A Measurement-based Framework for Modeling, Analysis and Control of Large-Scale Power Systems using Synchrophasors

Aranya Chakrabortty

Electrical & Computer Engineering Department
Texas Tech University

North Carolina State University, Raleigh, NC
26th April, 2010

Wide Area Measurements (WAMS)

- 2003 blackout in the Eastern Interconnection

→ EIPP (Eastern Interconnection Phasor Project)

→ NASPI (North American Synchrophasor Initiative)

Power System Research Consortium (PSRC, 2006-present)

- Rensselaer (Joe Chow, Murat Arcak)
- Virginia Tech (Yilu Liu)
- Univ. of Wyoming (John Pierre)
- Montana Tech (Dan Trudnowski)

Industry Members

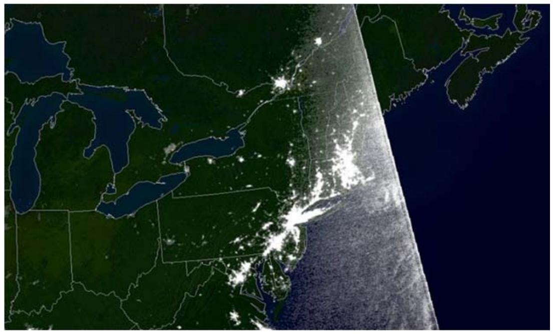
AEP, NEW YORK INDEPENDENT SYSTEM OPERATOR, ISO new england, FirstEnergy, pjm

- Technical Research (RPI)

1. Model Identification of large-scale power systems
2. Post-disturbance data Analysis
3. Controller and observer designs, robustness, optimization

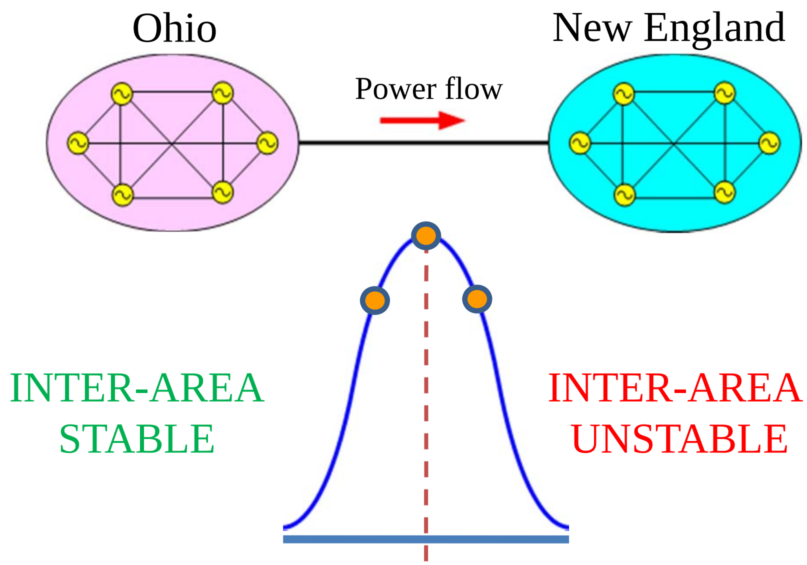
Main trigger: 2003 Northeast Blackout

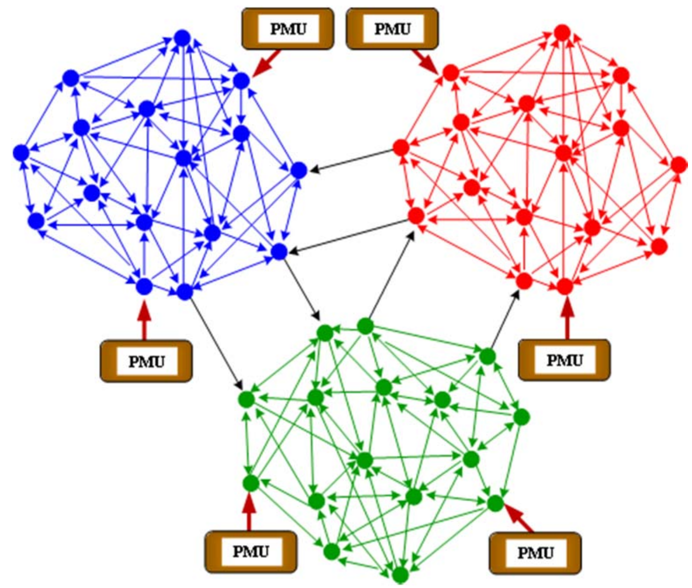
NYC before blackout



NYC after blackout

Hauer, Zhou & Trudnowsky, 2004
Kosterev & Martins, 2004

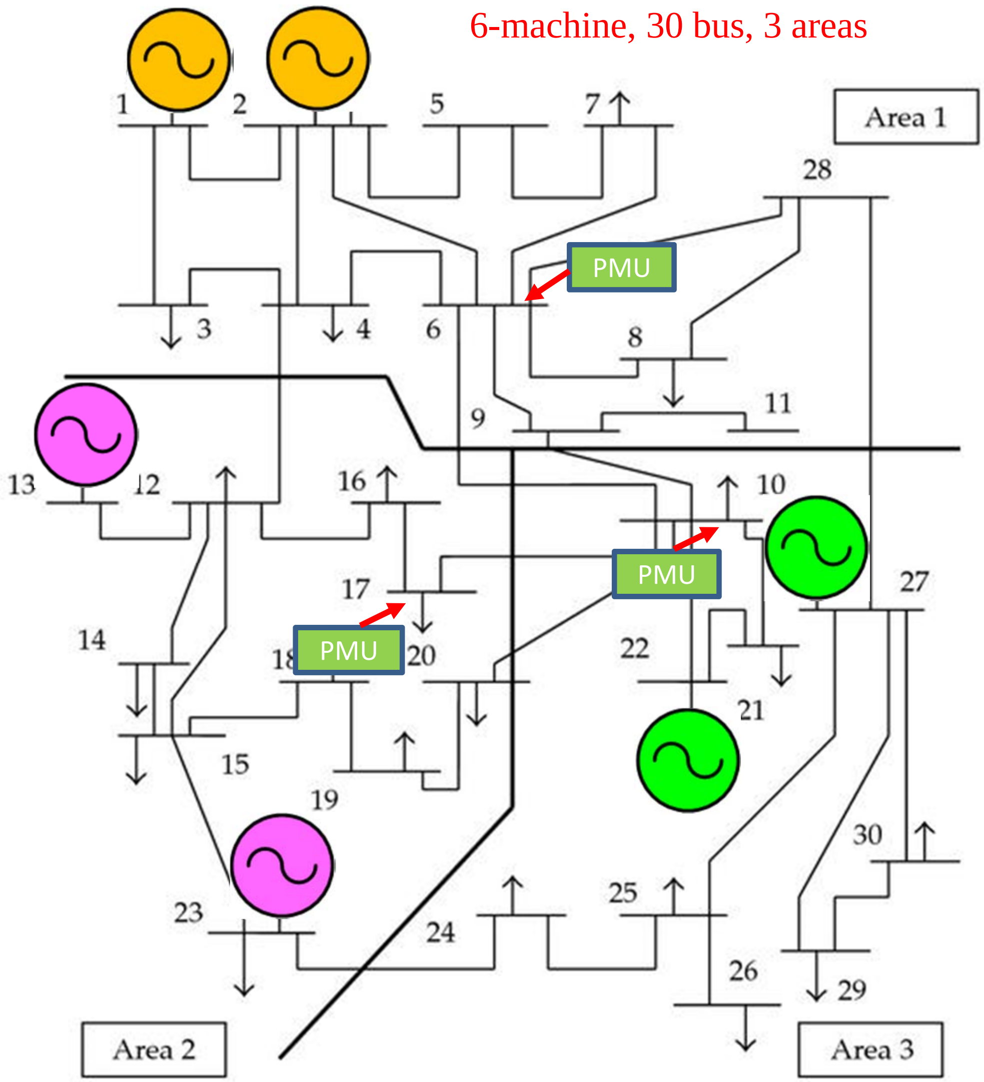




Lesson learnt:

1. Wide-Area Dynamic Monitoring is important
2. Clustering and aggregation is imperative

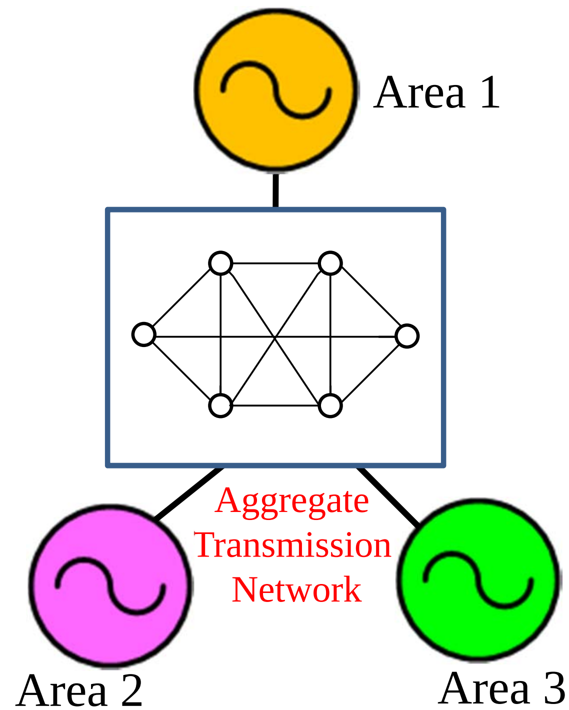
Model Aggregation using distributed PMU data



Problem Formulation:

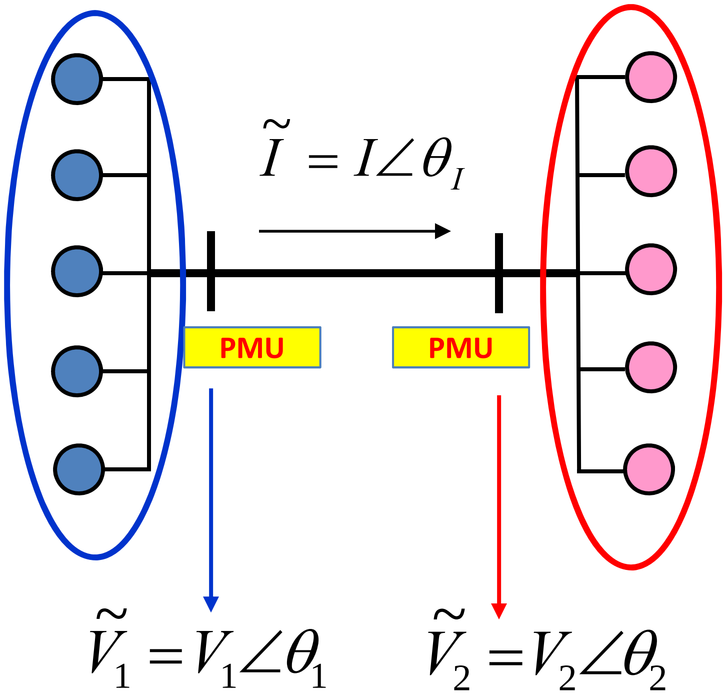
1. Model Reduction

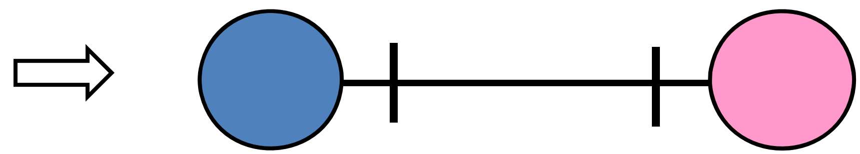
- How to form an aggregate model from the large system

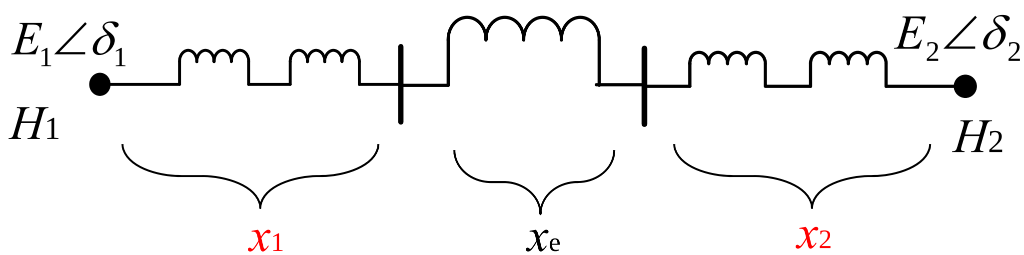


- *Chakrabortty & Chow* (2008, 2009, 2010), *Chakrabortty & Salazar* (2009, 2010)

Two-area Model Estimation







Problem:

How to estimate all parameters?

↓

x_1, x_2, H_1, H_2

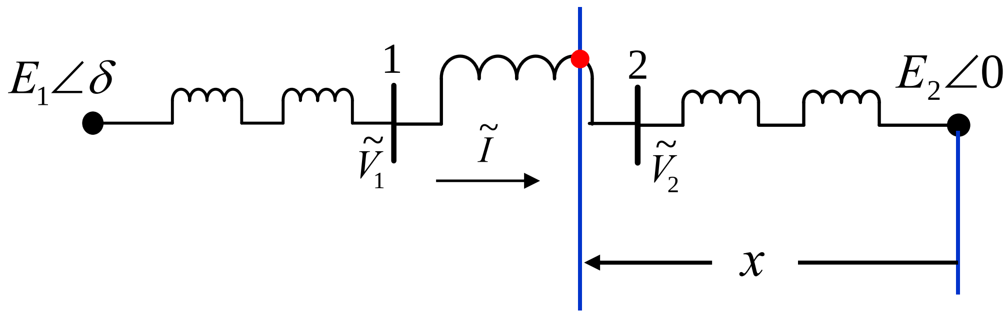
$$\dot{\delta} = \omega$$

$$2\frac{H_1 H_2}{H_1 + H_2}\dot{\omega} = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} - \frac{E_1 E_2}{(x_1 + x_e + x_2)}\sin\delta$$

Swing Equation

IME: Method *(Reactance Extrapolation)*

- **Key idea** : Amplitude of voltage oscillation at any point is a function of its electrical distance from the two fixed voltage sources.



$$\tilde{V}(x) = [E_2(1-a) + E_1 a\cos(\delta)] + j\,E_1 a\sin(\delta), \qquad a = \frac{x}{x_1 + x_e + x_2}$$

- Voltage magnitude : $V = |\tilde{V}(x)| = \sqrt{c + 2E_1E_2(a-a^2)\cos(\delta)}, \qquad c = (1-a)^2 E_2^{\,2} + a^2 E_1^{\,2}$

- Assume the system is initially in an equilibrium $(\delta_0, \omega_0 = 0, V_{ss})$:

$$\Delta V(x) = J(a,\delta_0)\Delta\delta$$

$$J(a,\delta_0) := \left.\frac{\partial V(a,\delta_0)}{\partial\delta}\right|_{\delta=\delta_0} = \frac{-E_1E_2}{V(a,\delta_0)}(a-a^2)\sin(\delta_0)$$

Reactance Extrapolation

$$\Delta V(x,t)V(a,\delta_0) = -E_1E_2\sin(\delta_0)\,(a-a^2)\Delta\delta(t)$$

can be computed from measurements at *x*

A

$$V_n(x,t) = A\,(a-a^2)\Delta\delta(t)$$

Note: *Spatial* and *temporal* dependence are separated

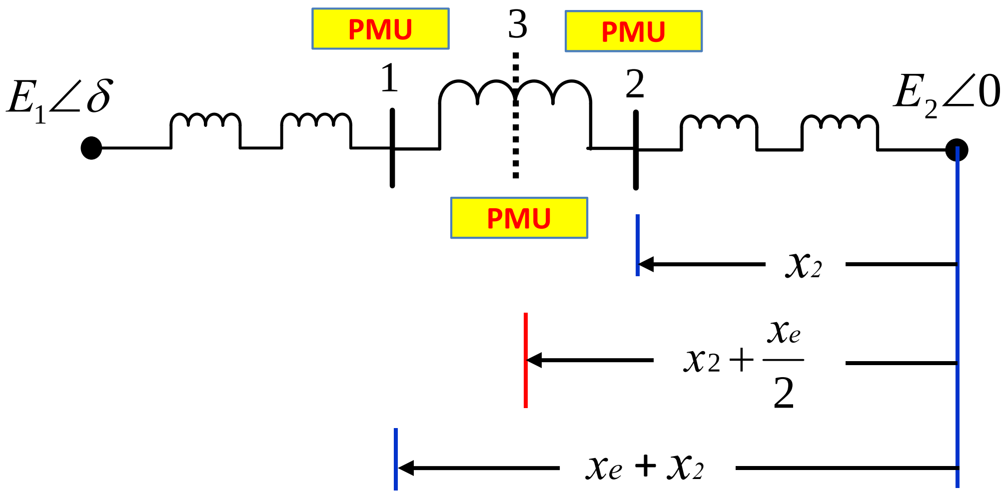
- Fix time: $t=t^*$

$$V_n(x,t^*) = A\,(a-a^2)\Delta\delta(t^*)$$

How can we use this relation to solve our problem?

Reactance Extrapolation

$$V_n(x,t^*) = A\,(a - a^2)\Delta\delta(t^*)$$



At Bus 2, $a_2 = \dfrac{x_2}{x_1 + x_e + x_2} \longrightarrow V_{n,\,Bus2} = A\,(a_2 - a_2^2)\Delta\delta(t^*)$

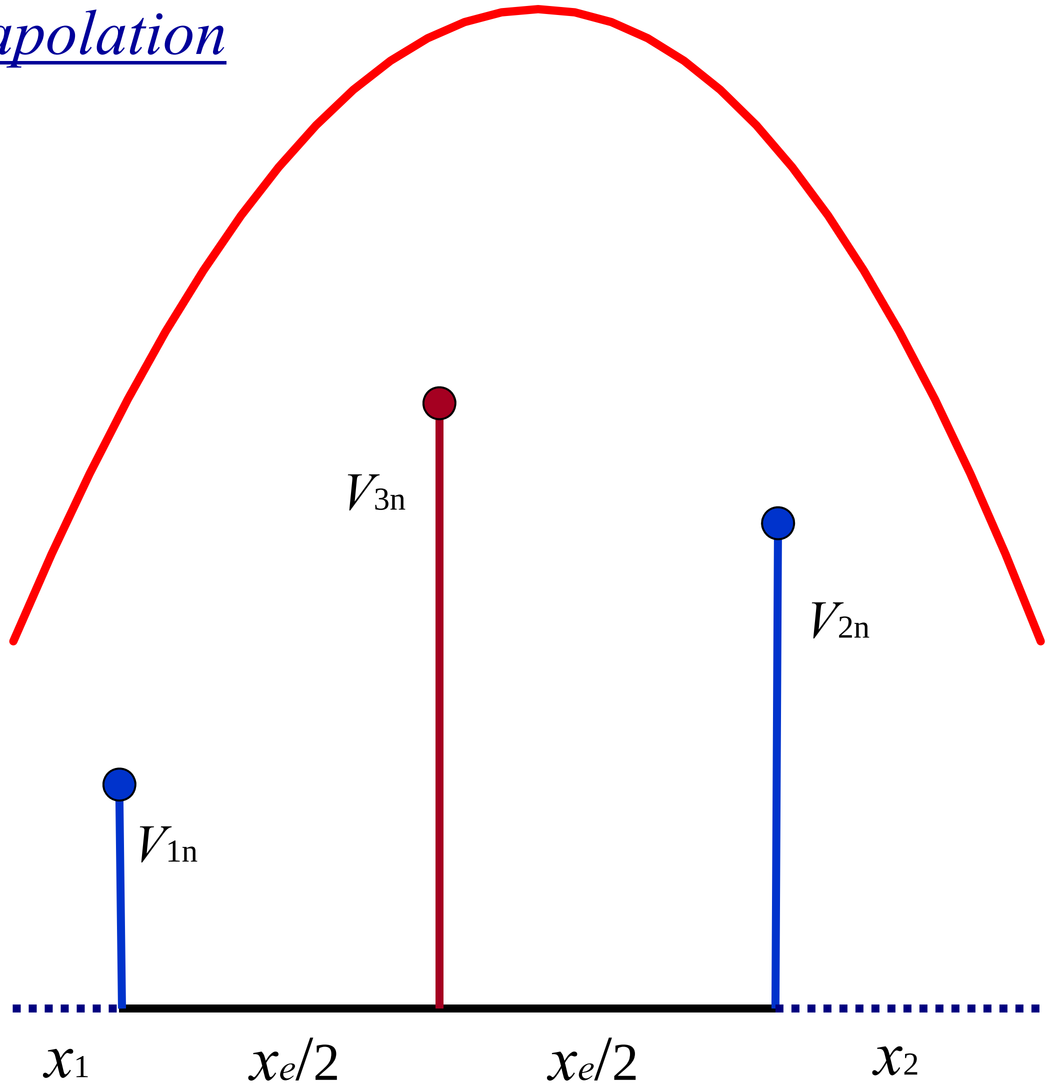
At Bus 1, $a_1 = \dfrac{x_e + x_2}{x_1 + x_e + x_2} \longrightarrow V_{n,\,Bus1} = A\,(a_1 - a_1^2)\Delta\delta(t^*)$

$$\frac{V_{n,\,Bus2}}{V_{n,\,Bus1}} = \frac{a_2(1-a_2)}{a_1(1-a_1)}$$

• Need one more equation
- hence, need one more measurement at a known distance $\longrightarrow \dfrac{V_{n,\,Bus3}}{V_{n,\,Bus1}} = \dfrac{a_3(1-a_3)}{a_1(1-a_1)}$

Reactance Extrapolation

$V_n(a) = A\, a\,(1 - a)$



Key idea: Exploit the spatial variation of phasor outputs

IME: Method *(Inertia Estimation)*

- From linearized model

$$f_s = \frac{1}{2\pi}\sqrt{\frac{E_1E_2\cos(\delta_0)\Omega}{2H(x_e + x_1 + x_2)}}$$

where f_s is the *measured* swing frequency and $H = \frac{H_1H_2}{H_1 + H_2}$

- For a second equation in H_1 and H_2, use *law of conservation of angular momentum*

$$2H_1\omega_1 + 2H_2\omega_2 = 2\int(H_1\dot{\omega}_1 + H_2\dot{\omega}_2)dt = \int(P_{m1} - P_{e1} + P_{m2} - P_{e2})\,dt = 0$$

$$\Rightarrow \boxed{\frac{H_1}{H_2} = -\frac{\omega_2}{\omega_1}}$$

- Reminiscent of Zaborsky's result

$$\frac{H_1}{H_2} = -\frac{\theta_2}{\theta_1}$$

- However, ω_1 and ω_2 are not available from PMU data,

→ Estimate ω_1 and ω_2 from the measured frequencies ξ_1 and ξ_2 at Buses 1 and 2

IME: Method *(Inertia Estimation)*

- Express *voltage angle* θ as a function of δ, and differentiate wrt time to obtain a relation between the machine speeds and bus frequencies:

$$\xi_1 = \frac{a_1\omega_1 + b_1(\omega_1+\omega_2)\cos(\delta_1-\delta_2) + c_1\omega_2}{a_1 + 2b_1\cos(\delta_1-\delta_2) + c_1}$$

$$\xi_2 = \frac{a_2\omega_1 + b_2(\omega_1+\omega_2)\cos(\delta_1-\delta_2) + c_2\omega_2}{a_2 + 2b_2\cos(\delta_1-\delta_2) + c_2}$$

where,

$$a_i = E_1^{\,2}(1-r_i)^2,\;\; b_i = E_1E_2r_i(1-r_i),$$

$$c_i = E_2^{\,2}r_i^{\,2}$$

- ξ_1 and ξ_2 are measured, and a_i, b_i, c_i are known from reactance extrapolation.
- Hence, we calculate ω_1/ω_2 to solve for H_1 and H_2 .

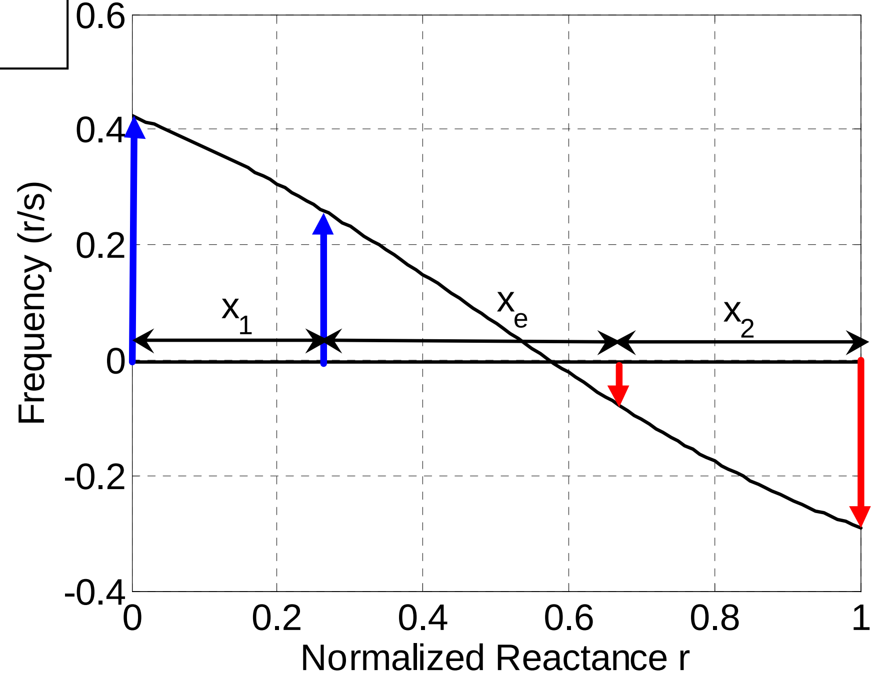
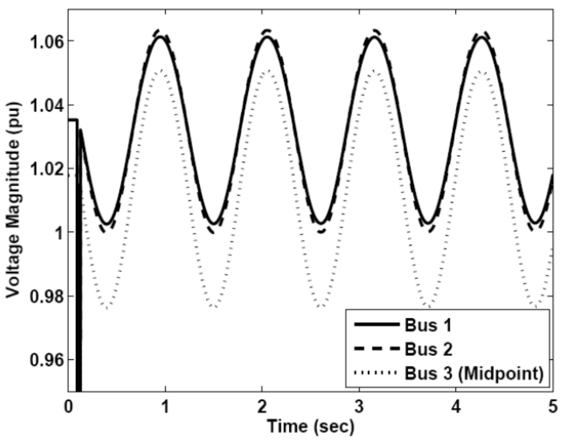


Illustration: 2-Machine Example

- Illustrate IME on classical 2-machine model ($r_e = 0$)
- Disturbance is applied to the system and the response simulated in MATLAB



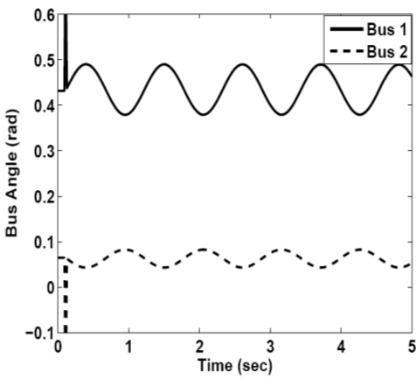
Voltage oscillations at 3 buses

$$V_{1m} = 0.0292 \quad V_{2m} = 0.0316 \quad V_{3m} = 0.0371$$
$$V_{1ss} = 1.0320 \quad V_{2ss} = 1.0317 \quad V_{3ss} = 1.0136$$
$$V_{1n} = 0.0301 \quad V_{2n} = 0.0326 \quad V_{3n} = 0.0376$$

IME Algorithm

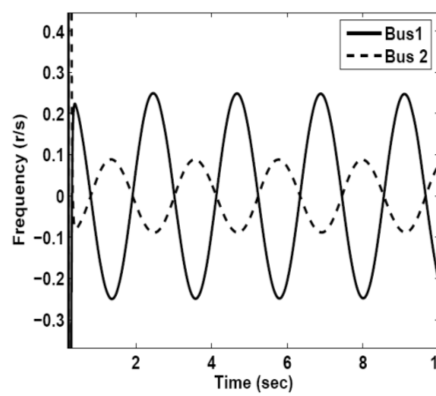
x_1 = 0.3382 pu
x_2 = 0.3880 pu

Exact values:
x_1 = 0.34 pu,
x_2 = 0.39 pu



Bus angle oscillations

$$G(s) = \frac{s}{sT+1}$$



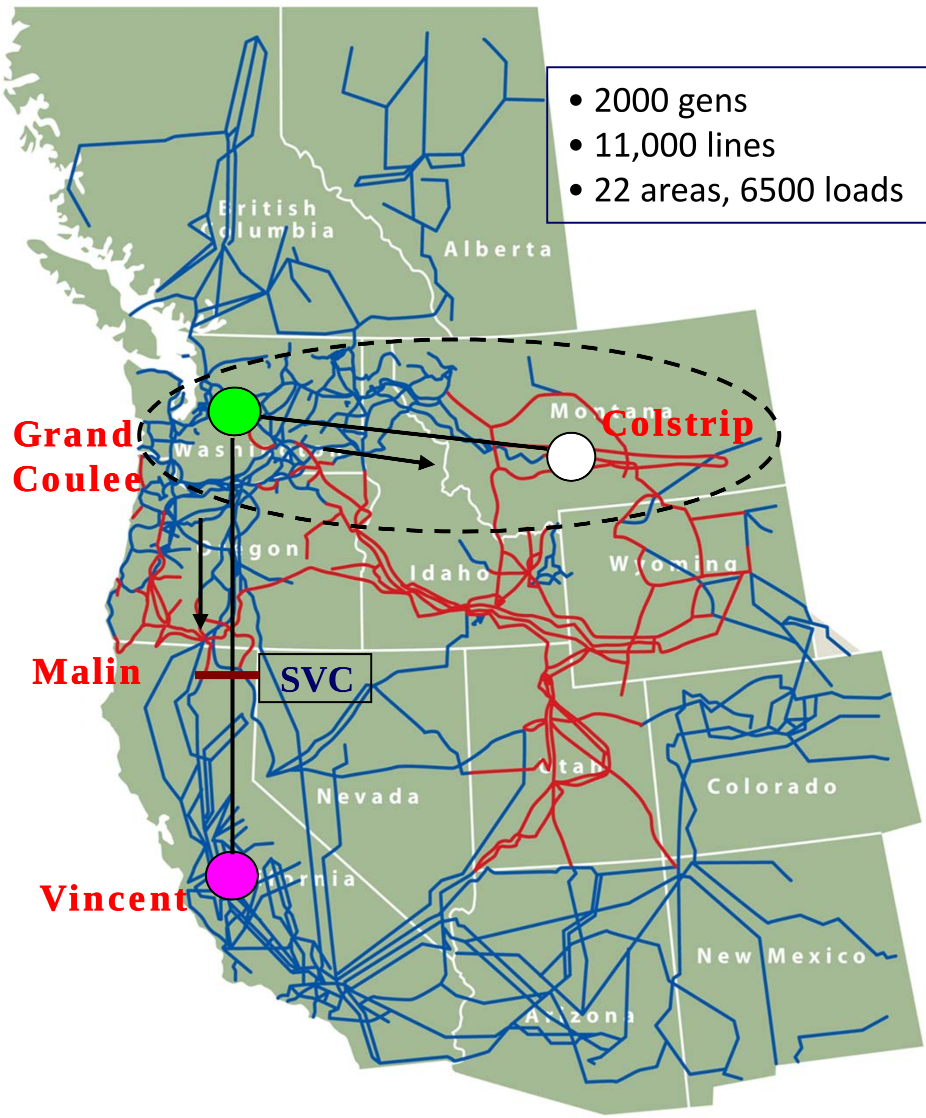
Bus frequency oscillations

IME

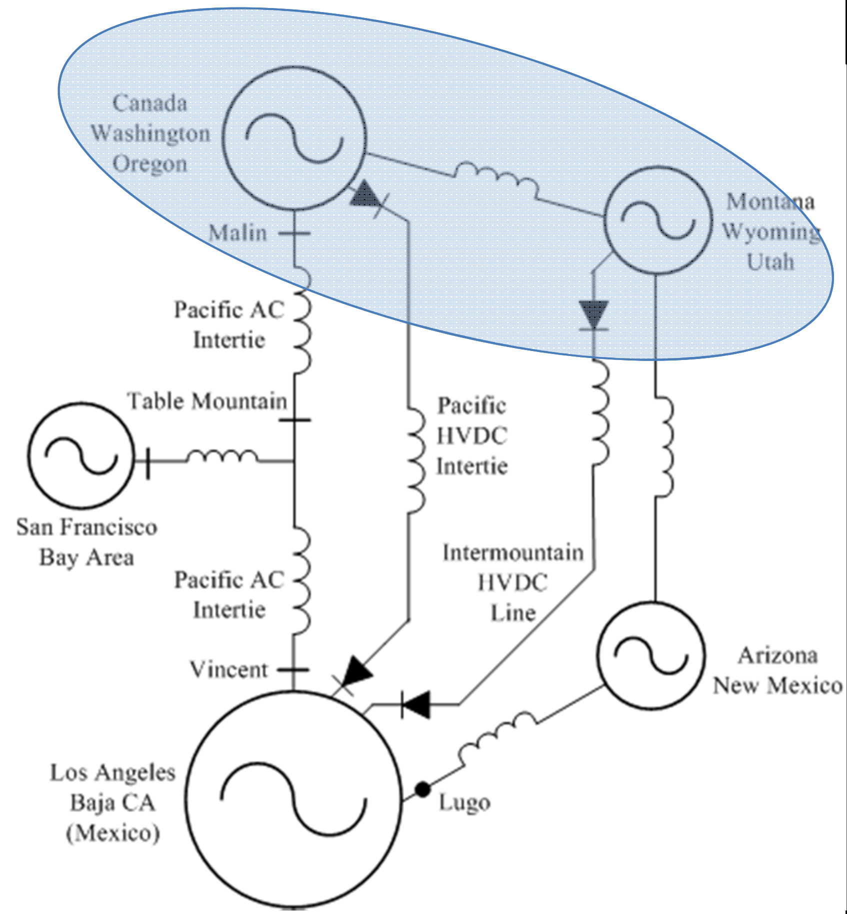
H_1 = 6.48 pu
H_2 = 9.49 pu

Exact values: H_1 = 6.5 pu,
H_2 = 9.5 pu

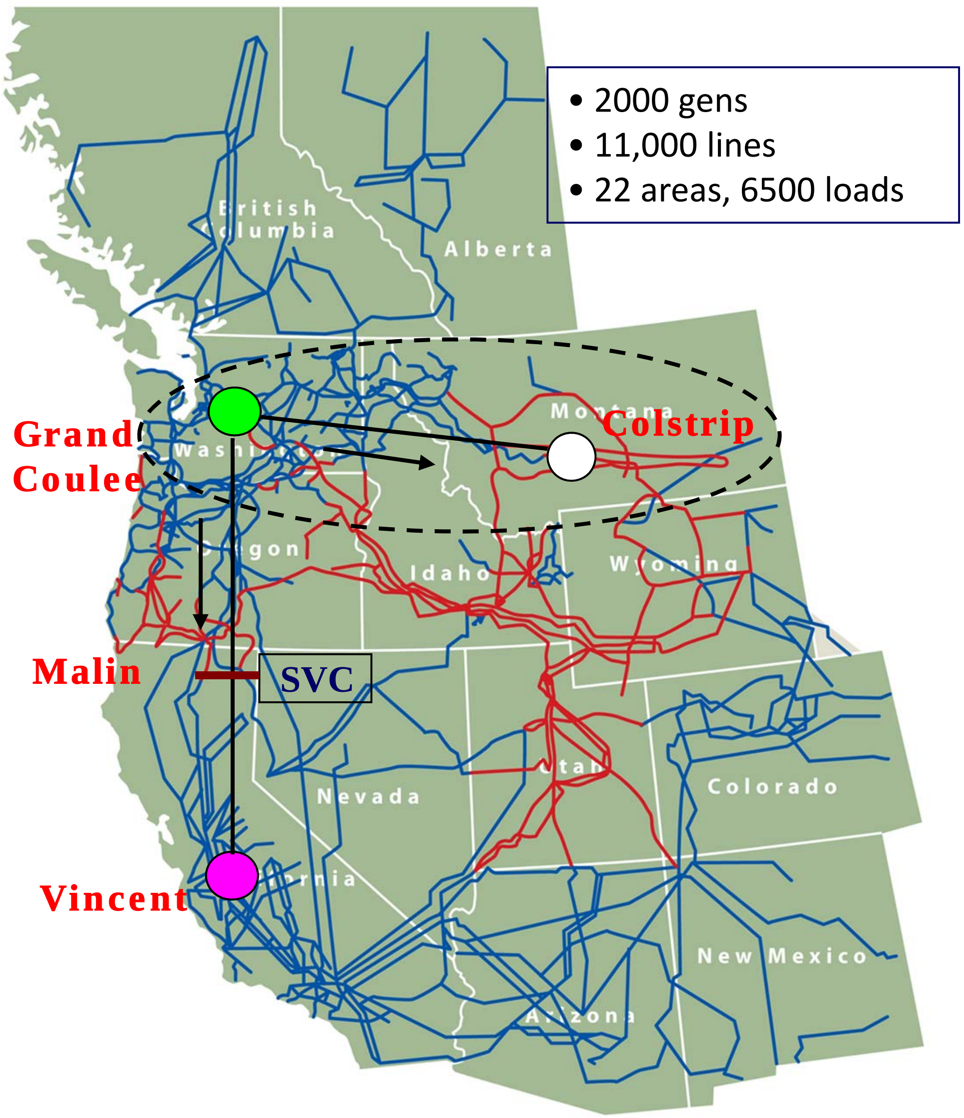
Application to WECC Data

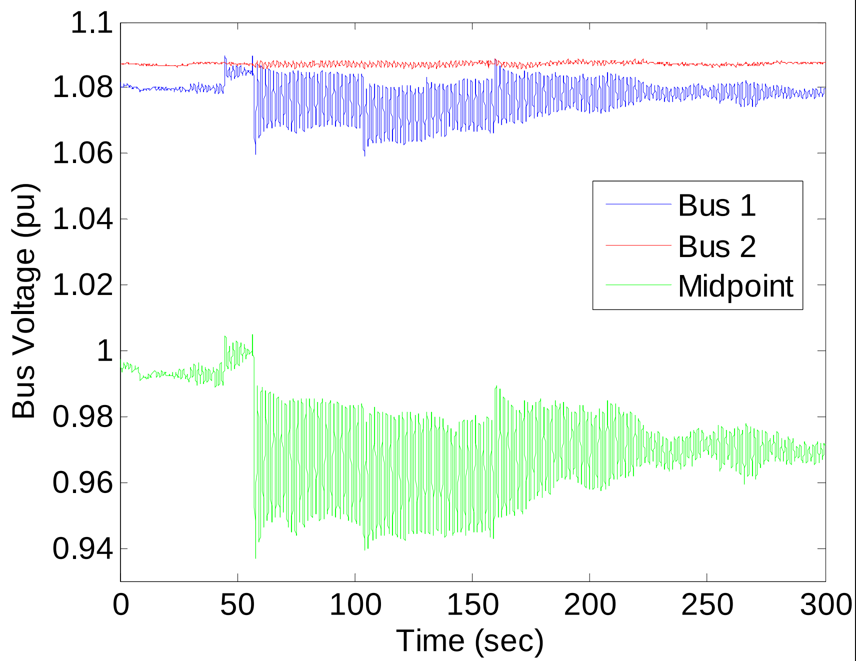


- 2000 gens
- 11,000 lines
- 22 areas, 6500 loads



Application to WECC Data

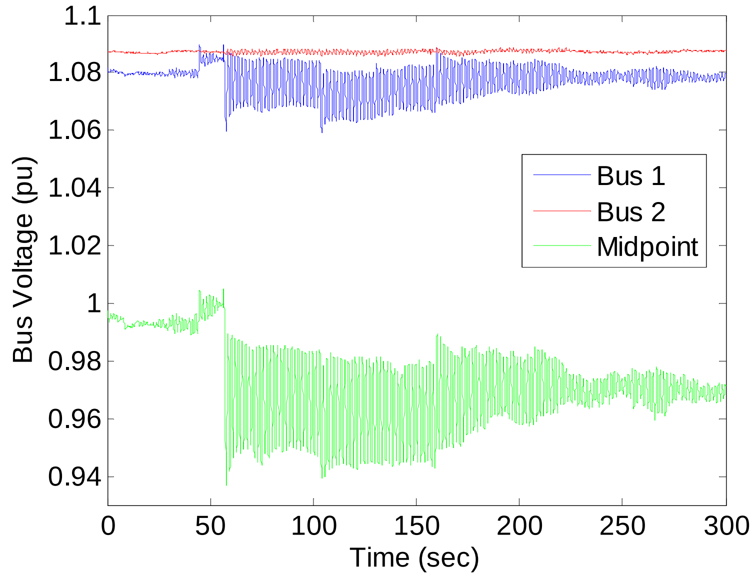




Needs processing to get usable data

- Sudden change/jump
- Oscillations
- Slowly varying steady-state (governer effects)

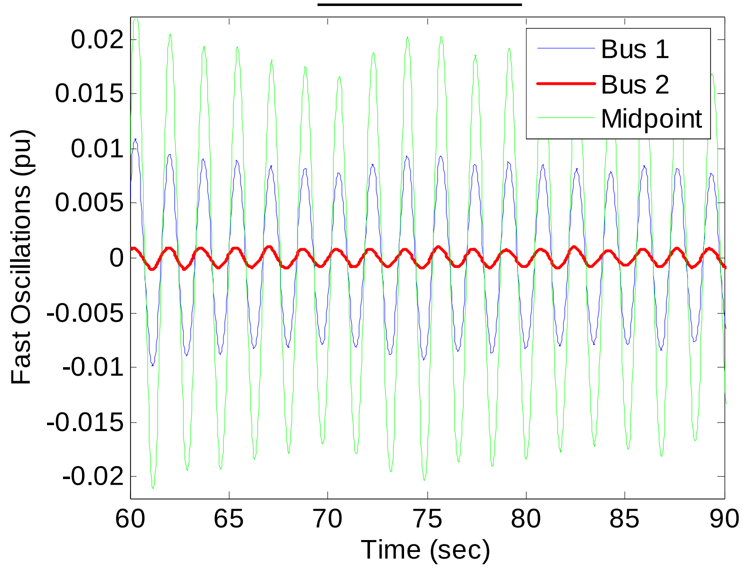
WECC Data



Band-pass Filter

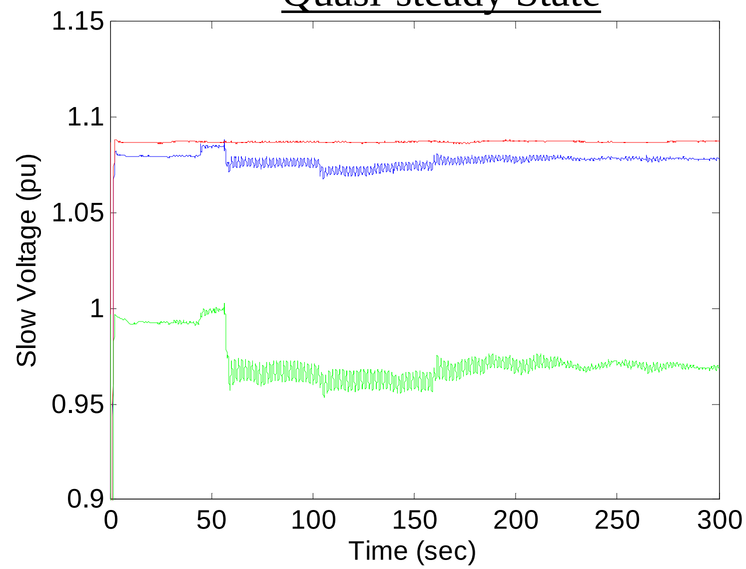
Choose pass-band covering typical swing mode range

Oscillations



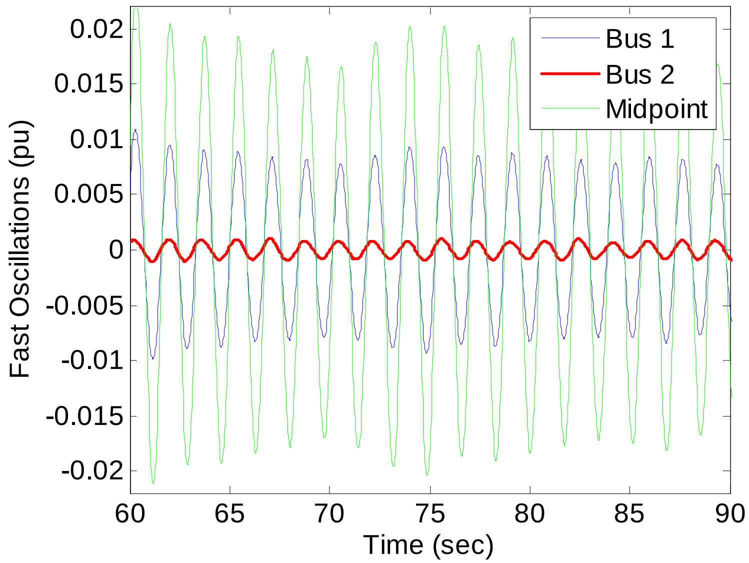
+

Quasi-steady State



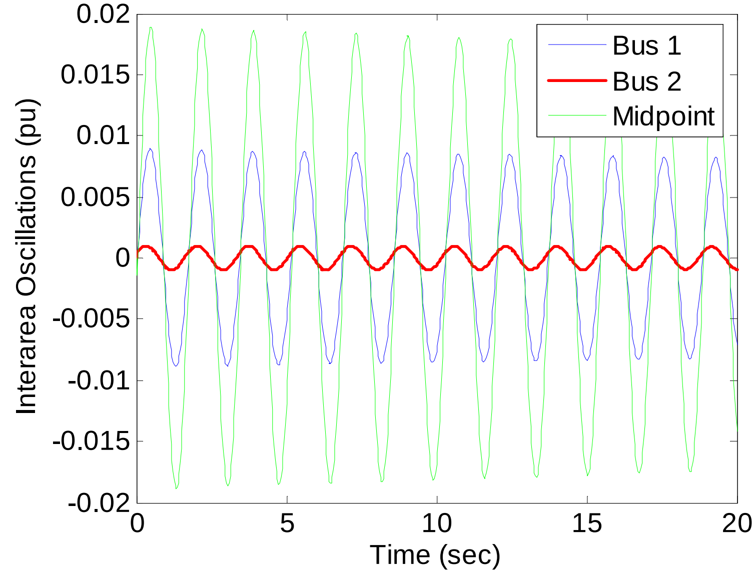
WECC Data

Oscillations

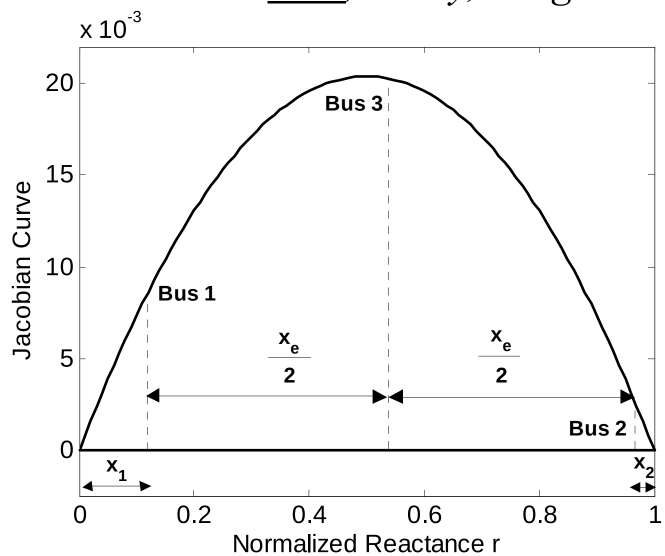


ERA

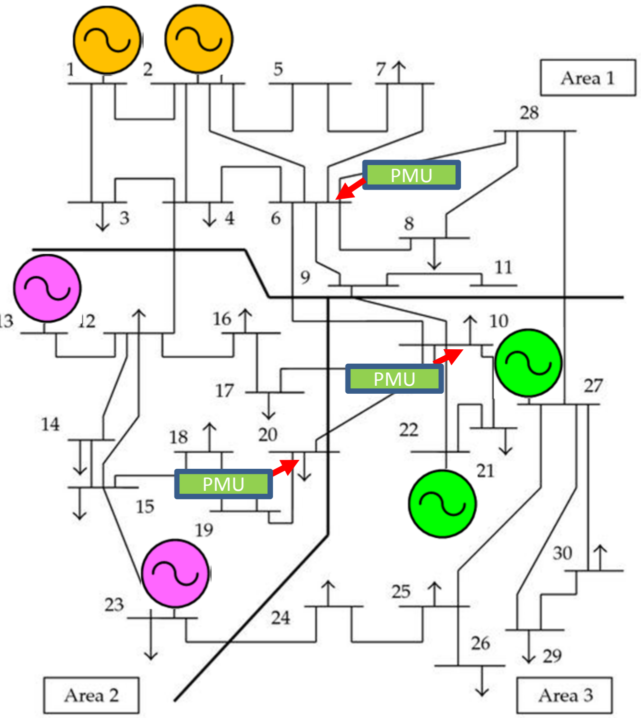
Interarea Oscillations



- Can use modal identification methods such as: *<u>ERA</u>, Prony, Steiglitz-McBride*

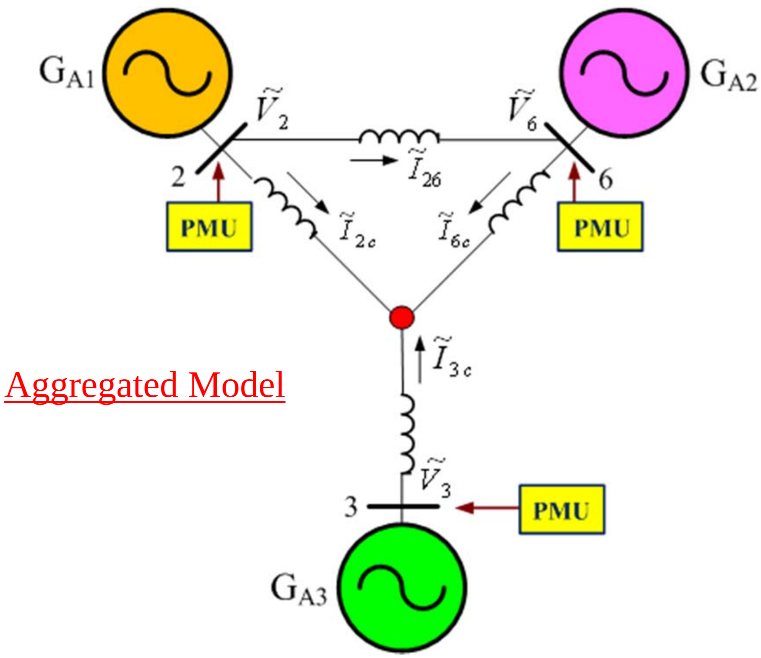


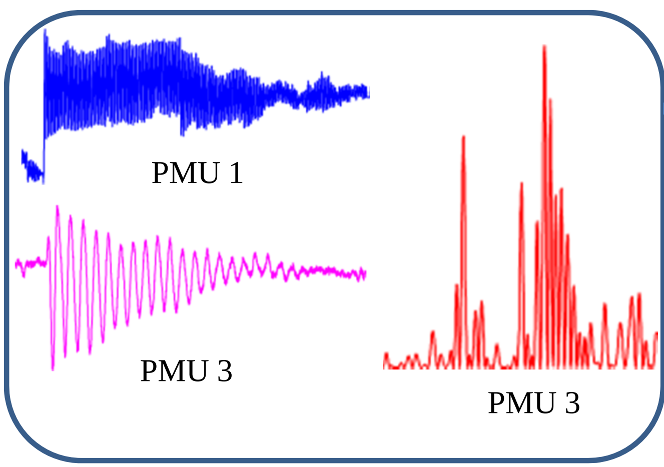
Application for Stability Assessment



Full Model

Aggregated Model



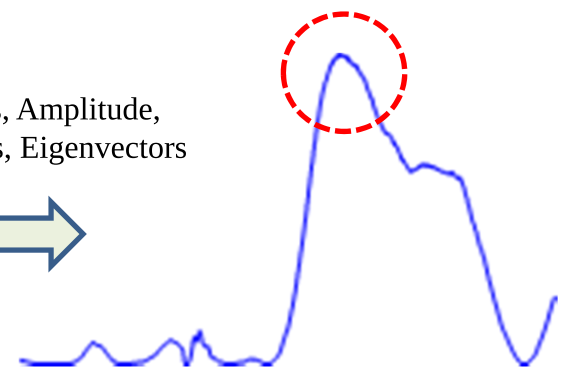


Signal Processing

+

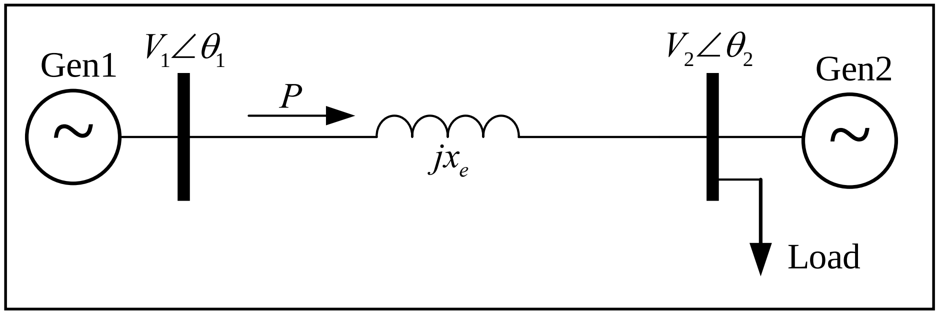
Aggregated Model

Modes, Amplitude, Residues, Eigenvectors



Metrics indicating the dynamic interaction between the areas

Energy Functions for Two-machine System



$$\dot{\delta} = \Omega\delta, \quad 2H\dot{\omega} = P_m - \frac{E_1E_2\sin(\delta)}{x_e'}$$

$$P = \frac{E_1E_2\sin(\delta)}{x_e'}$$

$$S = S_1 + S_2 = \sum_{j=1}^{n(n-1)/2} \int_{\delta_{ij}*}^{z_j} \psi_j(k)\,dk + \sum_{j=1}^{n} \frac{M_j}{2}\xi_j^2$$

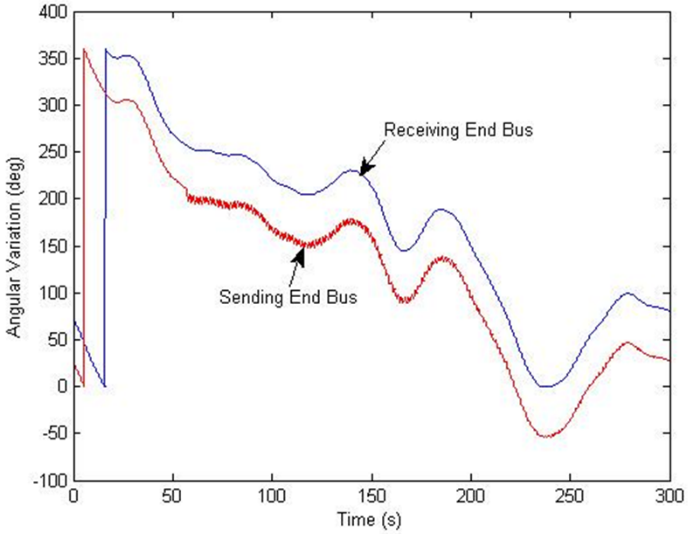
$$= \underbrace{\frac{E_1E_2}{x_e'}[\cos(\delta_{op}) - \cos(\delta) + \sin(\delta_{op})(\delta_{op} - \delta)]}_{\text{Potential Energy}} + \underbrace{H\omega^2}_{\text{Kinetic Energy}}$$

Using IME algorithm: x_e' E_1, E_2, $\delta = \delta_1 - \delta_2$, δ_{op}, $\omega = \omega_1 - \omega_2$ & H are computable from

x_e, V_1, V_2, $\theta = \theta_1 - \theta_2$, θ_{op}, $v = v_1 - v_2$ & ω_s

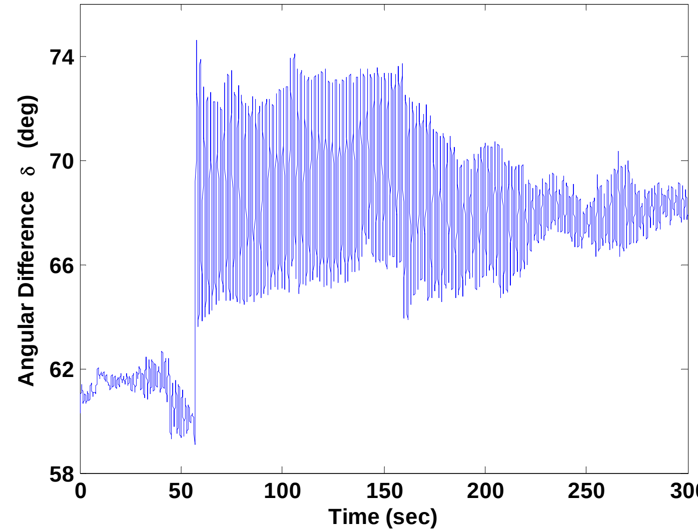
- Note : θ_{op} = *pre-disturbance angular separation*

Energy Functions for WECC Disturbance Event

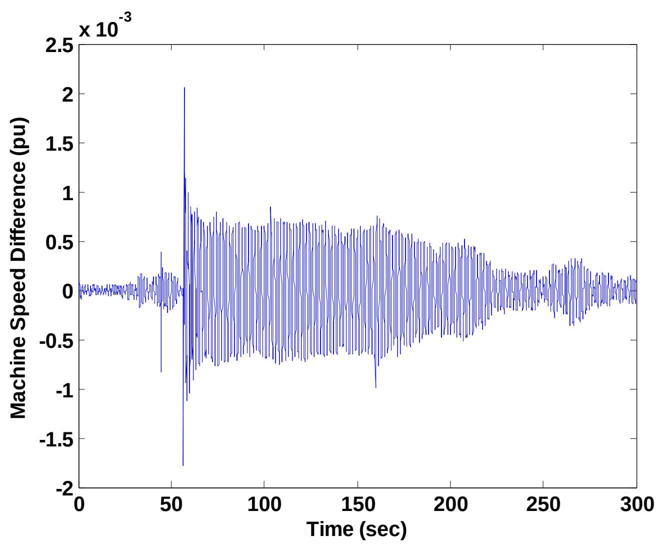


Sending End and Receiving End Bus Angles

IME



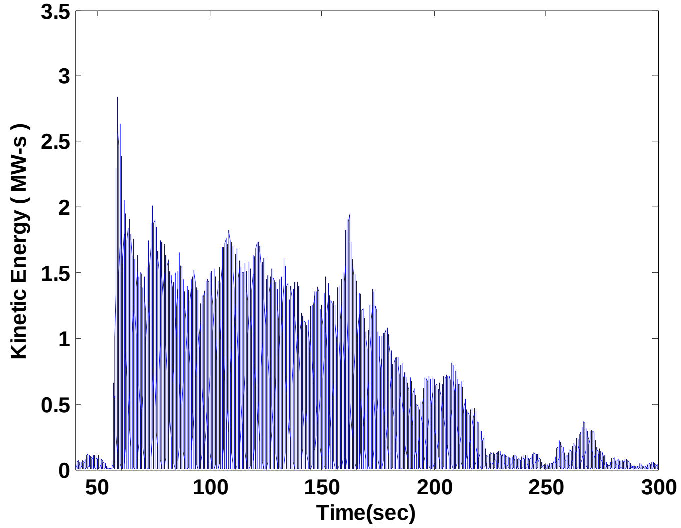
Angle difference between machine internal nodes



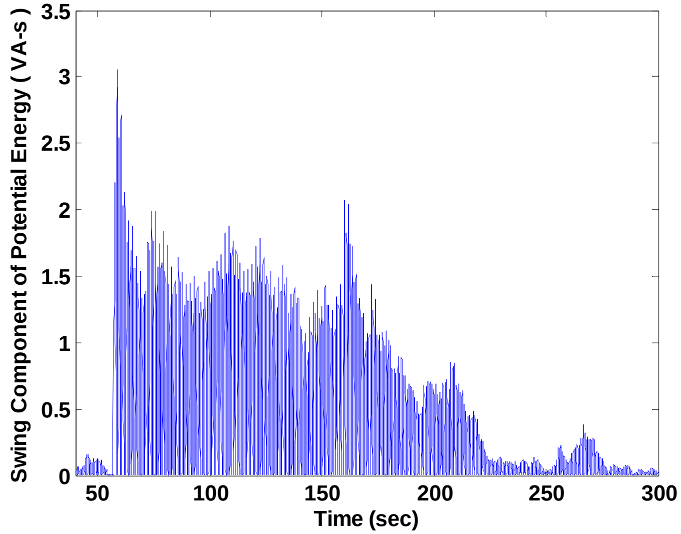
Machine speed difference

- *Chow* & *Chakrabortty* (2007)

Energy Functions for WECC Disturbance Event

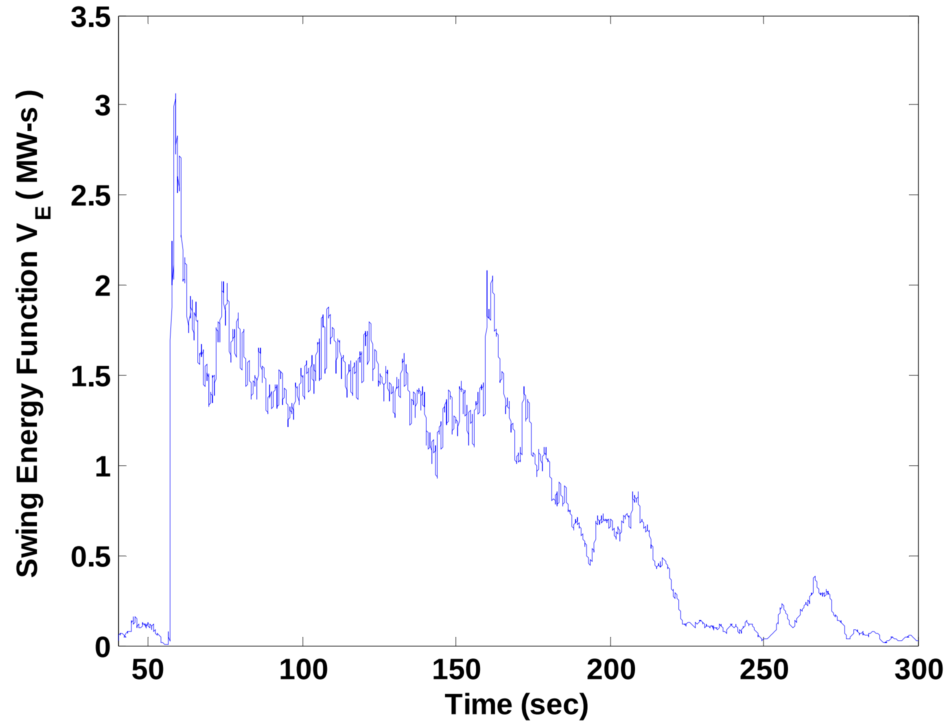


Kinetic Energy



Potential Energy

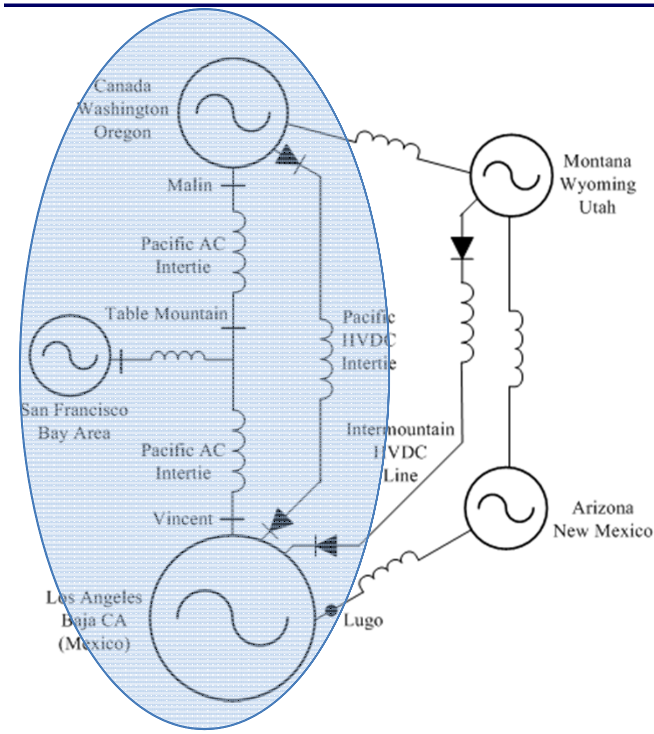
Total Energy = Kinetic Energy + Potential Energy

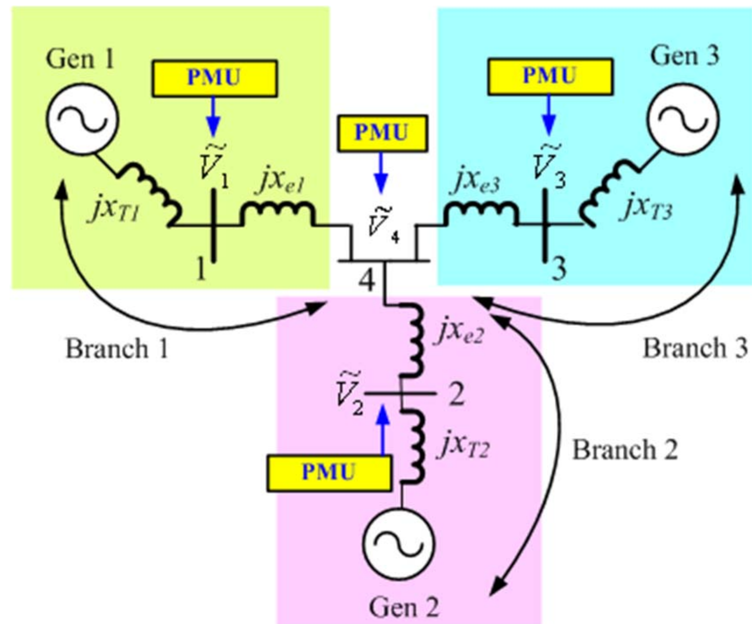


- Total energy decays exponentially – *damping stability*
- Total energy <u>does not</u> oscillate – *Out - of - phase osc.*

  – *Damped pendulum*

More than Two Areas: Pacific AC Intertie





Salient Points

- Current in each branch is different
- No *single* spatial variable *a*
- Derivations need to be done *piecewise* (each edge of the star)
- Two interarea modes/ relative states – δ_1 & δ_2

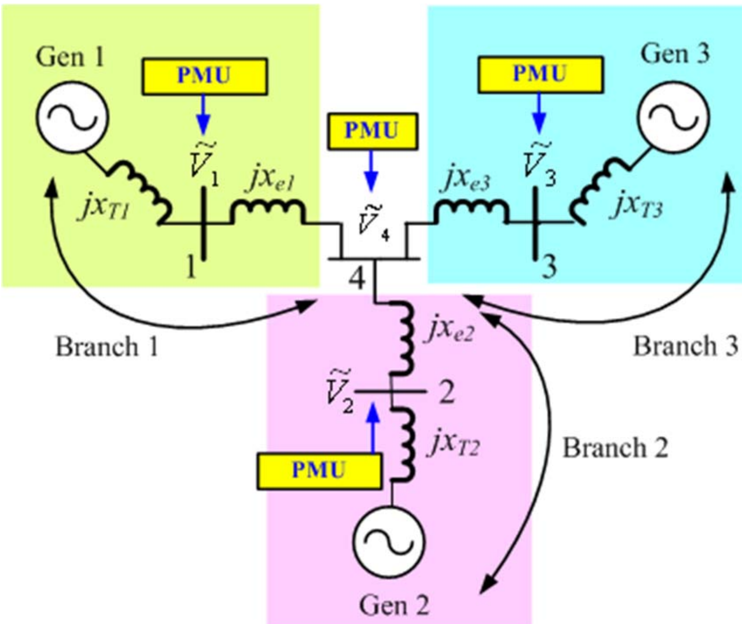
$$V_n = J_1(x)\Delta\delta_1(t) + J_2(x)\Delta\delta_2(t)$$

$$\frac{V_n^{\,3}}{V_n^{\,4}} = \frac{J_1^{\,3}(x)\Delta\delta_1(t^*) + J_2^{\,3}(x)\Delta\delta_2(t^*)}{J_1^{\,4}(x)\Delta\delta_1(t^*) + J_2^{\,4}(x)\Delta\delta_2(t^*)}$$

Time-space separation property lost!

- *Chakrabortty & Salazar* (2009, 2010)

Pacific AC Intertie



Solution – Use phase angle as a 2nd degree of freedom

$$\theta = \tan^{-1}\left(\frac{f_1(x)\sin(\delta_1) + f_2(x)\sin(\delta_2)}{f_1(x)\cos(\delta_1) + f_2(x)\cos(\delta_1) + f_3(x)}\right)$$

$$\Delta\theta(t) = S_1(x)\Delta\delta_1(t) + S_2(x)\Delta\delta_2(t)$$

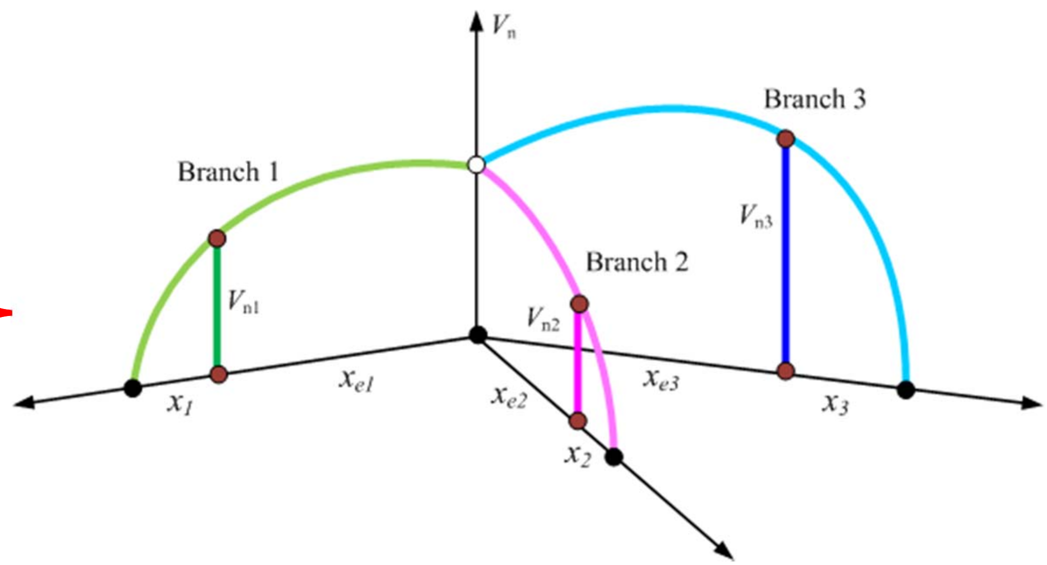
Measurable if a PMU is installed at that point

Voltage equation

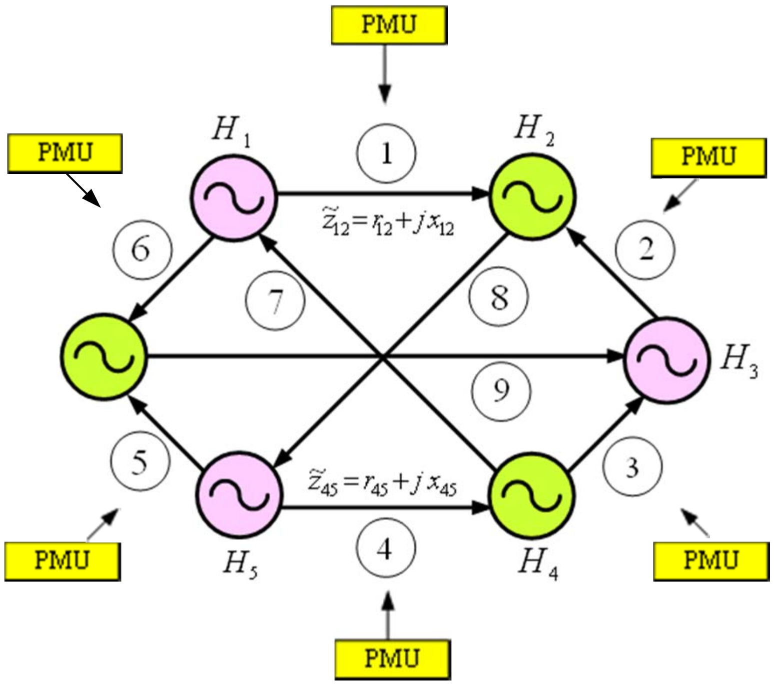
$$\frac{V_n^{\ 3}}{V_n^{\ 4}} = \frac{J_1^{\ 3}(x)\Delta\delta_1(t^*) + J_2^{\ 3}(x)\Delta\delta_2(t^*)}{J_1^{\ 4}(x)\Delta\delta_1(t^*) + J_2^{\ 4}(x)\Delta\delta_2(t^*)}$$

Phase equation

$$\frac{\Delta\theta^3}{\Delta\theta^4} = \frac{S_1^{\ 3}(x)\Delta\delta_1(t^*) + S_2^{\ 3}(x)\Delta\delta_2(t^*)}{S_1^{\ 4}(x)\Delta\delta_1(t^*) + S_2^{\ 4}(x)\Delta\delta_2(t^*)}$$



PMU Placement Problem



- If a tie-line has PMU's at both ends, what is the best point of measurement for identification?

Especially if the PMU data are noisy and unreliable?

Model : $\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ \mathcal{L} & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ \mathcal{E}_j \end{bmatrix} u$

Want to estimate : r_j, x_j, H_{1j}, H_{2j}

- For any edge j : $V_j(a_j, k) = \psi(a_j) \sum_{i=1}^{n} (A_{ji} m_i^{k-1} + A^*_{ji} \, m^*_i{}^{k-1}) \, u(k-1)$

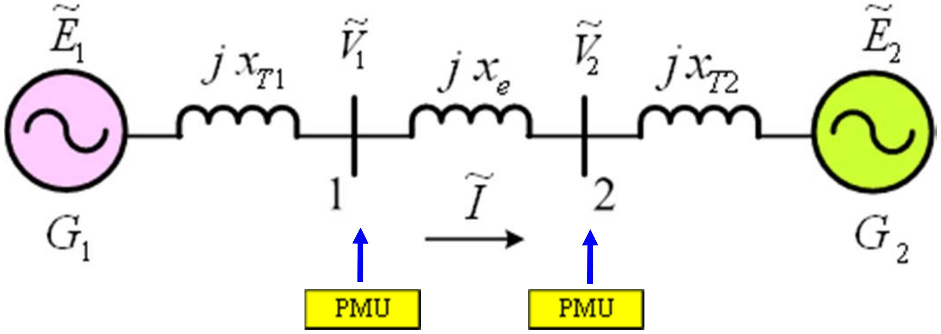
Spatial variable

- Stack up measurements & define : $H = \dfrac{\partial V(k)}{\partial(x_1, x_2)}$ $K = \dfrac{\partial V(k)}{\partial(H_1, H_2)}$ $\Rightarrow$ $J = \begin{bmatrix} HH^T & HK^T \\ KH^T & KK^T \end{bmatrix}$

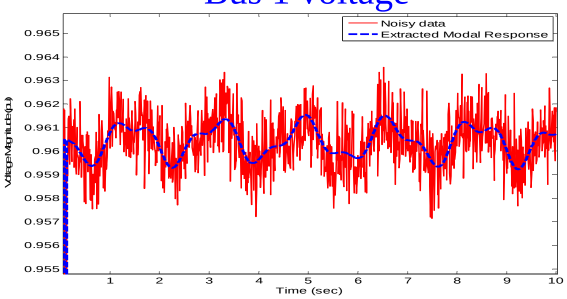
Fisher Information Matrix depends on a_j

- Problem statement: Find optimal a_j s.t. *Cramer-Rao Bound* is minimised

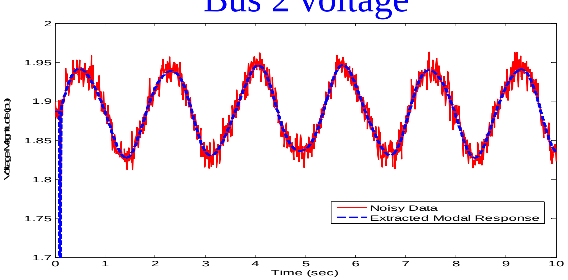
The PMU Allocation Problem



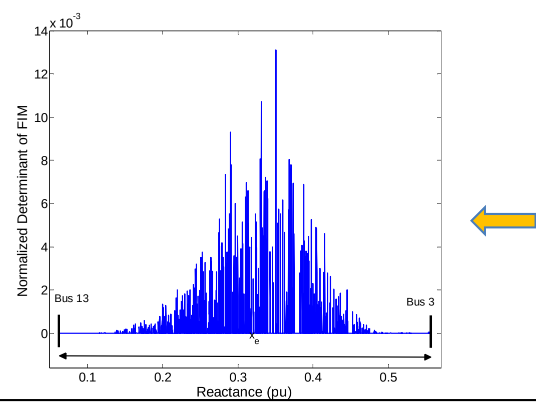
Bus 1 voltage



Bus 2 voltage



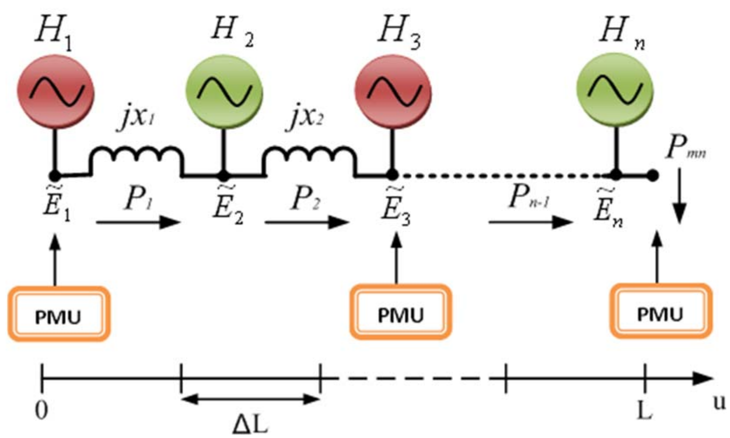
Parameters	Actual Values	Iteration 1	Iteration 1	Iteration 1	Iteration 1	Iteration 1
r	0.1	0.05	0.06	0.08	0.08	0.093
x	1	0.58	0.64	0.78	0.83	0.981
H_1	19	15.65	17.91	17.65	18.55	18.98
H_2	13	10.86	10.91	11.68	12.35	12.75
a	-	0.428	0.412	0.409	0.408	0.408



Spatial variation of
the determinant of the FIM

• *Chakrabortty & Sczodrak (2010)*

PMU based Disturbance Modeling



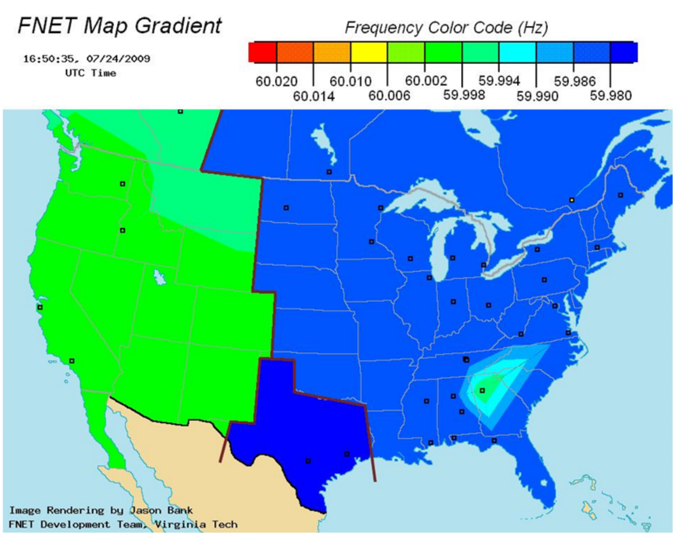
Hauer (1983), Thorp & Parashar (2003)

Swing Equation becomes a PDE

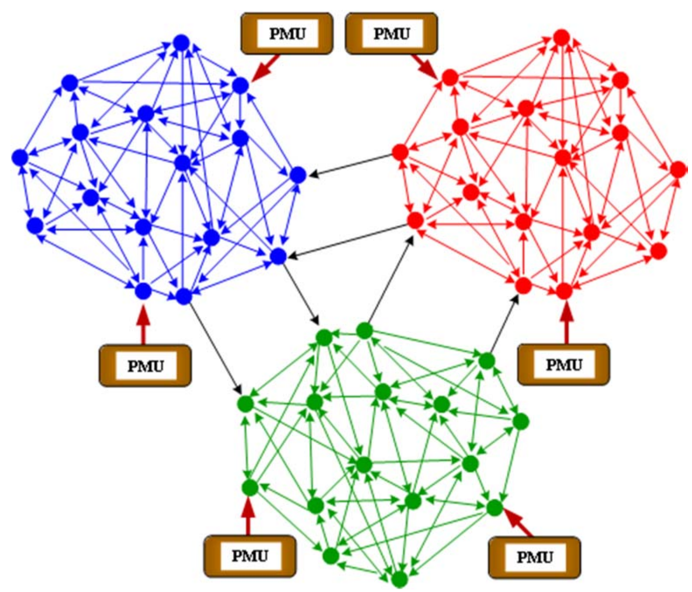
$$\frac{\partial^2 \delta}{\partial^2 t} = c^2 \frac{\partial^2 \delta}{\partial^2 x}$$

(Source behind the Frequency waves in FNET)

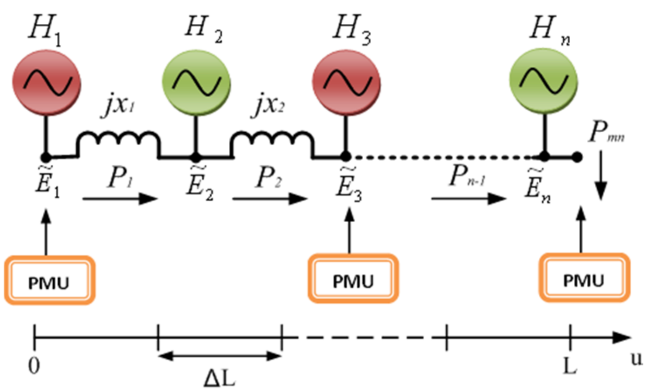
How to model for a network with given topology using PMUs ?



Frequency Distribution captured by FNET-FDRs



PMU based Disturbance Modeling

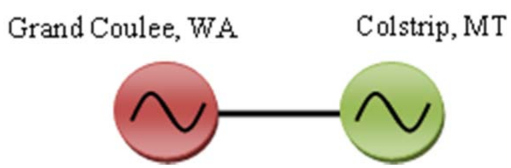


- Is there an analytical model for disturbance propagation in power networks?

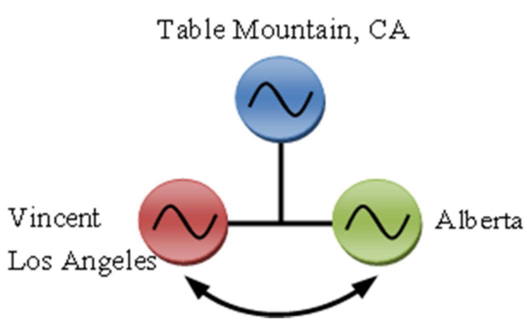
$$\left.\begin{aligned} 1 - p_i(t) &= \{(1 - p_i(t-1)) + \mu p_i(t-1)\}\xi_i(t), \;\; i = 1,2\ldots, N \\ \xi_i(t) &= \prod_{j \in N_i} (1 - \beta p_j(t-1)) \end{aligned}\right\}$$

→ Well-used Markovian Models in Computer Networks

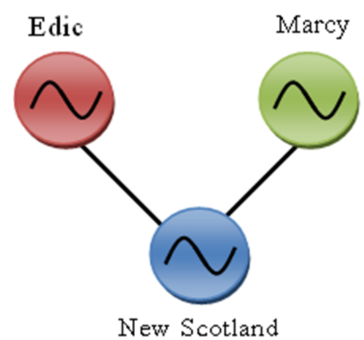
- How does mode propagation depend on network shape?



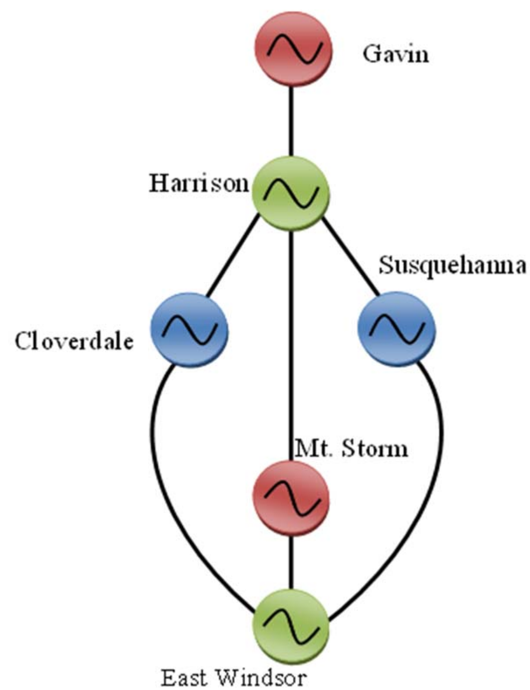
Radial



Cyclic



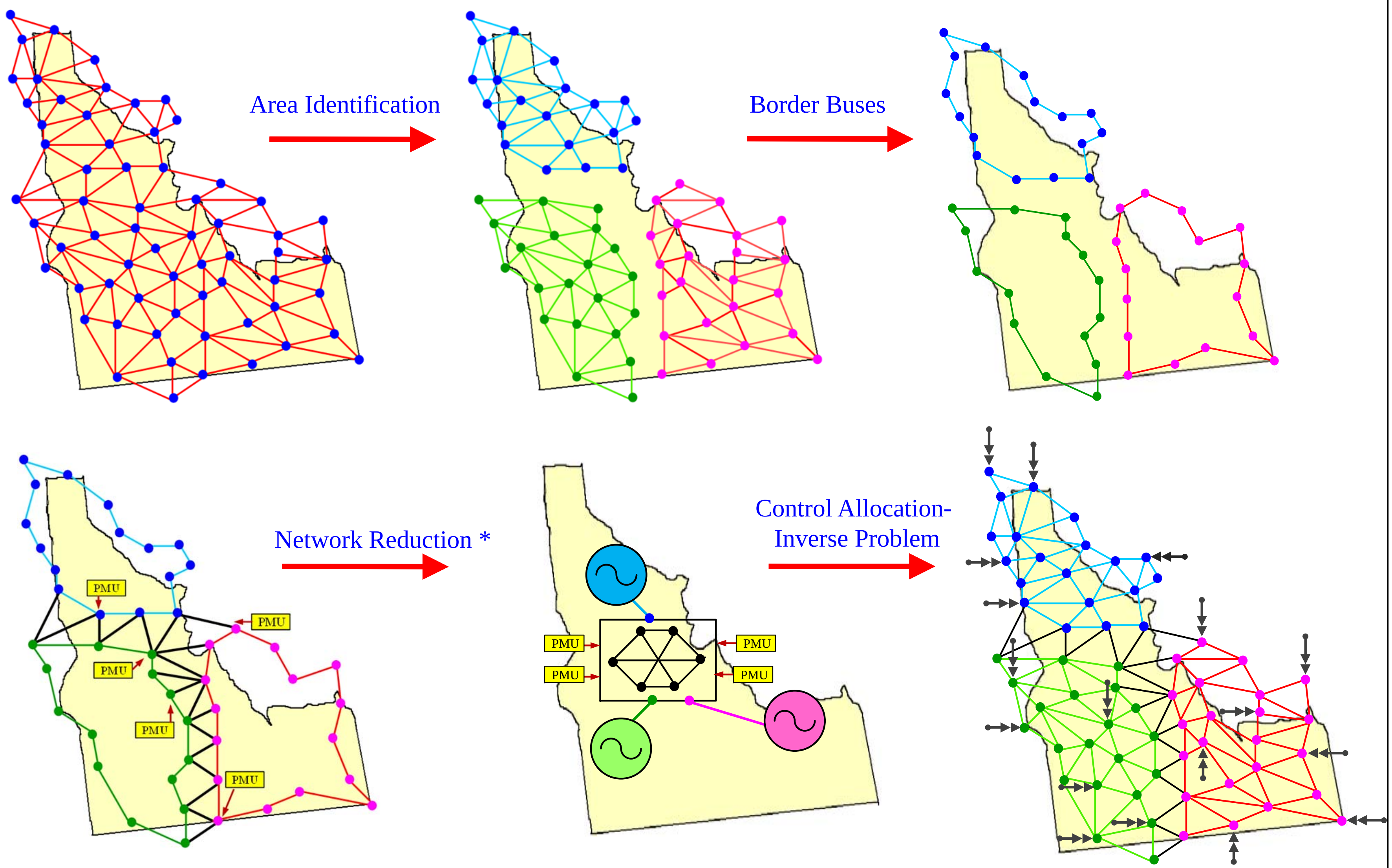
Star



Arbitrary

The Wide-area Control Problem

- Computational complexity sharply increases with number of areas

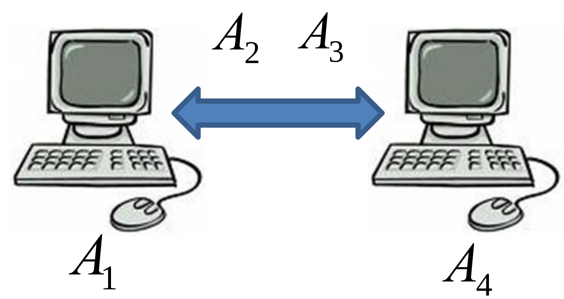


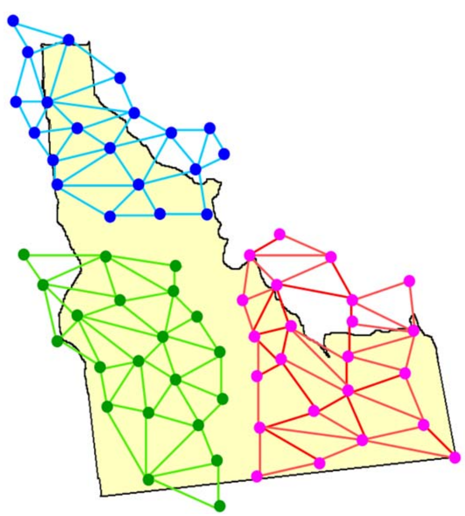
The Cyber-Physical Challenge

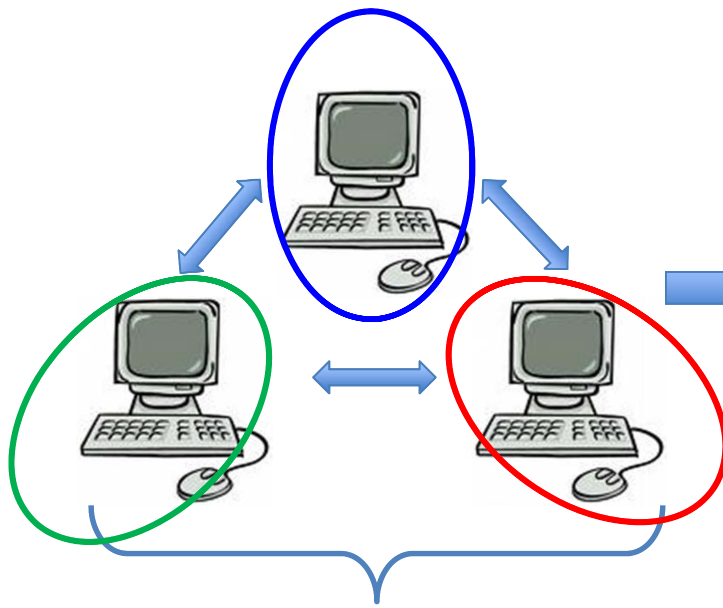
- Distributed Identification/Simulation:

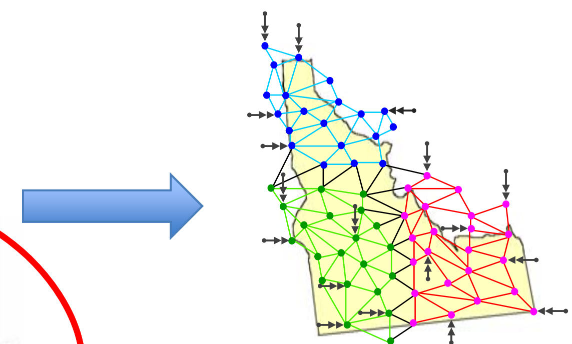
⟶ A number of computers solve assigned chunks of the system dynamics and Exchange information to update the state - coupling

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \qquad \dot{x} = Ax \qquad x = \begin{bmatrix} x_1 \\ \hline x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} X_1 \\ \hline X_2 \end{bmatrix}, \qquad \dot{x} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} x$$





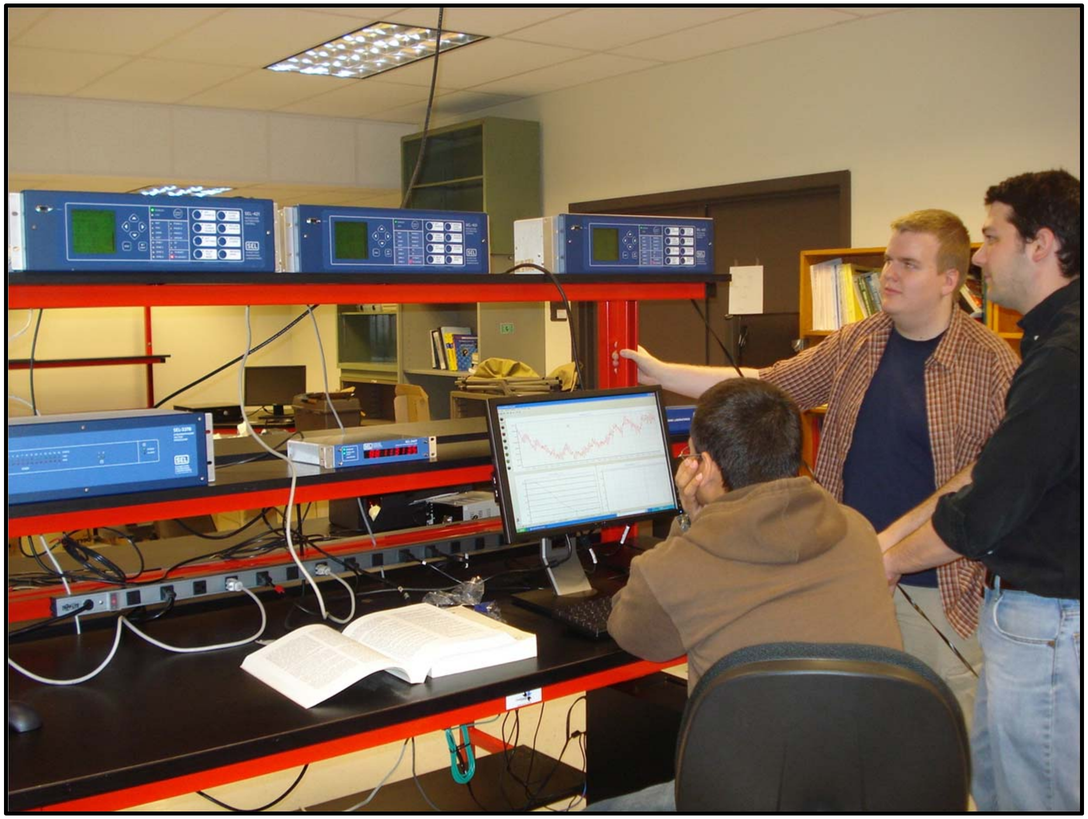




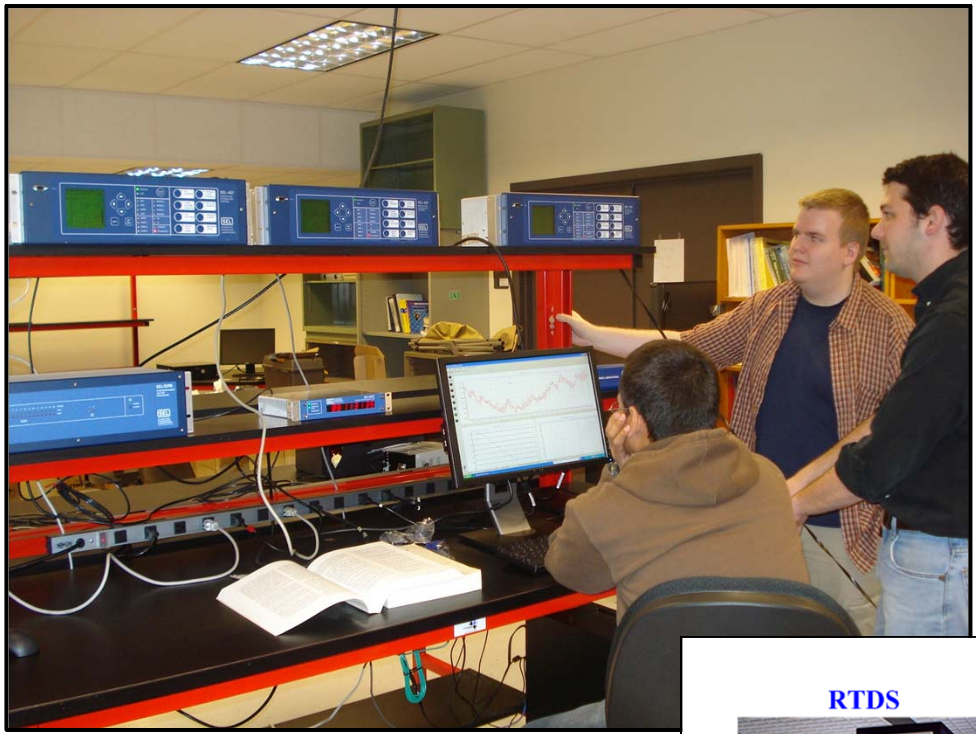
Actuation: Frequency feedback
FACTS: STATCOM, SVC

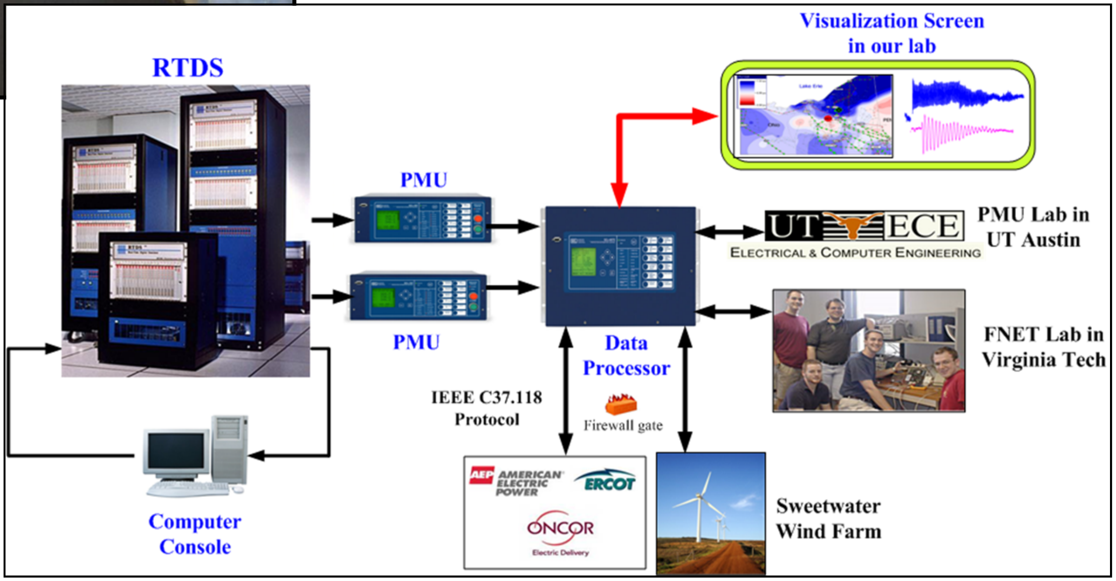
Exchange will depend on the connection graph

Phasor Lab



Phasor Lab





Visualization Screen
in our lab
RTDS
PMU
PMU
Data
Processor
PMU Lab in
UT Austin
UT ECE
ELECTRICAL & COMPUTER ENGINEERING
FNET Lab in
Virginia Tech
IEEE C37.118
Protocol
Firewall gate
AMERICAN ELECTRIC POWER
ERCOT
ONCOR
Electric Delivery
Sweetwater
Wind Farm
Computer
Console

Conclusions

1. WAMS is a tremendously promising technology for smart grid researchers
2. Different disciplines must merge
3. Plenty of new research problems – EE, Applied Math, Computer Science
4. Plenty of new engineering problems
5. Right time to think mathematically – Network theory is imperative
6. Right time to pay attention to the bigger picture of the electric grid
7. Needs participation of young researchers!
8. Promises to create jobs and provide impetus to the ARRA