Inelastic collisions of pulses in optical fibers

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Abstract

We present a comprehensive theory explaining interaction between two solitons from distant frequency channels in an optical fiber. The interaction may be viewed as an inelastic collision, in which energy is lost to continuous radiation due to non-zero third order dispersion. We derive a perturbation theory with two small parameters: the third order dispersion coefficient d_3, and the reciprocal of the inter-channel frequency difference $1/\Omega$. In the leading order the amplitude of the emitted radiation and the soliton's position shift are both proportional to $d_3/|\Omega|$. The accumulated effect of many collisions is a coherent frequency shift, proportional to d_3.

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Modern high speed optical fiber communication systems extensively use multi-frequency channel technology (wavelength division multiplexing - WDM, see e.g.[1]). One of the major limitations on the performance of WDM systems is caused by the nonlinear interaction of data signals from different channels. We investigate this phenomena using conventional optical solitons as an example of information carriers. In an ideal case soliton bit-patterns from different channels would not experience any distortion due to the elastic character of the soliton-soliton interaction. The effect of higher order dispersion breaks the ideal picture. In this inelastic case collisions between solitons from different frequency channels lead to emission of radiation, which is accompanied by shift in soliton position (soliton walk off from the assigned time slot), corruption of the soliton shape and other undesirable effects. Moreover, the radiation emitted due to collisions might, in its turn, lead to intra-channel interaction between solitons from the same frequency channel. Therefore, it is important to have a realistic estimation for the intensity of the radiation emitted, as well as for the change in the soliton parameters due to the inter-channel interaction.

In this letter we calculate the spectrum and intensity of the radiation emitted in the result of collision between solitons from different channels taking into account the effect of third order dispersion. (See Figure 1 for cartoon of the collision process.) We also calculate the change, induced by the collision, in the soliton parameters. To achieve the goal a (double) perturbation theory with respect to two small parameters is developed: the dimensionless third order dispersion coefficient d_3 and the reciprocal of the dimensionless inter-channel frequency difference $1/\Omega$. We find that the amplitude of the emitted radiation and the change in soliton positions are proportional to $d_3/|\Omega|$ and do not depend on the sign of Ω. The amplitude and the phase velocity do not change in the leading order of the perturbation theory. Accumulation in the soliton position shift due to many collisions leads to the frequency shift, $\propto d_3$.

Propagation of short wave packets through an optical fiber is described by the following modification of the nonlinear Schrödinger equation (see Ref.[2], p. 44)

$$i\partial_z \Psi + \partial_t^2 \Psi + 2|\Psi|^2\Psi = id_3\partial_t^3\Psi, \tag{1}$$

where z is the position along the fiber and t is the retarded time associated with the reference channel. Coefficients in front of the second-order dispersion term and the nonlinear Kerr term are re-scaled to unity and to the factor of two in Eq.(1) by a proper choice of time- and Ψ- units respectively[3]. Term which appears on the rhs of Eq.(1) accounts for the effect of third order dispersion (linear dependence of fiber chromatic dispersion on the wavelength of carrier frequency), with d_3 being a constant. Higher order terms (with higher temporal derivatives, and also accounting for other than given by the Kerr term types of nonlinearity) can be neglected in the majority of practical cases. Notice also that fiber losses in Eq. (1) are omitted. Eq.(1) applies to description of the inter-channel interaction of optical pulses in three different cases: (i) dispersion length, length of nonlinearity and characteristic distance of soliton interaction are much smaller than characteristic length of fiber losses[4]; (ii) fiber losses are compensated by in-line distributed optical amplifiers; and (iii) these losses are compensated by lumped optical amplifiers achieved by insertion of fiber spans with exponentially decreasing spatial dispersion profile[6] (dispersion tapered fibers).

It is important to mention that Eq.(1) is generic, as it explains simultaneous propagation through many frequency channels. Unlike in the degenerate case of $d_3 = 0$, Eq.(1) is not integrable. However, in many practical examples $d_3 \ll 1$, thus a perturbative calculation about the integrable $d_3 = 0$ limit is proper.

Single soliton solution of Eq.(1) with $d_3 = 0$ in a given frequency channel, characterized by a frequency shift Ω relative to a reference channel, is given by

$$\eta \frac{\exp[i\alpha + i\Omega(t-y) + i(\eta^2 - \Omega^2)z]}{\cosh[\eta(t-y-2\Omega z)]}, \tag{2}$$

where α, η and y stand for the soliton phase, amplitude and position, respectively. Assuming that $d_3 \ll 1$, we will be looking for perturbative solution of Eq. (1) in the form

$$\Psi_\Omega(t,z) = \left[\frac{\eta_\Omega}{\cosh(\tilde{\eta}_\Omega \tau_\Omega)} + g_\Omega(\tilde{\eta}_\Omega \tau_\Omega) + \dots\right] e^{i\chi_\Omega}, \tag{3}$$

where $\tau_\Omega = t - y_\Omega - 2\Omega(1 + 3d_3\Omega/2)z$, $\tilde{\eta}_\Omega = (1 + 3d_3\Omega)^{-1/2}\eta_\Omega$, and $\chi_\Omega = \alpha_\Omega + \Omega(t - y_\Omega) + [\eta_\Omega^2 - \Omega^2(1 + d_3\Omega)]\, z$. The first term on the rhs of Eq. (3) is the ideal single soliton solution, which accounts for the shift in the second order dispersion $\sim d_3\Omega$. (The shift is not necessarily small. The only limitation on $d_3\Omega$ is $d_3\Omega > -1/3$, which is the condition for the existence of a soliton solution in Eq. (1).) The second term in Eq. (3) is perturbative, $O(d_3)$. To calculate the term we adopt perturbation method introduced by Kaup in[7]. In Kaup's theory, a differential operator $\hat{L}_\eta$ is used to describe a linear perturbation around the ideal soliton solution. The complete system of eigen-functions of $\hat{L}_\eta$ includes continuous spectrum of delocalized modes, as well as four discrete localized modes, related to small changes in the four parameters of the soliton: Ω, α, η and y. We expand g_Ω in terms of the eigen-functions of $\hat{L}_\eta$ and calculate the coefficients of this expansion. Although the expansion contains contributions from both localized and unlocalized modes, the complete contribution is localized.(Explicit expression for g_Ω can be found in[9].)

Let us now describe collision between two solitons from different channels. For simplicity, and without any loss of generality, we choose one of the channels to be the reference one with $\Omega = 0$. One also assumes that for the second channel Ω is much larger than inverse width of the pulse (i.e. $\Omega \gg 1$ in the dimensionless units used in Eq.(3)). We are looking for a two-soliton solution of Eq.(1) in the form $\Psi_{two} = \Psi_0 + \Psi_\Omega + \Phi$, where Ψ_0 and Ψ_Ω

are single-soliton solutions of Eq. (1) in channels 0 and Ω, respectively, and Φ is a small correction due to collision. (It is straightforward to check that the exact two-soliton solution of Eq.(1) at $d_3 = 0$ turns into $\Psi_0 + \Psi_\Omega$ in the leading, i.e. accounting for $O(1/\Omega)$ terms, order.) One substitutes Ψ_{two} into Eq.(1) and calculates the correction Φ_0 to the soliton in the reference channel. Calculation of the correction Φ_Ω to the soliton in the Ω channel is similar. Since Φ_0 oscillates together with Ψ_0, and $\Omega \gg 1$, one neglects the exponentially small contributions from the terms rapidly oscillating with z. Then, the equation describing Φ_0 is

$$\left(\partial_z - i\hat{L}_\eta\right) \begin{pmatrix} \tilde{\Phi}_0 \\ \tilde{\Phi}_0^* \end{pmatrix} = 4i|\Psi_\Omega|^2 \begin{pmatrix} \tilde{\Psi}_0 \\ -\tilde{\Psi}_0^* \end{pmatrix}, \tag{4}$$

where $\tilde{\Phi}_0 \equiv \Phi_0 \exp(-i\chi_0)$ and $\tilde{\Psi}_0 \equiv \Psi_0 \exp(-i\chi_0)$. Vicinity (in z) of the collision event, $[z_0 - \tilde{z}/\Omega, z_0 + \tilde{z}/\Omega]$, where $\Omega \gg \tilde{z} \gg 1$, is naturally separated from the regions before and after collision. In the collision region $\tilde{\Phi}_0$ acquires a fast change with respect to z. Since for this region $\Delta z \sim 1/\Omega$, the $\partial_z \Phi_0$ and $|\Psi_\Omega|^2 \tilde{\Psi}_0$ terms give leading contributions to Eq. (4), while the $\hat{L}_\eta$ term in Eq.(4) can be neglected. After collision, i.e. at $z > z_0 + \tilde{z}/\Omega$, interaction between the two solitons becomes exponentially small, so that the term $|\Psi_\Omega|^2 \tilde{\Psi}_0$ can be neglected. Formally, separation of scales means that one can replace the rhs of Eq.(4) by $C\delta(z - z_0)$, where $\delta(z)$ is the Dirac-delta function and the constant C is simply the integral of the rhs of Eq.(4) over z. It results in a well-formulated Cauchi problem for $\tilde{\Phi}_0$, which is solved by projecting the source term (integral of the rhs of Eq.(4)) and $\tilde{\Phi}_0$ into the series with respect to the eigen function of $\hat{L}_\eta$. Coefficients of the expansion for $\tilde{\Phi}_0$ correspond to soliton parameters and intensity of the emitted radiation. The resulting equations for the expansion coefficients are first order ordinary differential equations with source terms being convolutions of the integral of the rhs of Eq.(4) with respective eigen-functions of $\hat{L}_\eta$.

(See Ref.[9] for complete table of integrals entering the problem.) The resulting $\tilde{\Phi}_0$, found in the form of expansion over the eigen-functions of $\hat{L}_\eta$, contains $O(1/\Omega)$ and $O(d_3/\Omega)$ terms. (It is straightforward to check that there are no $O(d_3)$ terms in $\tilde{\Phi}_0$.) Moreover, one finds that the only effect in the $O(1/\Omega)$ order of the theory is seen in the change of the soliton phase, described by $\Delta\alpha_0 \simeq 4\eta_\Omega(1+3d_3\Omega)^{1/2}[(1+3d_3\Omega/2)|\Omega|]^{-1}$. To calculate the radiation contribution and also change in other soliton parameters (e.g. soliton position) one turns to account for the next $O(d_3/\Omega)$ order of the theory.

One finds that the amplitude of radiation emitted by a soliton from the reference channel is proportional to $d_3/|\Omega|$. The total radiation energy $\mathcal{E}_0$, emitted by this soliton, is

$$\mathcal{E}_0 \simeq 0.16967\frac{\pi\eta_0^3\eta_\Omega^2(1+3d_3\Omega)d_3^2}{2(1+3d_3\Omega/2)^2\Omega^2}\,.$$

The soliton amplitude and phase velocity do not change in this order of the theory. (This later statement is consistent with the conservation law for the total energy, which requires $\eta_0 = 1 + O(d_3^2/\Omega^2)$. See also[8] for an example of similar situation.) The collision also leads to a change in the position of the soliton, which is given by

$$\Delta y_0 \simeq -2.710132\frac{\eta_\Omega(1+3d_3\Omega)^{1/2}d_3}{(1+3d_3\Omega/2)|\Omega|}\,.$$

Notice that the frequency difference Ω, enters the expressions for the position shift and the phase shift dependent on its absolute value only, $\sim 1/|\Omega|$. This is consistent with the fact that Eq. (1) is not invariant under the transformation $z \to -z$. The observation is especially important for the position shift, since it means that the shift will always be in the same direction, regardless of the Ω sign. Therefore, soliton in the reference channel acquires a frequency shift in the result of many collisions

$$\Delta\Omega_0 \simeq \frac{2s|\Omega|(1+3d_3\Omega/2)\Delta y_0}{T} = O(d_3)\,,$$

where T is the width of the time slot allocated for the solitons in channel Ω and s is the parameter describing the average fraction of occupied time slots, $0 < s < 1$. The fact that radiation amplitude and soliton position acquire shifts in the mixed, second order (with respect to d_3 and $1/\Omega$) can be explained in the following manner. For ideal solitons (i.e. for solitons unperturbed by d_3) the only effect of the collision is seen through a phase shift. Therefore, it is natural to expect that account for collision between non ideal solitons, perturbed by d_3, will be proportional to the product of the soliton shape distortion, estimated by $O(d_3)$, and the duration (in z) of the collision event, estimated by $O(1/\Omega)$. Hence, the combined effect is $\sim d_3/\Omega$, indeed.

Let us use our results to make some predictions for an optical fiber setup with distributed amplification compensating losses or with lumped amplification and dispersion tapered fibers. Taking $\eta_0 = 1$ and requiring that widths of the two solitons would be equal (bit-rates should be the same in all channels) one obtains $\eta_\Omega = (1 + 3d_3\Omega)^{1/2}$. Then, for the values specified in[3] one derives, $\mathcal{E}_0 \approx 1.33274 \times 10^{-5}$, for the fraction of the total radiation emitted by the soliton in the reference channel, and $\Delta y_0 = -0.0271$ for the soliton position shift, measured in units of the soliton width. For $s = 0.5$ and $T = 5$, $\Delta\Omega_0 = -0.1084$, corresponding to he dimensional frequency shift of -6.83×10^{10}Hz.

Even though the effect of a single collision is relatively small, the accumulated effect of multiple collisions of a soliton from the $\Omega = 0$-channel with many solitons from other frequency channels can be significant. We have already discussed one of the possible accumulative effects: the effect of the soliton (channel) frequency shift, which appears as a result of multiple shifts of the soliton position. One also finds that the total energy emitted by a soliton grows linearly with the number of collisions. Thus, for the parameters explained in Ref.[3], the average distance passed by the reference channel soliton until it experiences 10^4 collisions (that is the number correspondent to a noticeable loss of its energy: $10^4 \times \mathcal{E}_0 \approx 10^{-1}$) is $\sim$ 1000km. Notice also, that the radiation emitted in the result of multiple collisions can also lead to undesirable radiation-mediated interaction of the given soliton with other solitons in the same frequency channel.

We conclude by pointing out that this study opens new vistas for testing variety of inter-channel interaction phenomena. For example, the effect of four-wave mixing resulting from soliton collisions in case of three or more equally separated channels, can also be addressed. Inter-channel pulse interaction caused by Raman scattering is another example of phenomenon, interesting and significant to study.

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3. The dimensionless z in Eq. (1) is $z = x(\alpha P_0/2)$, where x is the actual position, P_0 is the peak soliton power, and α is the Kerr nonlinearity coefficient. The dimensionless retarded time is $t = \tau/\tau_0$, where τ is the retarded time and τ_0 is the soliton width. The spectral width ν_0 is given by $\nu_0 = 0.31483/\tau_0$, and the channel spacing by $\Delta\nu = \Omega\nu_0$. $\Psi = E/\sqrt{P_0}$, where E is the actual electric field. The dimensionless second and third order dispersion coefficients are given by $d = 1 = \beta_2/(\alpha P_0 \tau_0^2)$ and $d_3 = \beta_3/(3\alpha P_0 \tau_0^3)$, where β_2 and β_3 are the second and third order chromatic dispersion coefficients, respectively.

Consider, for example, the following set of experimental parameters: $\tau_0 = 0.5$ps, $\beta_2 = -1$ps^2/km, $\beta_3 = 0.1$ps^3/km, $\alpha = 10$W^{-1}km^{-1}, $P_0 = 0.4W$, $\Delta\nu = 6.30 \times 10^{11}$Hz, and the total energy of the soliton is 4×10^{-13}J. These values correspond to $d_3 = 0.066667$, and $\Omega = 10$.

4. Effects of fiber losses can be neglected (see for instance[5]) if values of $\Delta\nu$, β_2, β_3, and τ_0 satisfy the following two conditions: $\tau_0 \ll \sqrt{\beta_2/\gamma}$, and $\Delta\nu \gg (\gamma\tau_0^2)/\beta_3$.

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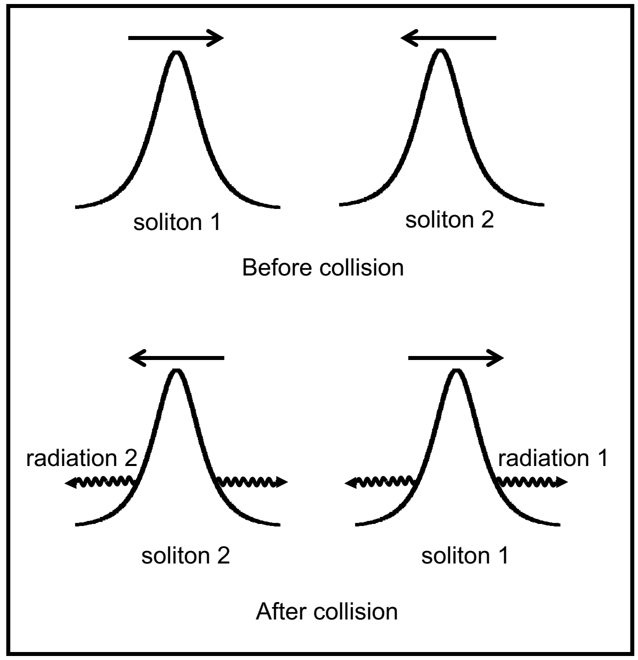
List of Figure Captions

Fig. 1. Schematic description of the collision between two solitons from different frequency channels.



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