Message passing for integrating and assessing renewable generation in a redundant power grid


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## Optimization \& Control Theory for Smart Grids

http://cnls.lanl.gov/~chertkov/SmarterGrids/ (or google "Chertkov" and follow "smart grid" link)

Information science foundations for the Smart Grid


## Optimization \& Control Theory for Smart Grids

Smart Grid as a National Grand Challenge
neopmetens tor smut onse




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Approach





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## Optimization \& Control Theory for Smart Grids




## design

 control stability
## Assessing renewable generation

- Intermittent renewable-sources-based generation destabilizes the grid. How to improve grid control schemes?
- If renewable sources produce power $x$, how much can be saved on the level of the firm generation?


## Improvement trough redundancy

- Build additional power lines and introduce switches (on / off = power line connected to / disconnected from the network)
- Redundancy must help to optimize both stability and efficiency - larger space to optimize over. But how much does redundancy help?


## Methodology:

- Approach A: Take a realistic power grid model and several computers and run simulations. Do again when details change ...
- Approach B (probabilistic + physicist way): Study behavior of simple abstract models that facilitate the analysis, and look for universal properties, dependencies and behavior. Model choice criteria (in physics): The simpler and richer the better.


## Our power grid model

- M producers, N=DM consumers
- Out of every D consumers $R$ have auxiliary lines


Consumer "i" consumes $x_{i}$ produces $z_{i}$

Producer "a" capability $y_{a}$
$M=4, \quad N=12, \quad D=3$

## Setting

Switch variables for power lines:

$$
\sigma_{i a}=0 / \sigma_{i a}=1
$$

Each consumer has exactly one line on.

## Constraints

## $\sum_{a \in \partial i} \sigma_{i a}=1 \quad$ Every consumer one connection

$\sum_{i \in \partial a}$
Producers not overloaded



Note that the final topology is a tree, hence the Kirchhoff's laws satisfied.

However, general power flow optimum cannot be worse than the tree case!

## Questions

Given $\left\{x_{i}\right\},\left\{z_{i}\right\},\left\{y_{a}\right\}$ can all the constraints be simultaneously satisfied? (Nobody overloaded.)

If yes, then how many satisfying configurations of the switches are there?
Is it easy to find one?

Answer: via Belief Propagation

## How does BP work?

Prob. that line "ia" is in state $O_{i a}$ conditioned

$$
\begin{array}{ll}
\psi_{\sigma_{i a}}^{a \rightarrow i} & \text { constraint on " } i \text { " is missing } \\
\chi_{\sigma_{i a}}^{i \rightarrow a} & \text { constraint on " } a \text { " is missing }
\end{array}
$$



Iterative "message passing" scheme

## Belief Propagation

- Distributed approximative way of:
- (a) computing the probability that a given switch is on or off.
- (b) estimating number of valid (not overloading) configurations.
- For large number of customers and producers (thermodynamic limit) - average analysis solvable.


## Example n. 1



Fraction $1 / 3$ of consumers produce amount $z$
Every consumer consumes random number in (0.9,1.1)

## Example n. 2



Every consumer produced a random number between $(0, z)$

## Example n. 3

## produced > consumed

$y / 3>1-f z$

amount $z$ is produced by fraction $f$ of consumers

## Conclusions and Perspectives

- Existence of SAT/UNSAT phase transition and regimes where higher penetration useful or futile.
- Redundancy + switches help renewable integrations. Belief propagation a tool of analysis but also distributed control algorithm.
- In physics: Study of toy models (and phase transitions) leads to qualitative understanding. Is that true also for the Smart Grid?
- Combine belief propagation with DC or AC power flow rules on a non-tree topology.


## References

- L. Zdeborová, A. Decelle, M. Chertkov; Phys. Rev. E 90, 046112 (2009).
- L. Zdeborová, S. Backhaus, M. Chertkov; in HICSS 43.



## Belief Propagation Equations

$$
\begin{aligned}
\chi_{1}^{i \rightarrow a} & =\frac{1}{Z^{i \rightarrow a}} \prod_{b \in \partial i \backslash a} \psi_{0}^{b \rightarrow i} \\
\chi_{0}^{i \rightarrow a} & =\frac{1}{Z^{i \rightarrow a}} \sum_{b \in \partial i \backslash a} \psi_{1}^{b \rightarrow i} \prod_{c \in \partial i \backslash a, b} \psi_{0}^{c \rightarrow i} \\
\psi_{1}^{a \rightarrow i} & =\frac{1}{Z^{a \rightarrow i}} \sum_{\sigma \partial a \backslash i a} \theta\left(y_{a}-w_{i}-\sum_{j \in \partial a \backslash i} \sigma_{j a} w_{j}\right) \prod_{j \in \partial a \backslash i} \chi_{\sigma_{j a}}^{j \rightarrow a} \\
\psi_{0}^{a \rightarrow i} & =\frac{1}{Z^{a \rightarrow i}} \sum_{\sigma_{\partial a \backslash i a}} \theta\left(y_{a}-\sum_{j \in \partial a \backslash i} \sigma_{j a} w_{j}\right) \prod_{j \in \partial a \backslash i} \chi_{\sigma_{j a}}^{j \rightarrow a}
\end{aligned}
$$

## Example n. 0

Everybody consuming random number between (mean-width/2) and (mean+width/2), epsilon - fraction of consumers with no demand.
$\forall a$
$\forall i$

## Asymptotic, but also algorithmic solution



