

# Stability-constrained Optimal Power Flows and Applications to Electricity Markets

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**ENGINEERING**

engineering.uwaterloo.ca

**Claudio Cañizares**

University of Waterloo  
Electrical & Computer Engineering

[ccanizar@uwaterloo.ca](mailto:ccanizar@uwaterloo.ca)

[www.power.uwaterloo.ca](http://www.power.uwaterloo.ca)

[www.wise.uwaterloo.ca](http://www.wise.uwaterloo.ca)

# Outline

- Motivation.
- OPF problems in electricity markets:
  - Security-constrained optimal power flows (SC-OPF).
  - SC-OPF energy market clearing and dispatch models:
    - AC SC-OPF model.
    - DC SC-OPF multi-period model
- Stability-Constrained OPFs:
  - Multi-objective voltage-stability-constrained (VSC) OPF.
  - Minimum singular value (MSV) VSC-OPF.
  - NN Security Boundary Constrained OPF.
- Conclusions.
- List of relevant publications.

# Motivation

- Optimal Power Flow (OPF) problem:
  - Obtain daily/hourly/minute generation schedules to minimize costs and losses subject to network constraints and limits (e.g. power flows, reactive power and voltage limits).
  - Linear (LP) and Nonlinear programming (NLP) problem, depending on the modeling of the constraints and limits.

# Motivation

- OPF applications to electricity markets :
  - Dispatch generation and loads to maximize “social benefit” or “social welfare”), i.e. minimize the difference between demand and supply bids.
  - In most markets, nodal-prices of electricity are also obtained from the optimization process.
- OPF problems have been successfully solved using a variety of well-known optimization techniques for large systems (e.g. Interior Point methods).

# Motivations

- Deregulation/privatization of electricity markets has increased the need for minimizing prices while meeting system security constraints.
- New SC-OPF problems are needed to address electricity market issues:
  - Objective is to produce secure and “cheaper” generation/load schedules.
  - Some “security” constraints should be replaced by constraints that better reflect system security.

# SC-OPF

- “Classical” NLP SC-OPF problem:

$$\begin{aligned} \text{Min.} \quad & \sum_{i=1}^{N_G} a_i P_{G_i}^2 + b_i P_{G_i} + c_i \\ \text{s.t.} \quad & F_{PF}(\delta, V, Q_G, P_G) = 0 \\ & P_{G_{\min}} \leq P_G \leq P_{G_{\max}} \\ & P_T(\delta, V) \leq P_{T_{\max}} \\ & I_T(\delta, V) \leq I_{T_{\max}} \\ & Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}} \\ & V_{\min} \leq V \leq V_{\max} \end{aligned}$$

# SC-OPF

where the nonlinear power flow equations  $F_{PF}(\delta, V, Q_G, P_G)$  have the general form (2 equations per bus  $i = 1, \dots, N$ ):

$$P_i - \sum_{k=1}^N V_i V_k [G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)] = 0$$

$$Q_i - \sum_{k=1}^N V_i V_k [G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k)] = 0$$

$$P_i = \begin{cases} P_{G_i} & \forall i \in \text{generators} \\ -P_{L_i} & \forall i \in \text{loads} \end{cases}$$

$$Q_i = \begin{cases} Q_{G_i} & \forall i \in \text{generators} \\ -Q_{L_i} & \forall i \in \text{loads} \end{cases}$$

$G_{ik}, B_{ik}, P_{L_i}, Q_{L_i} \rightarrow \text{constants}$

# SC-OPF

- The objective is to minimize generation costs.
- Grid “security” is represented in this model by:
  - Line power flows  $P_T$ , typically computed off-line using an N-1 contingency security criterion.
  - Current thermal limits  $I_T$ .
  - Bus voltage limits  $V$ .
- These types of problems have been solved successfully for large networks (thousands of constraints) using Interior Point methods.

# SC-OPF

- In electricity markets, the “typical” NLP SC-OPF model of a double auction market is:

$$\begin{aligned} \text{Max.} \quad & S_b = C_d^T P_d - C_s^T P_s \\ \text{s.t.} \quad & F_{PF}(\delta, V, Q_G, P_s, P_d) = 0 \\ & 0 \leq P_s \leq P_{s\max} \\ & 0 \leq P_d \leq P_{d\max} \\ & P_T(\delta, V) \leq P_{T\max} \\ & I_T(\delta, V) \leq I_{T\max} \\ & Q_{G\min} \leq Q_G \leq Q_{G\max} \\ & V_{\min} \leq V \leq V_{\max} \end{aligned}$$

# SC-OPF

where  $F_{PF}(\delta, V, Q_G, P_s, P_d)$  have the general form:

$$P_i - \sum_{k=1}^N V_i V_k [G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)] = 0$$

$$Q_i - \sum_{k=1}^N V_i V_k [G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k)] = 0$$

$$P_i = \begin{cases} P_{Go_i} + P_{s_i} & \forall i \in \text{generators} \\ -P_{Lo_i} - P_{d_i} & \forall i \in \text{loads} \end{cases}$$

$$Q_i = \begin{cases} Q_{G_i} & \forall i \in \text{generators} \\ -Q_{L_i} & \forall i \in \text{loads} \end{cases}$$

$G_{ik}, B_{ik}, P_{Go_i}, P_{Lo_i}, Q_{L_i} \rightarrow \text{constants}$

# SC-OPF

- This market model is basically an NLP SC-OPF that maximizes social benefit  $S_b$ , i.e. the difference between demand and supply bids.
- Nodal energy prices or LMPs are a byproduct of this optimization problem (the Lagrange multipliers of the active power components of  $F_{PF}$ ).
- It is not widely used by utilities yet, but some utilities (e.g. PJM) are using them for settlement purposes (e.g. determine Locational Marginal Prices or LMPs).
- In practice, market models based on LP models are more commonly used by system operators.

# SC-OPF

- For example, the following multi-period LP model may be used to clear these market (e.g. Ontario):

$$\text{Max. } S_b = \sum_{t=1}^N C_{d_t}^T P_{d_t} - C_{s_t}^T P_{s_t}$$

$$\text{s.t. } P_{d_{it}} - P_{s_{it}} - \sum_{j=1, j \neq i}^n P_{loss_{ijt}} - \sum_{j=1, j \neq i}^n P_{ijt} = 0 \quad \forall i, t$$

$$P_{ijt} - b_{ij}(\delta_{it} - \delta_{jt}) = 0 \quad \forall i, j, i \neq j, t$$

$$P_{T_t} = [P_{ijt}] \leq P_{T_t \max} \quad \forall t$$

$$0 \leq P_{s_t} \leq P_{s_t \max} \quad \forall t$$

$$0 \leq P_{d_t} \leq P_{d_t \max} \quad \forall t$$

$$P_{s_t} - P_{s_{t+1}} \leq R d_{s_t} \quad \forall t$$

$$P_{s_{t+1}} - P_{s_t} \leq R u_{s_t} \quad \forall t$$

# SC-OPF

where  $R_d$  and  $R_u$  represent the ramp-down and ramp-up constraints of the generators, respectively.

- Other temporal constraints are typically included in these types of models (e.g. operating reserves in Ontario).
- Power transfer limits are obtained off-line by means of ATC computations, considering thermal, voltage and stability limits.

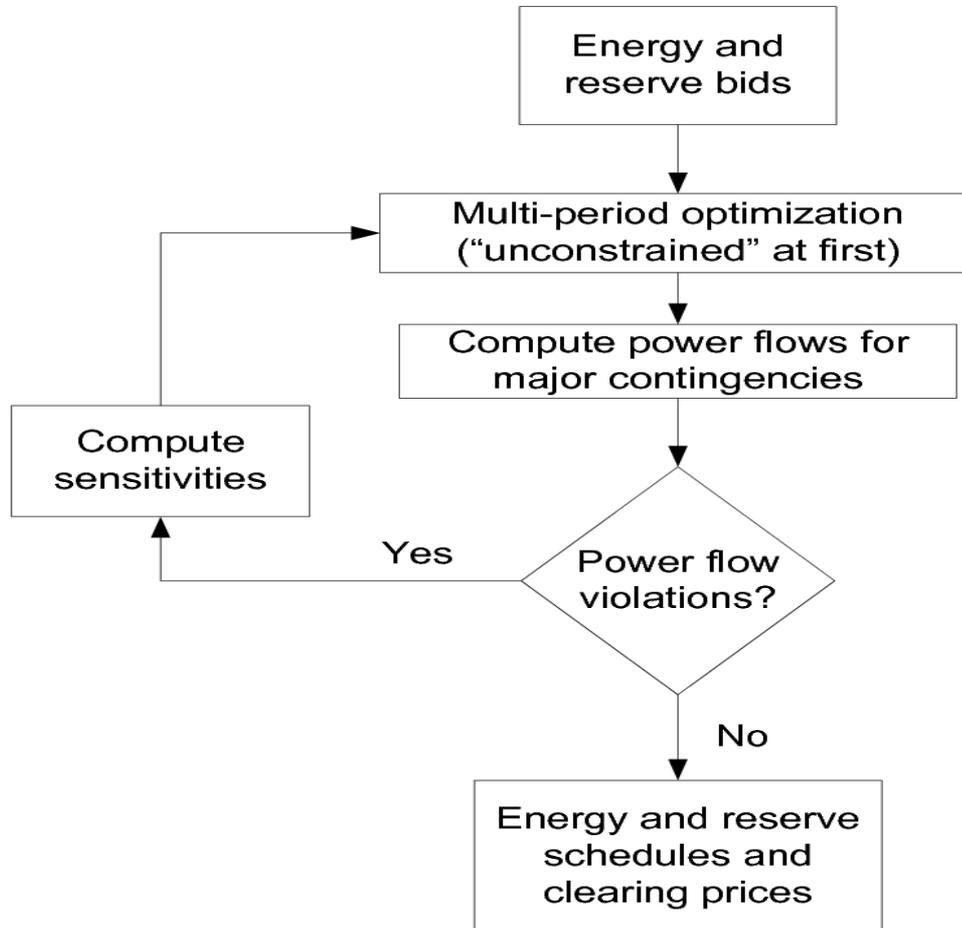
# SC-OPF

- Ontario market example:
  - Multi-period optimization with  $N = 5$ .
  - Solved every 5 min., for 3000+ buses and about 300 market participants.
  - Reserve bids are also considered: 10 spinning, 10 min. non-spinning, and 30 min.
  - Unconstrained solution defines uniform MCPs.

# SC-OPF

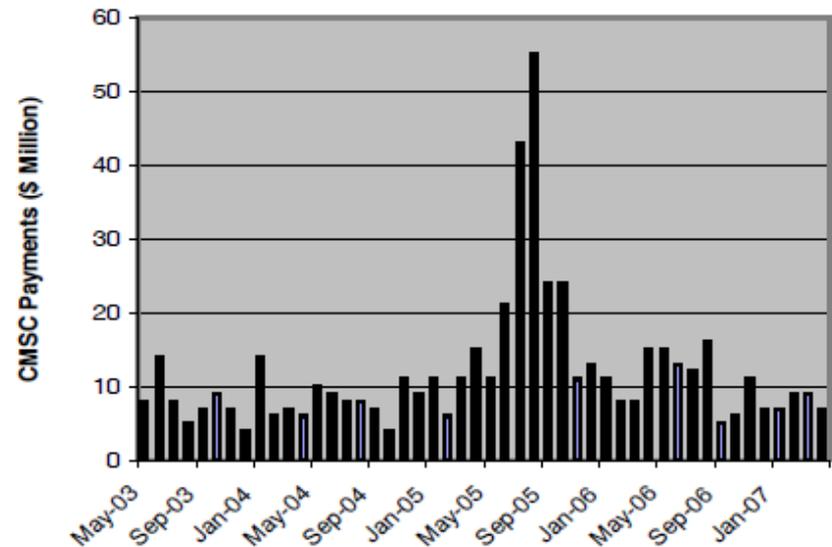
- Power-flow-based contingency analyses are used to check if unconstrained solution violate limits and to “dispatch” reactive power (set generator bus voltages).
- If security violations are encountered, “sensitivities” are used to add constraints to the OPF model and procedure is repeated until no violations are encountered.
- Final constrained solution defines “uplift” prices to be added to the MCP, and are used to determine the Congestion Management Settlement Credit (CMSC) payments.

# SC-OPF



# SC-OPF

- In Ontario, total CMSC payments made by the IESO over the period May 2003-April 2007 averaged \$11.77 million/month (H. Ghasemi and A. Maria, “Benefits of Employing an On-line Security Limit Derivation Tool in Electricity Markets,” in *Proc. IEEE-PES General Meeting, July 2008*):



# SC-OPF

- The line power flow limits vary with the solution of the auction; thus, fixed limits are not representative of system conditions, negatively affecting prices and system security.
- This has led to the development of stability constrained OPF models.

# Multi-objective VSC-OPF

- The objective is to maximize both social benefit and system “loadability”, i.e. voltage stability margins (VSM):

$$\text{Max. } (1 - w) \underbrace{(C_d^T P_d - C_s^T P_s)}_{S_b} + w \lambda_c$$

$$\text{s.t. } F_{PF}(\delta, V, Q_G, P_s, P_d) = 0$$

$$F_{PF_c}(\delta_c, V_c, Q_{G_c}, P_s, P_d, \lambda_c) = 0$$

$$0 \leq P_s \leq P_{s\max}$$

$$0 \leq P_d \leq P_{d\max}$$

$$\lambda_{c\min} \leq \lambda_c \leq \lambda_{c\max}$$

$$I_T(\delta, V) \leq I_{T\max}$$

$$Q_{G\min} \leq Q_G \leq Q_{G\max}$$

$$V_{\min} \leq V \leq V_{\max}$$

$$I_{T_c}(\delta_c, V_c) \leq I_{T_{c\max}}$$

$$Q_{G_{c\min}} \leq Q_{G_c} \leq Q_{G_{c\max}}$$

$$V_{c\min} \leq V_c \leq V_{c\max}$$

# Multi-objective VSC-OPF

where  $\lambda_c$  represents the VSM, and all  $c$  constraints correspond to the system at its “critical” point (max. VSM).

- By varying the weight  $w$  ( $0 < w < 1$ ), more or less stress can be put on security.
- In practice,  $w$  should be very small to avoid “undesirable” effects on the market power levels and prices.
- Problems with this technique:
  - Number of constraints are doubled.
  - LPMs are not directly a by-product of this model, given the objective function definition which mixes system costs with security.
  - No consideration for system dynamics.

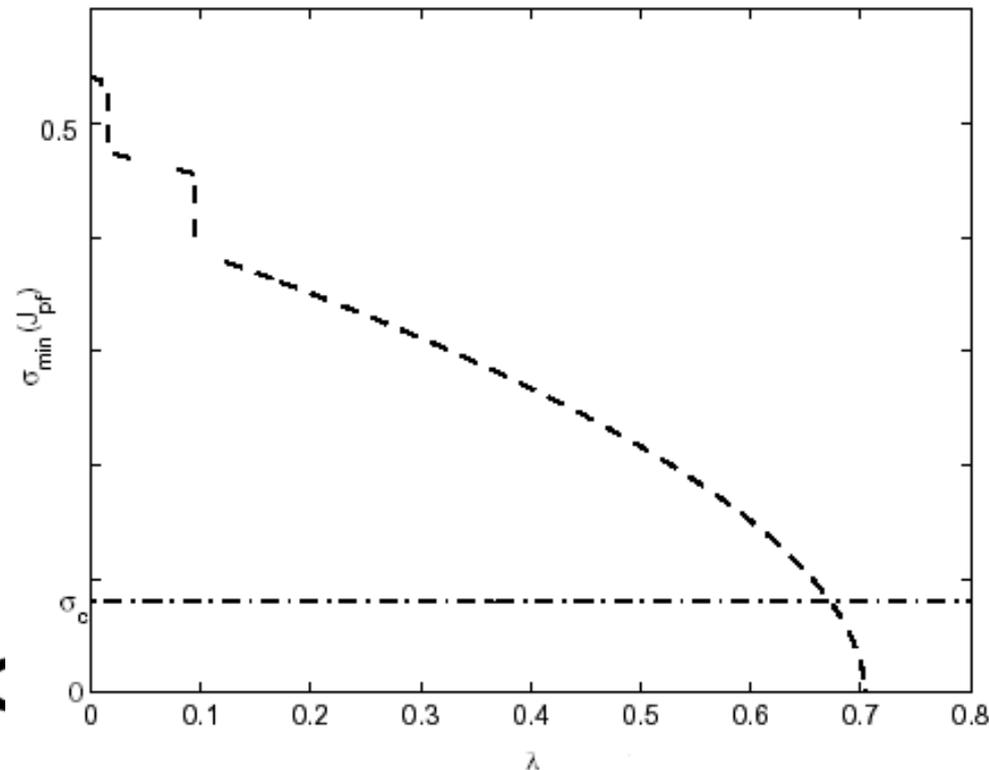
# MSV VSC-OPF

- The objective again is to maximize social benefit while guaranteeing a min. VSM:

$$\begin{aligned} \text{Max. } & S_b = C_d^T P_d - C_s^T P_s \\ \text{s.t. } & F_{PF}(\underbrace{\delta, V, Q_G, P_s, P_d}_{z,p}) = 0 \\ & \sigma_{\min}(\underbrace{D_z F_{PF}|_o}_{J_{PF}}) \geq \sigma_c \\ & 0 \leq P_s \leq P_{s\max} \\ & 0 \leq P_d \leq P_{d\max} \\ & I_T(\delta, V) \leq I_{T\max} \\ & Q_{G\min} \leq Q_G \leq Q_{G\max} \\ & V_{\min} \leq V \leq V_{\max} \end{aligned}$$

# MSV VSC-OPF

- where  $\sigma_{\min}(J_{PF})$  is a VS index that becomes zero at a singularity point of the power flow Jacobian:



# MSV VSC-OPF

- This is an NLP problem with an implicit constraint solved as follows:

$$\text{Min. } S(\chi)$$

$$\text{s.t. } F(\chi) = 0$$

$$\underline{H} \leq H(\chi) \leq \overline{H}$$

$$\underline{\chi} \leq \chi \leq \overline{\chi}$$

- Interior point solution approach:

$$\text{Min. } S(\chi) - \mu_s \sum_i (\ln s_i + \ln q_i)$$

$$\text{s.t. } F(\chi) = 0$$

$$-s - q + \overline{H} - \underline{H} = 0$$

$$-H(\chi) - q + \overline{H} = 0$$

$$s \geq 0, \quad q \geq 0$$

# MSV VSC-OPF

– Lagrange-Newton method:

$$L_{\mu}(u) = S(\chi) - \mu_s \sum_i (\ln s_i + \ln q_i) - \rho^T F(\chi) \\ - \varsigma^T (-s - q + \bar{H} - \underline{H}) - \tau^T (-H(\chi) - q + \bar{H})$$

$$\Rightarrow \nabla_u L_{\mu}(u) = 0$$

– The solution procedure requires finding the Hessian:

$$\nabla_{\chi}^2 L_{\mu}(u) = \nabla_{\chi}^2 S(\chi) - \rho^T \nabla_{\chi}^2 F(\chi) + \tau^T \nabla_{\chi}^2 H(\chi)$$

# MSV VSC-OPF

- Since  $H(\chi)$  has an implicit constraint, to obtain  $\nabla^2_{\chi} H(\chi)$ :

$$\frac{\partial^2 \sigma_{min}(J_{PF})}{\partial \chi_i \partial \chi_j} \Big|_{\chi^* + \Delta \chi} \approx \frac{\frac{\partial \sigma_{min}(J_{PF})}{\partial \chi_i} \Big|_{\chi^*} - \frac{\partial \sigma_{min}(J_{PF})}{\partial \chi_i} \Big|_{\chi^* + \Delta \chi}}{\Delta \chi_j}$$

based on approximations that are obtained from the properties of the singular value:

$$\Delta \sigma_{min}(J_{PF}) \approx U_1^T D_z^2 F_{PF}|_* \Delta z V_1$$

$$\Delta \sigma_{min}(J_{PF}) \approx -U_1^T D_z^2 F_{PF}|_* D_z F_{PF}|_*^{-1} D_p F_{PF}|_* \Delta p V_1$$

# MSV VSC-OPF

- Some system dynamics (oscillatory stability) can be represented in the VSC-OPF problem by replacing the MSV  $\sigma_{min}(J_{PF})$  constraint with the singular value of a dynamic Jacobian.
- Problems with this model:
  - Proposed handling of implicit constraint yields approximate solutions.
  - High computational costs.

# MSV VSC-OPF

- Replacing the implicit MSV constraint with an explicit representation based on singular value decomposition (SVD):

$$\text{Max. } S_b = C_d^T P_d - C_s^T P_s$$

$$\text{s.t. } F_{PF}(\delta, V, Q_G, P_s, P_d) = 0$$

$$u_n^T J_{PF}(\delta, V) w_n \geq \sigma_c$$

$$0 \leq P_s \leq P_{s\max}$$

$$0 \leq P_d \leq P_{d\max}$$

$$I_T(\delta, V) \leq I_{T\max}$$

$$Q_{G\min} \leq Q_G \leq Q_{G\max}$$

$$V_{\min} \leq V \leq V_{\max}$$

# MSV VSC-OPF

- $J_{PF}$  is an “invariant” sub-Jacobian of  $D_z F_{PF}$ .
- Solution is iterative:

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**Algorithm:** Solution of the VSC-OPF using a SVD

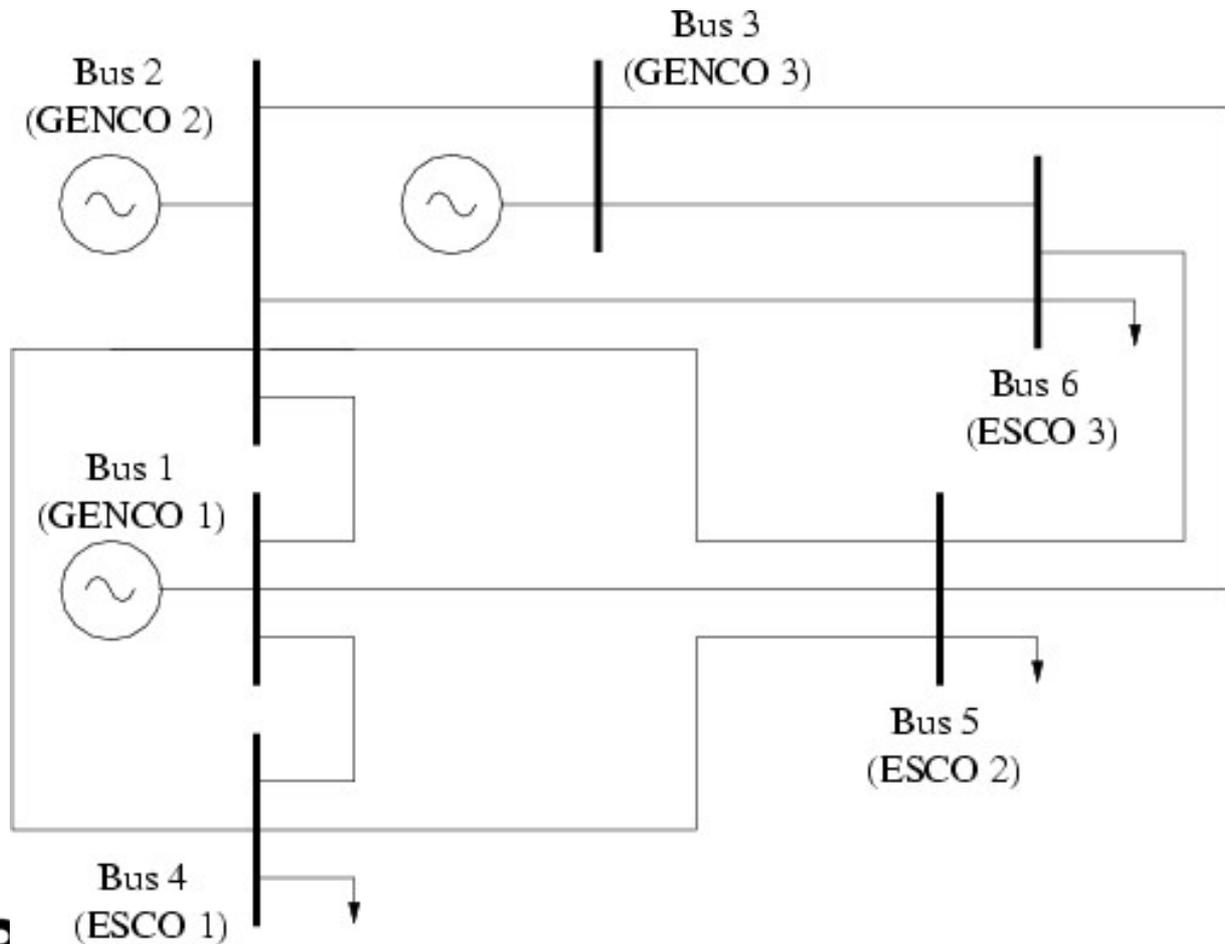
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```
begin
   $\sigma_c \leftarrow$  Off-line VS study
   $k \leftarrow 1$ 
   $(\delta^*, V_L^*, V_G^*) \leftarrow$  VSC-OPF (“relaxed”)
   $(u_n, \sigma_n, w_n) \leftarrow$  SVD( $J_{PF}|_*$ )
  if  $\sigma_n \geq \sigma_c$  then
    end
  else
    repeat
       $k \leftarrow k + 1$ 
       $(\delta^*, V_L^*, V_G^*) \leftarrow$  VSC-OPF
       $(u_n, \sigma_n, w_n)^{(k)} \leftarrow$  SVD( $J_{PF}|_*$ )
    until  $\sigma_n^{(k)} \geq \sigma_c$ 
  end
   $(\delta^*, V_L^*, Q_G^*, V_G^*, P_S^*, P_D^*) \rightarrow$  Optimum
end
```

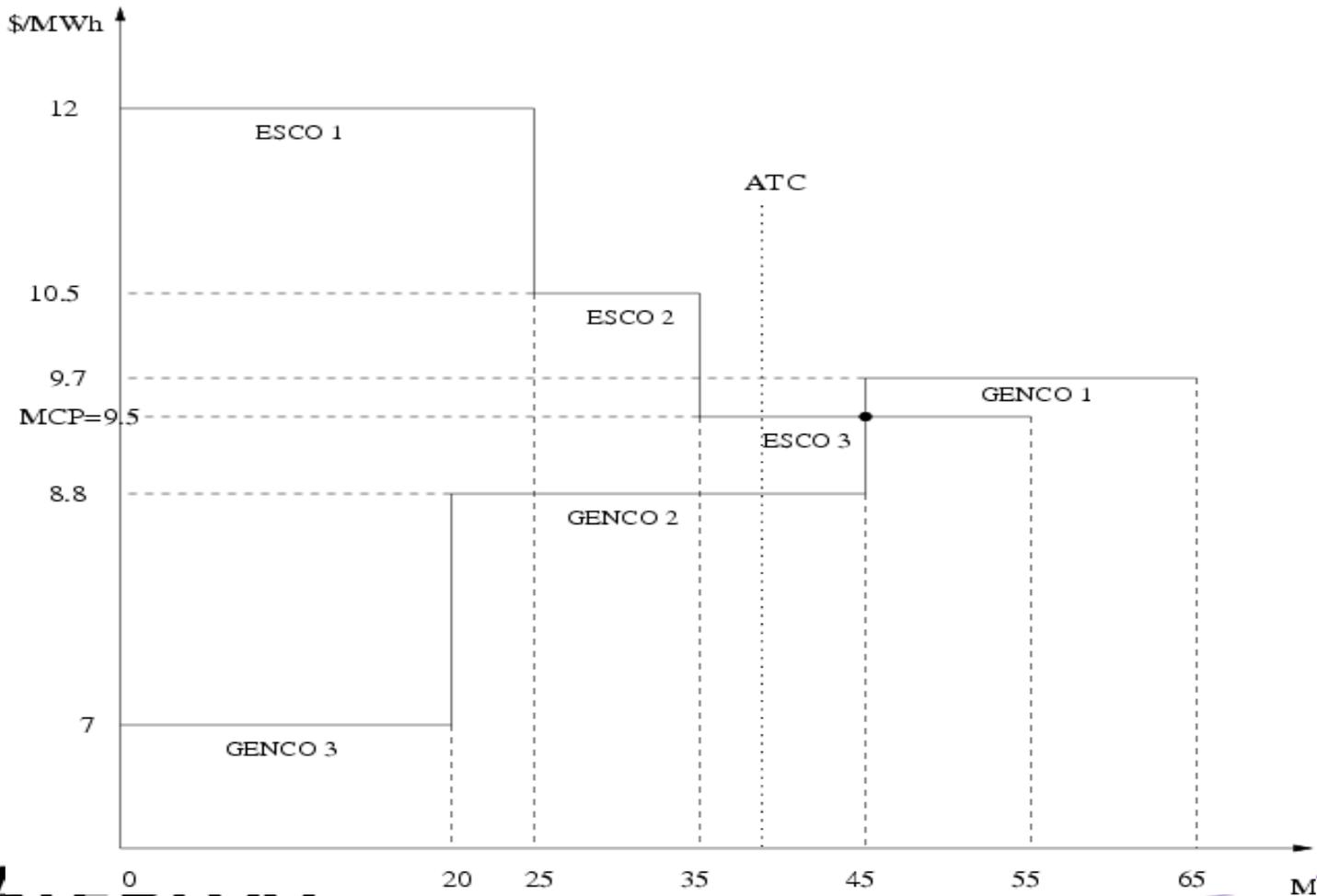
# MSV VSC-OPF

- Observations:
  - Relatively easy to implement.
  - Faster to solve and more robust.
  - MSV constraint is properly enforced as opposed to previous solution method.
  - It requires an iterative solution process, where appropriate  $(u_n, w_n)$  are calculated at each iteration  $k$  ( $k \leq 3$  for all the test cases studied).
- In principle, oscillatory stability limits could be accounted for in the MSV security constraint using a dynamic Jacobian, but not other dynamic phenomena.

# VSC-OPF Example

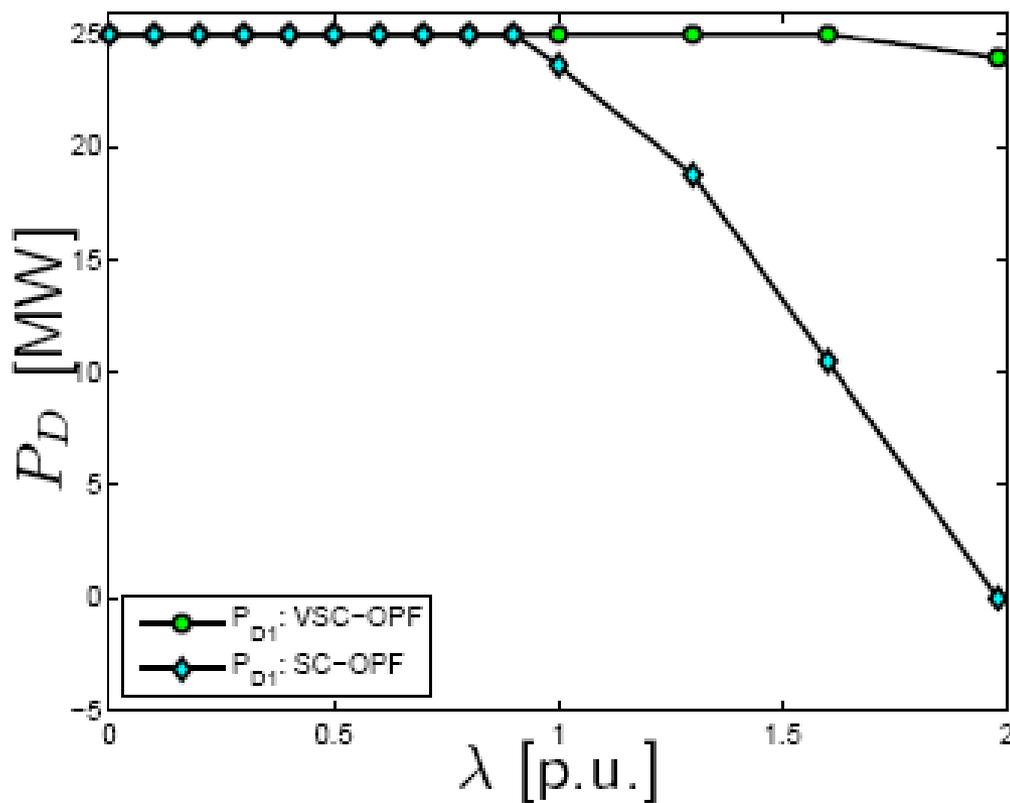


# VSC-OPF Example



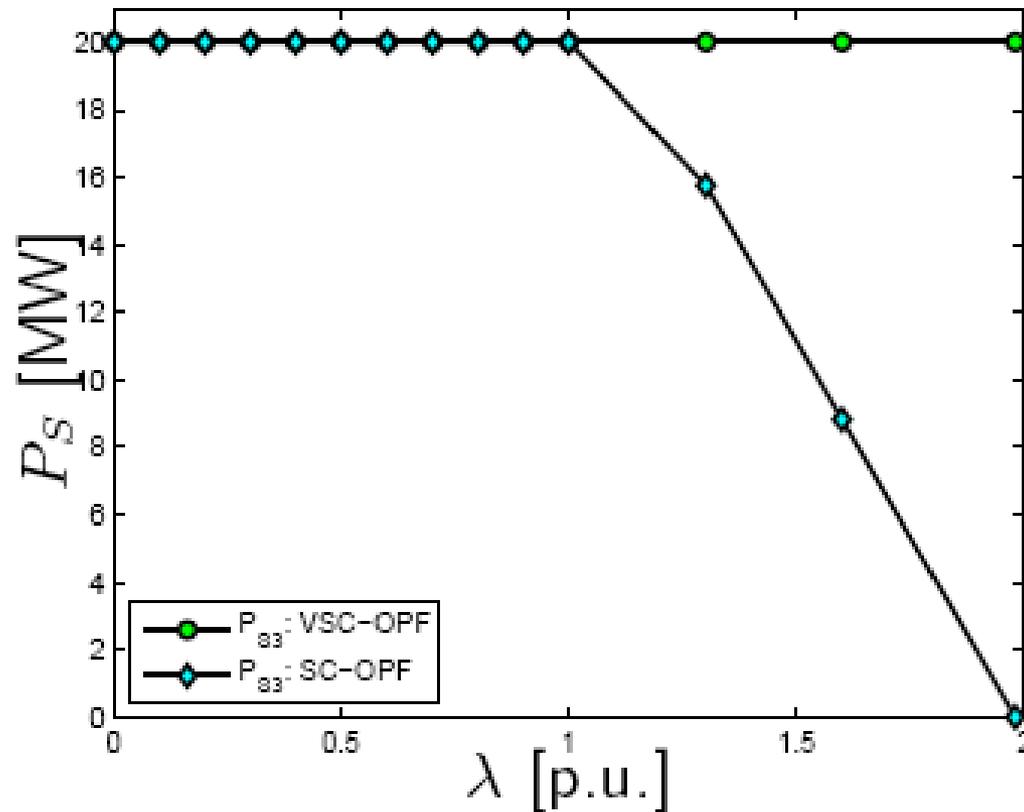
# VSC-OPF Example

- ESCO 1 power as loading (represented by the loading factor  $\lambda$ ) increases:



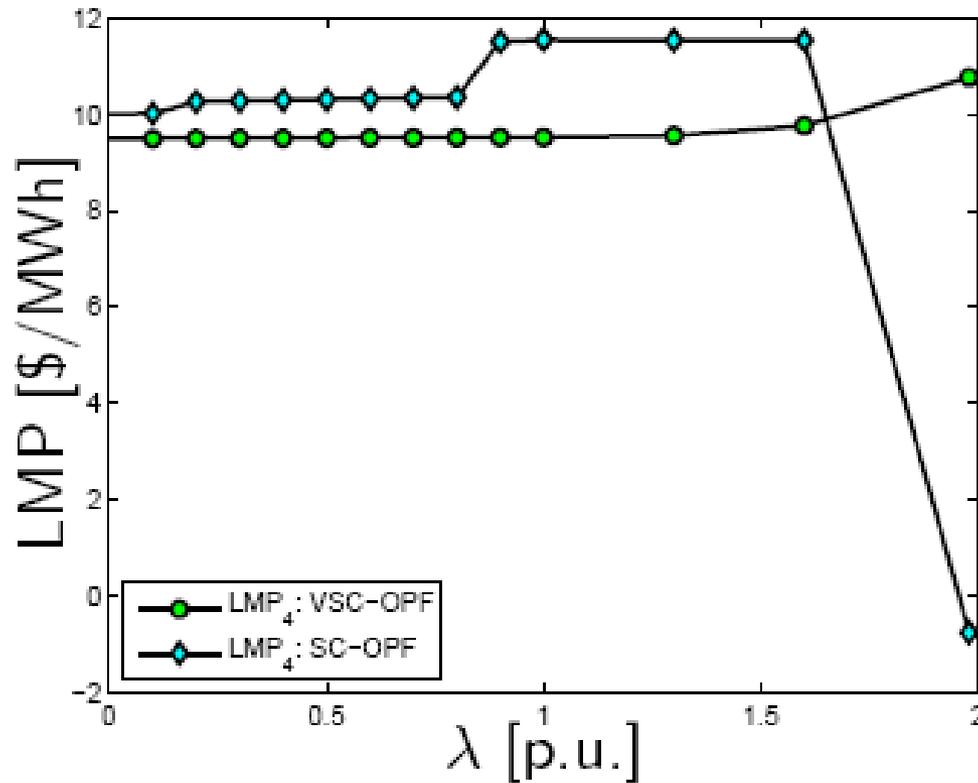
# VSC-OPF Example

- GENCO 3 power as loading increases:



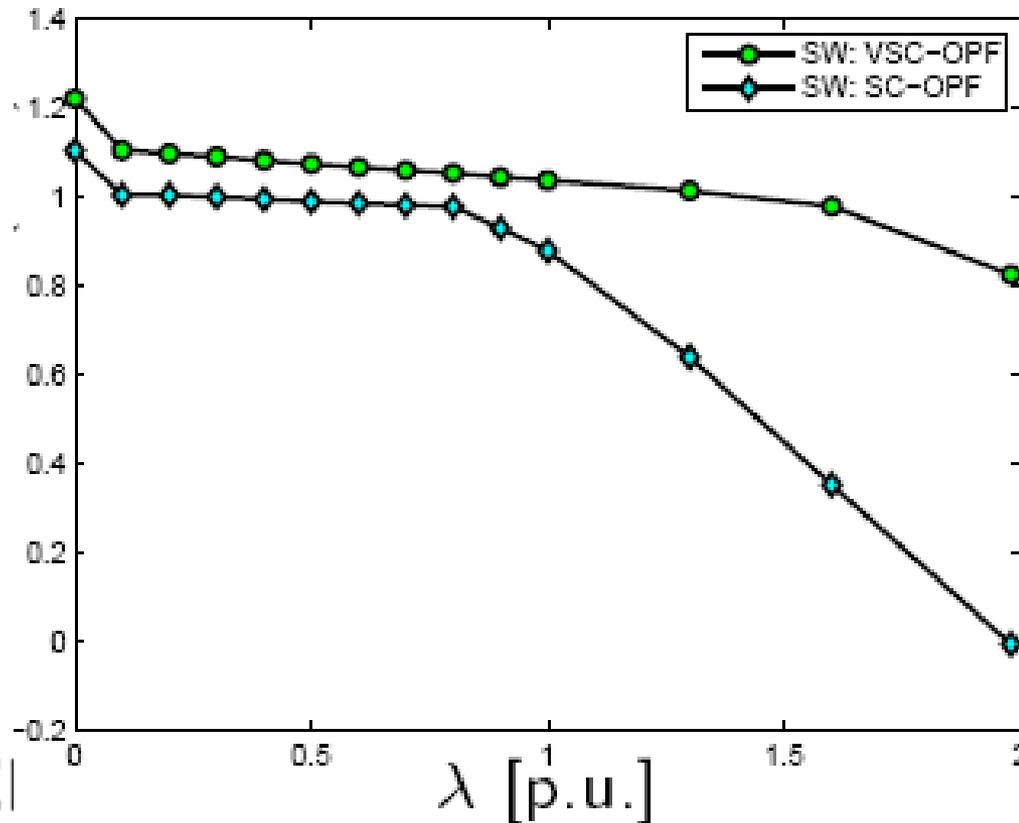
# VSC-OPF Example

- LMP at Bus 4 as loading increases:



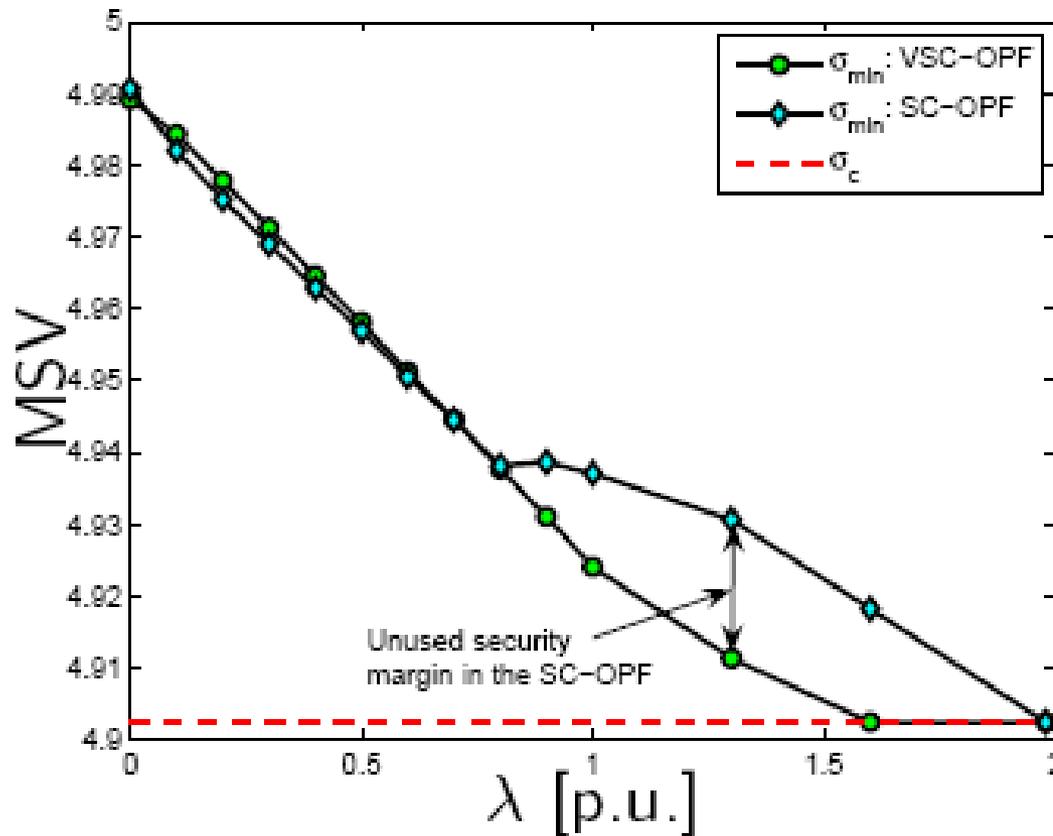
# VSC-OPF Example

- Social benefit  $S_b$  as loading increases:



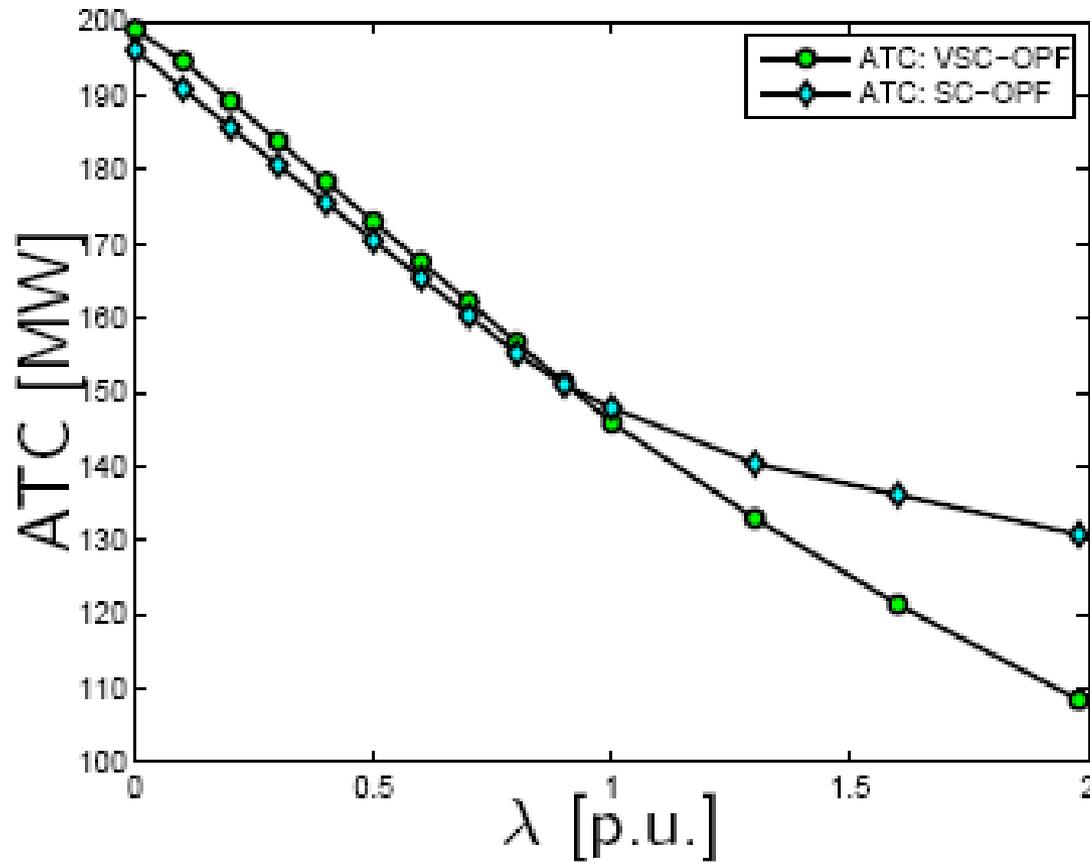
# VSC-OPF Example

- MSV as loading increases:



# VSC-OPF Example

- ATC as loading increases:



# VSC-OPF Example

- Better operating conditions:
  - Higher voltages.
  - Lower losses.
  - Higher VSM.
- Better market conditions:
  - Lower nodal-prices.
  - Higher transaction levels.

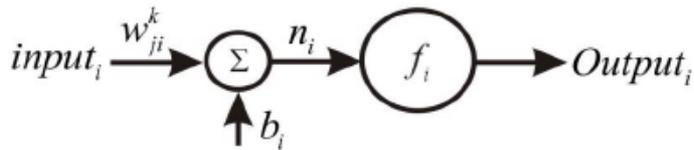
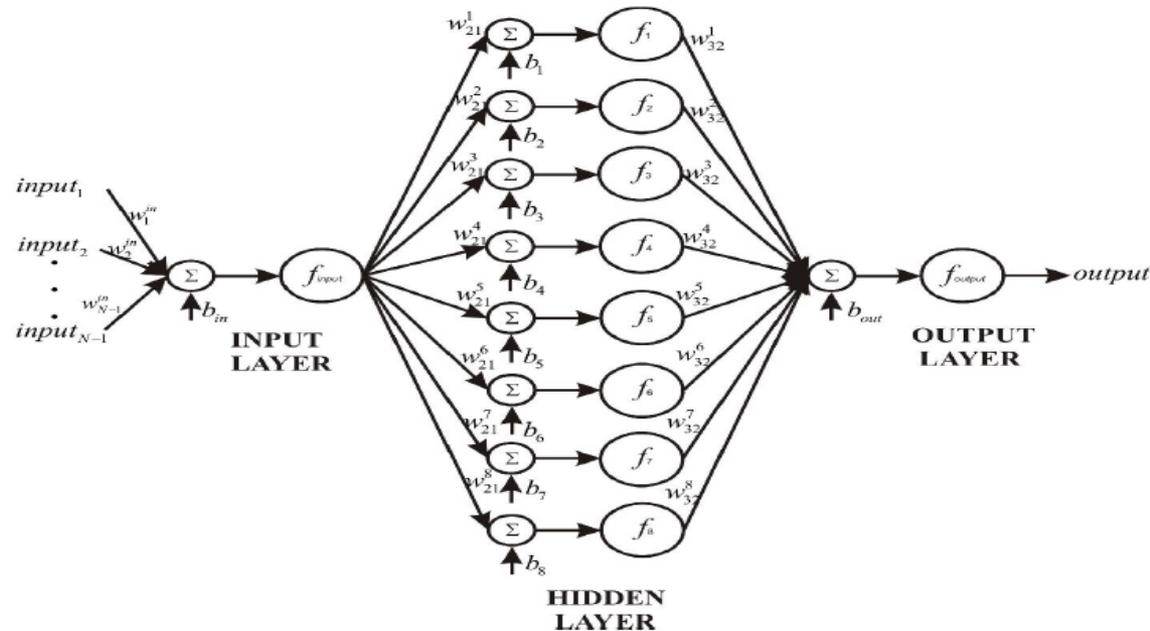
# NN SBC-OPF

- In this model, the security constraint is represented in this case by a NN-based security boundary:

$$\begin{aligned} \text{Max.} \quad & S_b = C_d^T P_d - C_s^T P_s \\ \text{s.t.} \quad & F_{PF}(\delta, V, Q_G, P_s, P_d) = 0 \\ & F_{NN}(P_s, P_d) \geq c \\ & 0 \leq P_s \leq P_{s\max} \\ & 0 \leq P_d \leq P_{d\max} \\ & I_T(\delta, V) \leq I_{T\max} \\ & Q_{G\min} \leq Q_G \leq Q_{G\max} \\ & V_{\min} \leq V \leq V_{\max} \end{aligned}$$

# NN SBC-OPF

- The NN SB is obtained using a BPNN:

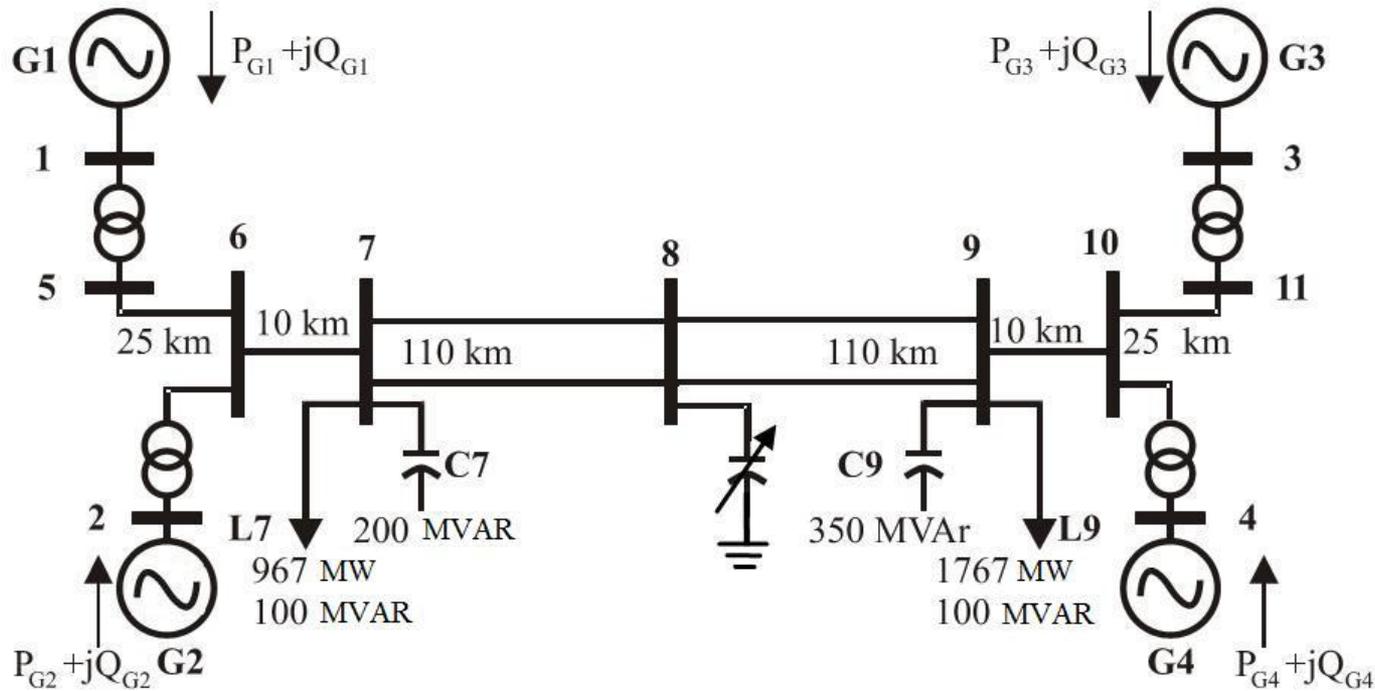


$$Output_i = f_i(input_i w_{ji}^k + b_i) = f_i(n_i)$$

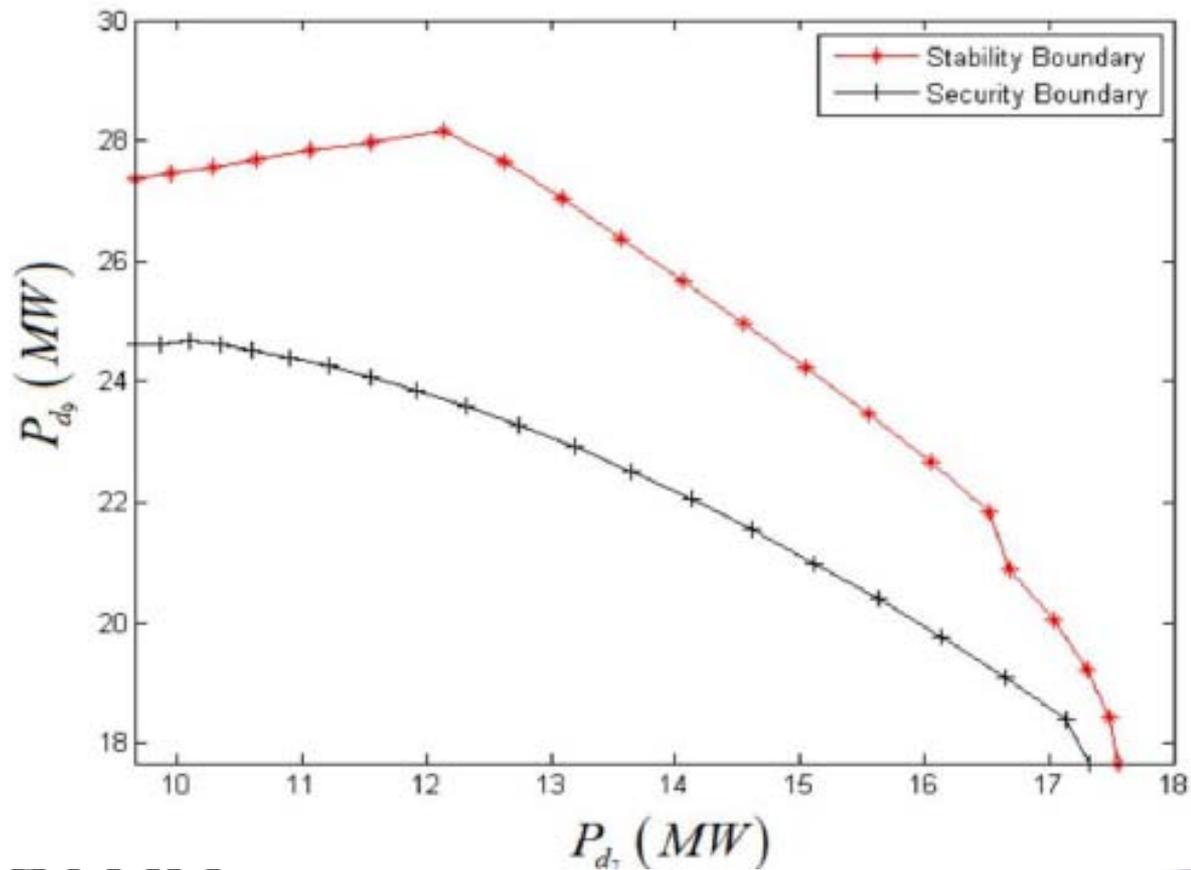
$$f_i(n_i) = \frac{2}{1 + 2^{-2n_i}} - 1$$

# NN SBC-OPF

- Example of an NN security and stability boundary for a 2 area system with respect to load increases in both areas:

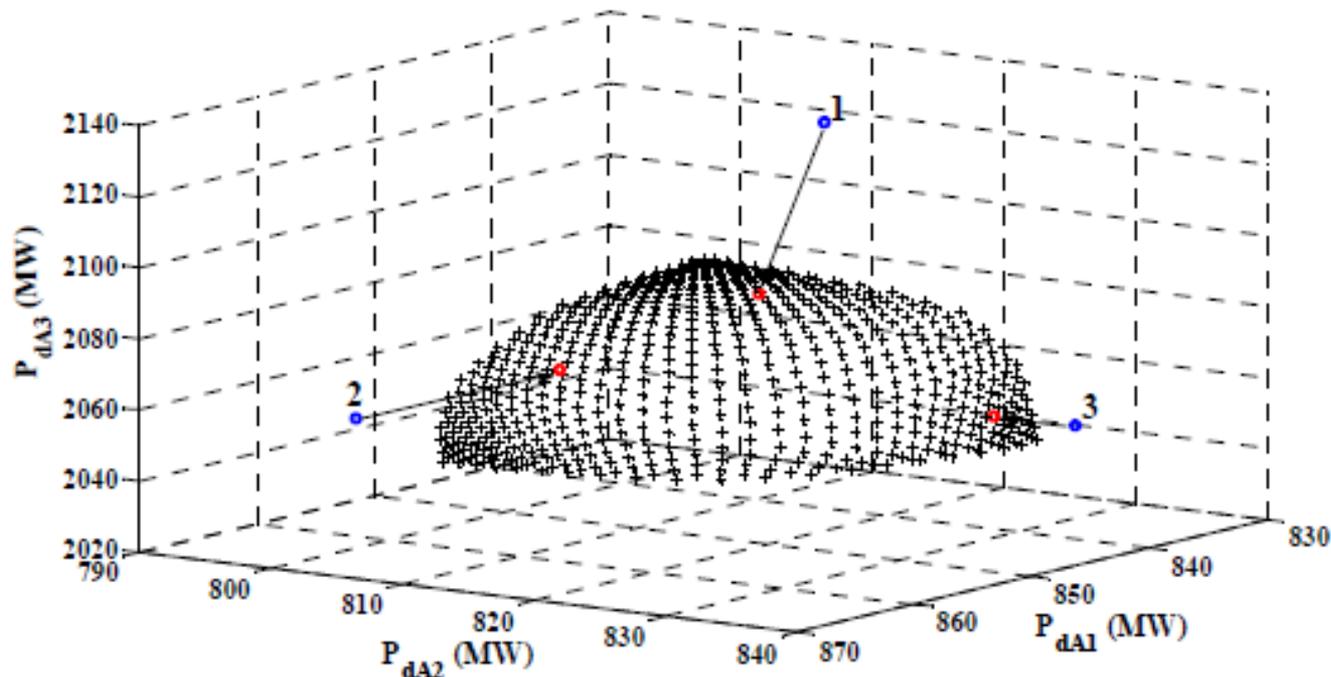


# NN SBC-OPF



# NN SBC-OPF

- Example of an NN security boundary for a 3 area system (118-bus IEEE benchmark system) with respect to load increases in all areas:



# NN SBC-OPF

- Security constraint representation using an NN-based SB in a load curtailment OPF model:

$$\text{Min } S_b = C_s^T P_s - C_d^T \Delta P_d$$

$$\text{s.t. } F_{PF}(\delta, V, Q_g, P_s, P_d, Q_d) = 0$$

$$0 \leq P_s \leq P_{s \max}$$

$$Q_{s \min} \leq Q_s \leq Q_{s \max}$$

$$V_{\min} \leq V \leq V_{\max}$$

$$\lambda_\ell - \sum_{k=1}^8 f_{k_m} \left( (\hat{\lambda}^T w_m^{in} + b_{in_m}) w_{21_m}^k + b_{k_m} \right) w_{32_m}^k + b_{out_m} \leq 0 \quad \forall m = 1, \dots, G$$

$$\Delta P_{dj} \leq 0 \quad \forall j = 1, \dots, N$$

$$\Delta P_{dj} = (\lambda_j - \lambda_{j0}) P_{dj0} \quad \forall j = 1, \dots, N$$

$$= (\alpha d_j - \alpha_0 d_{j0}) P_{dj0}$$

$$Q_{dj} = \tan(\varphi_j) P_{dj} \quad \forall j = 1, \dots, N$$

$$0 \leq d_j \leq 1 \quad \forall j = 1, \dots, N$$

$$\sum_{j=1}^N d_j = 1$$

$$\alpha \geq 0$$

# NN SBC-OPF

- Similarly, for the more realistic multi-period DC-OPF model:

$$\text{Min. } J = \sum_{t=1}^T \sum_{i=1}^{N_G} C_{i,t} P_{G_i,t}$$

$$\text{s.t. } P_{G_k,t} - P_{L_k,t} - \sum_{k=1, k \neq m}^N P_{km,t}^{\text{loss}} - \sum_{k=1, k \neq m}^N P_{km,t} = 0 \quad \forall k, m \in \text{Bus}, \forall t$$

$$P_{km,t} - b_{km} (\delta_{k,t} - \delta_{m,t}) = 0 \quad \forall k, m, k \neq m, \forall t$$

$$P_{G_i}^{\min} \leq P_{G_i,t} \leq P_{G_i}^{\max} \quad \forall i \in \text{Gen.}, \forall t$$

$$P_{G_i,t} - P_{G_i,t+1} \leq R_{DN_i} \quad \forall i \in \text{Gen.}, \forall t$$

$$P_{G_i,t+1} - P_{G_i,t} \leq R_{UP_i} \quad \forall i \in \text{Gen.}, \forall t$$

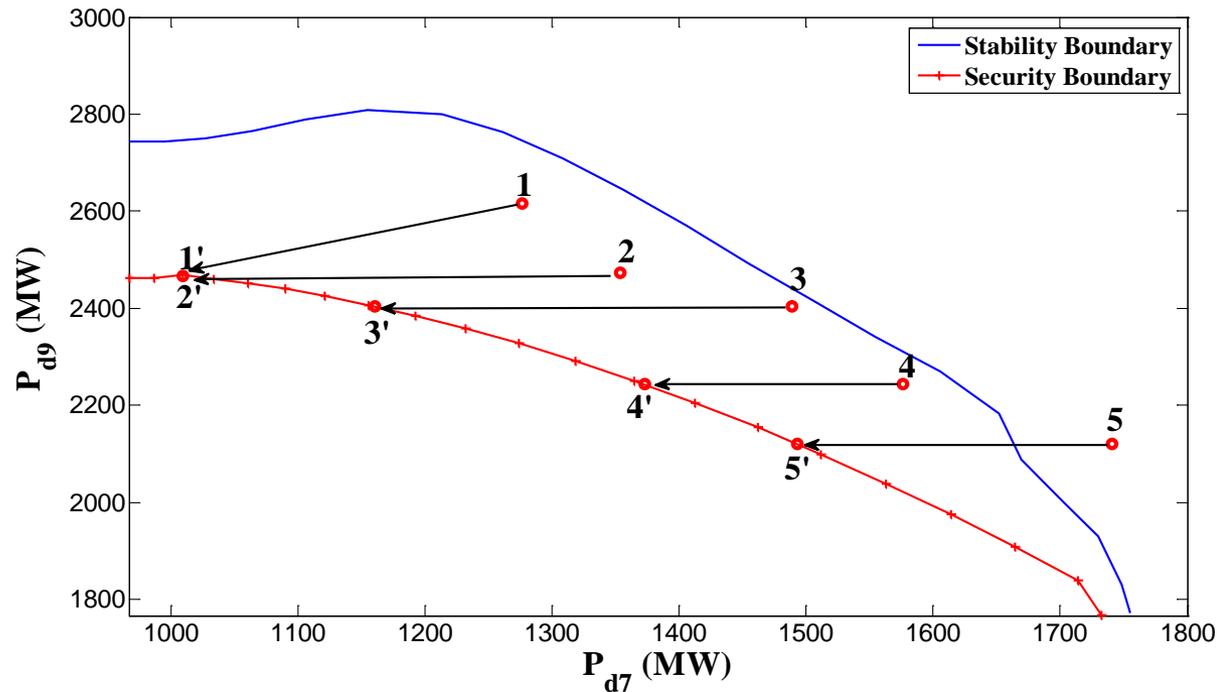
$$K_{G_\ell,t}^c - f_{PWL}(\hat{K}_{G_\ell,t}) \geq 0 \quad \forall t$$

# NN SBC-OPF

- Observations:
  - The NN SB constraint accounts for all system dynamics and an N-1 contingency criterion, but has limited number of input variables.
  - The NN SB function changes with system conditions.
  - The optimization problem is of similar complexity than an SC-OPF, since the SB is represented using a relatively simple and well-defined nonlinear function.

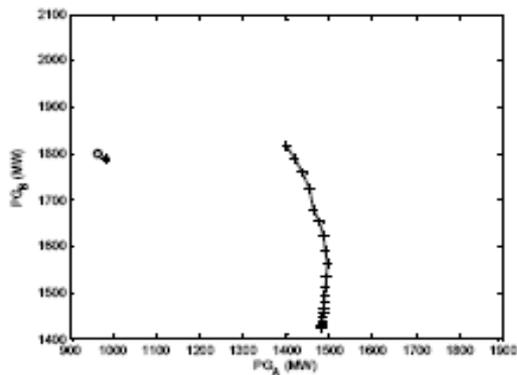
# NN SBC-OPF

- For the 2-area system, the load-curtailment OPF model yields:

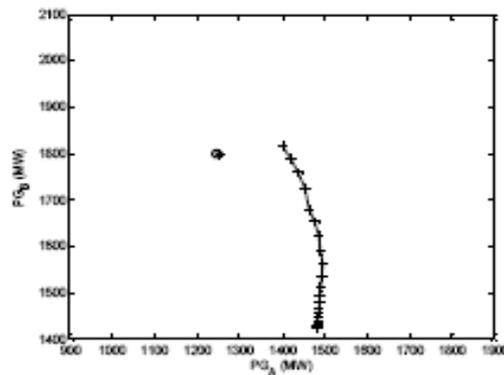


# NN SBC-OPF

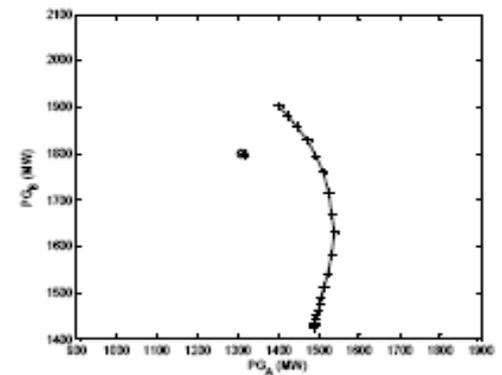
- For the 2-area system, the multi-period SBC-DC-OPF model yields:



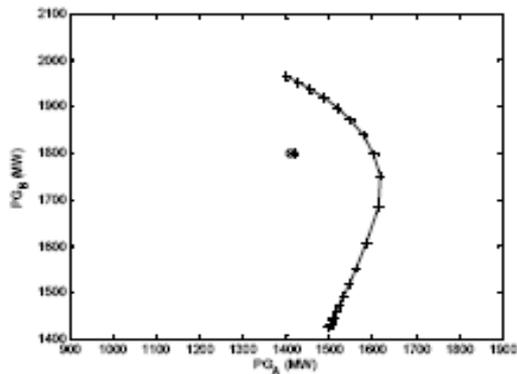
(a)



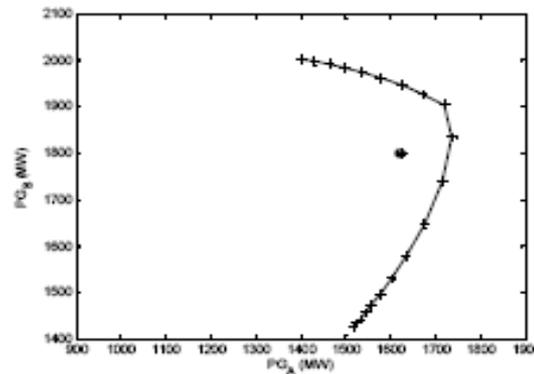
(b)



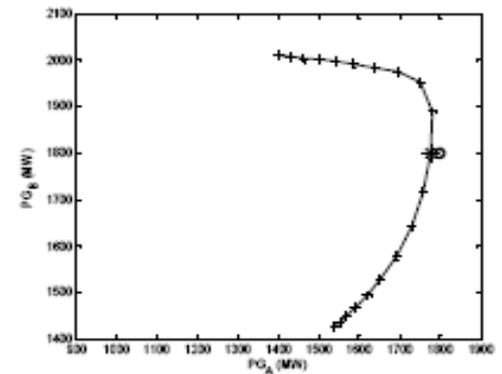
(c)



(d)



(e)



(f)

— SB

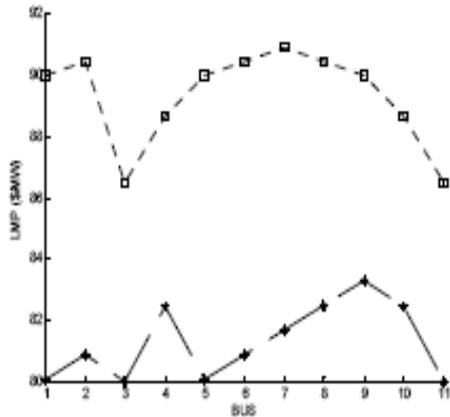


○ SC-DC-OPF solution

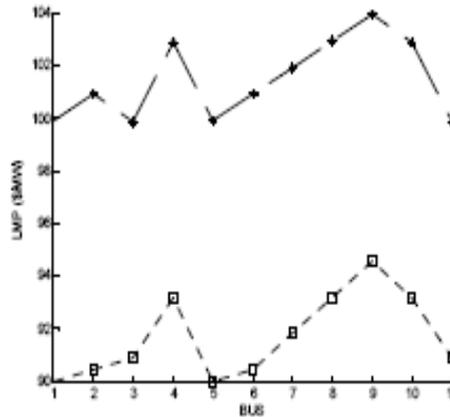


+ SBC-DC-OPF solution

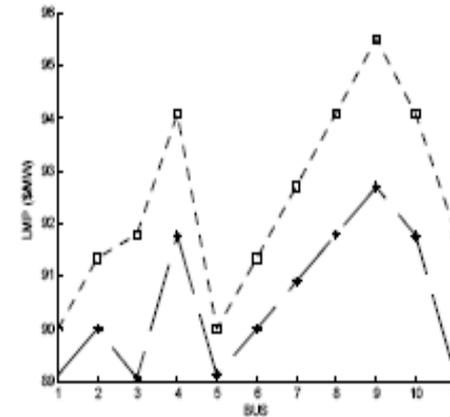
# NN SBC-OPF



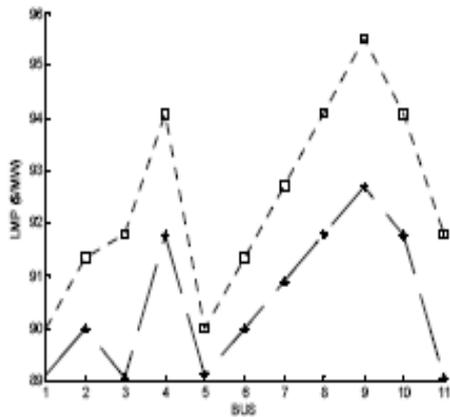
(a)



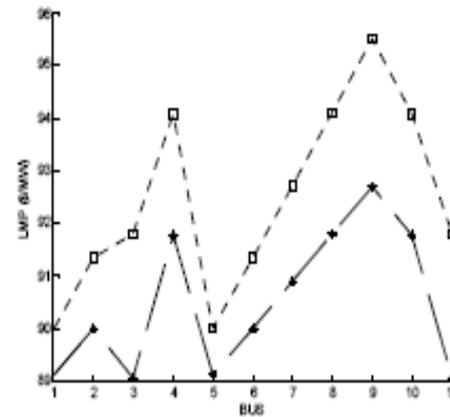
(b)



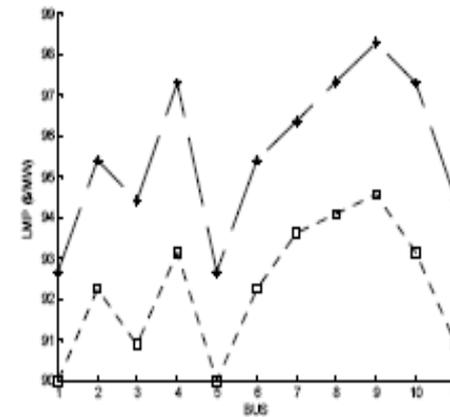
(c)



(d)



(e)



(f)

----- LMPs from SBC DC-OPF

----- LMPs from SC DC-OPF

# Conclusions

- Proper representation of system security in market clearing and dispatch mechanisms leads to better market and system conditions.
- Proposed methods so far are somewhat impractical:
  - Large NLP problems (several thousand constraints and variables), and solutions should be obtained in 1-2 min.
  - Convergence issues in some of these methods need further study, considering that global optimum values are not a great concern.

# Conclusions

- Current work:
  - Linearizing the SB constraint in the NN SBC-DC-OPF.
  - Developing a new VS index with “better” behavior than the MSV index, to replace the MSV constraint in the VSC-OPF model.

# Relevant Publications

- F. Milano, C. A. Cañizares, and M. Ivernizzi, “Multi-objective Optimization for Pricing System Security in Electricity Markets,” *IEEE Transactions on Power Systems*, Vol. 18, No. 2, May 2003, pp. 596-604.
- S. K. M. Kodsi and C. A. Cañizares, “Application of a Stability-constrained Optimal Power Flow to Tuning of Oscillation Controls in Competitive Electricity Markets,” *IEEE Transactions on Power Systems*, vol. 22, no. 4, November 2007, pp. 1944-1954.
- R. J. Avalos, C. A. Cañizares, and M. Anjos, “Practical Voltage-Stability-Constrained Optimal Power Flow,” *Proc. IEEE-PES General Meeting*, invited paper, Pittsburgh, PA, July 2008
- V. J. Gutierrez-Martinez, C. A. Cañizares, C. R. Fuerte-Esquivel, A. Pizano-Martinez, and X. Gu, “Neural-Network Security-Boundary Constrained Optimal Power Flow,” accepted to appear in *IEEE Transactions on Power Systems*, March 2010.
- C. Battistelli, C. A. Cañizares, M. Chehreghani, V. J. Gutierrez-Martinez, and C. R. Fuerte-Esquivel, “Practical Security-Boundary-Constrained Dispatch Models for Electricity Markets,” *Proc. Power Systems Computation Conference*, submitted November 2010.