Instanton approach for codes without/with loops

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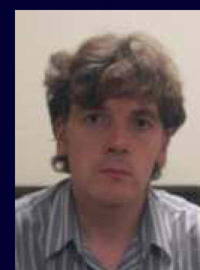

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Outline

- Basics
- Instanton method
- Codes without loops
- Bit-error-rate (BER)
- Dodecahedron code
- 4D cube code
- 4D cube code — BER
- Conclusions

Basics

Transmitter $\longrightarrow$ Encoder $\longrightarrow$ Channel $\longrightarrow$ Decoder $\longrightarrow$ Receiver

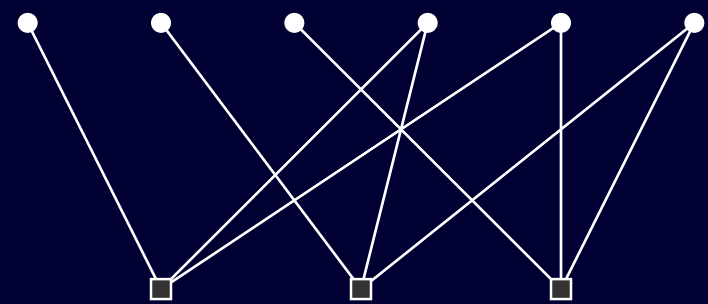
$\uparrow$

LDPC code:

Parity check matrix

$$\hat{\mathsf{H}} = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

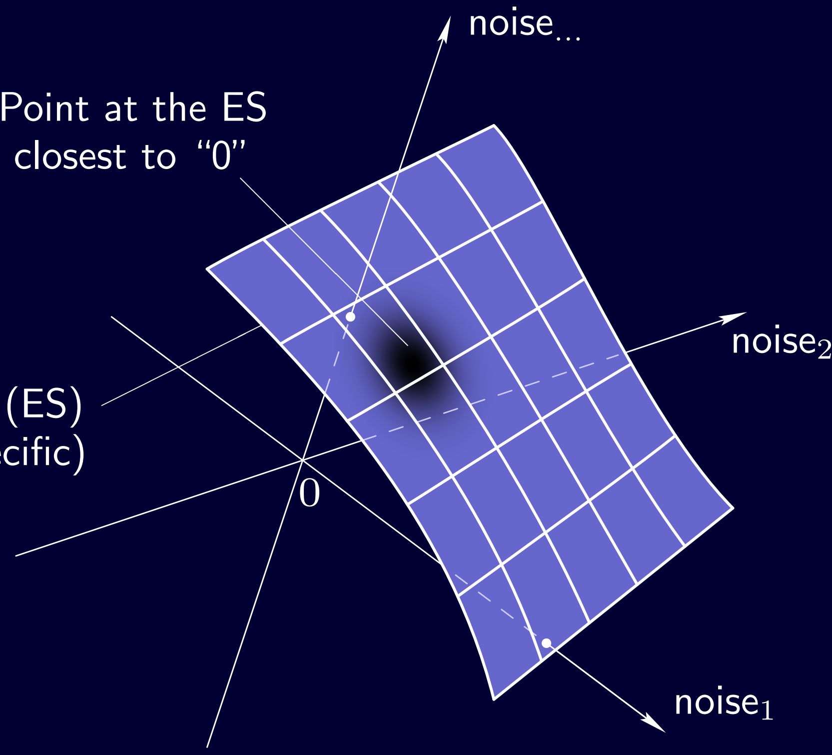
Tanner graph



MAP, Symbol-to-symbol MAP: too costly

Iterative decoding (Message passing, Believe propagation):
Gallager (1963), Pearl (1988), MacKay (1999), Bethe (1935)

"Instanton" method



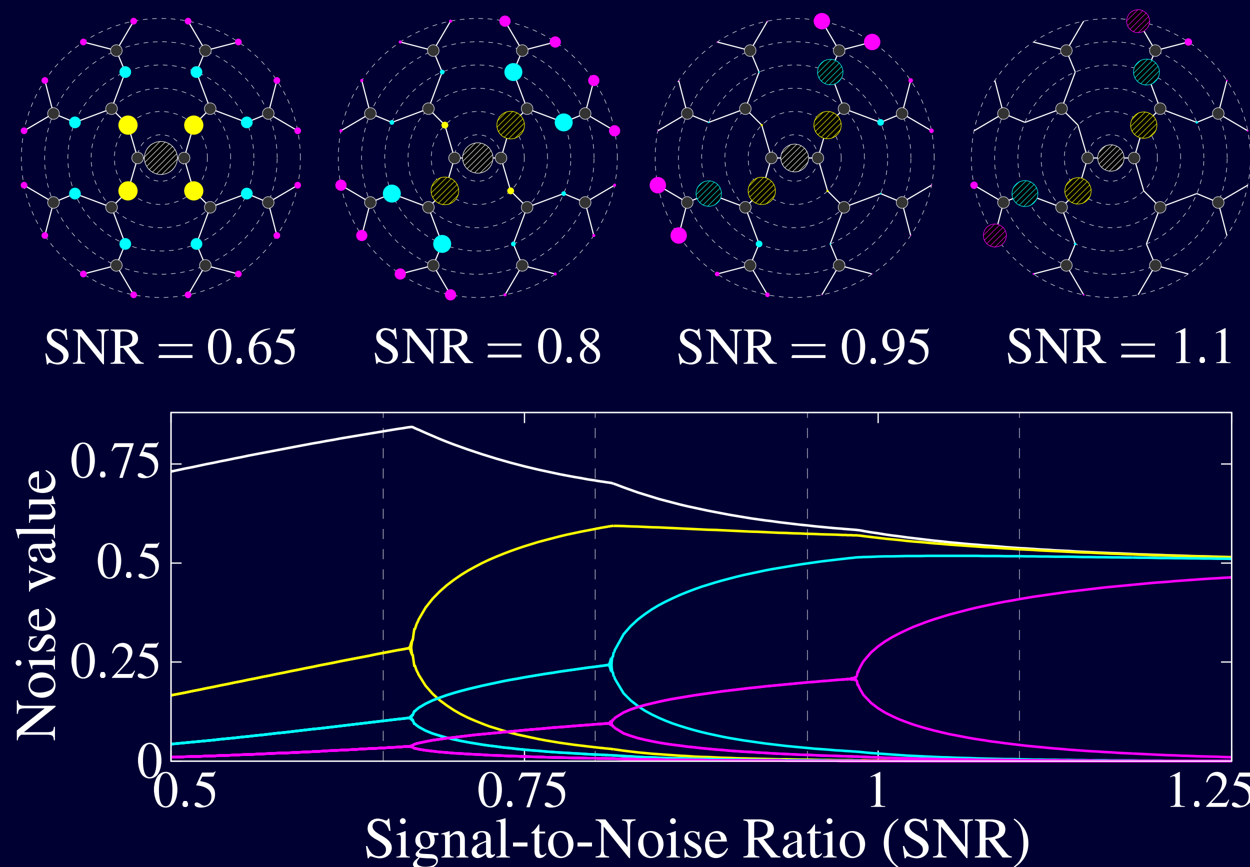
$$\mathsf{BER} = \int d(\mathsf{noise})\, \mathsf{WEIGHT}(\mathsf{noise})$$

$$\mathsf{BER} \sim \mathsf{WEIGHT}\begin{pmatrix} \text{instanton conf} \\ \text{of the noise} \end{pmatrix}$$

$$\begin{matrix} \text{instanton conf} \\ \text{of the noise} \end{matrix} = \begin{matrix} \text{Point at the ES} \\ \text{closest to "0"} \end{matrix}$$

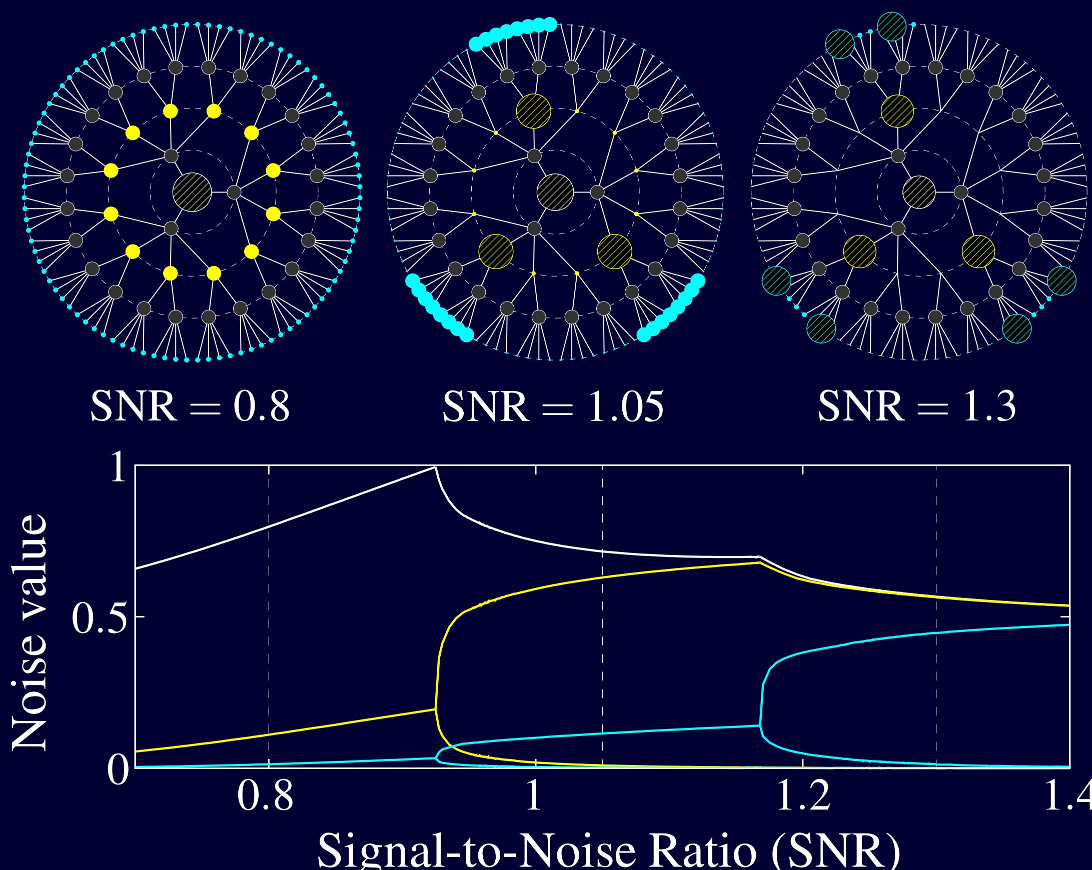
Saddle-point method
Method of steepest descent

“Optimal” noise configuration



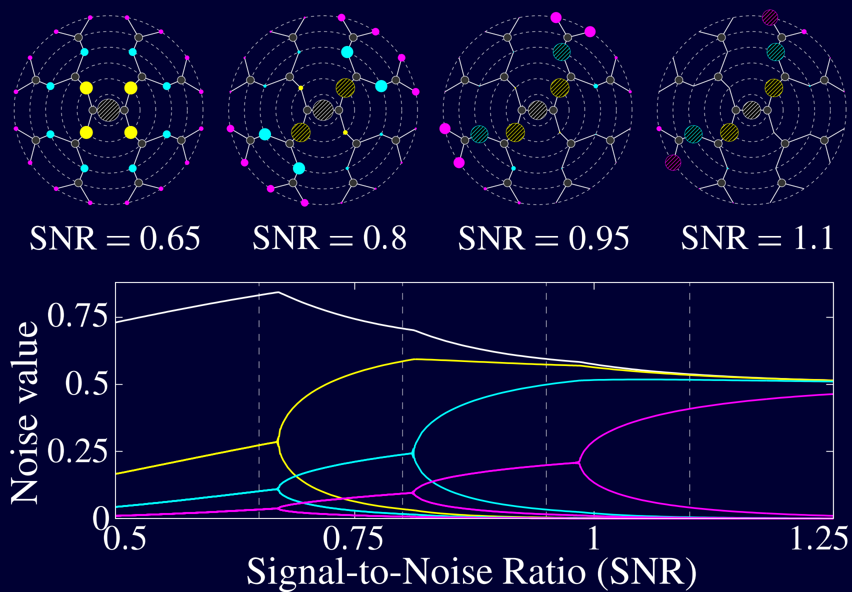
$m = 2,\ l = 3,\ n = 3$

“Optimal” noise configuration

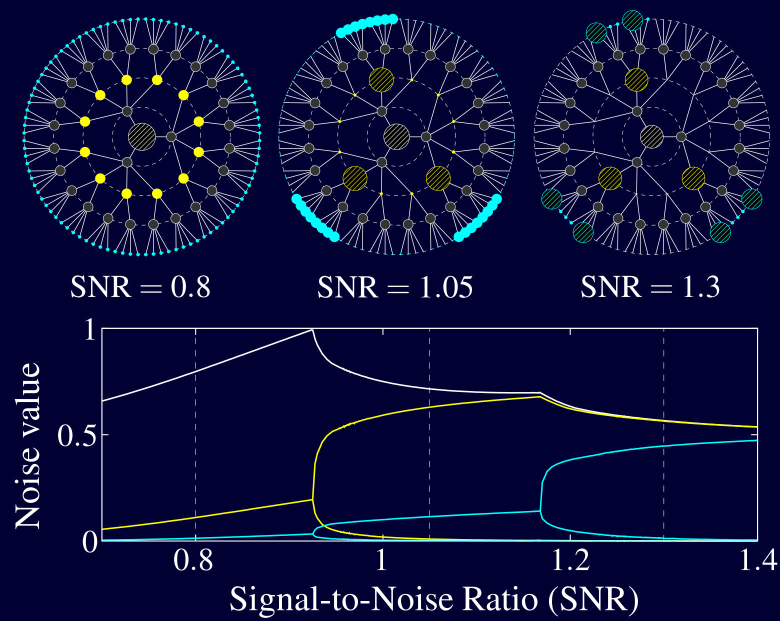


$m = 3,\ l = 5,\ n = 2$

“Optimal” noise configuration



$m = 2,\ l = 3,\ n = 3$



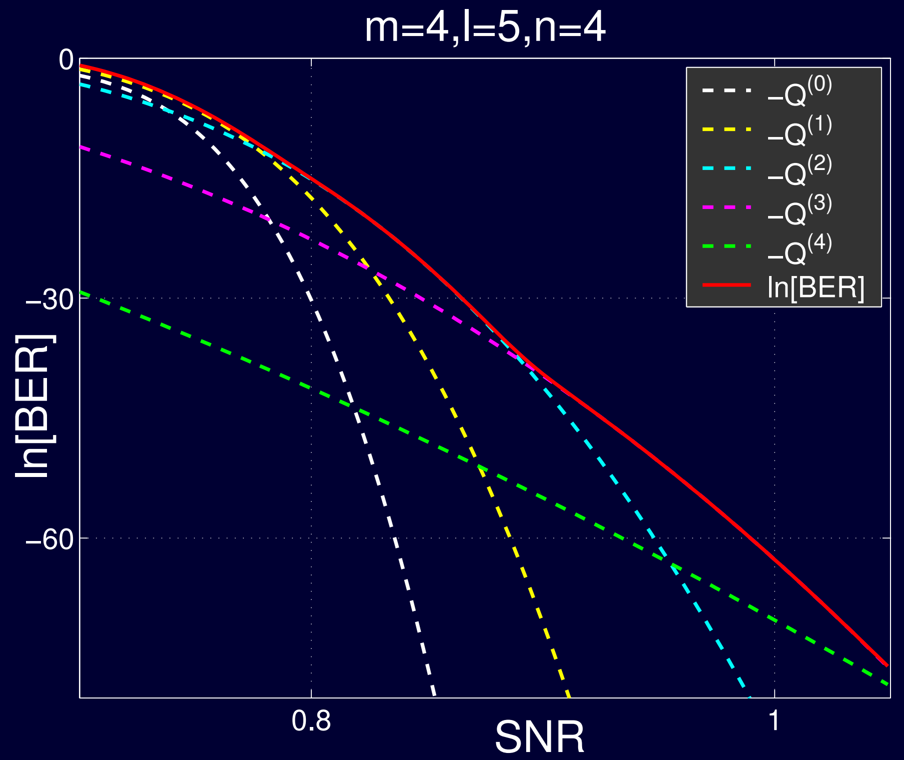
$m = 3,\ l = 5,\ n = 2$

Different symmetry noise configurations and bifurcation picture.
The area of circles on Tanner graph $\propto$ the value of the noise.

Bit-Error-Rate

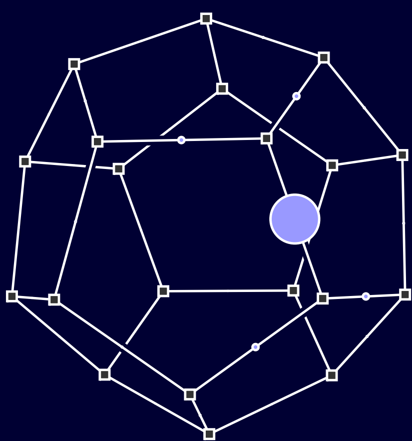
$Q^{(n)}$ — length^2 of the n^{th} solution for "optimal" noise configuration.

$Q^{(n+1)} = Q^{(n)}$ — transition points

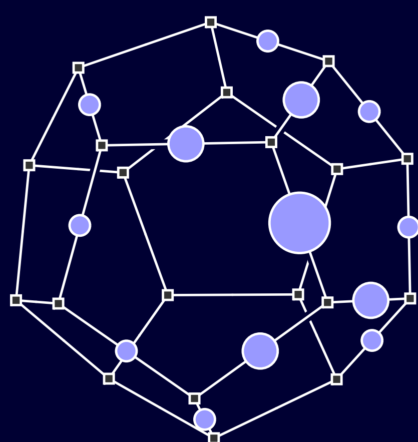


Error floor is due to the change of "optimal" noise configuration with SNR

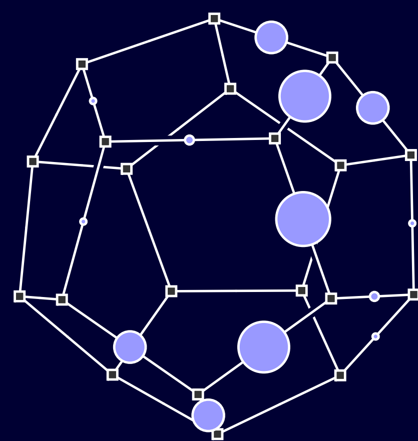
“Optimal” noise configuration



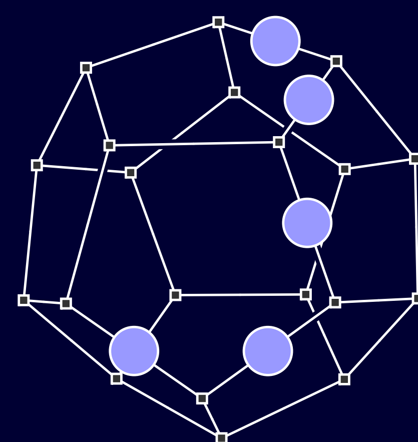
SNR = 0.125



SNR = 0.721



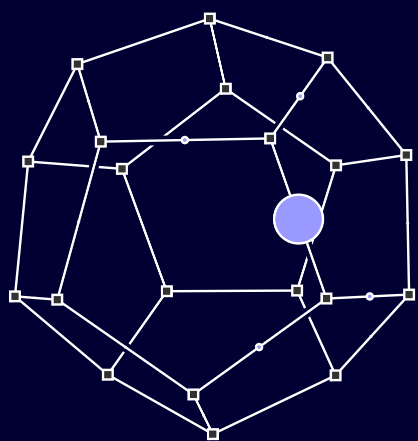
SNR = 0.926



SNR = 2.

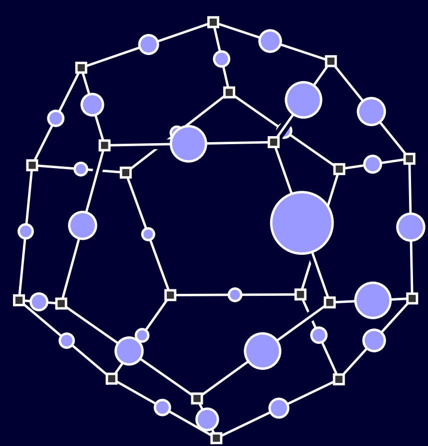
Iterative decoding, 2 iterations

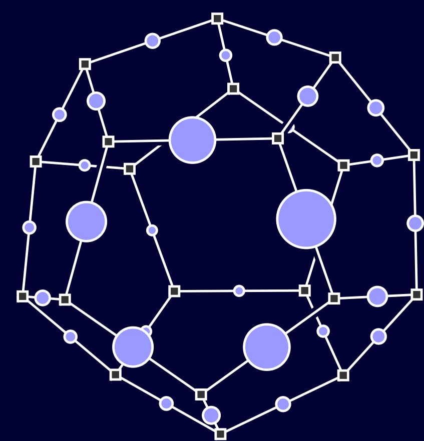
“Optimal” noise configuration



SNR = 0.125

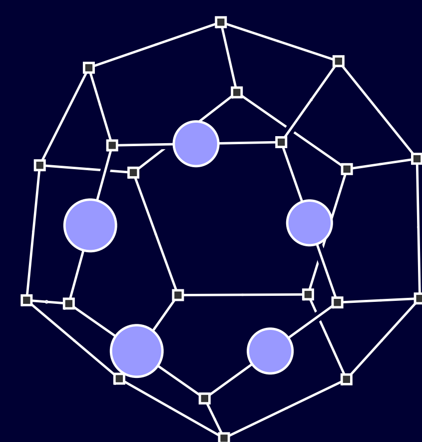
SNR = 0.6





SNR = 0.614

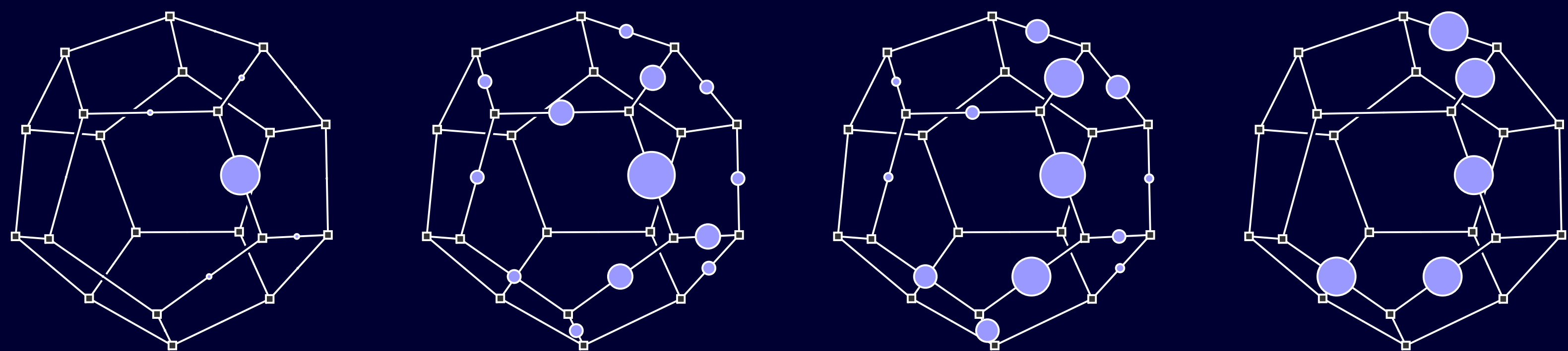
SNR = 2.



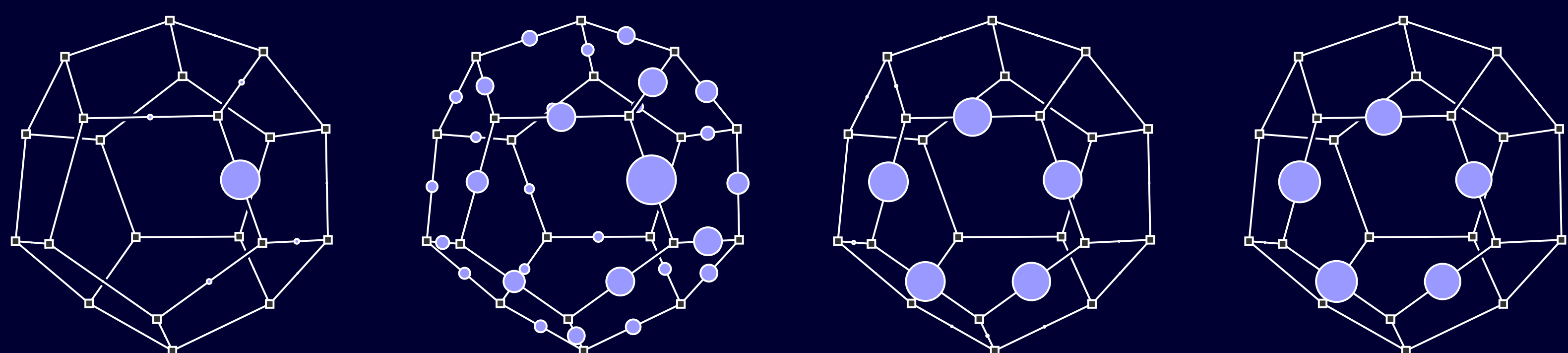
Iterative decoding, 8 iterations

“Optimal” noise configuration

2 iterations

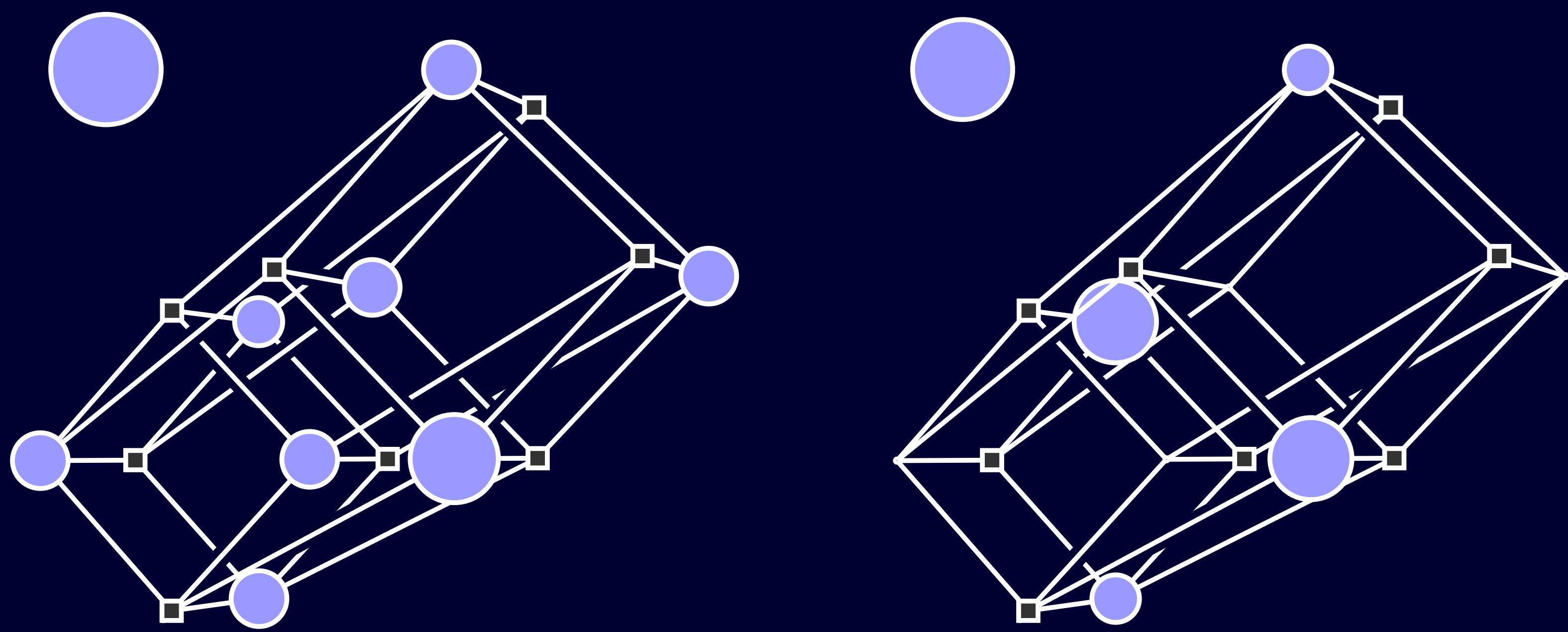


$\mathrm{SNR} = 0.125$ $\mathrm{SNR} = 0.6$ $\mathrm{SNR} = 0.824$ $\mathrm{SNR} = 2.$



8 iterations

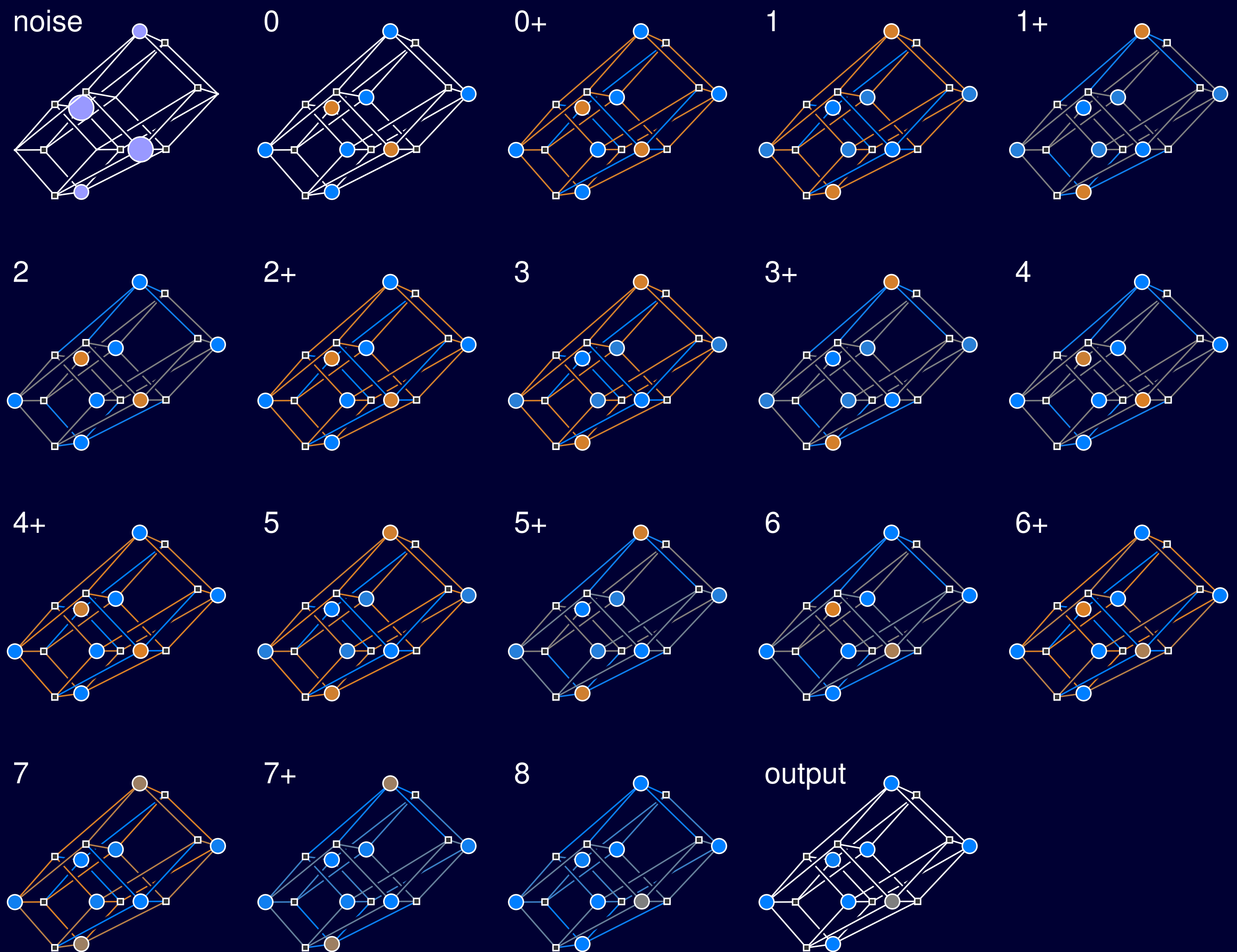
“Optimal” noise configuration



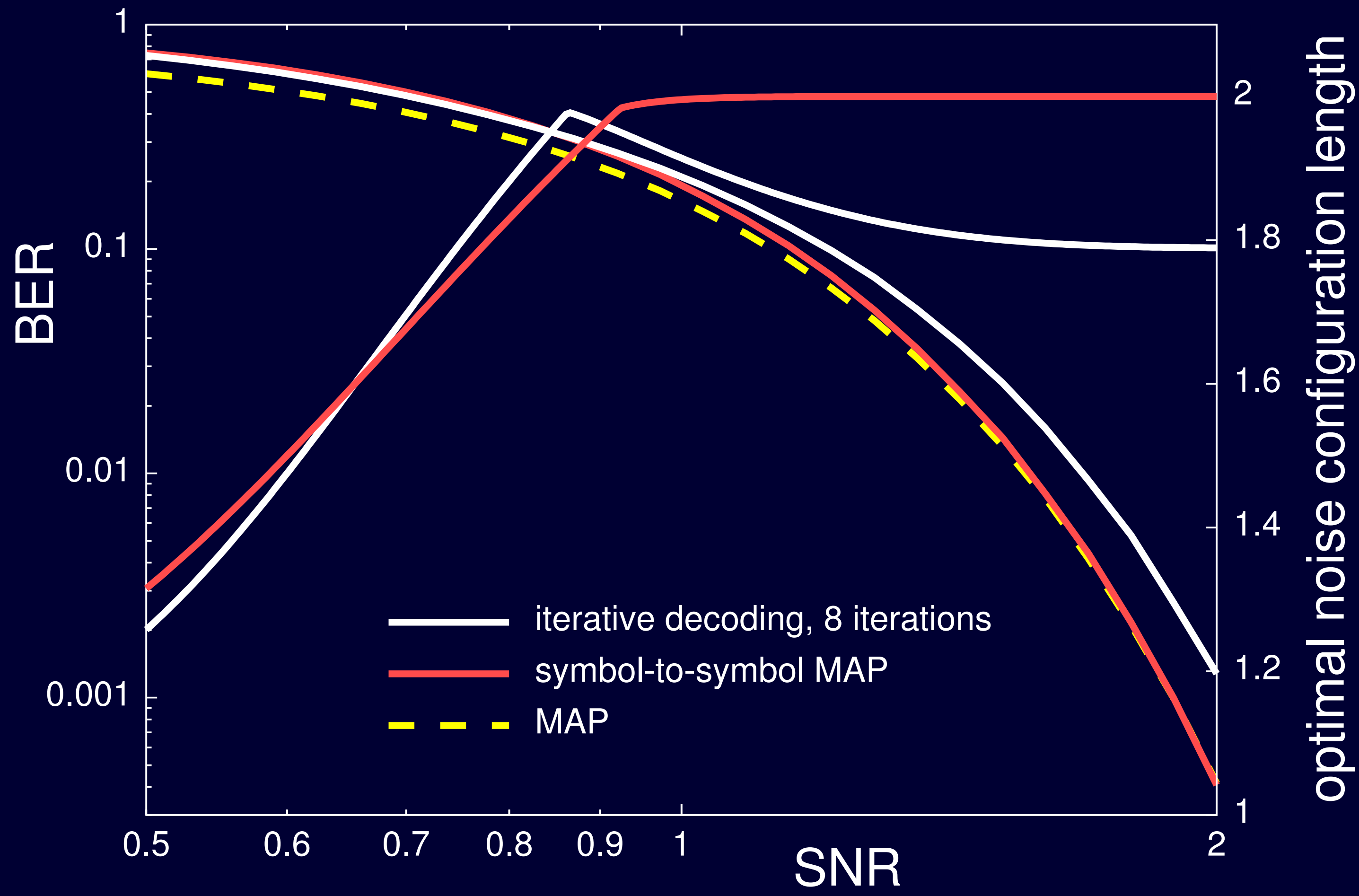
SNR = 0.861 SNR = 2.

Iterative decoding, 8 iterations

Iterations dynamics



Bit-Error-Rate



Conclusions

- While Signal-to-Noise Ratio (SNR) passes certain values, the symmetry of "optimal" noise configuration changes. There could be several bifurcations for one code.
- At low SNR the optimal noise configutations are localized on Tanner graph.
- If the cycles in the Tanner graph of the code are long enough, and the number of iterations is not so large, the bifurcation picture from a tree code is correct at low SNR.
- Even if the volume of the vicinity of the instanton that contributes to error probability is not known, the position of instanton gives the main part of the error probability logarithm (the only thing one actually wants to know).
- The bifurcations lead to flattening of the error probability *vs.* SNR curve, that provides an insight to error-floor phenomenon.
- The length of "optimal" noise configration could decrease with SNR.

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