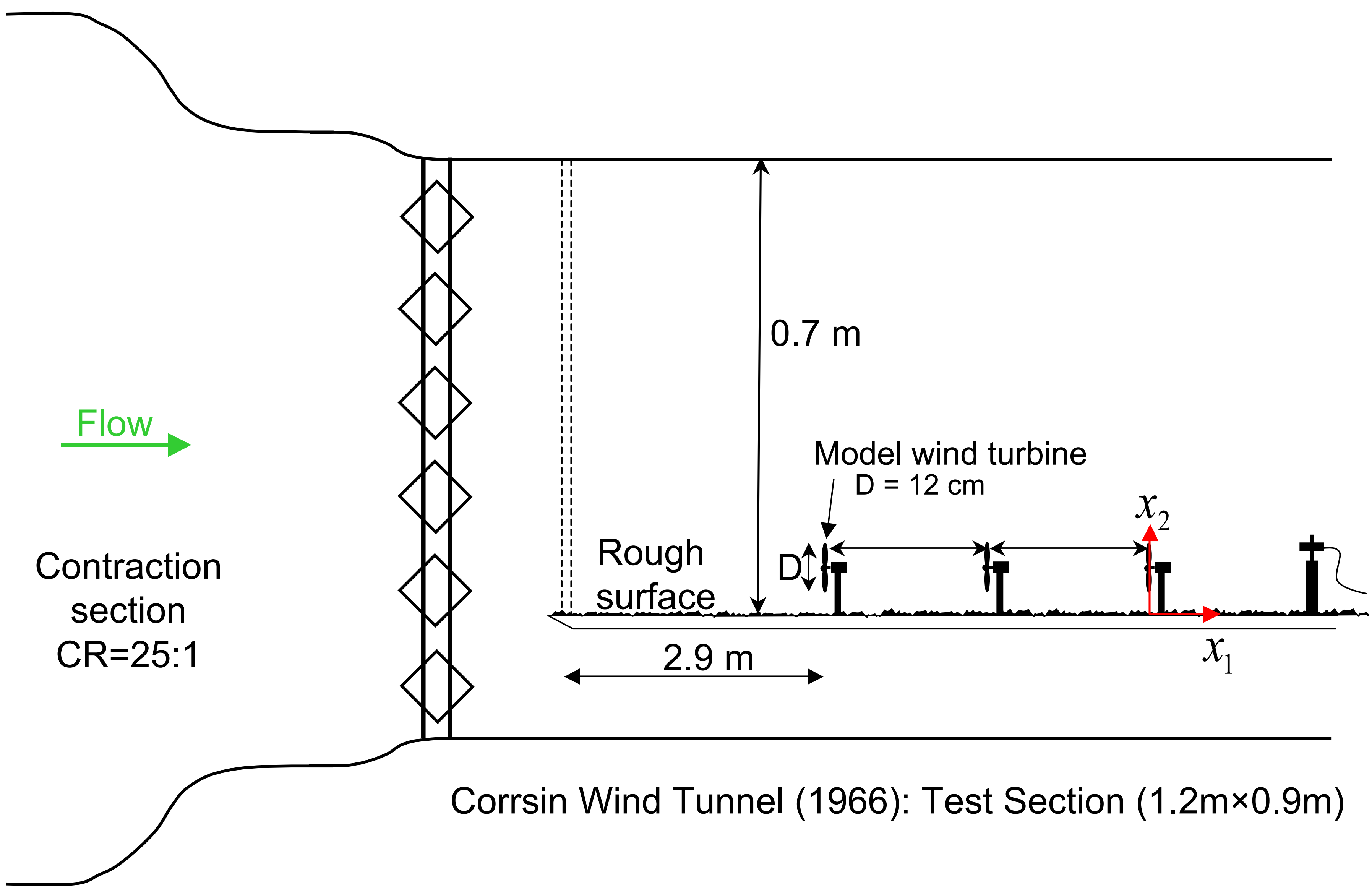




At common s ~ 7D, 10-20% suboptimal
possible reason for "array underperformance" ?

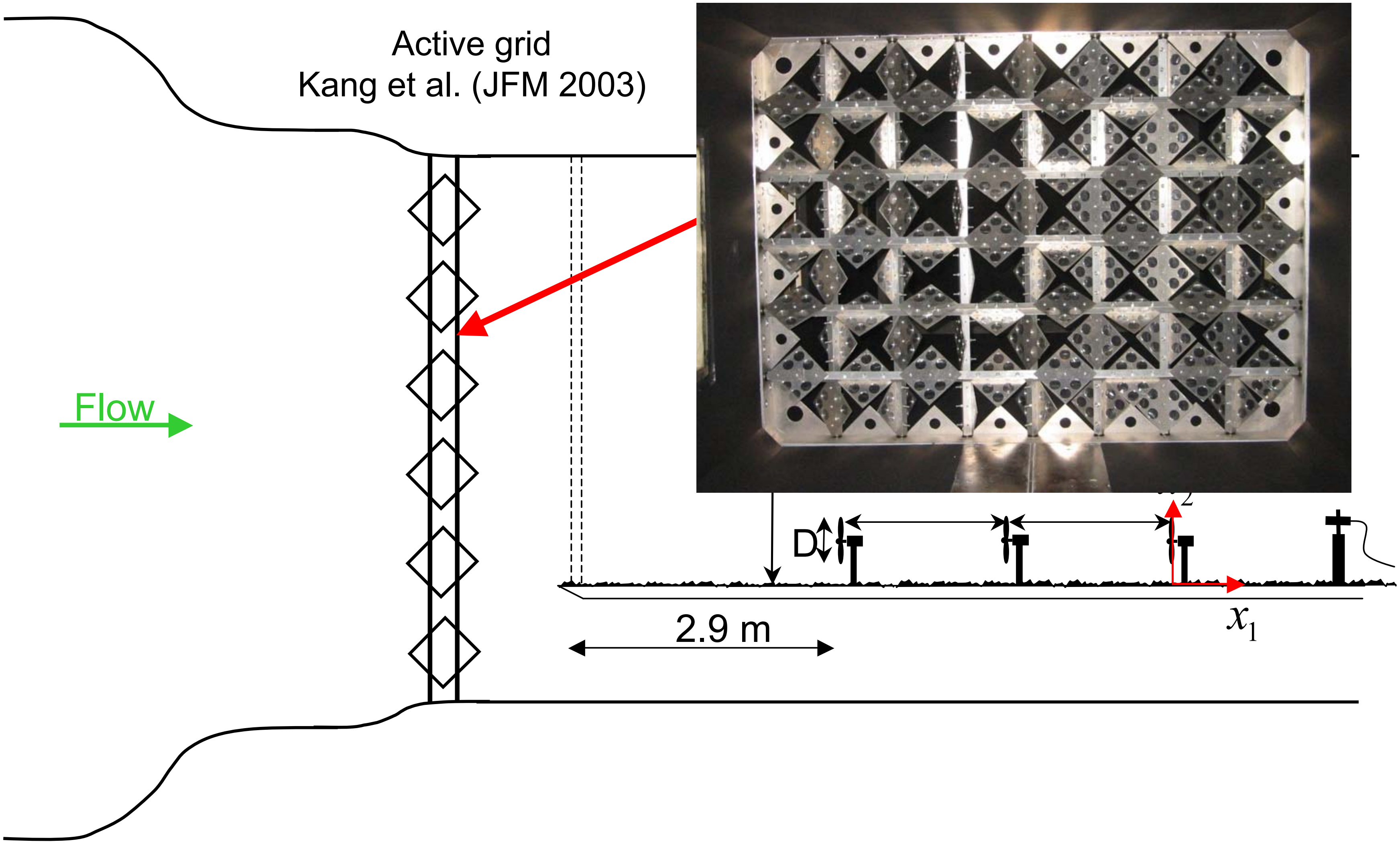
Meyers & Meneveau, 2010
(preprint, submitted to Wind Energy)

Wind-tunnel measurements: mechanics of vertical KE entrainment??

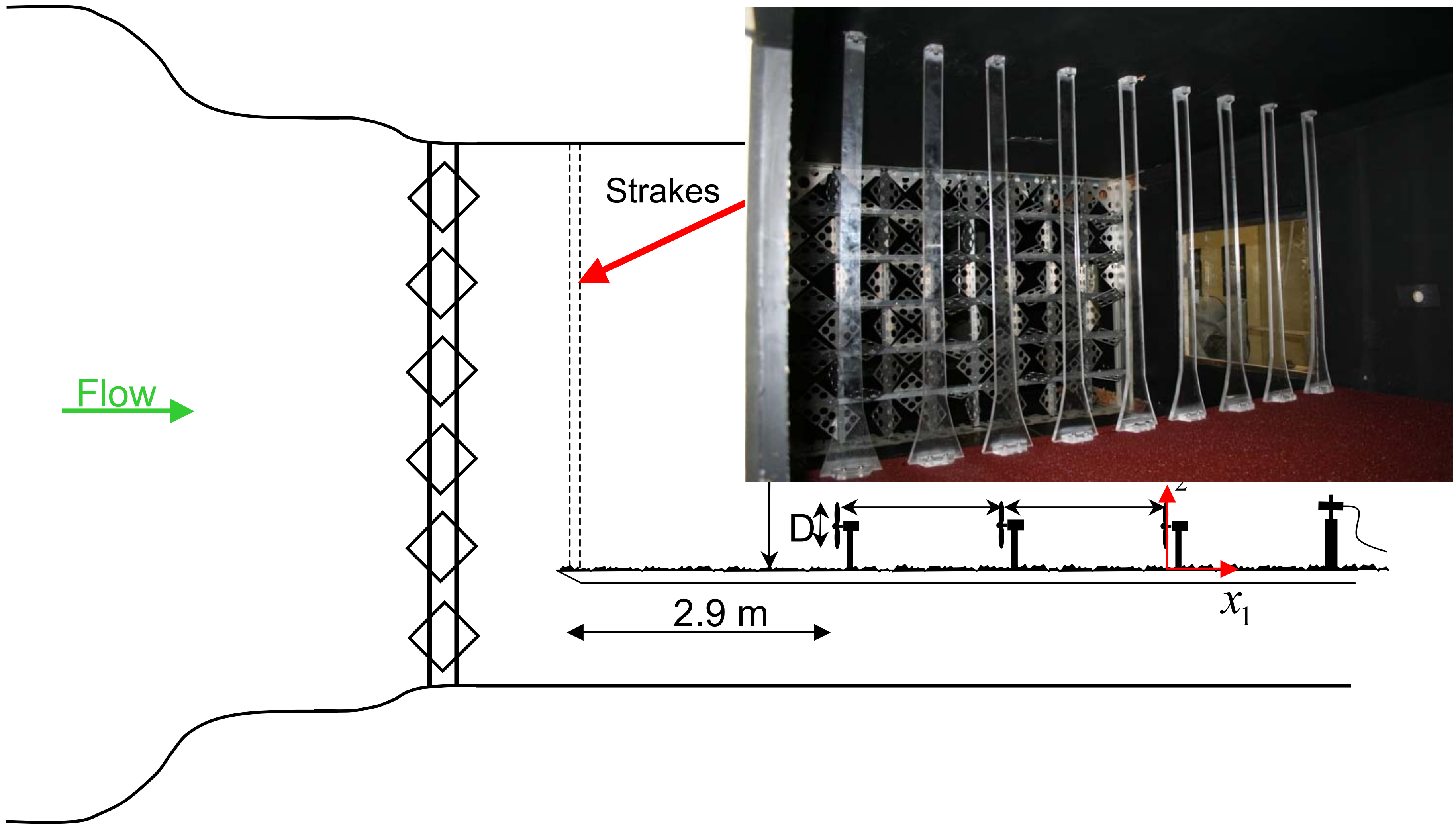


Corrsin Wind Tunnel (1966): Test Section (1.2m×0.9m)

Wind-tunnel measurements: mechanics of vertical KE entrainment??

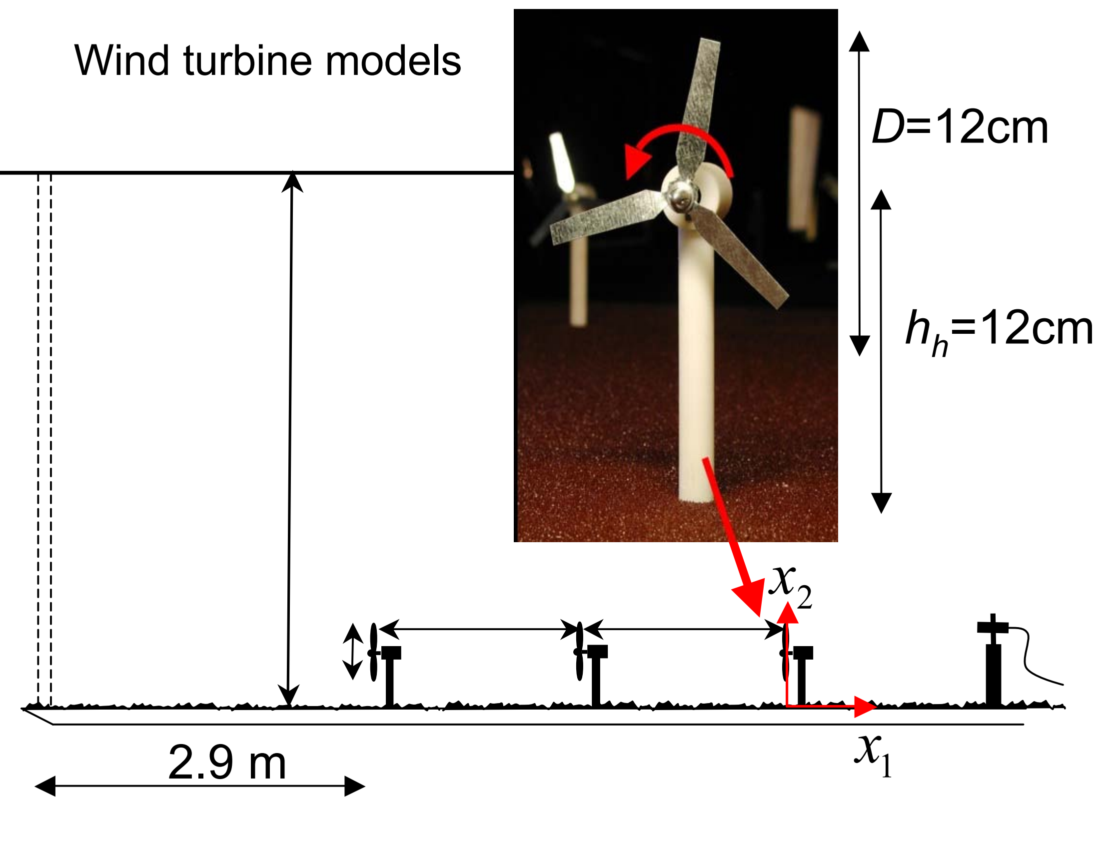


Wind-tunnel measurements

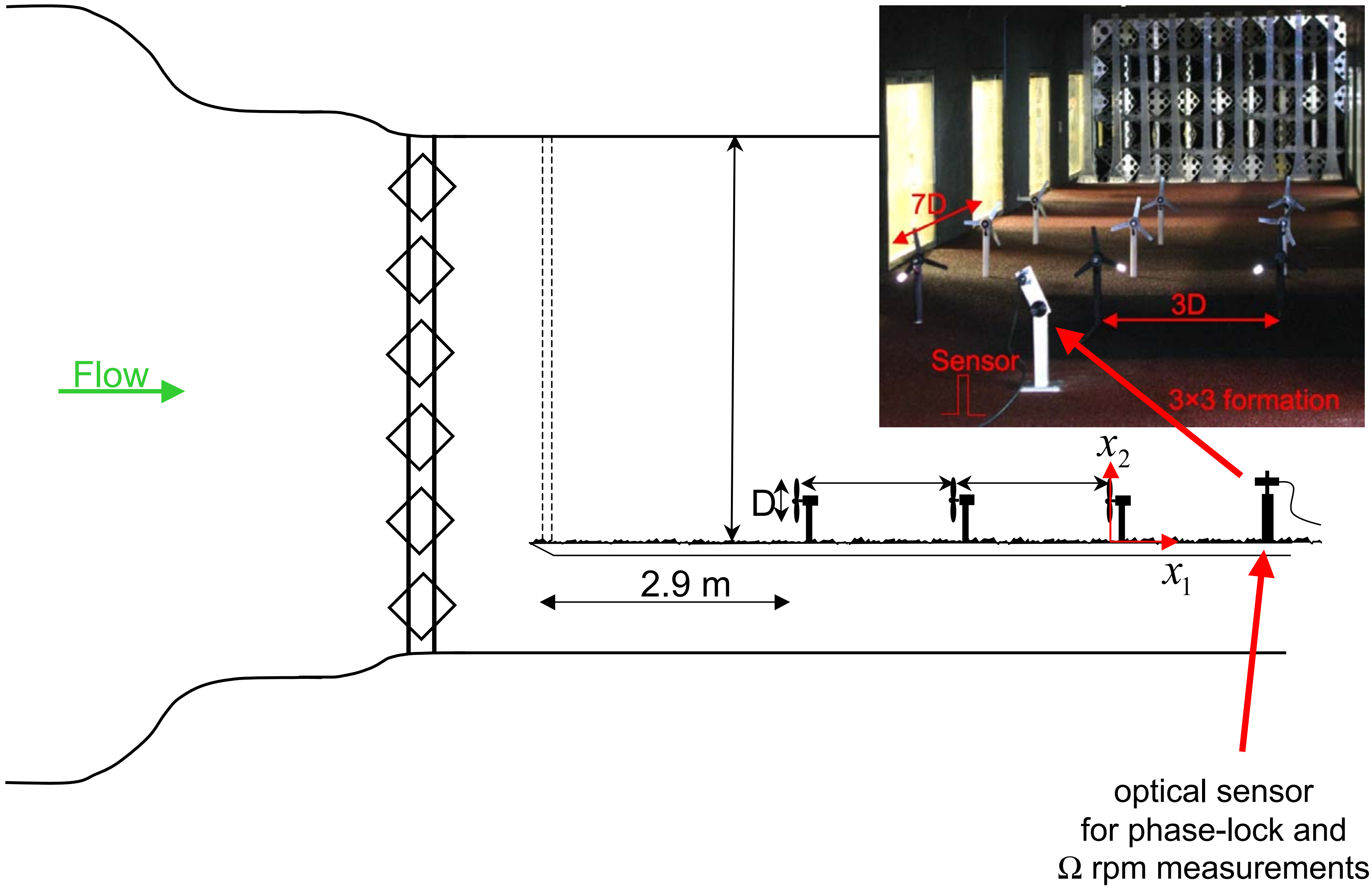


Wind-tunnel measurements

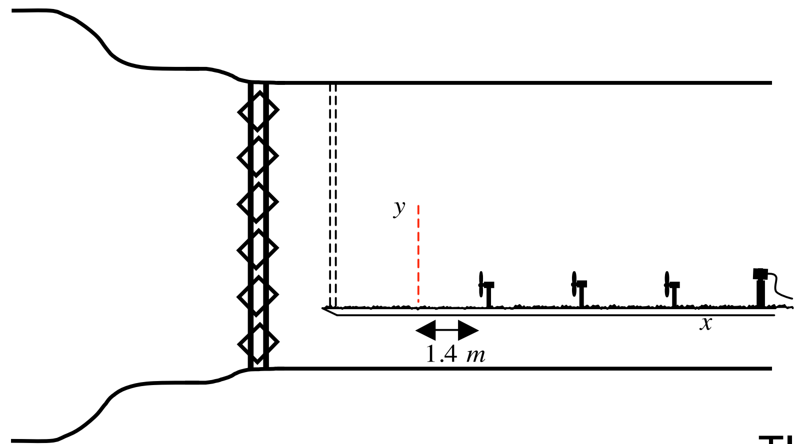
- **Turbine Models**
  - –Scaled down 850 times from typical real life length scales (real diameters of 100 m scaled down to 12 cm).
- **Rotors**
  - –Made from G28 galvanized sheet metal
  - –Twisted 1.1 degrees per cm, from 15º at the root to 10º at the tip
  - –Tip speed ratio, $\lambda = V_{tip}/U_{hub}$ is 5
  - –Rotate at 4800 RPM
- **Tower**
  - – Height of 12 cm
  - – Constructed using rapid prototyping

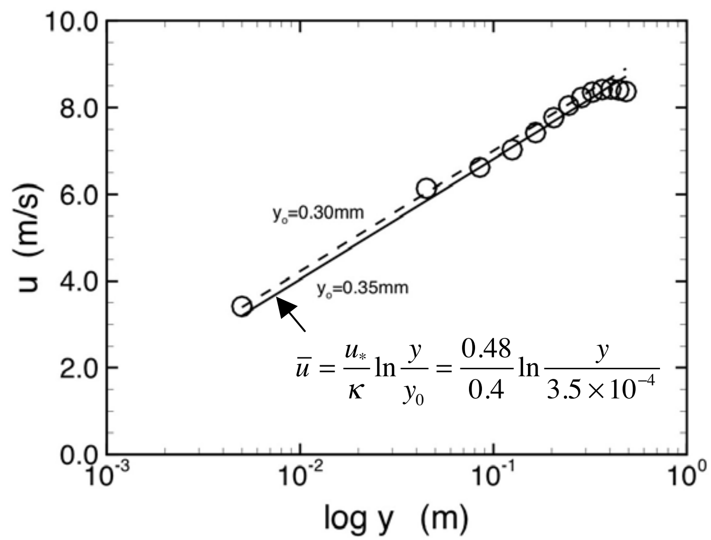


Wind-tunnel measurements

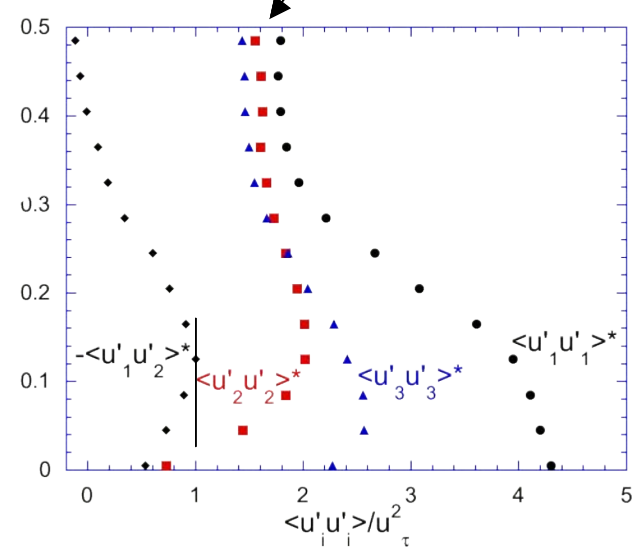


Hot-wire inlet profiles





$TI_\infty \sim 6\%$

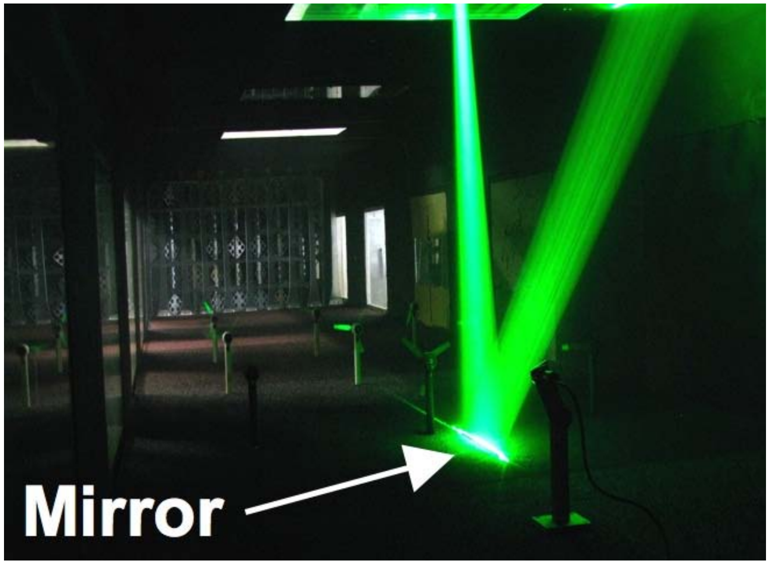


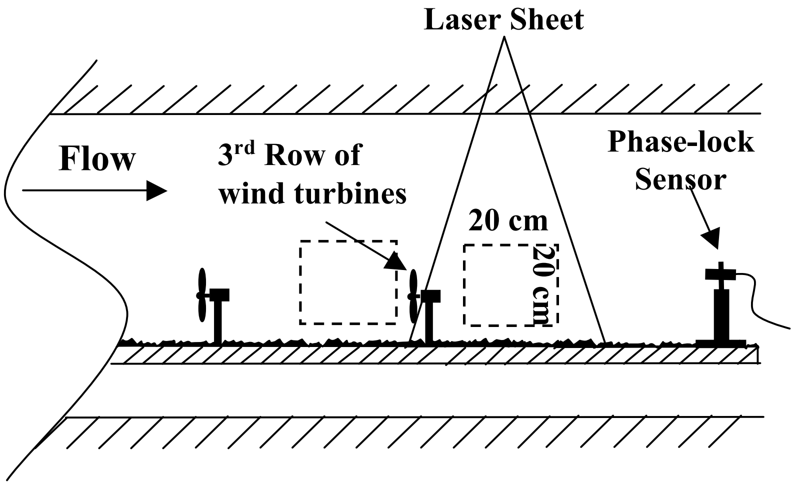
$u_* = 0.48$ m/s

Stereo-PIV system

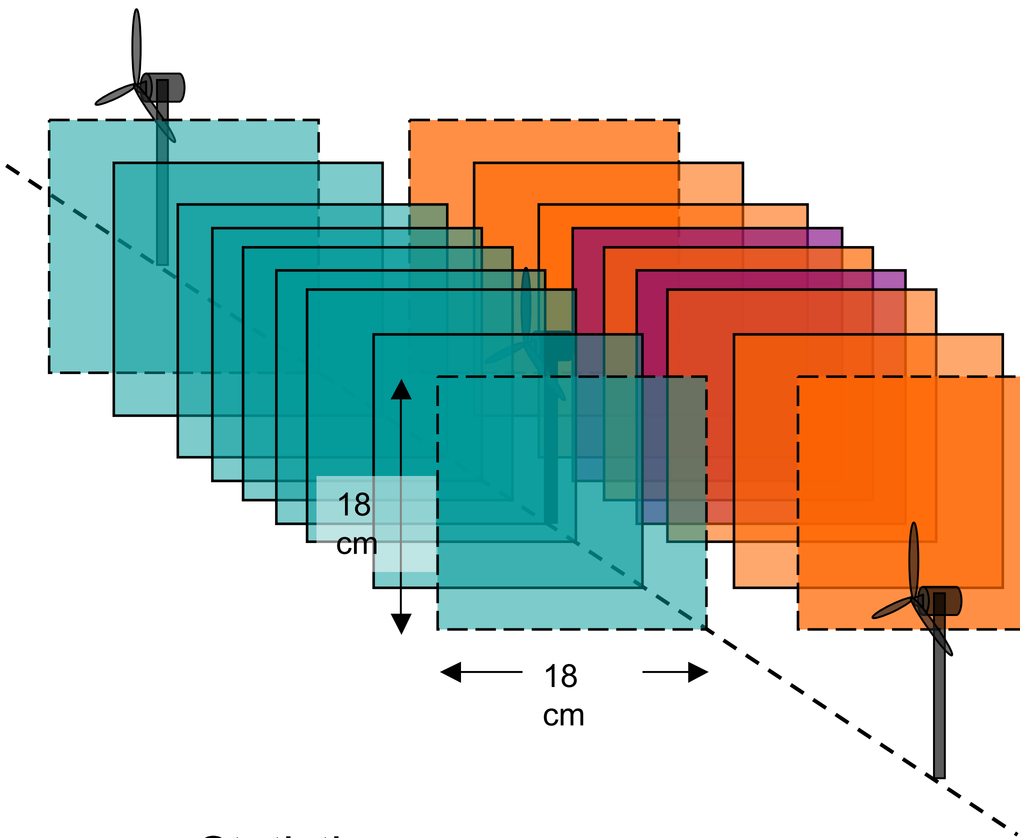
TSI System with:

- Double pulse Nd:YAG laser(120 mJ/pulse)
  - Laser sheet thickness of 1.2 mm
  - Time between pulses of 50 ms
  - Optical sensor external trigger for phase lock measurements
- Two high resolution cross/auto correlation digital CCD cameras with
  - a frame rate of 16 frames/sec.
  - Interrogation area of 20 cm by 20 cm

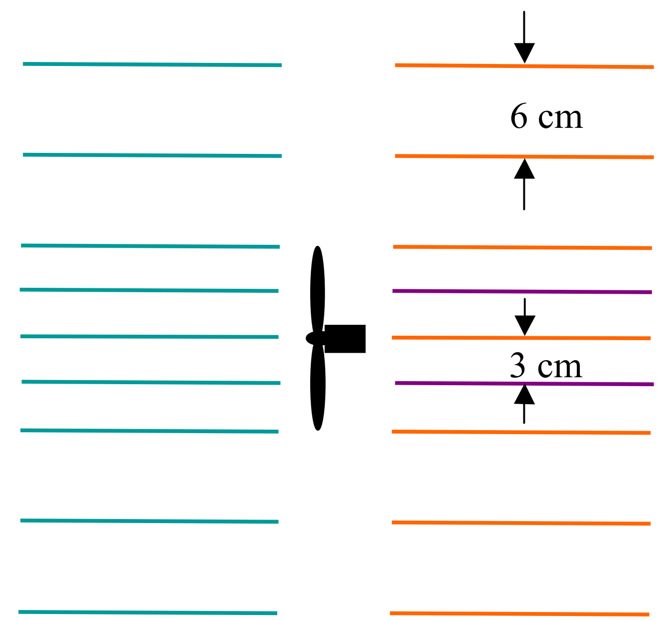




PIV data planes:



Top view:



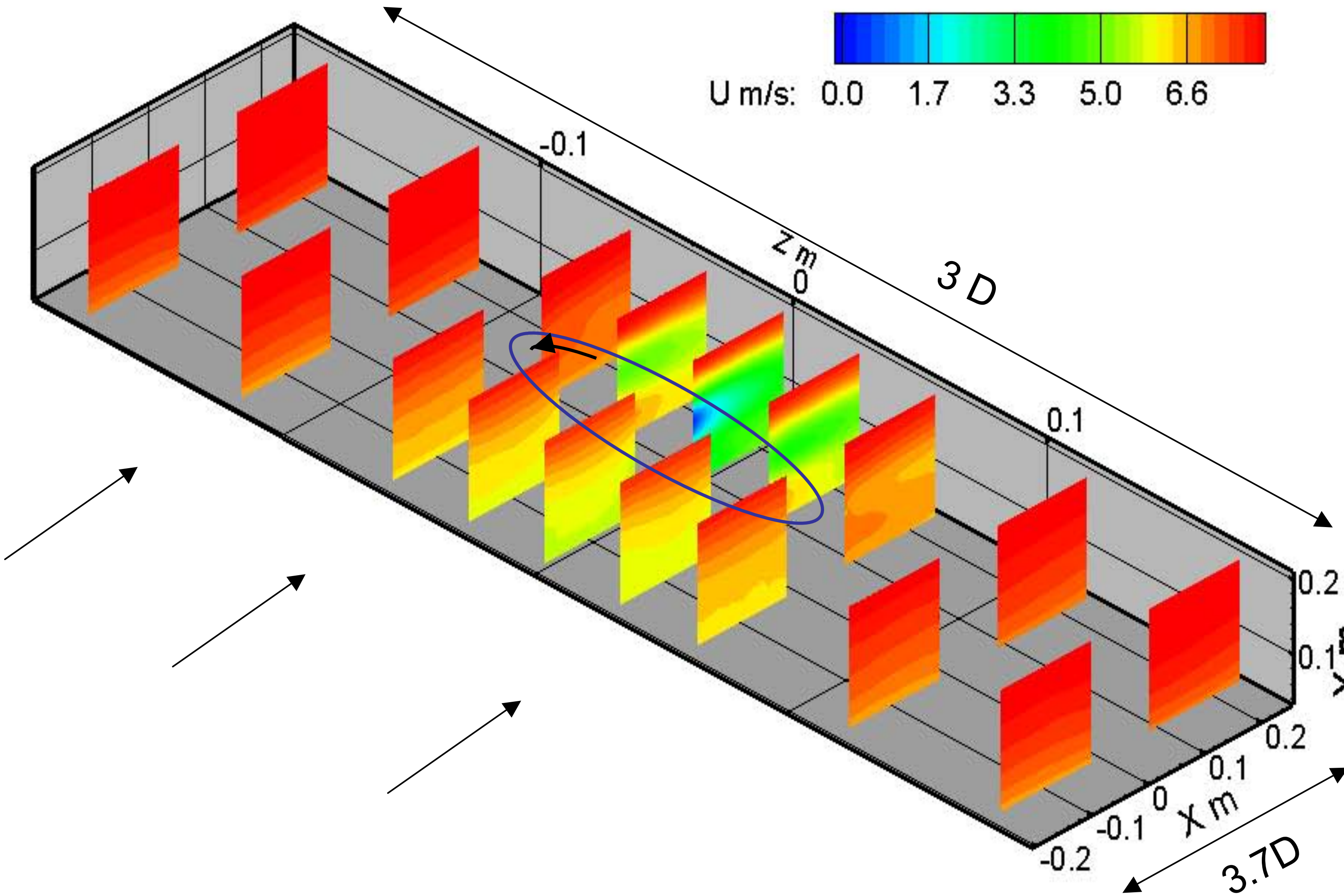
Statistics:

2000 vector maps for each front plane
12000 samples each back plane (6 phase-locked cases)

Wind-tunnel measurements

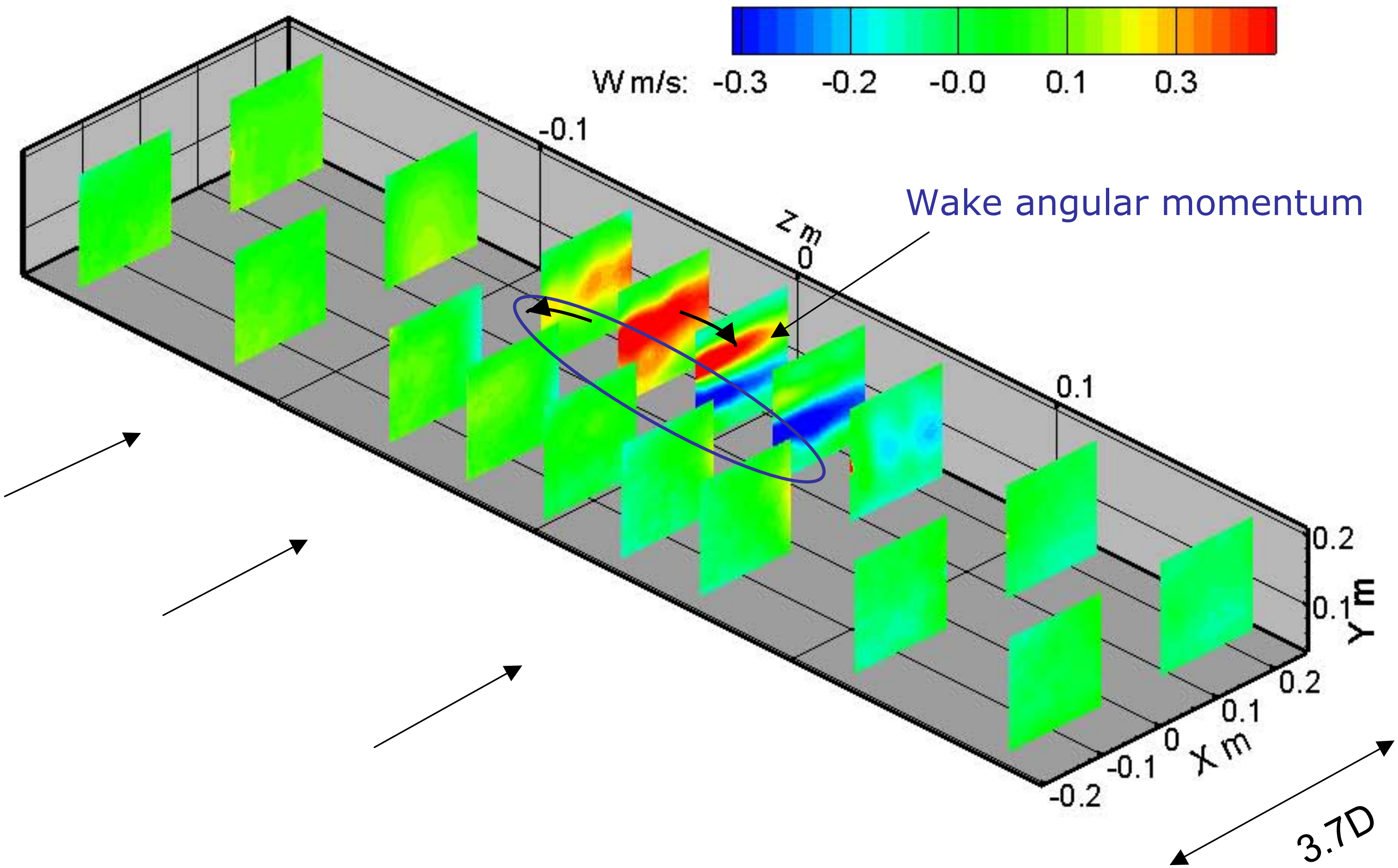
Velocity maps:

Mean streamwise velocity



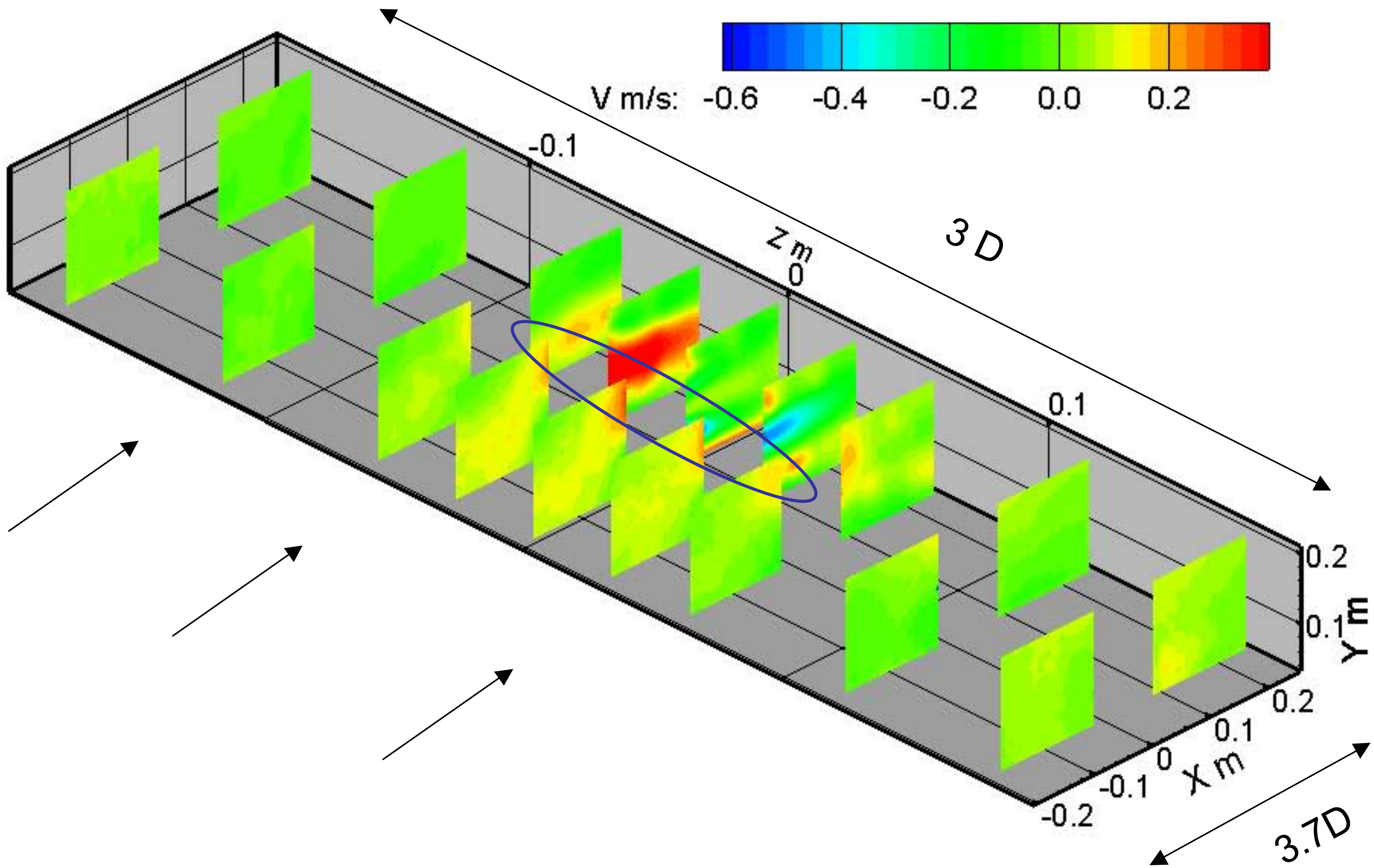
Velocity maps:

Mean transverse velocity



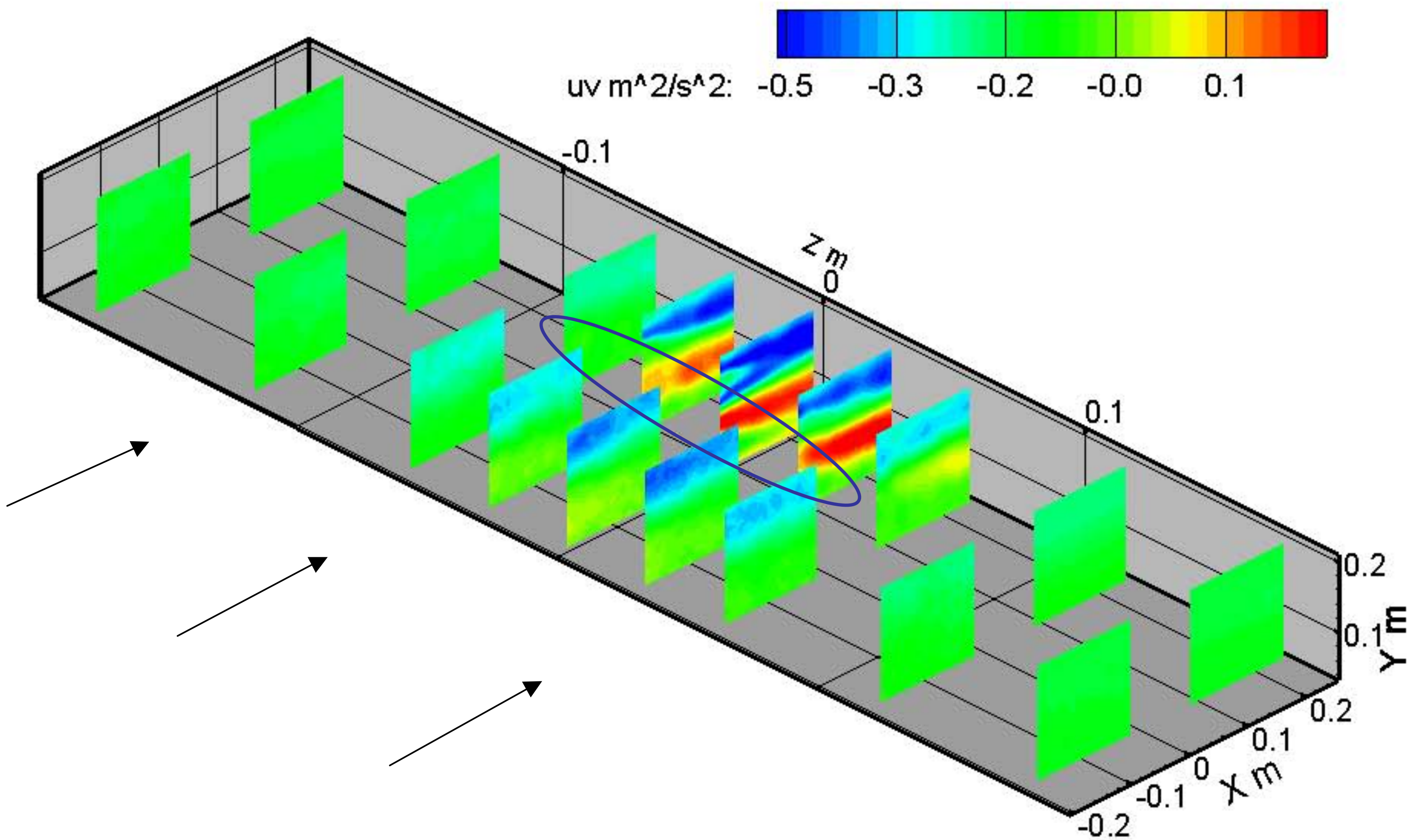
Velocity maps:

Mean vertical velocity



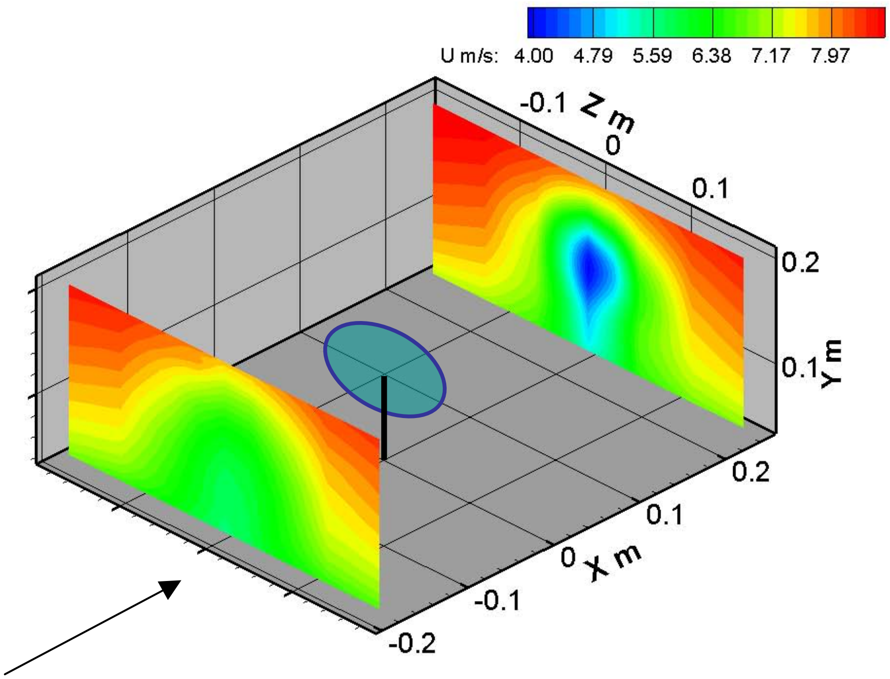
Velocity maps:

(negative) Reynolds shear stress

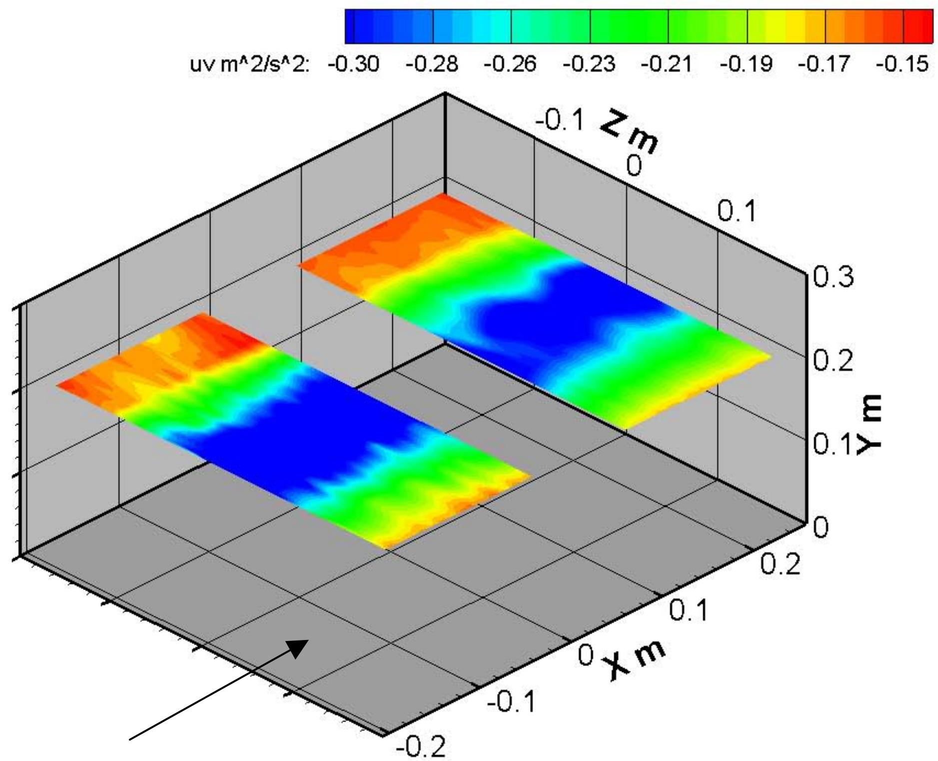


Cross-stream maps:

Streamwise velocity



(negative) Reynolds shear stress



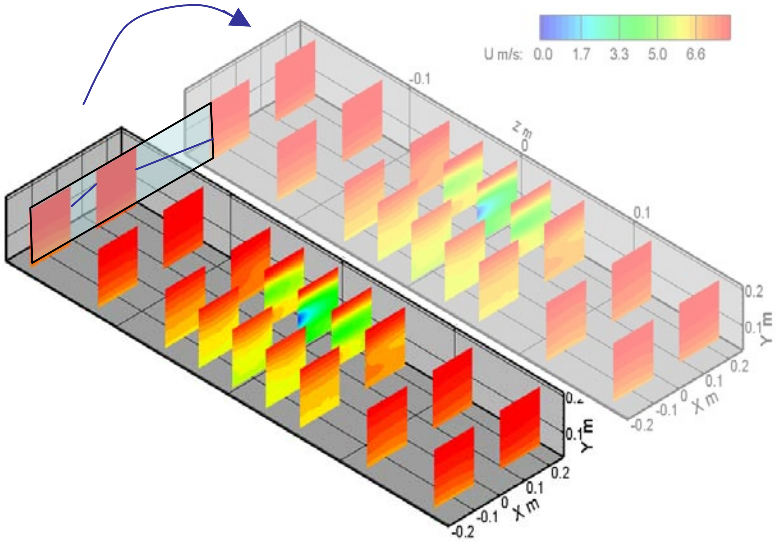
Measured induction factor:

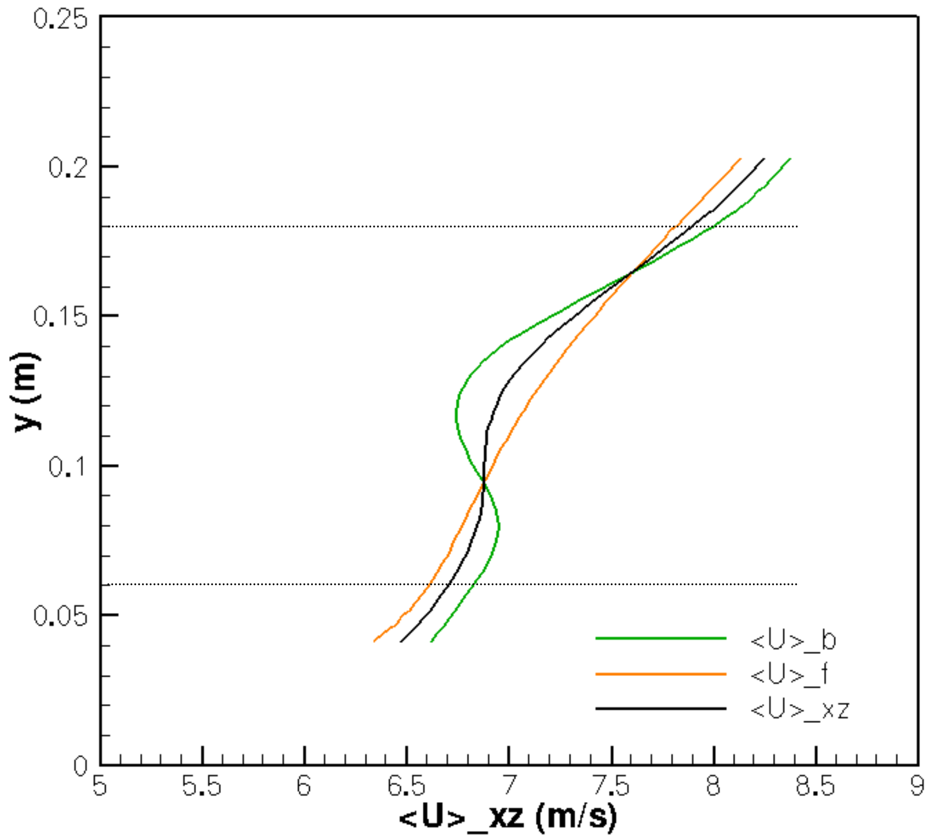
$$a = \frac{1}{2}\left(1 - \frac{U_{back}}{U_{front}}\right) \approx 0.087$$

Horizontally (canopy) averaged profiles:

Mean velocity
(a) in front
(b) in back
(c) overall

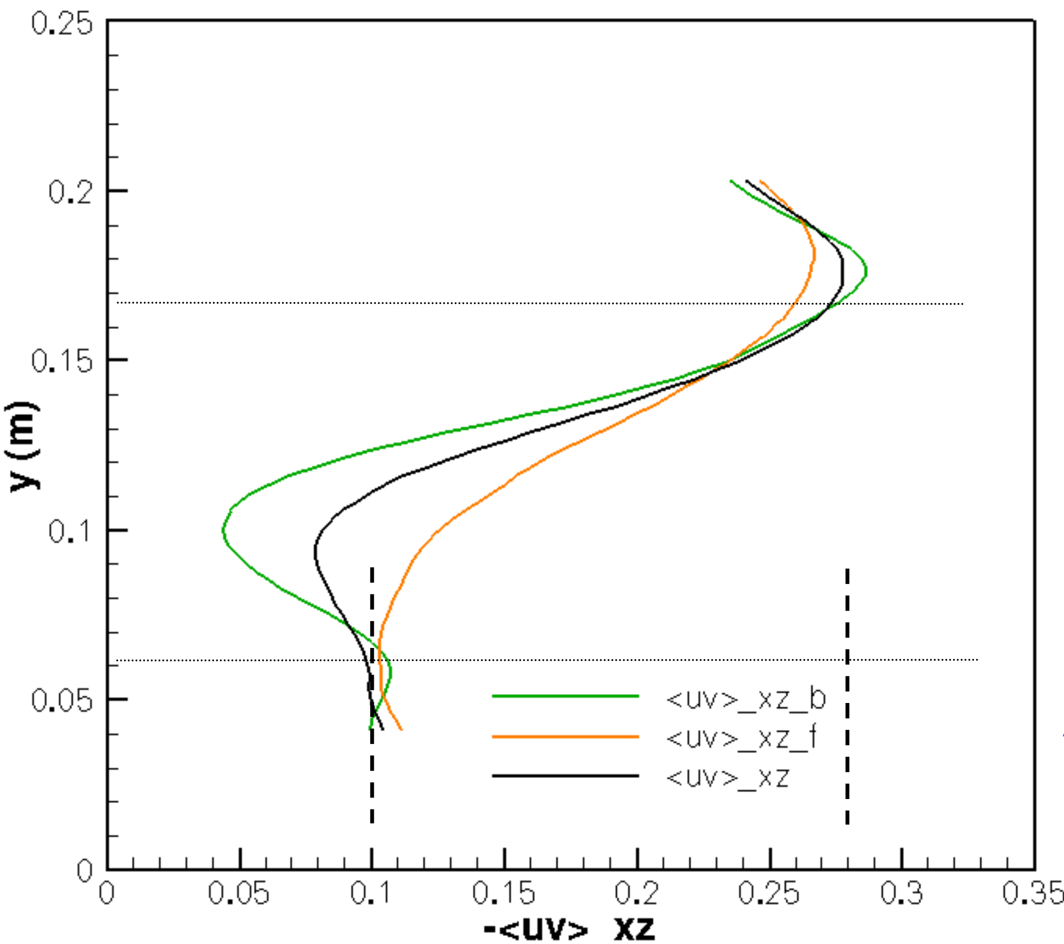
"copy" -
assume periodic
+ linear interpolation





Horizontally (canopy) averaged profiles:

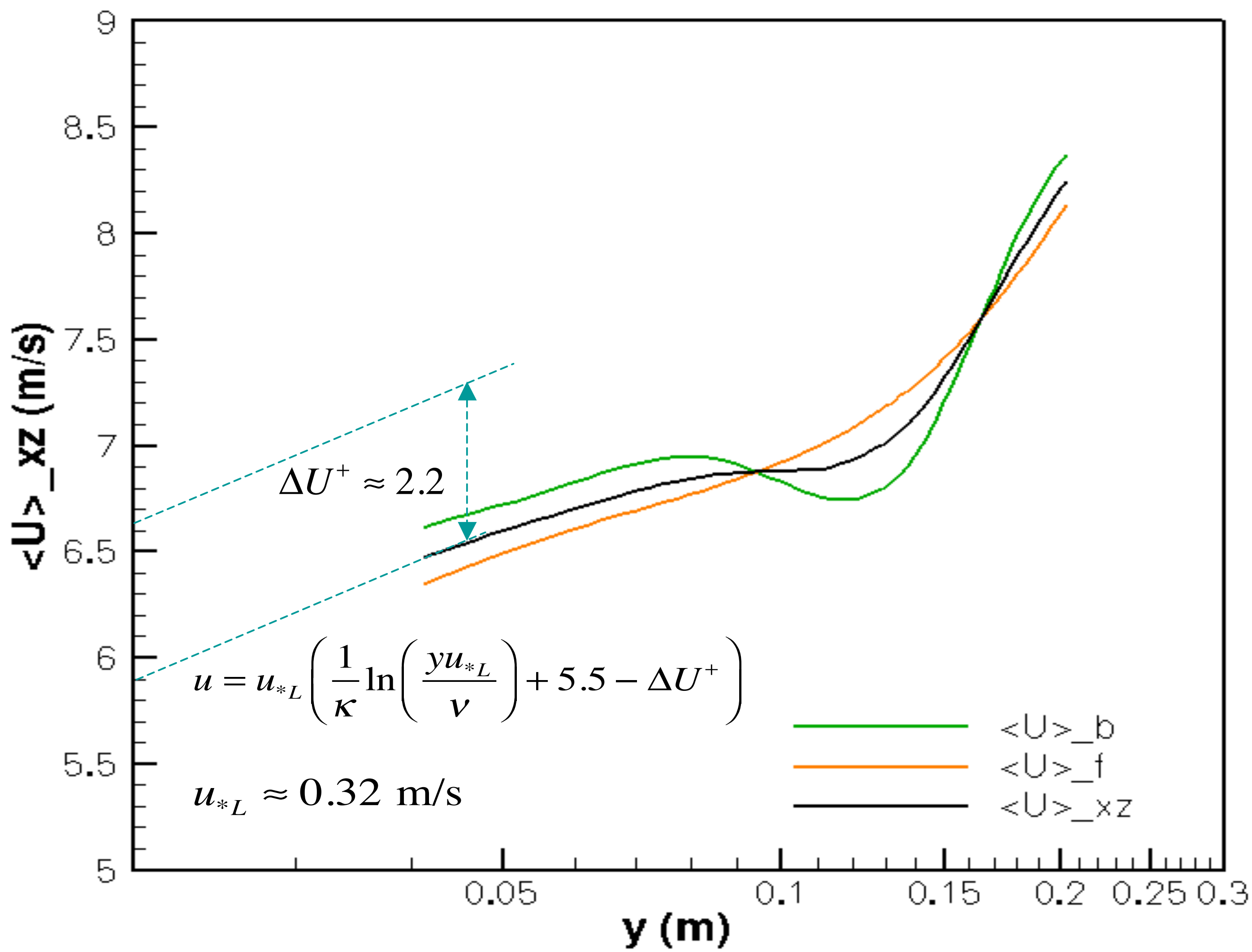
Horizontally averaged Reynolds stresses:



Cal et al.: J. Renewable And Sustainable Energy **2**, 2010

$u_{*L} \approx \sqrt{0.1} \approx 0.32$ m/s $\qquad u_{*H} \approx \sqrt{0.28} \approx 0.53$ m/s

Horizontally (canopy) averaged profiles:



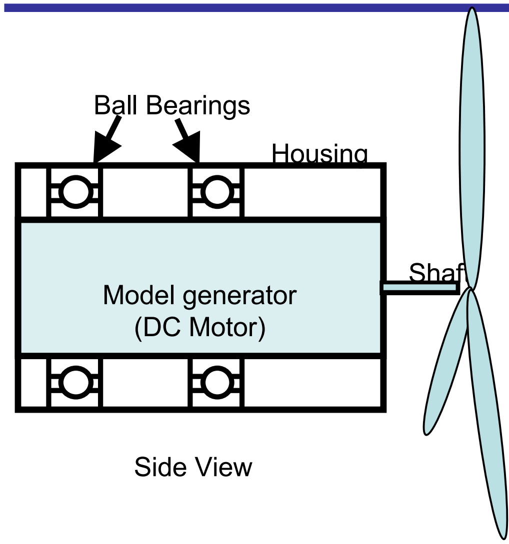
Power extraction:

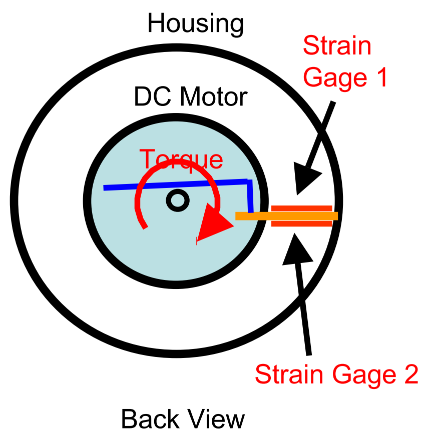
$$P = \frac{1}{2}\rho C_P \frac{\pi}{4} D^2 U_R^3$$

$$C_{P-ideal} = 4a(1-a)^2 = 4 \cdot 0.087(1-0.087)^2 \approx 0.29$$

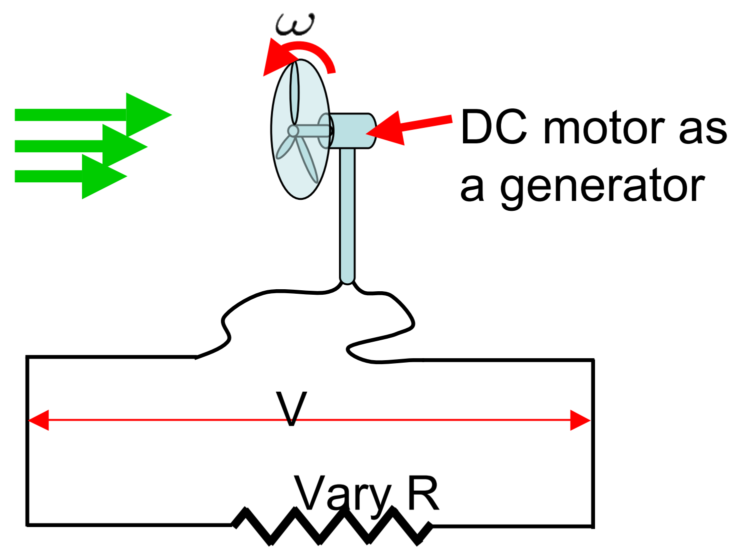
$$P = \frac{1}{2} \cdot 1.2 \cdot 0.29 \cdot \frac{\pi}{4} \cdot 0.12^2 \cdot 6.24^3 \approx 0.47\ W$$

Direct torque measurements:

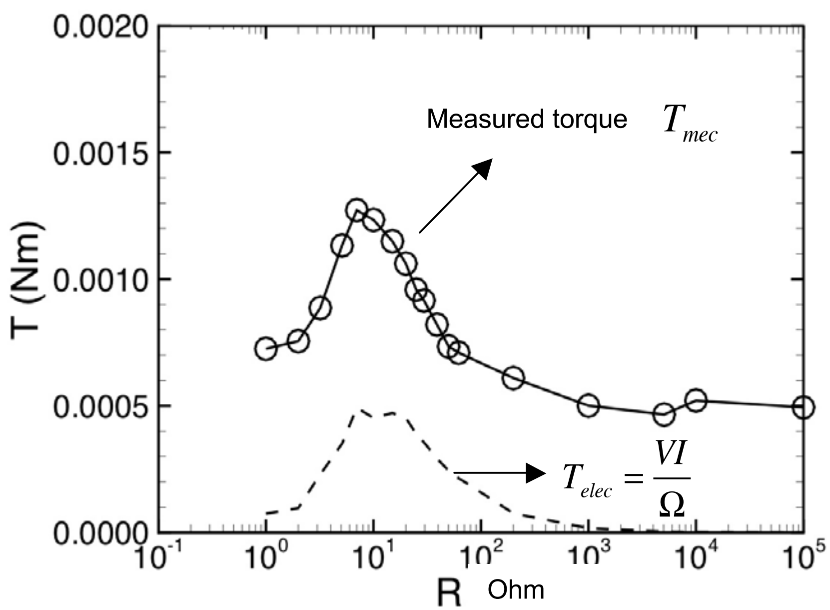




Thanks to Duane Dennis (High-School junior at Baltimore Polytech Inst.) for his help with these experiments



Direct torque measurements:



Therefore at 4800 rpm

$$P^{real} = T^{meas} \cdot \Omega = 0.34 \text{ W}$$

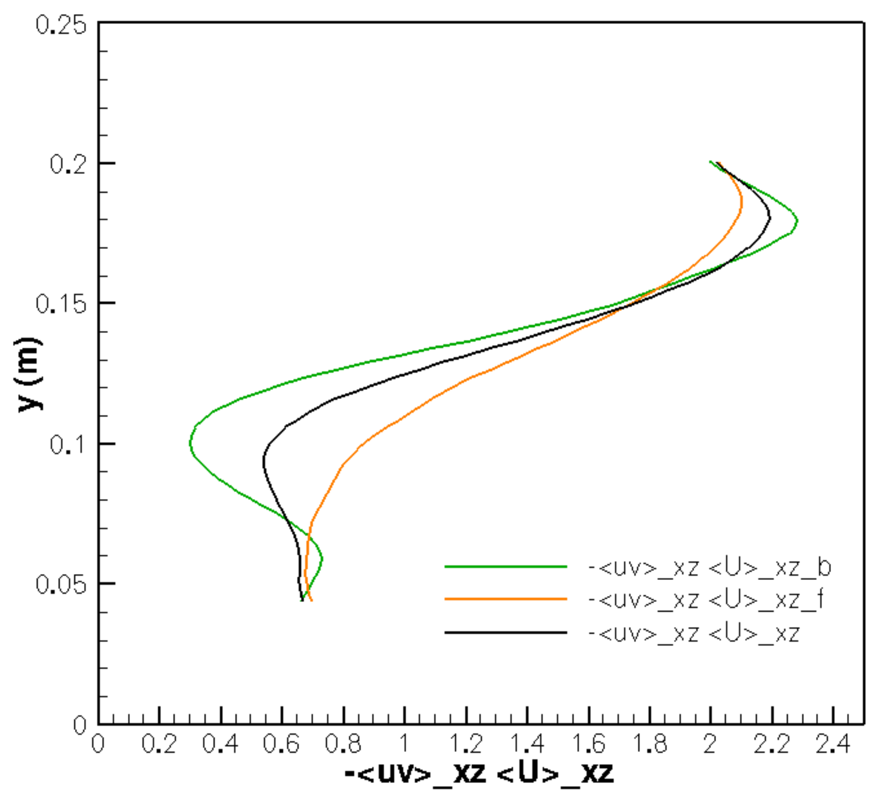
72% of ideal 0.47 W for given *a*

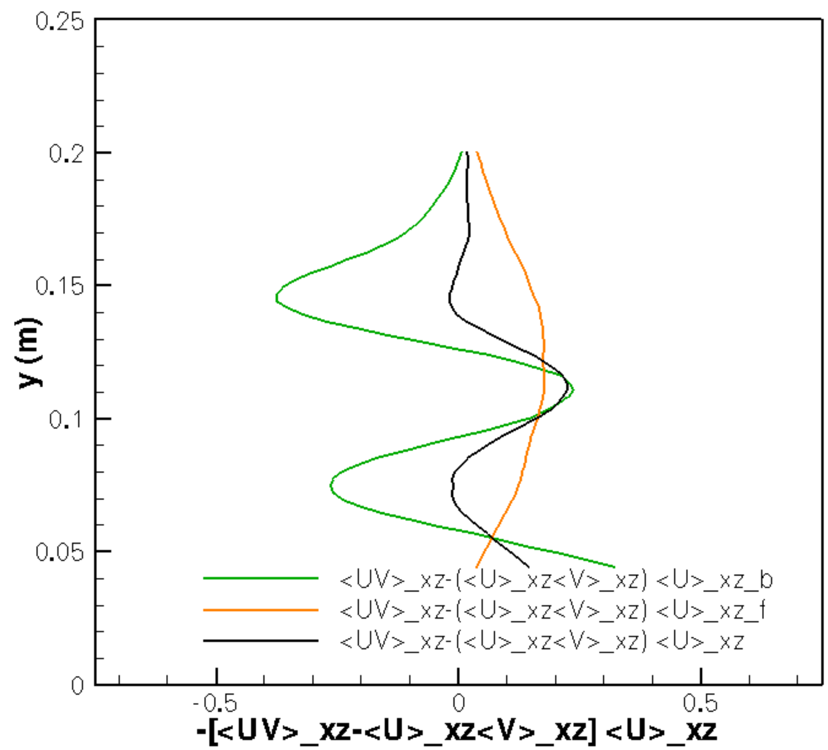
$$\Rightarrow \text{C}_{\text{P-real}} = \frac{P^{real}}{\frac{1}{2}\rho\pi R^2 U_R^3} \approx 0.21$$

About 50% of real-life wind turbines

Horizontally averaged profiles - kinetic energy terms:

$$\frac{d\,\frac{1}{2}\langle u\rangle_{xz}^2}{dt} = -\varepsilon_{turb} - \varepsilon_{canop} - \frac{d}{dy}\left(\left\langle\overline{u'v'}\right\rangle_{xz}\langle u\rangle_{xz} + \left\langle\overline{u}''\overline{v}''\right\rangle_{xz}\langle u\rangle_{xz}\right) - \langle u\rangle_{xz}\frac{1}{\rho}\frac{dp_\infty}{dx} - P_T(y)$$

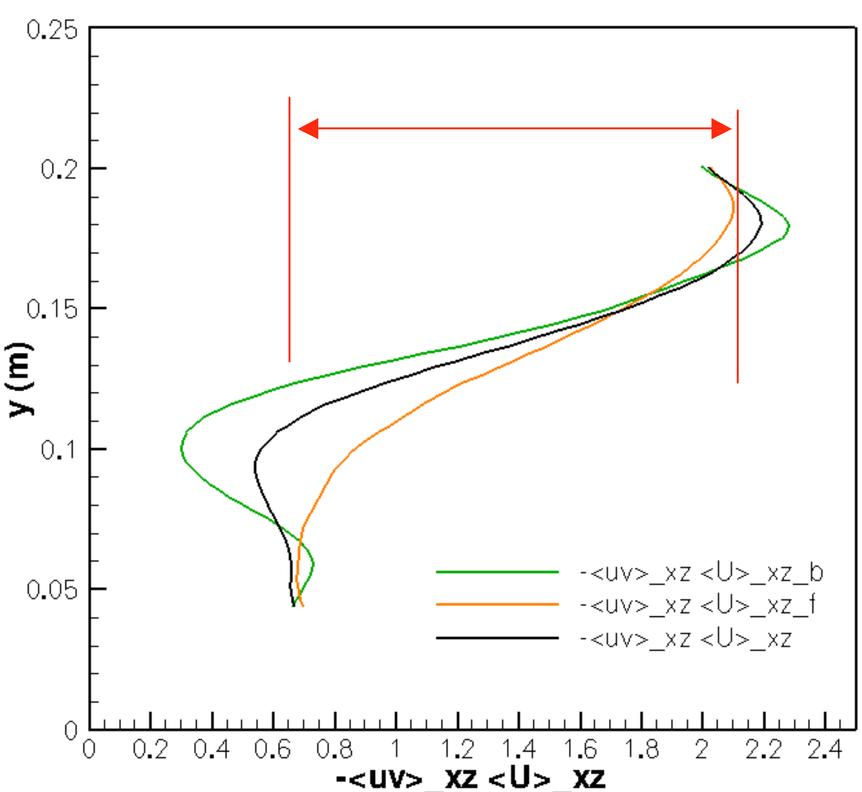




Small except in back

Horizontally averaged profiles - kinetic energy terms:

$$\frac{d\,\frac{1}{2}\langle u\rangle_{xz}^2}{dt} = -\varepsilon_{turb} - \varepsilon_{canop} - \frac{d}{dy}\left(\langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz} + \langle\overline{u}''\overline{v}''\rangle_{xz}\langle u\rangle_{xz}\right) - \langle u\rangle_{xz}\frac{1}{\rho}\frac{dp_\infty}{dx} - P_T(y)$$



$$\langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz}(y_{top}) - \langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz}(y_{bottom}) \approx 1.4 \quad \frac{W/m^2}{(kg/m^3)}$$

$$\Downarrow$$

$$P^{turb-flux} = 1.4\rho A$$

$$P^{turb-flux} = 1.4\times 1.2\times(3\times 0.12)(7\times 0.12)$$

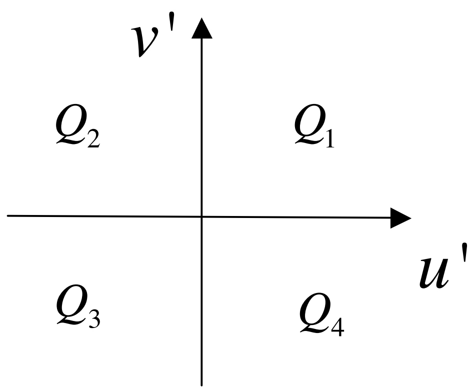
$$P^{turb-flux} = 0.51\ W$$

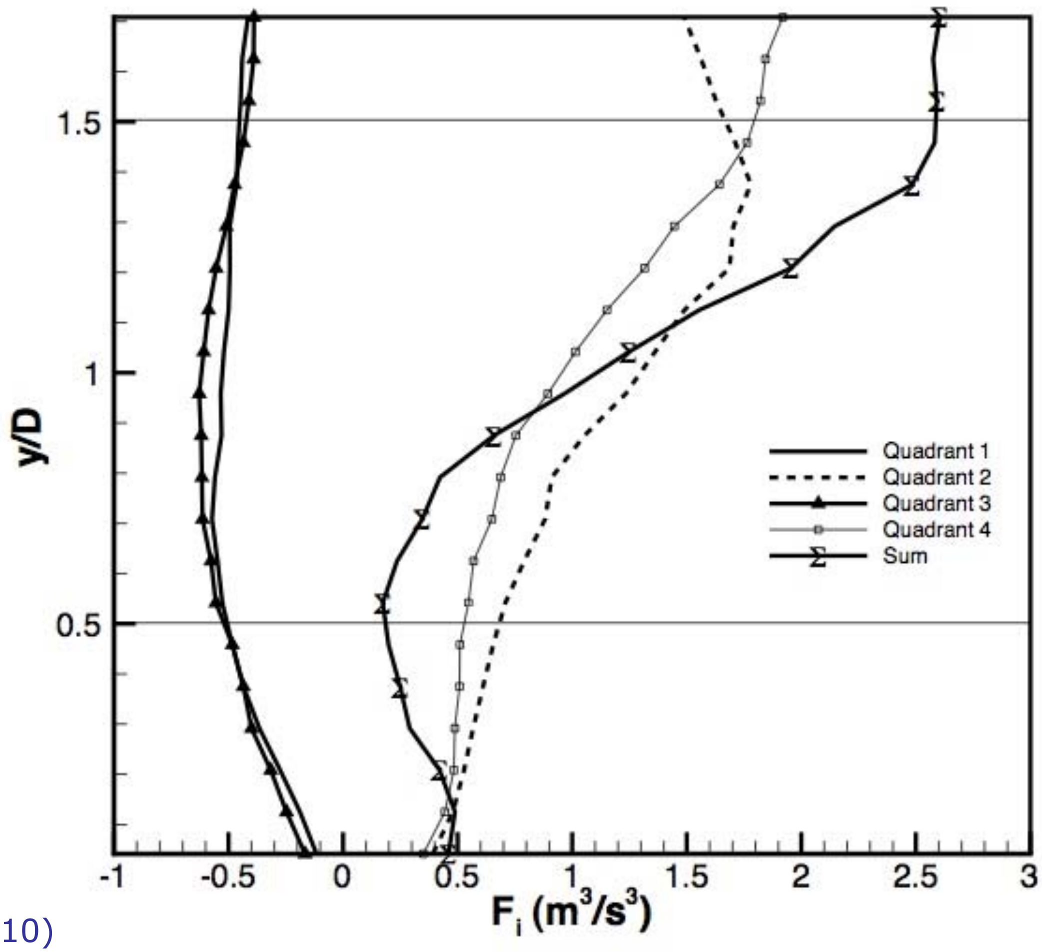
Analysis consistent with view that kinetic energy extracted by turbine (0.34W) is delivered vertically by turbulence fluxes (0.51W)
(rest goes into dissipation, etc…)

Quadrant and spectral analysis of vertical KE entrainment:

$$F_i = -\langle u'v' | Q_i \rangle \langle u \rangle (y)$$

"Quadrant Analysis":
contributions from u' > or < 0 v'> or < 0





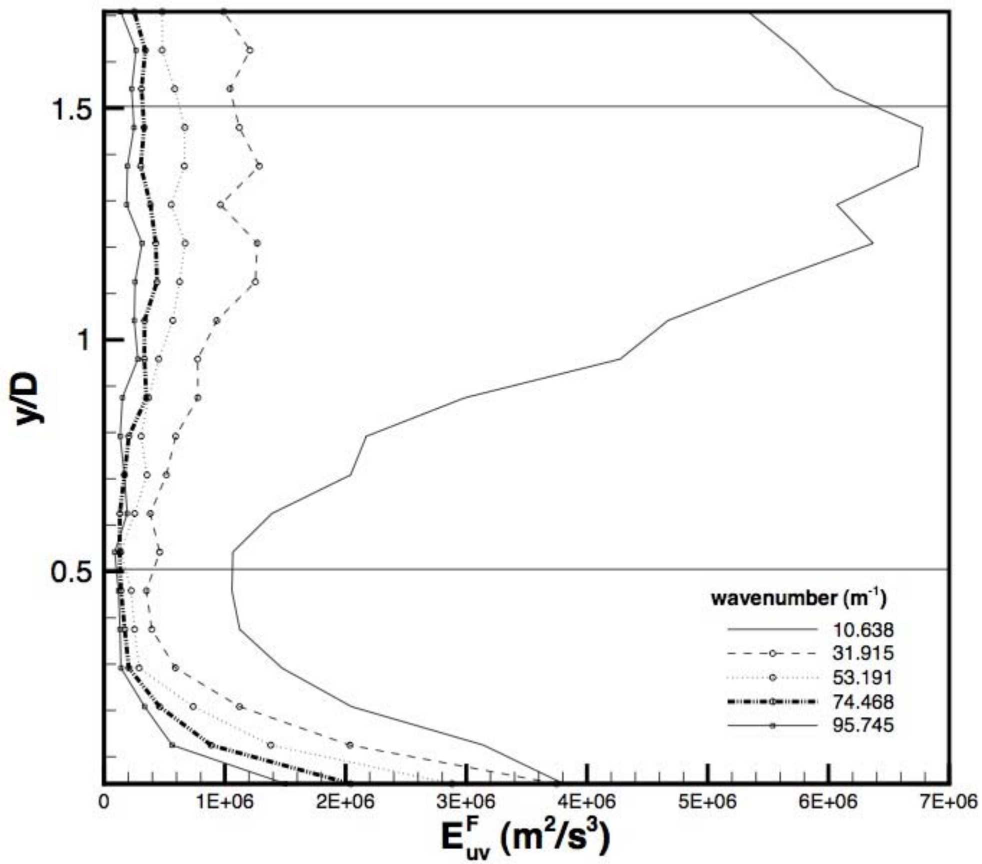
Main conclusion:
Q_2 (ejections) +
Q_4 (sweeps) dominant

From: Gibson, Cal & Meneveau (2010)

Quadrant and spectral analysis of vertical KE entrainment:

$$F = -\langle u(y) \rangle \int \Phi_{uv}(k_x, y) dk_x$$

"Co-spectral Analysis"

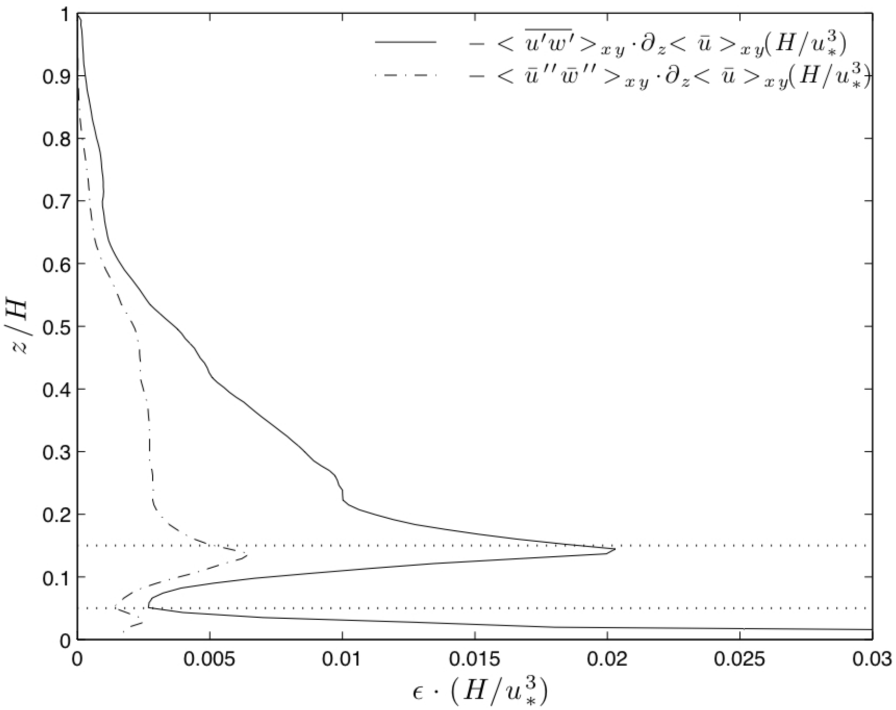


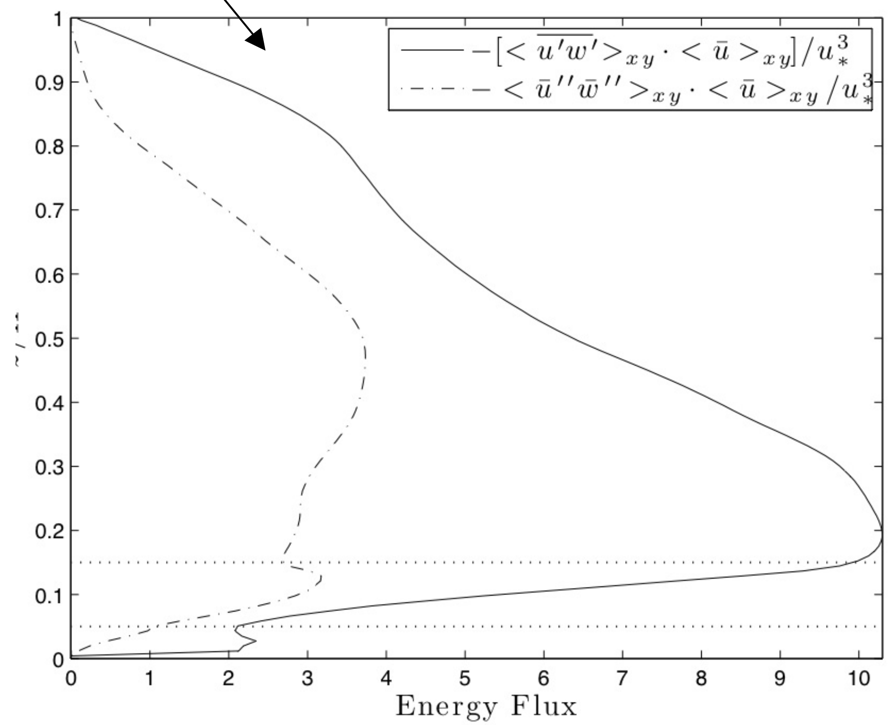
Main conclusion:
Large scales dominate entrainment process
(~ π/k ~ 10-30 cm = 1-3 D)

From: Gibson, Cal & Meneveau (2010)

LES: Horizontally averaged profiles - kinetic energy terms

$$\frac{d\frac{1}{2}\langle u\rangle_{xy}^2}{dt} = -\varepsilon_{turb} - \varepsilon_{canop} - \frac{d}{dz}\left(\langle \overline{u'w'}\rangle_{xy}\langle u\rangle_{xy} + \langle \bar{u}''\bar{w}''\rangle_{xy}\langle u\rangle_{xy}\right) - \langle u\rangle_{xy}\frac{1}{\rho}\frac{dp_\infty}{dx} - P_T(z)$$





Conclusions:

- LES and data so far confirms that overall, boundary layer momentum theory (friction velocity, thickness length-scales, momentum loss, etc..) gives valuable insight and parameterizations of dynamics.

- LES confirms existence of 2 log-laws below and above WT region

- We have extended Frandsen model for effective roughness height, with improved agreement with LES results.

- The notion of the fully developed WTABL is a useful theoretical notion, otherwise mix of BL + wake, scale separation, 3-D complex flow complicates statistical treatment and data analysis.

- How much momentum transfer caused by mean velocity dispersive stresses?
  we found them small in experiment
  we found them large in LES

- Energetics and profiles show that kinetic energy is delivered to turbine mostly by turbulence (both LES and experiments confirm this).

- Expand experiments with model rotors with higher induction factor, vary λ...

- Model for effective roughness height (R) can be used to design optimal power extraction per cost (terrain and # of WT)