Soft annealing: A new approach to difficult computational problems

Error-correcting codes:
In their Statistical Mechanics formulation there are two mathematically equivalent formulations.

- One in terms of the source letters

$$J_i = (-1)^{\sum_j G_{ij} u_j} = C^i_{k_1 \cdots k_i} \sigma_{k_1} \cdots \sigma_{k_i}$$

$$H^{source}(\vec{\sigma}) = -\ln P^{source}(\vec{\sigma} | \vec{J}^{out}) =$$

$$-\sum_i h_i C^i_{k_1 \cdots k_i} \sigma_{k_1} \cdots \sigma_{k_i}$$

$$h_i = \frac{1}{2} \log \frac{q(J_i^{out} | 1)}{q(J_i^{out} | -1)}$$

- One in terms of the code letters

$$(-1)^{\sum_j H_{lj}x_j} = 1 \;\rightarrow M^l_{k_1 \cdots k_l} J_{k_1} \cdots J_{k_l} = 1$$

$$P^{code}(\vec{J}|\vec{J}^{out}) = c \prod_l \delta(M^l_{k_1 \cdots k_l} J_{k_1} \cdots J_{k_l}; 1)$$

$$\exp\left(\sum_i h_i J_i\right)$$

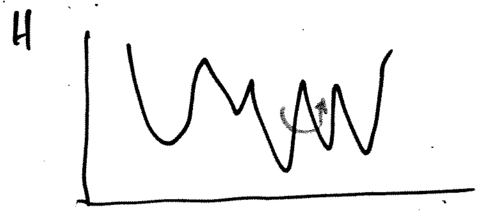
$$-H^{code}(\vec{J}) = \ln P^{code}(\vec{J}|\vec{J}^{out}) =$$

$$u \sum_l M^l_{k_1 \cdots k_l} J_{k_1} \cdots J_{k_l} + \sum_i h_i J_i$$

EMPIRICAL OBSERVATION

Easier to minimize H^{code} rather than H^{source} despite the fact that the number of variables is much smaller in H^{code}.

Intuitive explanation: In a larger dimensional space it is possible to go around barriers to find the minimum. Optimization easier in the large space.



Well known that there exists a large class of systems which are particularly hard to study both analytically and numerically

- disordered systems (spin-glasses, random fields)

- difficult optimization problem

- error correction codes ...

large number of local minima separated by large barriers

Proposal

- Imbed the original system into a new larger system

- Impose necessary constrains to reduce the large system to the original system

- replace the hard constrains with soft constrains

example of spin models in a three dimensional cubic lattice
Hamiltonian I want to study:

$$-H^0 = \sum_{x,y,z} J^x(x,y,z)\sigma(x,y,z)\sigma(x+1,y,z) +$$

$$J^y(x,y,z)\sigma(x,y,z)\sigma(x,y+1,z)$$

$$+J^z(x,y,z)\sigma(x,y,z)\sigma(x,y,z+1)$$

Introduce the new Hamiltonian:

$$-H^{new} = \sum_{x,y,z} J^x(x,y,z)\sigma^x(x,y,z)\sigma^x(x+1,y,z)+$$

$$J^y(x,y,z)\sigma^y(x,y,z)\sigma^y(x,y+1,z)$$

$$+J^z(x,y,z)\sigma^z(x,y,z)\sigma^z(x,y,z+1)+$$

$$u(\sigma^x(x,y,z)\sigma^y(x,y,z)+\sigma^x(x,y,z)\sigma^z(x,y,z)+$$

$$\sigma^y(x,y,z)\sigma^z(x,y,z))$$

H^{new} contains three times more spins than H^0 every $\sigma(x,y,z)$ of $H^0 \rightarrow \sigma^x(x,y,z)$, $\sigma^y(x,y,z)$, $\sigma^z(x,y,z)$ of H^{new}

The ferromagnetic coupling u couples together the three types of spin on every lattice site

- $u = 0 \Rightarrow H^{new}$ reduces to $3L^2$ decoupled one dimensional chains

- $u \to \infty \Rightarrow \sigma^x(x,y,z) = \sigma^y(x,y,z) = \sigma^z(x,y,z)$ and H^{new} reduces to H^0

H^{new} is called the soft constrained model

If H^0 has a phase transition at $\beta = 1/T = \beta_c$, we expect the phase diagramme of H^{new} to be two dimensional.

the points on this critical line to be on the same universality class

Spin Glasses

I studied the Edwards Anderson model on a cubic lattice with periodic boundary conditions $J = \pm 1$ independent random variables

The phase transition studied by N. Kawashima and A.P. Young, *Phys. Rev. B* **53** R484 (1996) see also E. Marinari, G. Parisi and J.J. Ruiz-Lorenzo, in *Spin Glasses and Random Fiels*, edited by A.P. Young

$\beta_c = .90$ $\nu = 1.7 \pm .3$

Simulations of the soft version of the model keeping the ratio $u = v/\beta$ fixed to $u = .75$
$L = 8$, 12, 14 and $L = 16$, 1280 samples per size

There is phase transition in the soft model for $\beta = .96$. This critical point belongs to the same universality class as the hard model. The low temperature phases are very similar in both models.

IS IT EASIER TO SIMULATE H^{new} ?

measure the spin autocorrelation functions

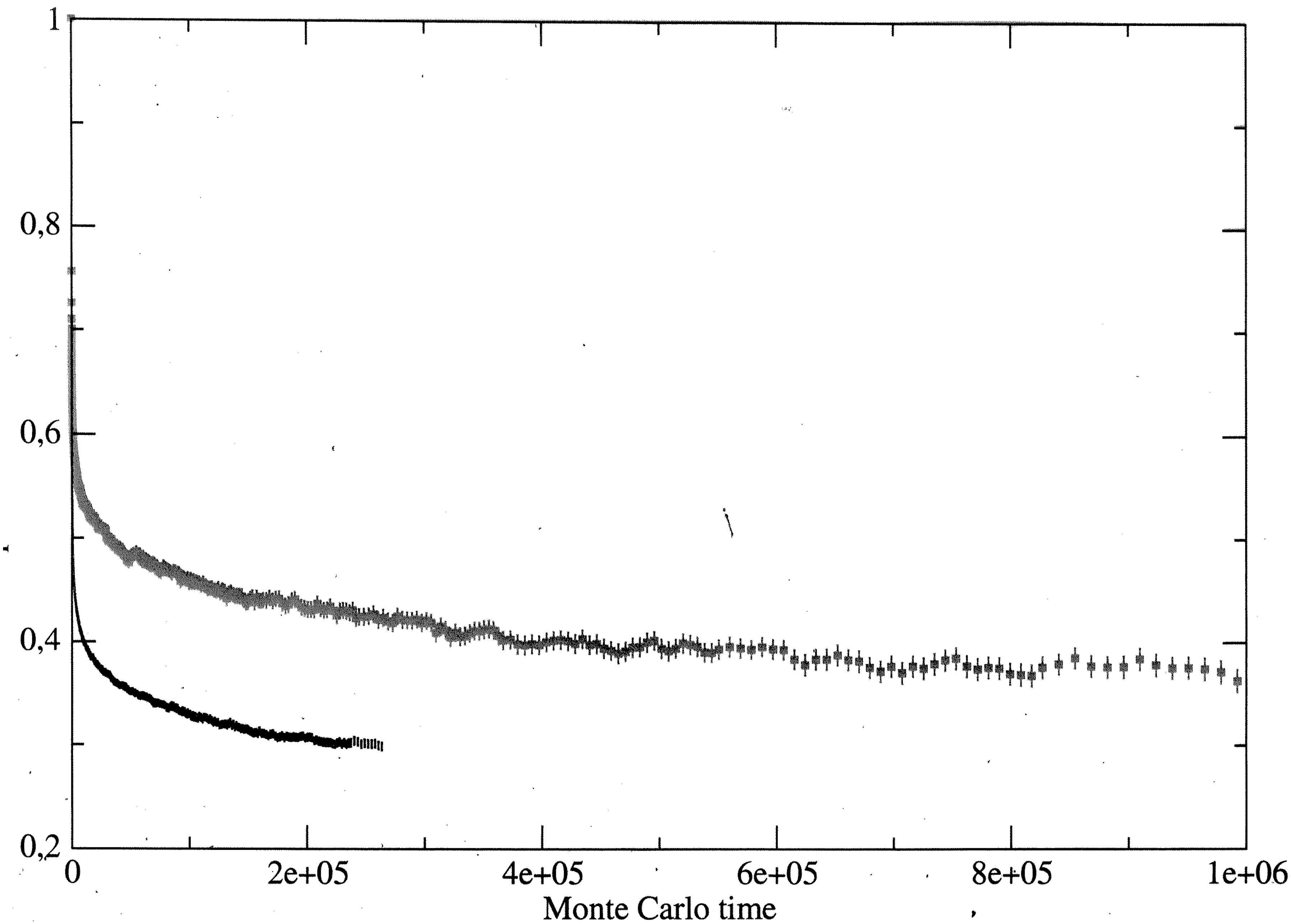
$$C^h(t) = (1/L^3) \sum_i \overline{< \sigma_i(t_0)\sigma_i(t_0+t) >}$$

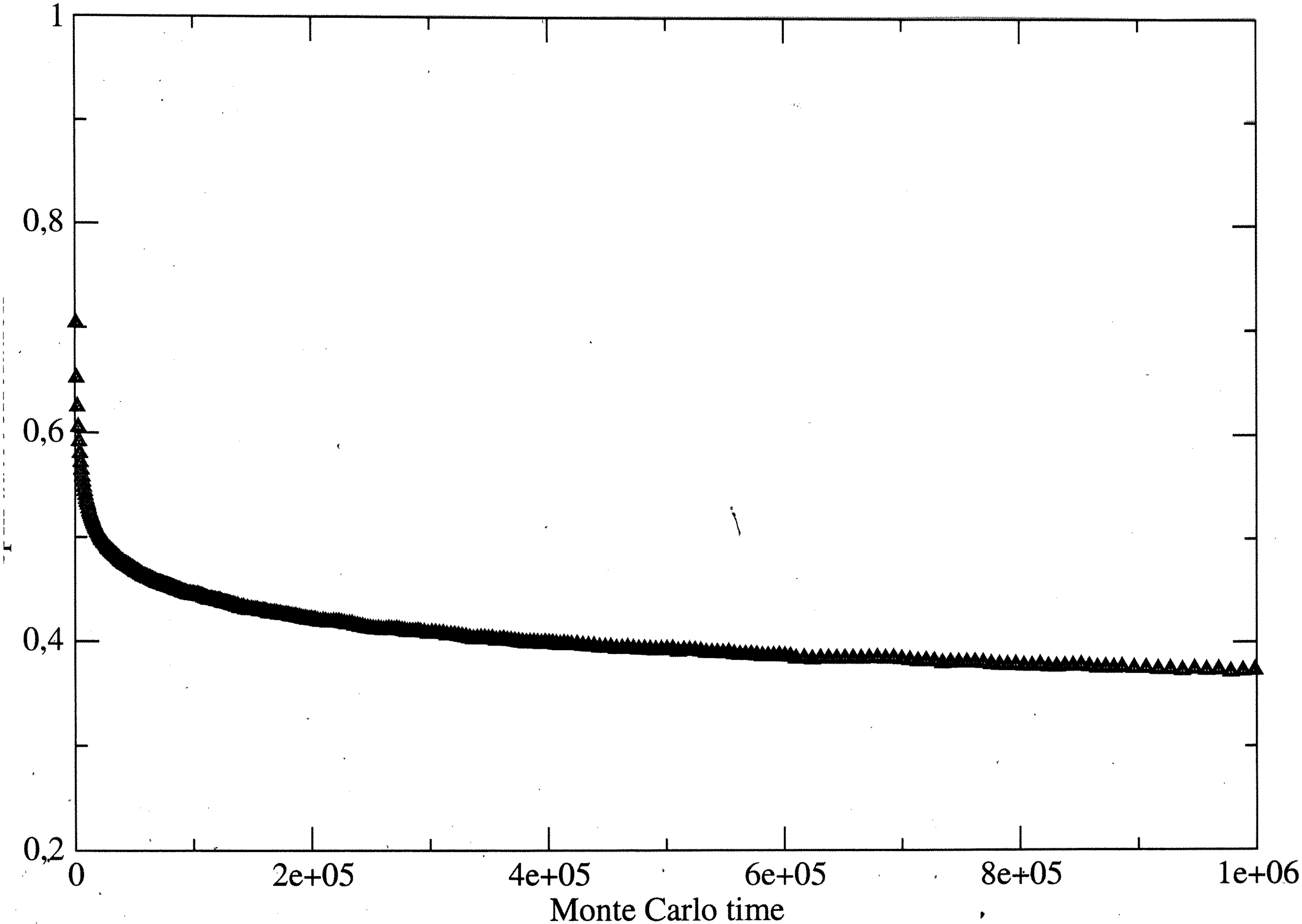
$$C^s(t) = (1/3L^3) \sum_i \overline{< \sigma_i^x(t_0)\sigma_i^x(t_0+t) >+}$$

$$\overline{< \sigma_i^y(t_0)\sigma_i^y(t_0+t) + \sigma_i^z(t_0)\sigma_i^z(t_0+t) >}$$

$C^s(t)$ decays much faster than $C^h(t)$ I measure $t^h(c,\beta)$ and $t^s(c,\beta)$ after which C^s and C^h fall below the value c

$C^h(t^h(c,\beta)) = c$ and $C^s(t^s(c,\beta)) = c$


1
0,8
0,6
0,4
0,2
0
2e+05
4e+05
6e+05
8e+05
1e+06
Monte Carlo time


1
0,8
0,6
0,4
0,2
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2e+05
4e+05
6e+05
8e+05
1e+06
Monte Carlo time

I found at $\beta = \beta_c$ and $c = .40$

- $L = 8 : t^h/t^s \;=\; 7.7$
- $L = 12 : t^h/t^s \;=\; 28$
- $L = 16 : t^h/t^s \;=\; 39$

rescale time in C^s by a factor t^h/t^s

The gain in very large in the soft model
larger for larger volumes
not clear whether t^h/t^s obeys some kind of finite size scaling
softening not unique procedure
interesting to apply the method also to other difficult optimization problems.