Market Model and Algorithmic Design for Demand Response in Power Networks

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Demand response

Use incentive mechanisms such as real-time pricing to induce customers/appliances to shift usage or reduce (even increase) consumption

Demand response techniques

- Smart appliances responding to price/event signals
- Load shifting technologies such as storage
- Peak-eliminating techniques such as distributed generation or simply turning off appliances
- …

Enabler

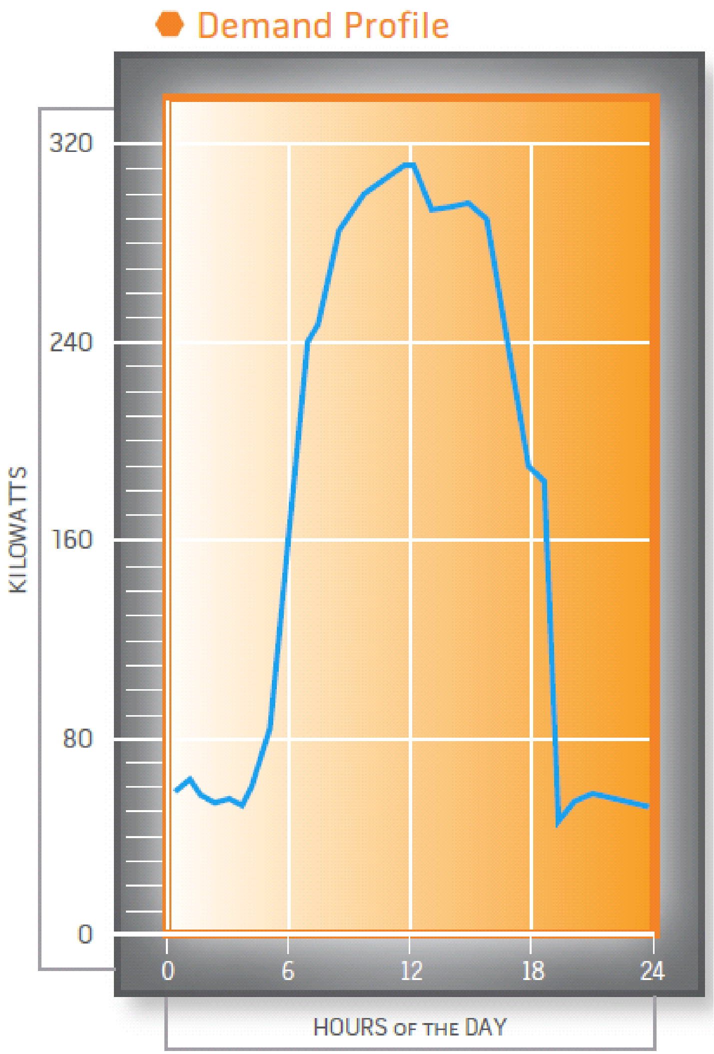
Smart grid

- Timely two-way communications between customers and utility companies
- Individual customers and appliances are empowered with certain computing capability
- High speed WAN allows real-time and global monitoring at control centers
- High performance computing allows faster control decisions

Outline

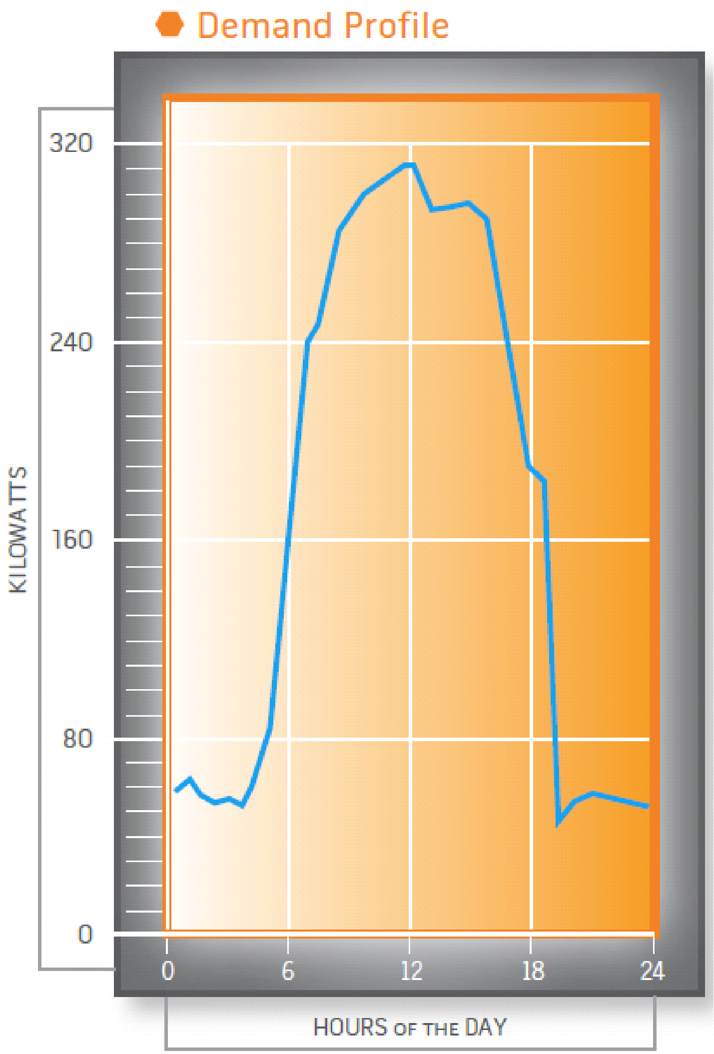
- Motivation for demand response
- Main issues in demand response design
- Demand response: Match the supply
- Demand response: Shape the demand

Time-varying demand



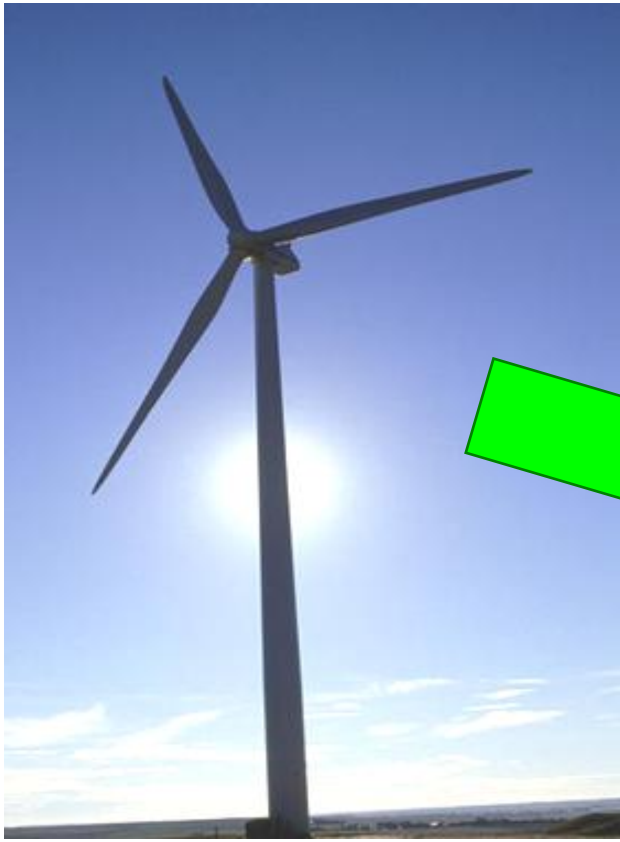
- Electricity demand is highly time-varying
- Provision for peak load
  - Low load factor
    - US national load factor is about 55%
  - Underutilized
    - 10% of generation and 25% of distribution facilities used less than 5% of the time

Source: DoE, Smart Grid Intro, 2008

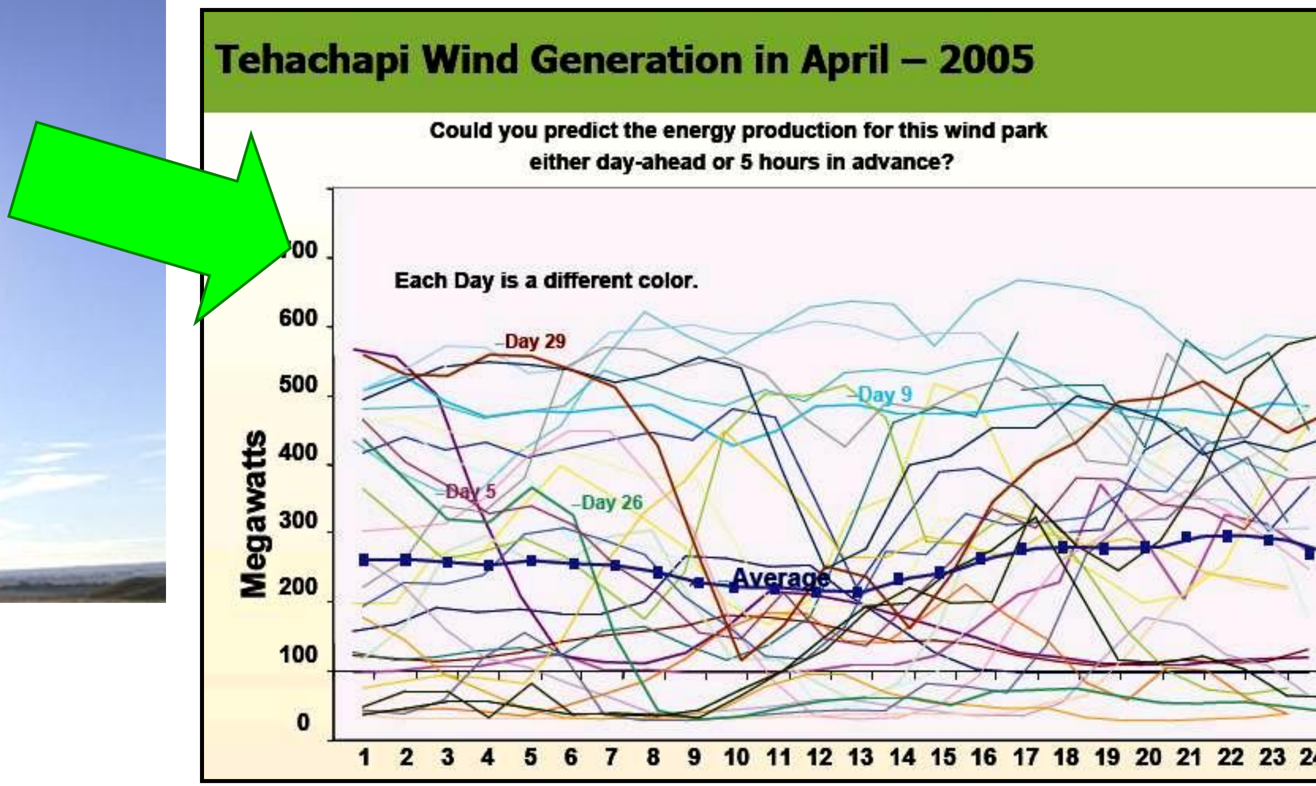


- Shape the demand
  - Reduce peak load
  - Flatten load profile
- Benefits
  - Lower generation cost
  - Larger safety margin
  - Reduce or slow down the need for new generation and distribution infrastructure

Uncertainty of renewables



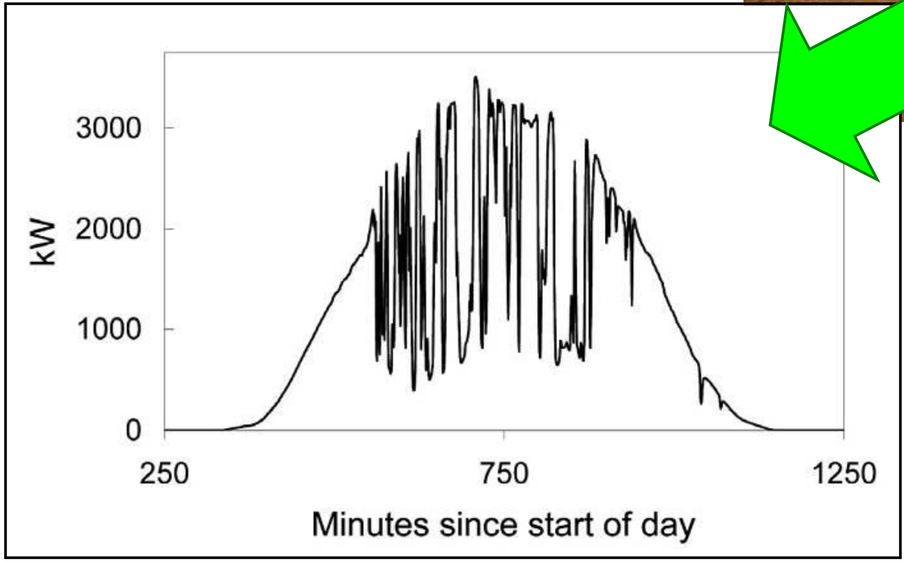
change at timescale of minutes



Source: Rosa Yang

Uncertainty of renewables

change at timescale
of seconds



Source: Rosa Yang

Dealing with uncertainty

- Reduce uncertainty by
  - Aggregating supply types
  - Aggregating over space
  - Aggregating over time (but large-scale storage is currently not available)

- Accommodate uncertainty
  - Reliability as resource to trade off
    - Optimize risk tolerance
  - Match time-varying supply (demand response)

Outline

- Motivation for demand response
- **Main issues in demand response design**
- Demand response: Match the supply
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Main challenge

Matching supply and demand

- Market challenge
  - achieve efficient and economic generation, delivery, and consumption
- Engineering challenge

electricity must be consumed at the moments it is generated

Overall structure

generation

wholesale market

retail market

utility company

customers

- ❑Bilateral contracts
- ❑Auction market
  - ❑day-ahead
  - ❑real-time
  - ❑ancillary service

Main issues

The role of utility as an intermediary

- Play in multiple wholesale markets to provision aggregate power to meet demands
- Resell, with appropriate pricing, to end users
- Provide two important values
  - Aggregate demand at the wholesale level so that overall system is more efficient
  - Absorb large uncertainty/complexity in wholesale markets and translate them into a smoother environment (both in prices and supply) for end users.

How to quantify these values and price them in the form of appropriate contracts/pricing schemes?

Main issues

Utility/end users interaction ← our focus

- Design objective
  - Welfare-maximizing, profit-maximizing, …
- Distributed implementation
- Real-time demand response

The impact of distribution network

- i.e., put in physical network (Kirkoff Law, and other constraints)
- How does it change the algorithm and optimality
- Can we exploit radial structure of distribution network

Retail market

Retail (utility-user) essentially uses fixed prices

- Tiered, some time-of-day

Demand response will (likely) use real-time pricing to better manage load

How should utility company design real-time retail prices to optimize demand response?

The basics of supply and demand

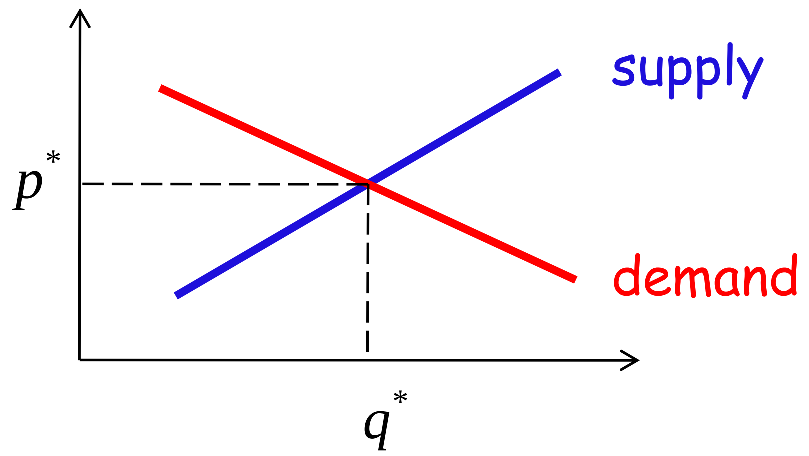
- Supply function: quantity supplied at given price

$$q = S(p)$$

- Demand function: quantity demanded at given price

$$q = D(p)$$

- Market equilibrium: (q^*, p^*) such that $q^* = S(p^*) = D(p^*)$
  - No surplus, no shortage, price clears the market



Competitive vs oligopolistic markets

- Competitive market: no market participant is large enough to have market power to set the price
  - Price-taking behavior
  - e.g., individual residential customers

- Oligopolistic market: (a few) market players can influence and be influenced by the actions of others
  - Price-anticipating behavior
  - e.g., large commercial customers

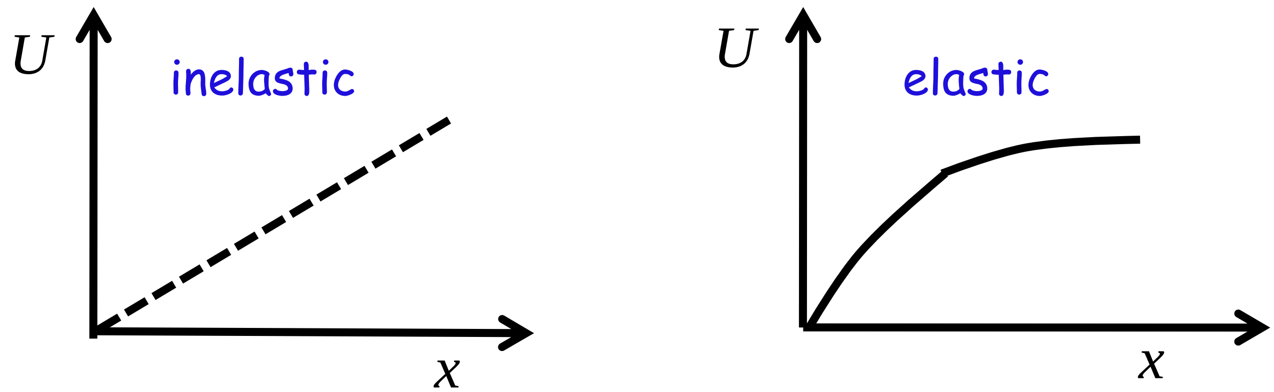
Utility function

- Given the set X of possible alternatives, a function

$$U : X \rightarrow R$$

is a utility function representing preference relation among alternatives, if for all $x, y \in X$,

$$"x \text{ is at least as good as } y" \Leftrightarrow \mathrm{U}(x) \geq U(y)$$



- To use utility function to characterize preferences is a fundamental assumption in economics

Outline

- Motivation for demand response
- Main issues in demand response design
- **Demand response: Match the supply**
- Demand response: Shape the demand

Problem setting

- Supply deficit (or surplus) on electricity: d
  - weather change, unexpected events, …
- Supply is inelastic
  - because of technical reasons such as supply friction

Problem: How to allocate the deficit/surplus among demand-responsive customers?

- load (demand) as a resource to trade

Supply function bidding

- Customer $i \in N$ load to shed: q_i
- Customer i supply function (SF):

$$q_i(b_i, p) = b_i p$$

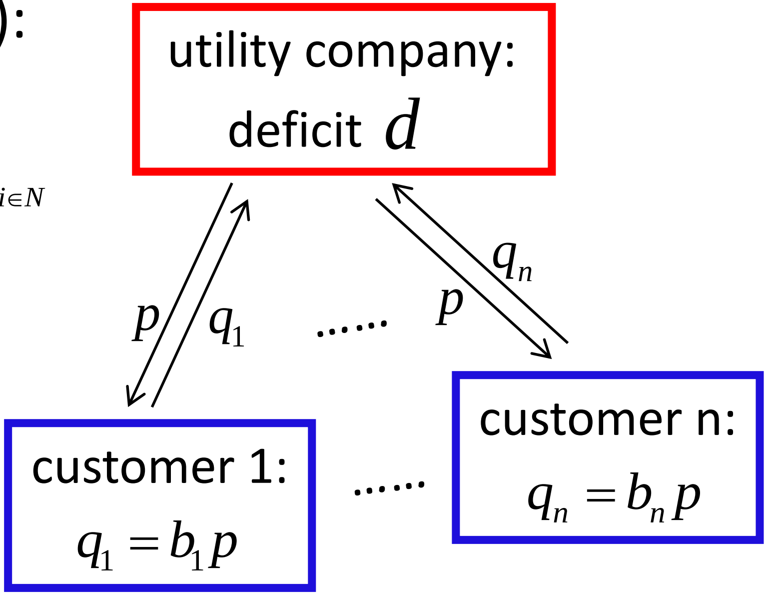
  - parameterized by $b_i \geq 0$; $b \triangleq (b_i)_{i \in N}$
  - the amount of load that the customer is committed to shed given price p

- Market-clearing pricing:

$$\sum_i q_i(b_i, p) = d$$

$$\Downarrow$$

$$p = p(b) \triangleq d \, / \sum_i b_i$$



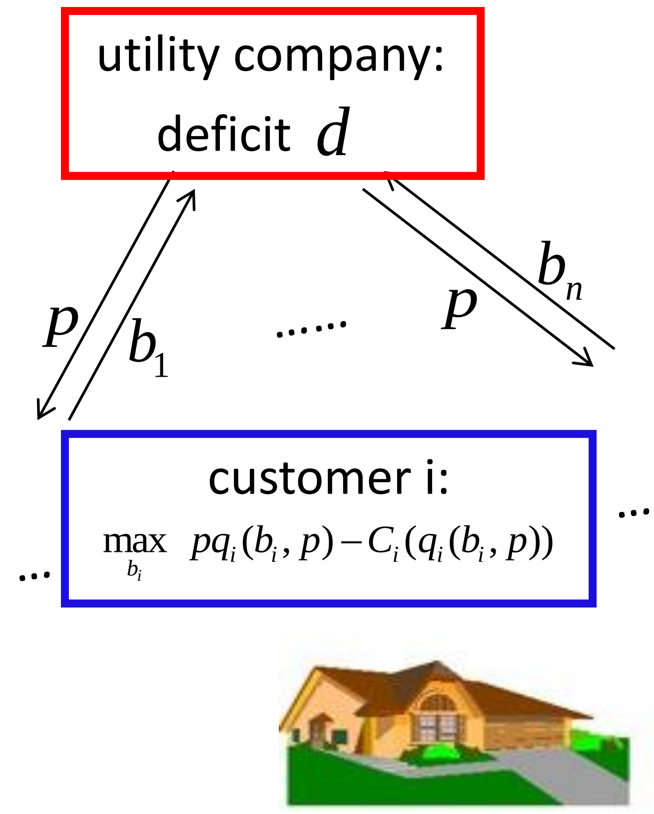
Parameterized supply function

- Adapts better to changing market conditions than does a simple commitment to a fixed price or quantity (Klemper & Meyer ’89)
  - widely used in the analysis of the wholesale electricity markets
  - Green & Newbery ‘92, Rudkevich et al ‘98, Baldick et al ‘02, ‘04, …
- Parameterized SF
  - easy to implement
  - control information revelation
  - …

Optimal demand response

- Customer i cost (or disutility) function: $C_i(q_i)$
  - continuous, increasing, and strictly convex
- Competitive market and price-taking customers
- Given price p, each customer i solves

$$\max_{b_i} \; pq_i(b_i, p) - C_i(q_i(b_i, p))$$



Competitive equilibrium

- **Definition**: A competitive equilibrium (CE) is defined as a tuple $\{(b_*^*)_{i\in N}, p^*\}$ such that

$$b_i^* = \arg\max_{b_i \geq 0} p^* q_i(b_i, p^*) - C_i(q_i(b_i, p^*)),\ \forall i$$

$$\sum_i q_i(b_i^*, p^*) = d$$

- **Theorem:** There exist a unique CE. Moreover, the equilibrium is efficient, i.e., maximizes social welfare

$$\max_{q_i} \ -C_i(q_i) \ \text{ s.t. } \sum_i q_i = d$$

Proof

Show the equilibrium condition is the optimality condition (KKT) of the optimization problem

Equilibrium

$$b_i^* = \arg\max_{b_i \geq 0} p^* q_i(b_i, p^*) - C_i(q_i(b_i, p^*)), \ \forall i$$

$$\sum_i q_i(b_i^*, p^*) = d$$

Social Welfare optimization

$$\max_{q_i} \ -C_i(q_i) \ \text{ s.t. } \sum_i q_i = d$$

$$(p^* - C_i'(q_i(b_i^*, p^*)))(b_i - b_i^*) p^* \leq 0, \quad \forall b_i \geq 0$$

$$\sum_i q_i(b_i^*, p^*) = d$$

$$(p^* - C_i'(q_i^*))(q_i - q_i^*) \leq 0, \quad \forall q_i \geq 0$$

$$\sum_i q_i^* = d$$

Iterative supply function bidding

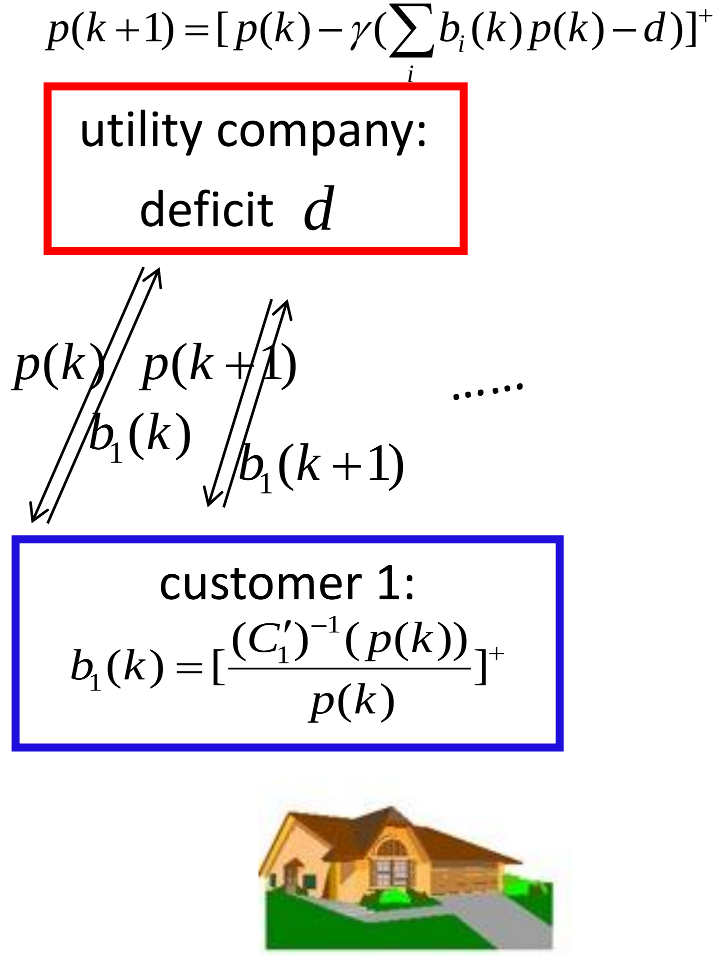
- Upon receiving the price information, each customer i updates its supply function

$$b_i(k) = \left[\frac{(C_i')^{-1}(p(k))}{p(k)}\right]^+$$

- Upon gathering bids from the customers, the utility company updates price

$$p(k+1) = [\,p(k) - \gamma(\sum_i b_i(k)\,p(k) - d)]^+$$

- Requires
  - timely two-way communication
  - certain computing capability of the customers



Strategic demand response

- Oligopoly market and price-anticipating customer

$$p = p(b) \triangleq d \,/ \sum_i b_i$$

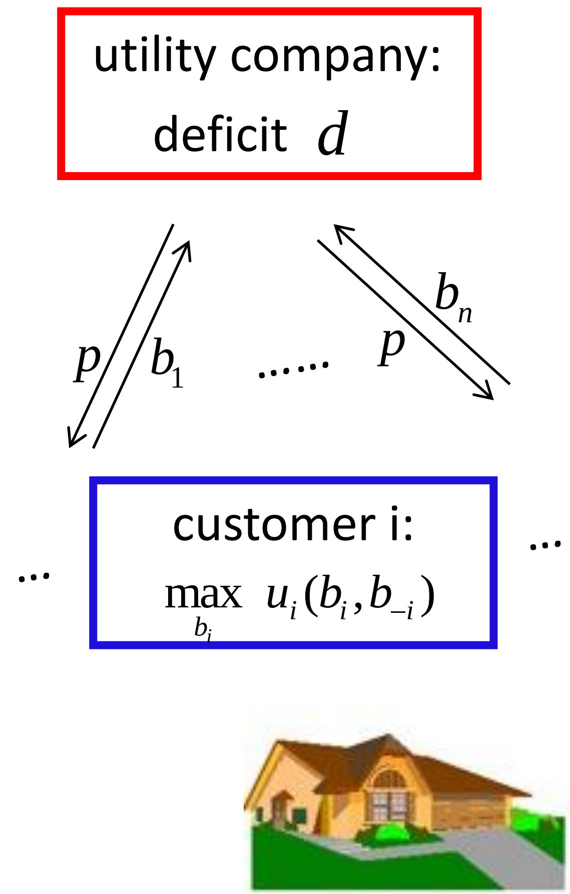
- Given others' supply functions b_{-i}, each customer i solves

$$\max_{b_i} \; u_i(b_i, b_{-i})$$

with

$$u_i(b_i, b_{-i}) = p(b) q_i(b_i, p(b)) - C_i(q_i(b_i, p(b)))$$

It is a game



Game-theoretic equilibrium

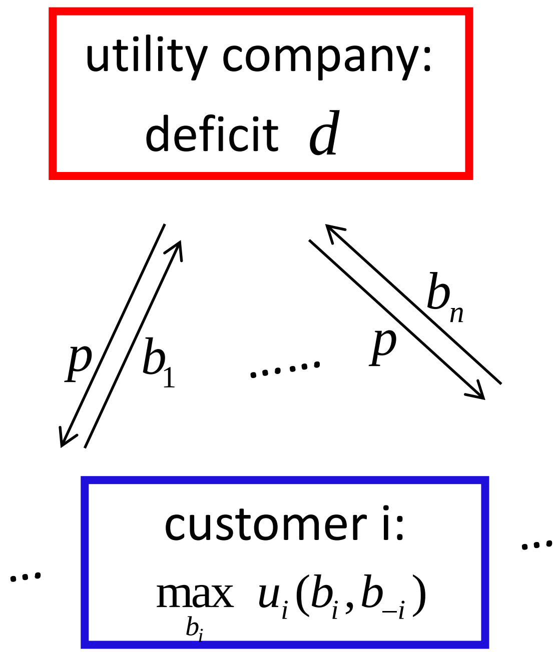
- **Definition**: A supply function profile b^* is a Nash equilibrium (NE) if, for all customers i and $b_i \geq 0$,

$$u_i(b_i^*, b_{-i}^*) \geq u_i(b_i, b_{-i}^*).$$

- **Theorem**: There exists a unique NE when the number of customers is larger than 2. Moreover, the equilibrium solves

$$\max_{0 \leq q_i \leq d/2} \; -D_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d$$

$$D_i(q_i) = \left(1 + \frac{q_i}{d - 2q_i}\right) C_i(q_i) - \int_0^{q_i} \frac{d}{(d\text{-}2x_i)^2} C_i(x_i)\, dx_i$$



Proof

Show the equilibrium condition is the optimality condition (KKT) of the optimization problem.

Nash Equilibrium

$$\max_{b_i} \; pq_i(p(b),p) - C_i(q_i(p(b),p))$$

$$= d^2 b_i / (\Sigma_j b_j)^2 - C_i(db_i / \Sigma_j b_j)$$

$$\sum_i q_i(b_i, q) = d$$

Optimization

$$\max_{q_i} \; -D_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d$$

$$D_i(q_i) = (1 + q_i / d - 2q_i) C_i(q_i)$$

$$- \int_0^{q_i} d / (d - 2x_i)^2 C_i(x_i) dx_i$$

$$\left(p^* - \left(1 + \frac{q_i^*}{d-2q_i^*}\right) C_i'(q_i^*)\right)(b_i p^* - q_i^*) \leq 0$$

$$\forall b_i \geq 0$$

$$\sum_i b_i^* p^* = d$$

$$p^* = \frac{d}{\sum_i b_i^*}; \; q_i^* = b_i^* p^*$$

$$\left(p^* - \left(1 + \frac{q_i^*}{(d-2q_i^*)}\right) C_i'(q_i^*)\right)(q_i - q_i^*) \leq 0,$$

$$\forall q_i \geq 0$$

$$\sum_i q_i^* = d$$

$$p^* > 0$$

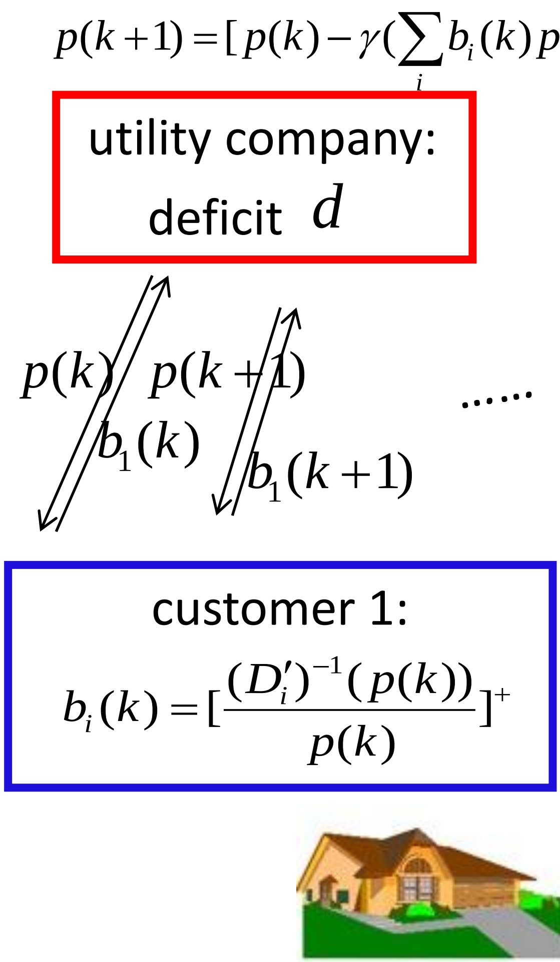
Iterative supply function bidding

- Each customer i updates its supply function

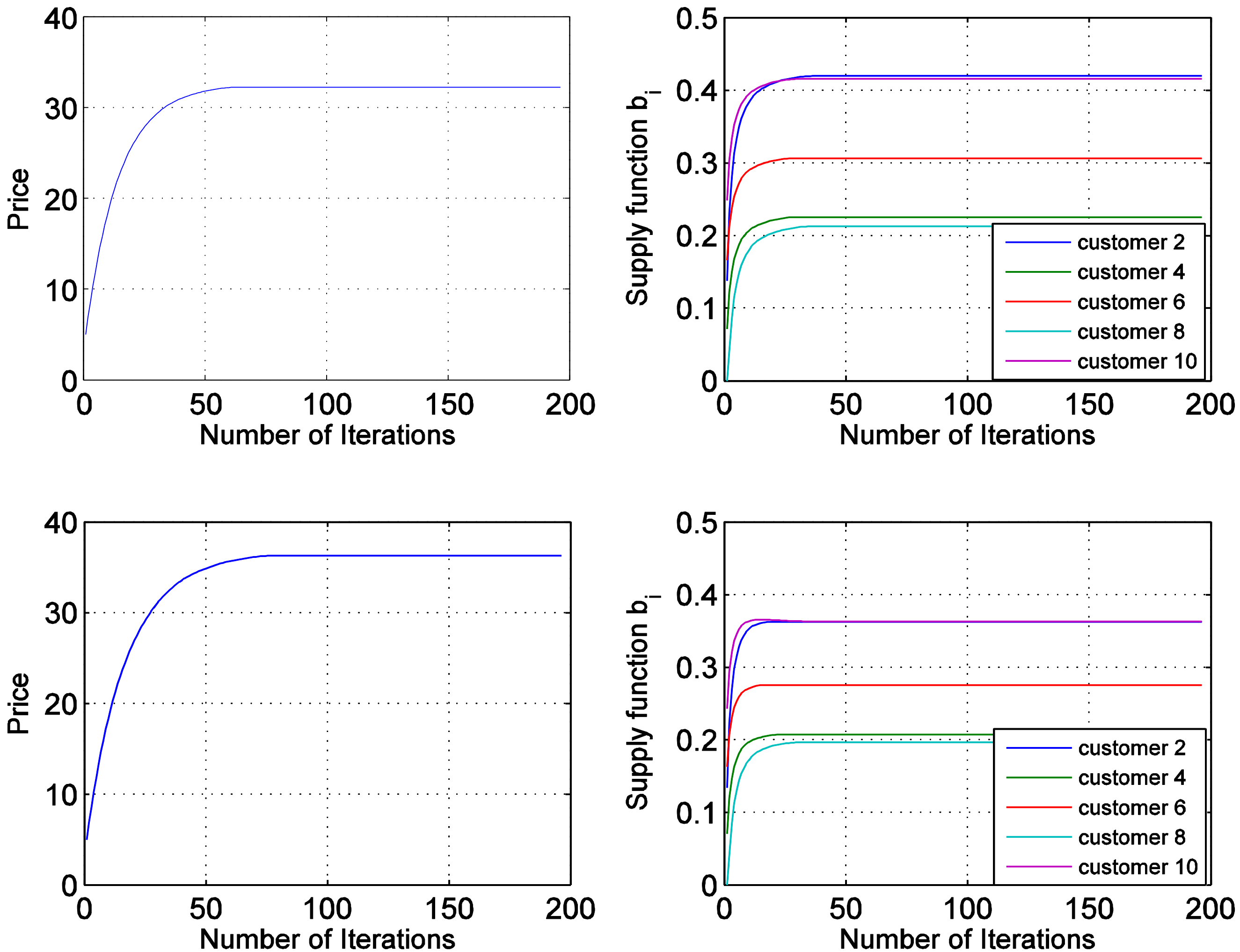
$$b_i(k) = [\frac{(D_i')^{-1}(p(k))}{p(k)}]^+$$

- The utility company updates price

$$p(k+1) = [p(k) - \gamma(\sum_i b_i(k)p(k) - d)]^+$$



Numerical example



Optimal supply function bidding (upper panels) v.s. strategic bidding (lower panels)

Outline

- Motivation for demand response
- Main issues in demand response design
- Demand response: Match the supply
- **Demand response: Shape the demand**

Problem setting

- Load is deferrable and reducible
- Subject to various constraints, depending on the types of appliances
  - minimal/maximal load over certain period of time
  - minimal/maximal load at each time
  - battery has finite capacity and usage-dependent cost
  - …

Problem: How to shape deferrable load over certain period of time, so as to reduce peak, flatten load profile and even conserve energy?

Customer-side model (abstract)

- Each customer i, each of the appliances $a \in \mathrm{A}_i$:
  - Load at time t : $q_{i,a}(t)$; define: $q_{i,a} \triangleq \left(q_{i,a}(t)\right)_{t\in T}$
  - Load constraint: $q_{i,a} \in C_{i,a}$
  - Total load at time t : $Q_i(t) = \sum_a q_{i,a}(t) + r_i(t)$
  - Utility: $U_{i,a}\left(q_{i,a}\right)$
  - The appliances divided into 4 categories

- Energy Storage: one battery for each customer i
  - Load at time t: $r_i(t)$; define $r_i \triangleq \left(r_i(t)\right)_{t\in T}$

    positive means charging

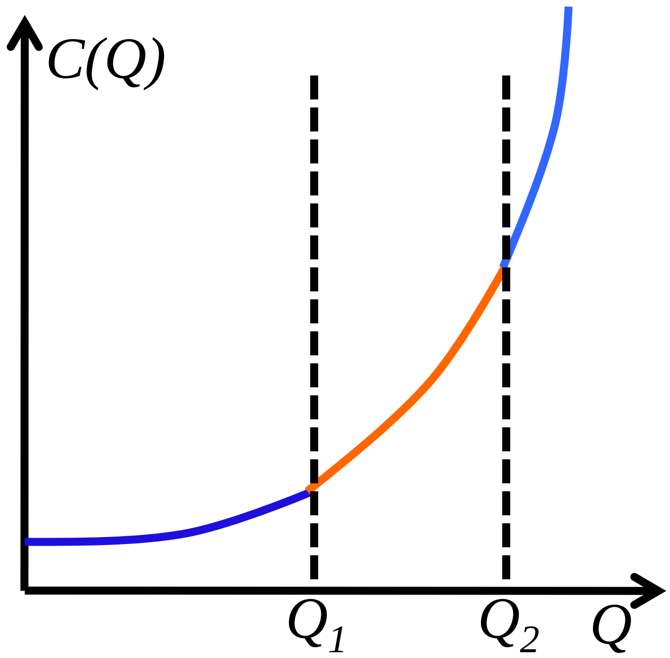
    negative means discharging
  - Load constraints: $r_i \in R_i$
  - Cost function: $D_i\left(r_i\right)$

Utility-side model

- The utility company incurs cost $C(Q)$ when the supply is Q
  - convex, with a positive, increasing marginal cost

- Piecewise quadratic cost functions

$$C(Q) = \begin{cases} c_1 Q^2 + b_1 Q + a_1; & 0 \le Q \le Q_1 \\ c_2 Q^2 + b_2 Q + a_2; & Q_1 \le Q \le Q_2 \\ \vdots & \\ c_m Q^2 + b_m Q + a_m; & Q_{m-1} \le Q \end{cases}$$



with

$$c_m > c_{m-1} > \cdots > c_1 > 0$$

Utility-side model

Objective: induce customers' consumption to maximize **social welfare**

$$\max_{q,r} \sum_i \left(\sum_{a \in A_i} U_{i,a}\left(q_{i,a}\right) - D_i(r_i) \right) - \sum_t C\left(\sum_i Q_i(t) \right)$$

$$\text{s.t.} \quad q_{i,a} \in C_{i,a}$$

$$r_i \in R_i$$

$$0 \le Q_i(t) \le Q_i^{\max}$$

proof of conception, to see how effective real-time pricing can be

Utility-customer interaction

- Utility sets prices $p \triangleq (p(t))_{t\in T}$ to induce customer behaviors
- Customer i maximizes his own **net benefit**

$$
\begin{aligned}
\max_{q_i, r_i} \quad & \sum_a U_{i,a}\left(q_{i,a}\right) - D_i(r_i) - \sum_t Q_i(t)\,p(t) \\
\text{s.t.} \quad & q_{i,a} \in C_{i,a} \\
& r_i \in R_i \\
& 0 \le Q_i(t) \le Q_i^{\max}
\end{aligned}
$$

price-taking

Market equilibrium

- **Definition**: The prices and customer demands $(p^*, q_{i,a}^*, r_i^*)$ is in equilibrium if $(q_{i,a}^*, r_i^*)$ maximizes the social-welfare, and also maximizes customer i net benefit for given price p^*.

- **Theorem**: There exists an equilibrium $(p^*, q_{i,a}^*, r_i^*)$. Moreover, the equilibrium price $p^*(t) = C'(\sum_i Q_i^*(t))$.
  - follow from the welfare theorem and imply that setting the price to be the marginal cost of power is optimal
  - similar proof

Customer-side model (appliances)

Air Conditioner
Refrigerator
Etc

Utility function:

$$U_{i,a}\left(q_{i,a}\right)=\sum_t U_{i,a}\left(T_{i,a}(t),T_{i,a}^{comf}\right)$$

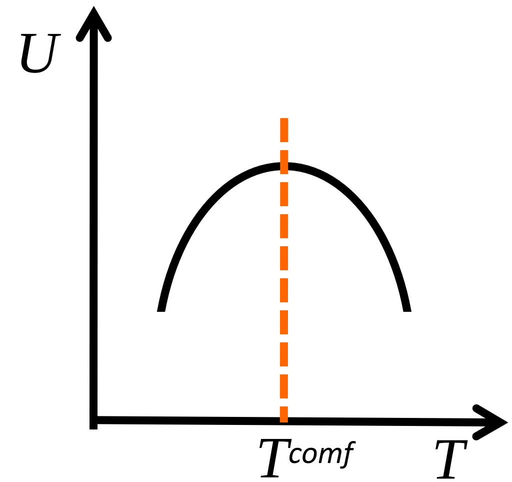
temperature

Constraints:

$$T_{i,a}^{\min} \le T_{i,a}(t) \le T_{i,a}^{\max}$$

$$T_{i,a}(t) = g(T_{i,a}(t-1), q_{i,a}(t))$$

$$0 \le q_{i,a}(t) \le q_{i,a}^{\max}(t)$$

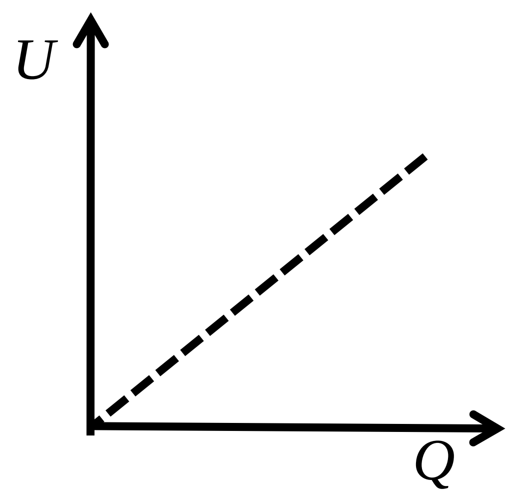


PHEV
Washer
Etc

Utility function: $U_{i,a}\left(q_{i,a}\right)=U_{i,a}\left(\sum_t q_{i,a}(t)\right)$

Constraints: $0 \le q_{i,a}(t) \le q_{i,a}^{\max}(t)$

$$Q_{i,a}^{\min} \le \sum_t q_{i,a}(t) \le Q_{i,a}^{\max}$$

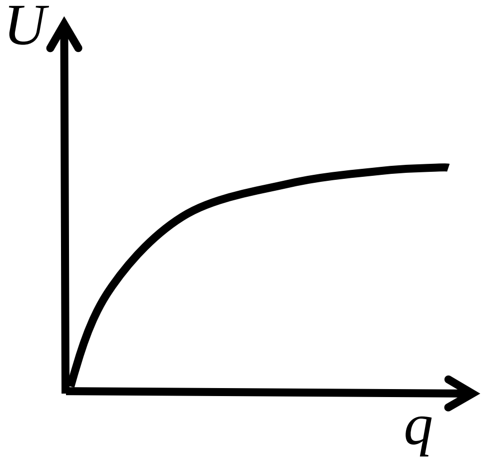


Customer-side model (appliances)

Lighting

Utility function: $U_{i,a}\left(q_{i,a}\right) = \sum_t U_{i,a}\left(q_{i,a}(t), t\right)$

Constraints: $0 \le q_{i,a}(t) \le q_{i,a}^{\max}(t)$



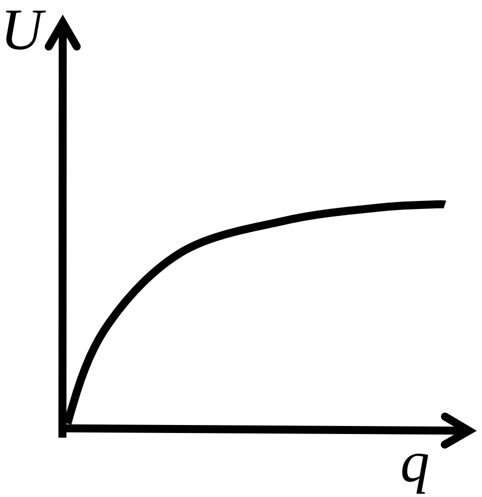
Entertainment

Utility function: $U_{i,a}\left(q_{i,a}\right) = \sum_t U_{i,a}\left(q_{i,a}(t), t\right)$

Constraints: $0 \le q_{i,a}(t) \le q_{i,a}^{\max}(t)$

$$Q_{i,a}^{\min} \le \sum_t q_{i,a}(t) \le Q_{i,a}^{\max}$$

a crude model



Customer-side model (Battery)

Cost function:

$$D_i(r_i) = \eta_1 \sum_t r_i^2(t) - \eta_2 \sum_t r_i(t) r_i(t+1) + \eta_3 \sum_t (\min\{B_i(t) - \delta B_i, 0\})^2$$

charging & discharging

charging -dis cycles

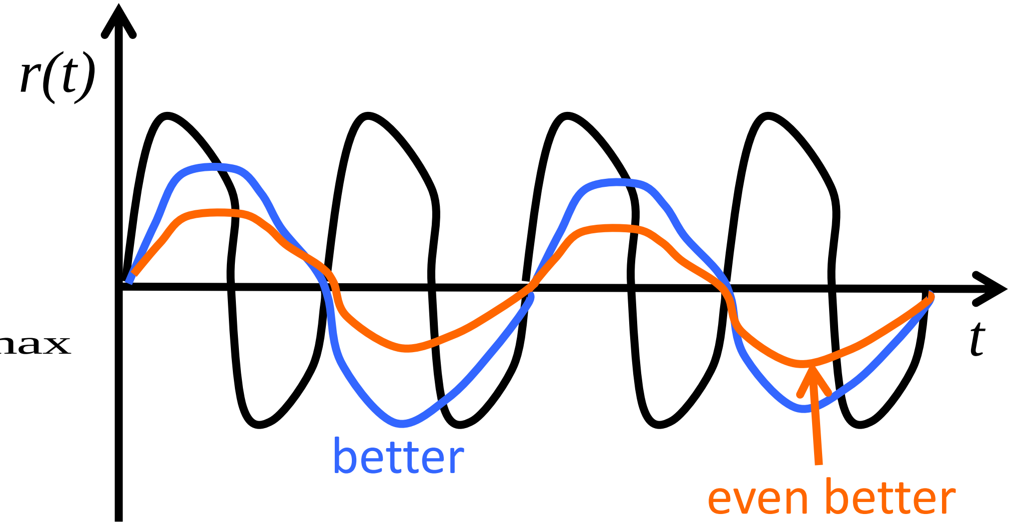
deep discharging

Constraints:

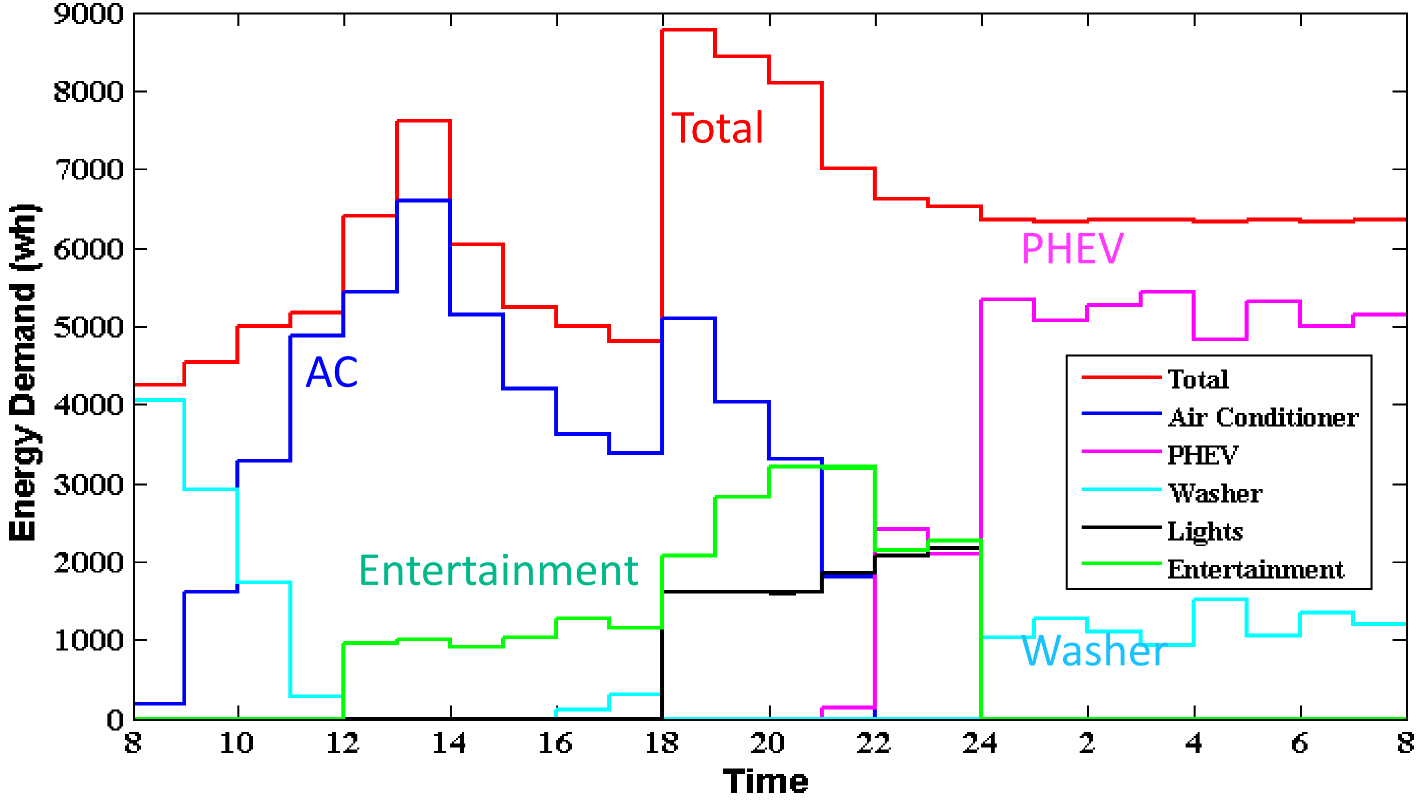
$$0 \le B_i(t) \le B_i$$

$$B_i(T) \ge \gamma_i B_i$$

$$r_i^{\min} \le r_i(t) \le r_i^{\max}$$

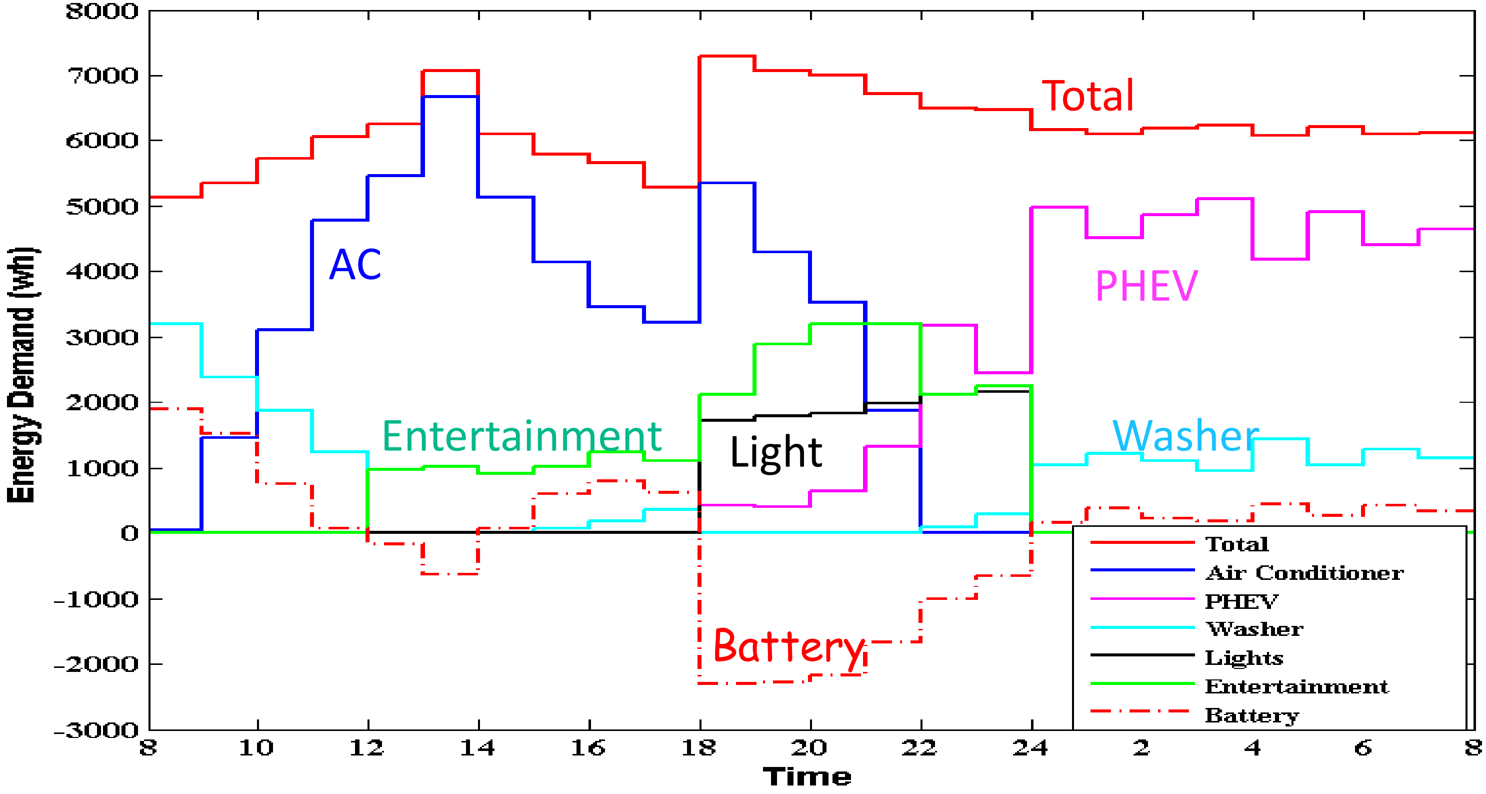


Numerical example: no battery

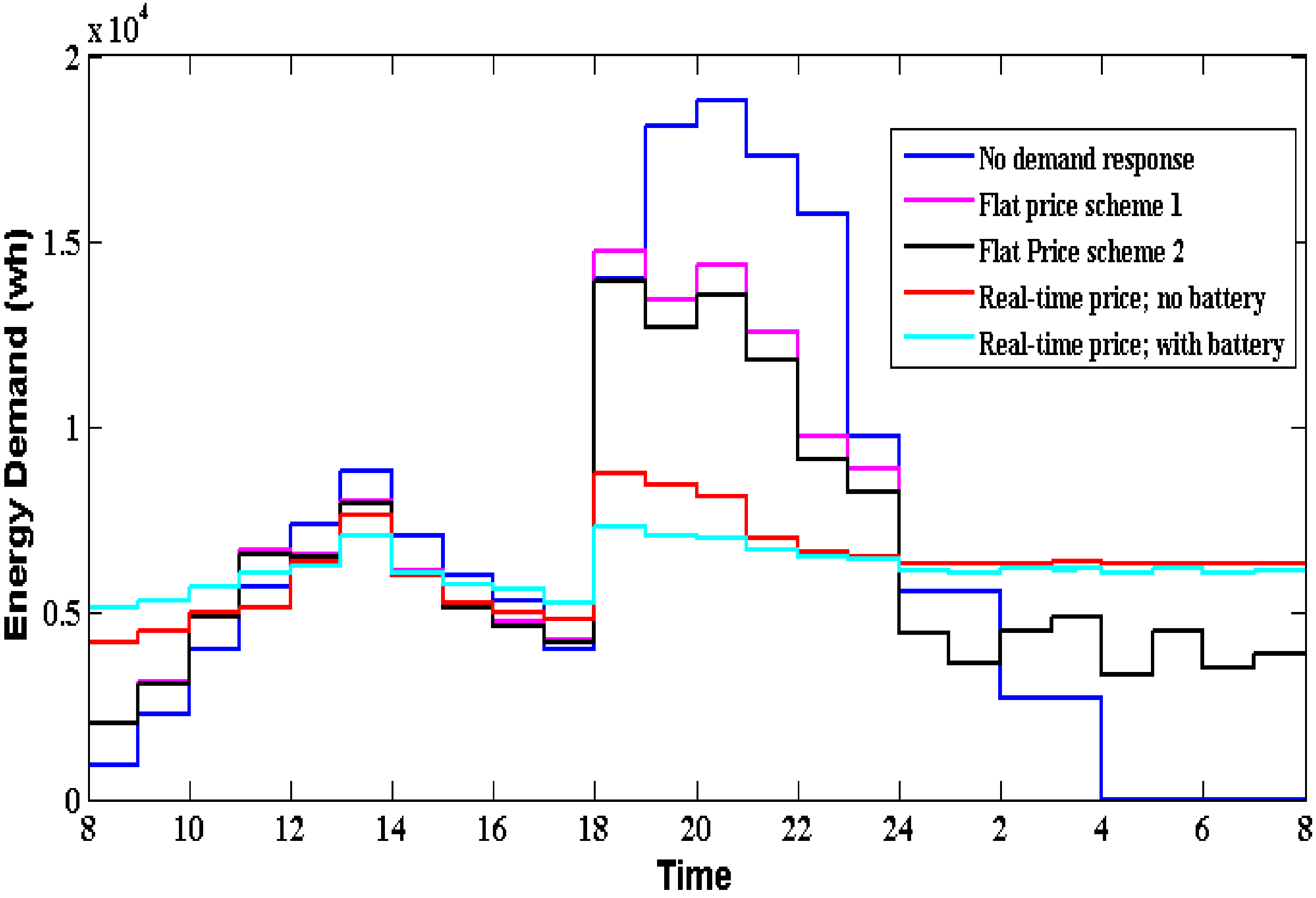


4 households with people at home all the day;
4 with no person at home during day time

Numerical example: with battery



Numerical example



Numerical experiments

$$\text{load factor} = \frac{\text{average load}}{\text{peak load}}$$

Concluding remarks

- Demand response: Match the supply
  - iterative supply function bidding (competitive vs oligopolistic)
- Demand response: Shape the demand
  - Real-time pricing based on marginal cost is “ideally” very effective
- Future work: extend the models to study the afore-mentioned issues in demand response design
  - Current focus: real-time demand response; coordinated control with Volt/Var

References

- *Two Market Models for Demand Response in Power Networks*, L. Chen, N. Li, S. Low and J. Doyle, IEEE SmartGridComm, 2010. http:\\cds.caltech.edu\~chen\papers\DemandResponse.pdf
- *Optimal Demand Response Based on Utility Maximization in Power Networks*, L. Chen, N. Li and S. Low, IEEE PESGM 2011. http:\\cds.caltech.edu\~chen\papers\ODemandResponse.pdf

Thanks