

# Cascading dynamics of power grid networks

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- Motivation: basic facts about cascading failures
- Dynamical model
- Results: effect of load fluctuations (arXiv:1012.0815)
- Results: mitigation techniques (arXiv:1104.4558)
- Path forward

## Cascades in power grids:

- About 20 large blackouts ( $> 300MW$ ) happen every year.
- Annual damage to US economy around \$100 billions.
- Power-law probability distribution, i.e. enhanced probability of large cascades.
- Will get only worse: more demand, intermittent generation.

## Unlike epidemic cascades

- Non-local response (failure of one component affect all the network).
- Hierarchical (i.e. not scale-free) networks.

Potential flow on the graphs:

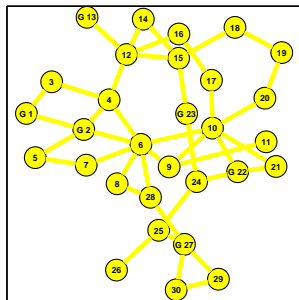
- $p_{ij}$  - flow over the edge  $(ij)$ :

$$x_{ij} p_{ij} = \theta_i - \theta_j$$

- $g_i$  - generator nodes (sources)
- $d_i$  - load nodes (sinks)

Kirchchoff Law:

$$\sum_{j \sim i} p_{ij} = \begin{cases} g_i, & i \in \mathcal{G}_g \\ -d_i, & i \in \mathcal{G}_d \\ 0, & i \in \mathcal{G}_0 \setminus (\mathcal{G}_g \cup \mathcal{G}_d) \end{cases}$$



Each line and generator have limited capacities:

$$|p_{ij}| < p_{ij}^{max}$$

$$0 < g_i < g_i^{max}$$

- When the line is overloaded, it is tripped by protective relay.
- If several lines are overloaded, the sequence of tripping is chosen randomly.
- Generators adjust their outputs to the total demand in the system
- When the capacity of generator is exceeded, its output saturates

# Algorithm: load fluctuations

Loads are assumed to be random and increase in linear fashion:

$$d_i = d_i^0 + \dot{d}_i t.$$

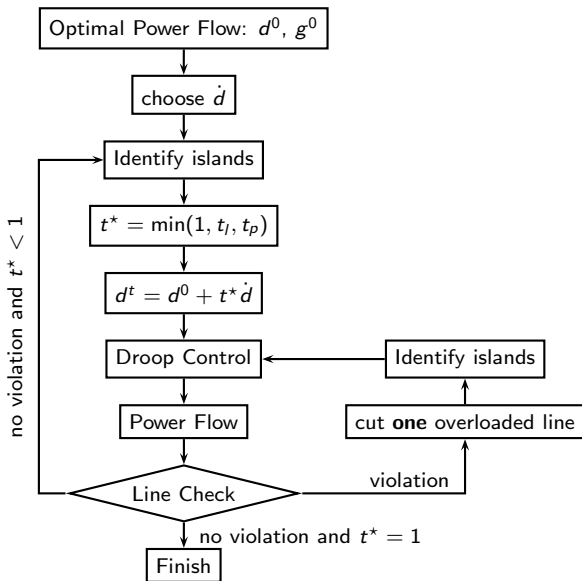
with  $d_i^0$  chosen from optimal power flow solution and  $\dot{d}_i$  being an i.i.d random variable with variance  $\Delta \cdot d_i^0$ .

At each step the moment of next overloading event is calculated:

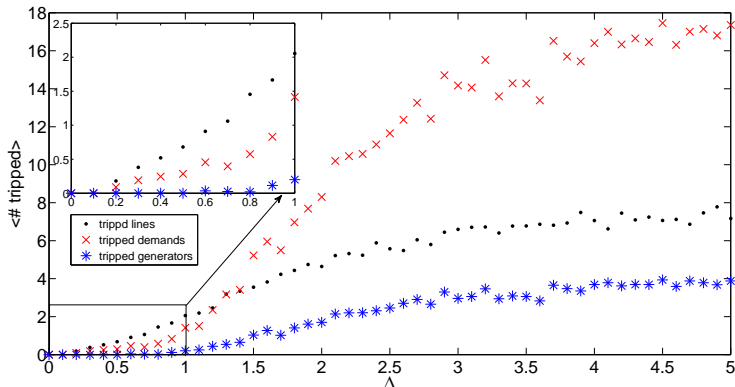
$$t^* = \min(1, t_{gen}, t_{line})$$

$$t_{gen} = \min_{i \in \mathcal{G}_g} \left( \frac{g_i^{max} - g_i^0}{\dot{g}_i} \right)$$

$$t_{line} = \min_{(i,j)} \left( \frac{p_{ij}^{max} - p_{ij}^0}{\dot{p}_{ij}} \right)$$



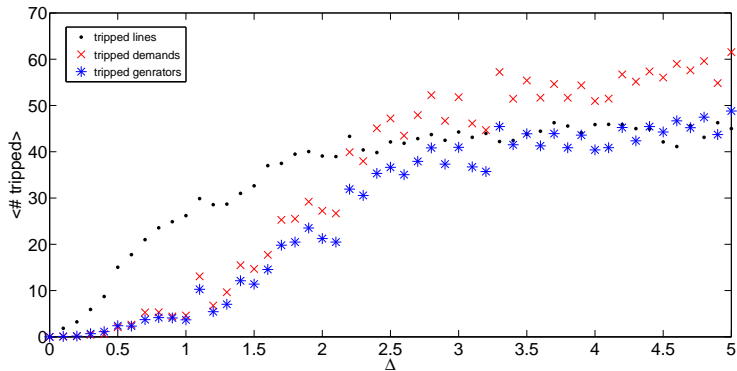
# Results: 30-bus IEEE network



Results of simulations for 30-bus IEEE network.



# Results: 118-bus IEEE network

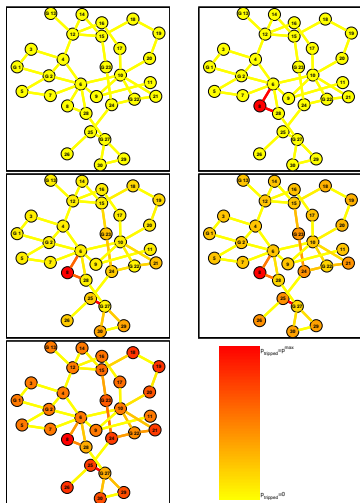


Results of simulations for 118-bus IEEE network.

# Results: interpretation

Several regimes of cascades:

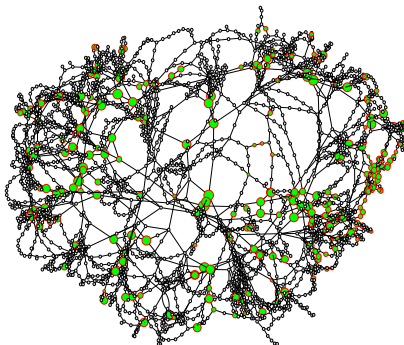
- Phase 0: Load fluctuations do not cause any failures
- Phase 1: Some lines are tripped, along with few loads.
- Phase 2: Few generators islanded
- Phase 3: Multiple islands, Large portion of loads unserved



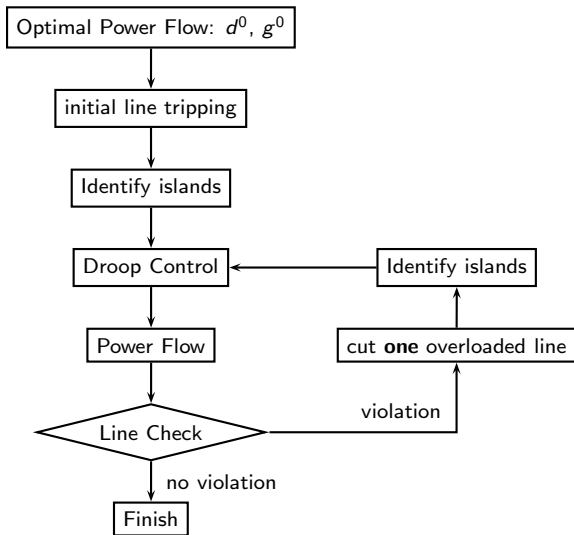
# Cascade mitigation

## Key differences

- The sequence of the line tripping events can be now controlled
- Cascade is initiated by two random line trippings
- No load fluctuations
- Simulations performed on Polish grid model ( 3000 components)



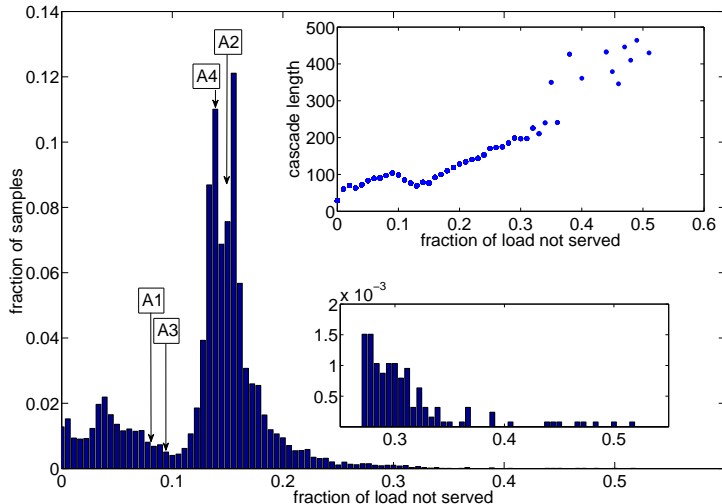
# Algorithm



Several heuristic were analyzed.

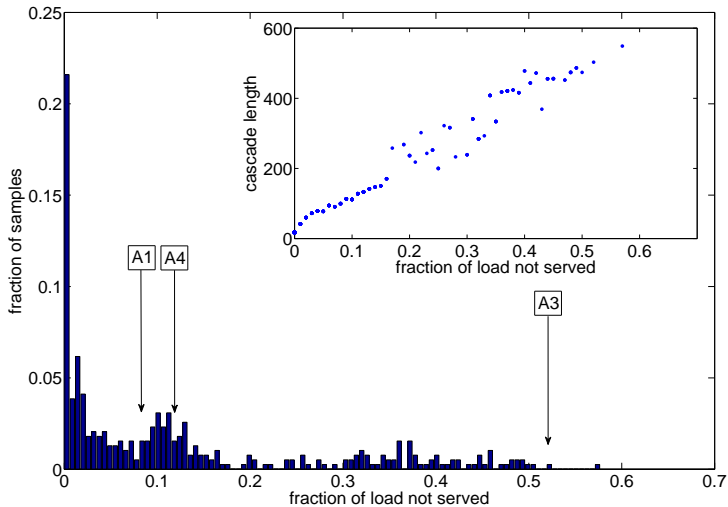
- A1** Trip the overloaded line,  $(i, j)$ , with the minimal current power flow,  $(i, j) = \operatorname{argmin}_{(i, j) \in \mathcal{L}_O} |p_{ij}|$ ;
- A2** Trip the overloaded line,  $(i, j)$ , with the maximal current power flow,  $(i, j) = \operatorname{argmax}_{(i, j) \in \mathcal{L}_O} |p_{ij}|$ ;
- A3** Trip the overloaded line,  $(i, j)$ , with the minimal relative overload,  $(i, j) = \operatorname{argmin}_{(i, j) \in \mathcal{L}_O} |p_{ij}| / p_{ij}^{\max}$ ;
- A4** Trip the overloaded line,  $(i, j)$ , with the maximal relative overload,  $(i, j) = \operatorname{argmax}_{(i, j) \in \mathcal{L}_O} |p_{ij}| / p_{ij}^{\max}$ ;

# Results: line 44



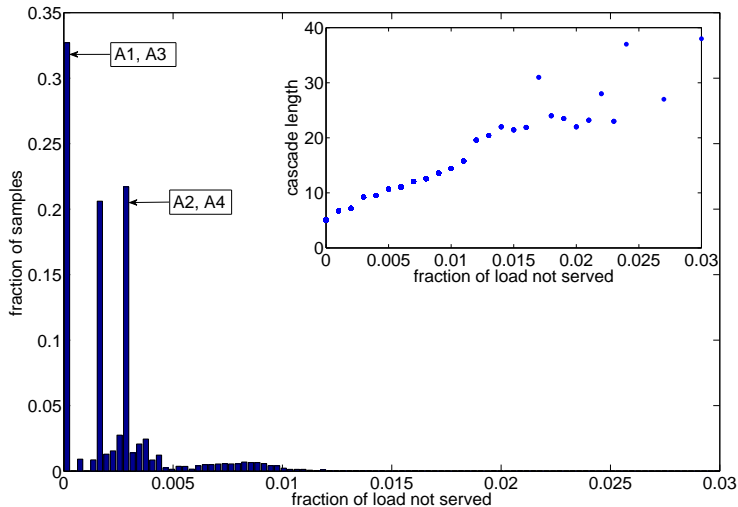
Distribution of outcomes with line 44 tripped initially.

# Results: line 3 and 29



Distribution of outcomes with lines 3 and 29 tripped initially

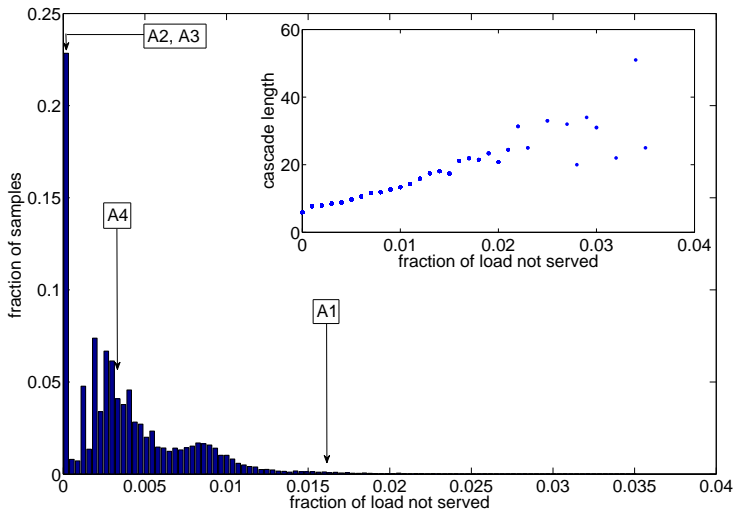
# Results: line 2832



Distribution of outcomes with line 2832 tripped initially



# Results: line 102



Distribution of outcomes with line 102 tripped initially

## Key points:

- Several scenarios of cascade dynamics.
- Broad distribution of outcomes: opportunity for (smart) control.
- Simple heuristics can help, but are from optimal.

## Other directions:

- Dynamic programming/optimization based approaches to control.
- Power-laws of cascade size distribution and network structure.
- More realistic models of power grid dynamics.