Superposition and Nonlocality in Bohmian Mechanics

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Pictures of quantum mechanics

Fundamental pictures of quantum mechanics:

- **Heisenberg (1925)** $\Rightarrow$ Operators ("black box")
- **Schödinger (1926)** $\Rightarrow$ Deterministic wave fields
- **Feynman (1948)** $\Rightarrow$ Classical-like paths and waves

Quantum system = wave

Why trajectory pictures of quantum mechanics?

Why trajectory pictures of quantum mechanics?

Particle distributions behave as waves …

(Born's statistical interpretation of quantum mechanics)

… but individual particles behave as individual point-like particles!

Is there any chance to understand quantum-mechanical processes and phenomena as in classical mechanics,
i.e., in terms of *exact* (non approximate) and *well-defined trajectories* in configuration space
(where *real* experiments take place)?

Why trajectory pictures of quantum mechanics?

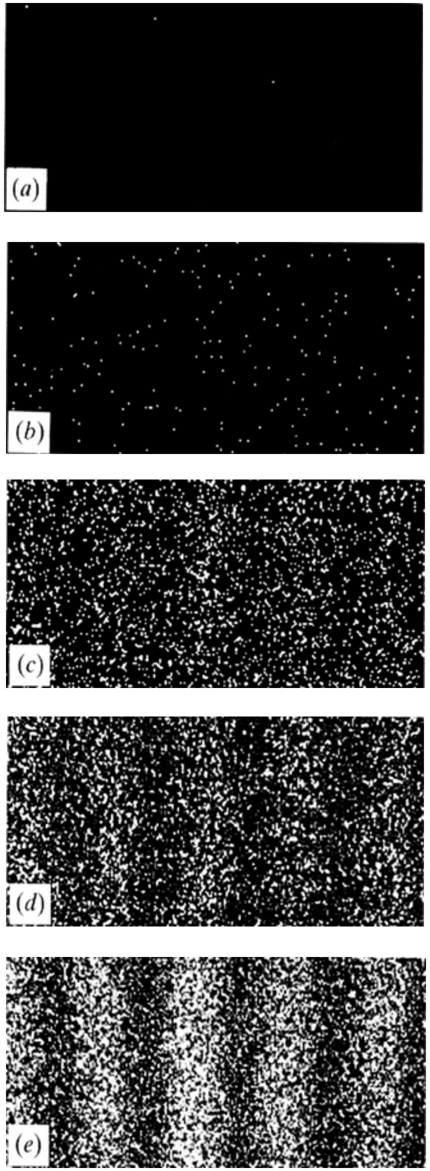
Particle distributions behave as waves …

(Born’s statistical interpretation of quantum mechanics)

… but individual particles behave as individual point-like particles!

Explaining both behaviors
within the same theoretical framework
is precisely the reason why
trajectory pictures of quantum mechanics
are needed and/or desirable

Trajectory pictures of quantum processes



BOHMIAN MECHANICS

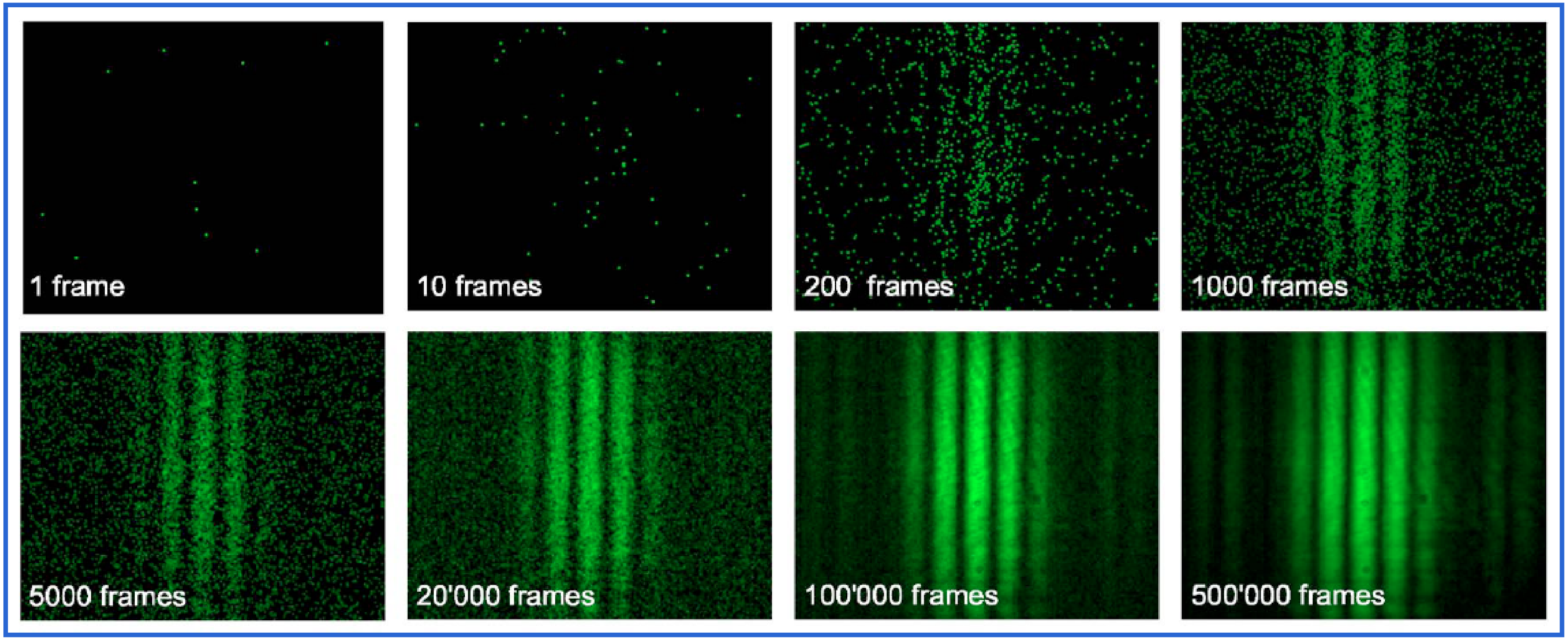
$$
\left.\begin{array}{l}
i\hbar \dfrac{\partial \Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\nabla^2\Psi + V\,\Psi \\[2ex]
\Psi = R\, e^{iS\hbar}
\end{array}\right\}
\longrightarrow
\left\{\begin{array}{l}
\dfrac{\partial S}{\partial t} + \dfrac{(\nabla S)^2}{2m} + V - \dfrac{\hbar^2}{2m}\dfrac{\nabla^2 R}{R} = 0 \\[2ex]
\dfrac{\partial R^2}{\partial t} + \nabla\cdot\left(R^2\dfrac{\nabla S}{m}\right) = 0
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
i\hbar \dfrac{\partial \Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\nabla^2\Psi + V\,\Psi \\[2ex]
\vec{p} = \nabla S
\end{array}\right.
$$

Demonstration of single-electron buildup of an interference pattern

Tonomura, Endo, Matsuda, Kawasaki and Ezawa, *Am. J. Phys.* **57**, 117 (1989)

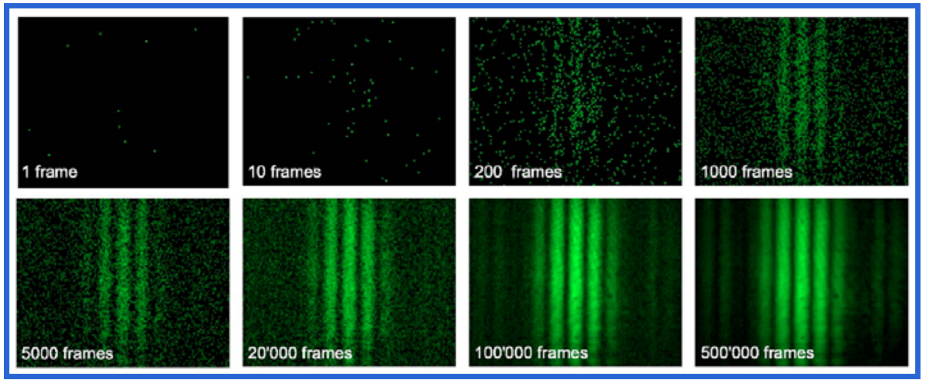
Trajectory pictures of quantum processes



The wave-particle duality of light: A demonstration experiment

Dimitrova and Weis, *Am. J. Phys.* **76**, 137 (2008)

Trajectory pictures of quantum processes



time-averaged EM energy flux: $\vec{S} = \frac{1}{2}\mathrm{Re}\left(\vec{E} \wedge \vec{H}^*\right)$

$$\vec{E} = \left(E_x, E_y, 0\right)$$

$$\vec{H} = \left(0, 0, H_z\right)$$

BOHMIAN MECHANICS

$$\vec{J} = \frac{\hbar}{m}\mathrm{Im}\left(\Psi^*\nabla\Psi\right) = \frac{\hbar}{2mi}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right)$$

$$\rho = \Psi^*\Psi$$

$$\frac{d\vec{r}}{dt} = \frac{\vec{J}}{\rho}$$

"MAXWELL-BOHMIAN" MECHANICS

$$\vec{S} = \frac{\lambda}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\,\mathrm{Im}\left(H_z^*\nabla H_z\right) = \frac{\lambda}{8\pi i}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(H_z^*\nabla H_z - H_z\nabla H_z^*\right)$$

$$U = \frac{1}{4}\left(\varepsilon_0\vec{E}\cdot\vec{E} + \mu_0\vec{H}\cdot\vec{H}\right)$$

$$\frac{d\vec{r}}{d\tau} = \frac{\vec{S}}{U} \longrightarrow \frac{dy}{dx} = \frac{S_y}{S_x} = \frac{\mathrm{Im}\left(H_z^*\dfrac{\partial H_z}{\partial y}\right)}{\mathrm{Im}\left(H_z^*\dfrac{\partial H_z}{\partial x}\right)}$$

Trajectory aspects of electromagnetic waves: A prescription to determine photon paths

Davidovic, Sanz, Arsenovic, Bozic and Miret-Artés, *Europhys. Lett.* (submitted, 2008); arxiv:quant-ph/0805.3330

Some nice features of Bohmian mechanics

- **Conceptually, Bohmian mechanics is as simple as classical mechanics (particles are always regarded as particles).**

- **Unlike other interpretations based on classical and/or semiclassical trajectories, those arising from Bohmian mechanics are fully grounded on quantum-mechanical/dynamical rules of motion.**

- **Bohmian quantum trajectories evolve in the (real) configuration space, where real experiments take place (this is an advantage with respect to other alternative quantum trajectory formalisms, e.g., complex quantum trajectories).**

- **The ensemble dynamics describes the quantum flux allowing, at the same time, to monitor the behavior of each individual particle, something which is forbidden in standard time-dependent wave-packet techniques.**

- **The statistical predictions of standard quantum mechanics are also obtained, without violating the uncertainty and complementarity principles, which have a simple explanation (meaning) within the Bohmian framework.**

A “completeness” diagram of dynamics

$$\begin{tikzcd} \text{Classical Mechanics} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & \text{Bohmian Mechanics} \arrow[d, leftrightarrow] \\ \text{Statistical Mechanics} \arrow[r, leftrightarrow] & \text{Quantum Mechanics} \end{tikzcd}$$

The discussion in this talk

- Superposition

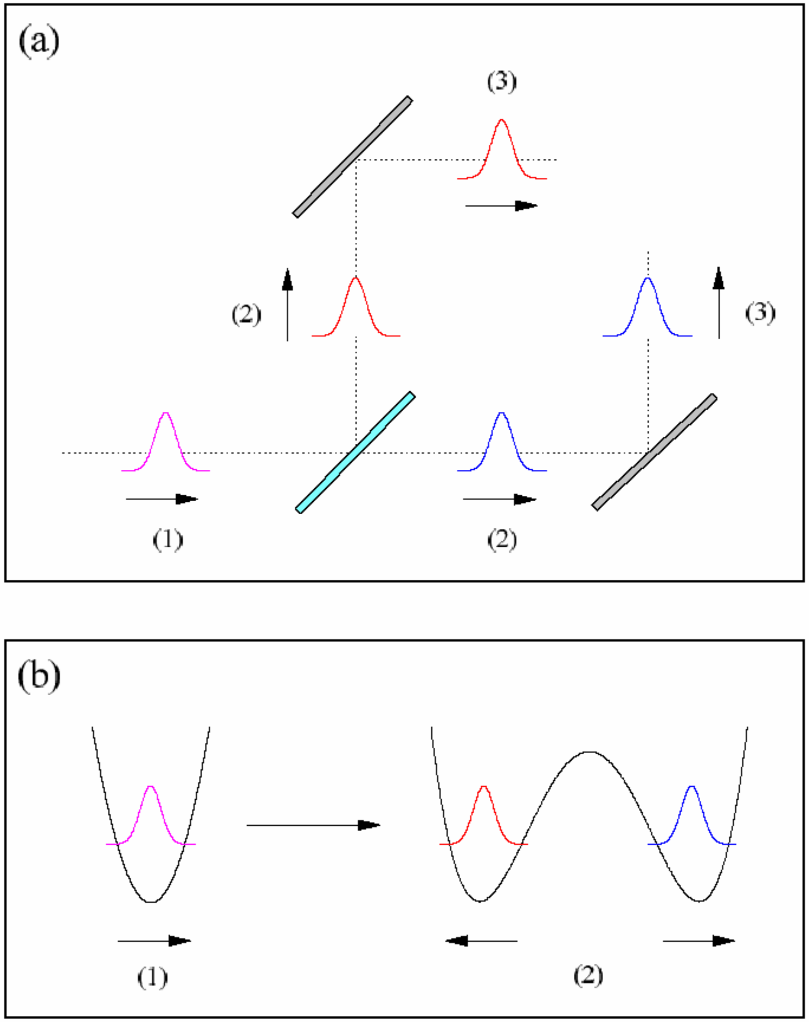
- Nonlocality

- Contextuality

The discussion in this talk

- Wave-packet collisions and interference effective potentials
- Slit systems: from simple slit arrays to the Talbot effect
- Quantum fractals and fractal quantum trajectories
- Decoherence and reduced quantum trajectories

The superposition principle revisited



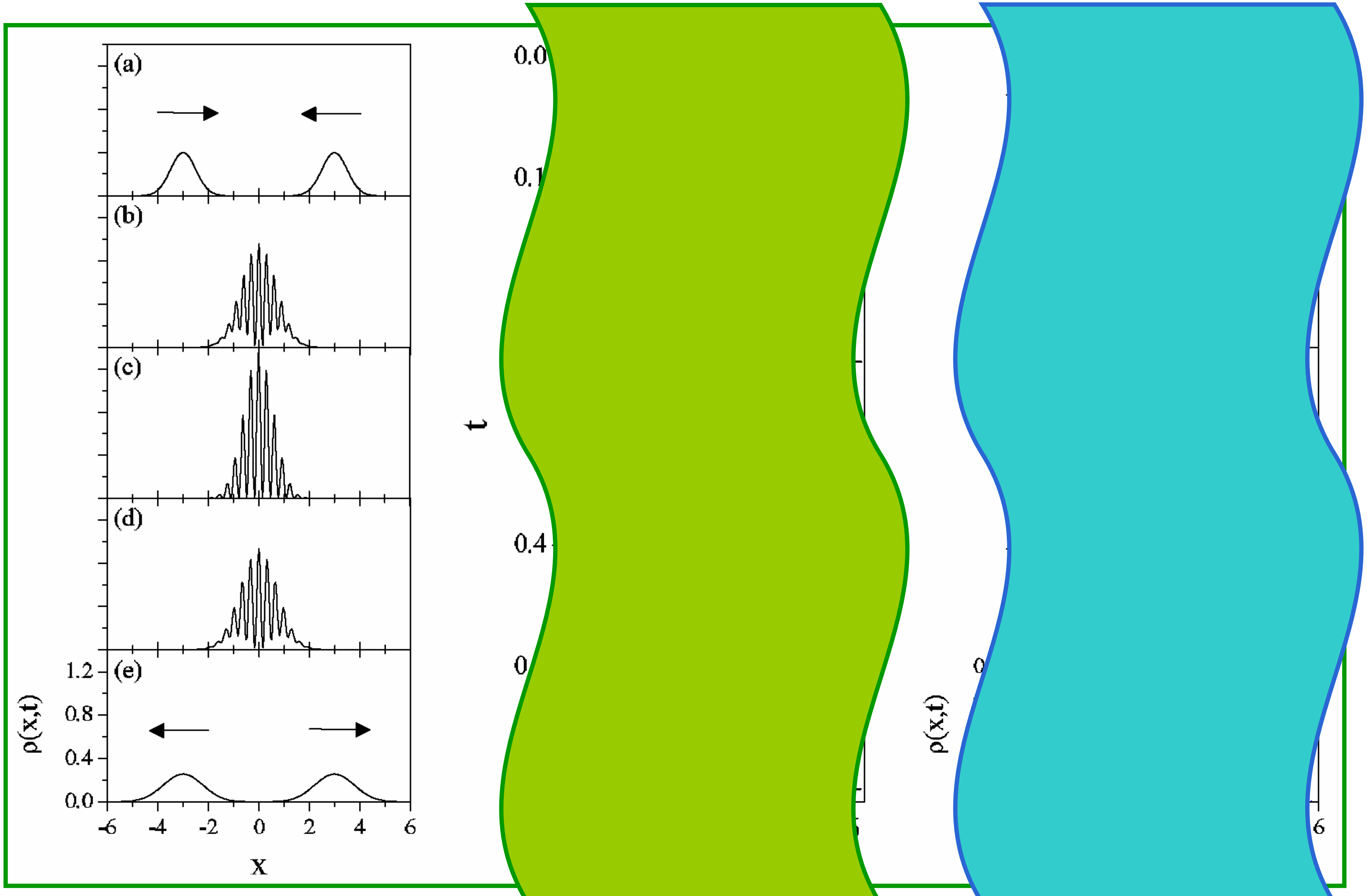
The superposition principle revisited

superposition principle

$$\begin{cases} i\hbar \dfrac{\partial \Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\nabla^2\Psi + V\Psi & \longleftrightarrow \quad \textbf{holds} \\[2ex] \dot{\vec{r}} = \dfrac{\nabla S}{m} = \dfrac{\hbar}{2im}\dfrac{\Psi^*\nabla\Psi - \Psi\nabla\Psi^*}{\Psi^*\Psi} & \longleftrightarrow \quad \textbf{doesn't hold} \end{cases}$$

NODAL PROBLEM

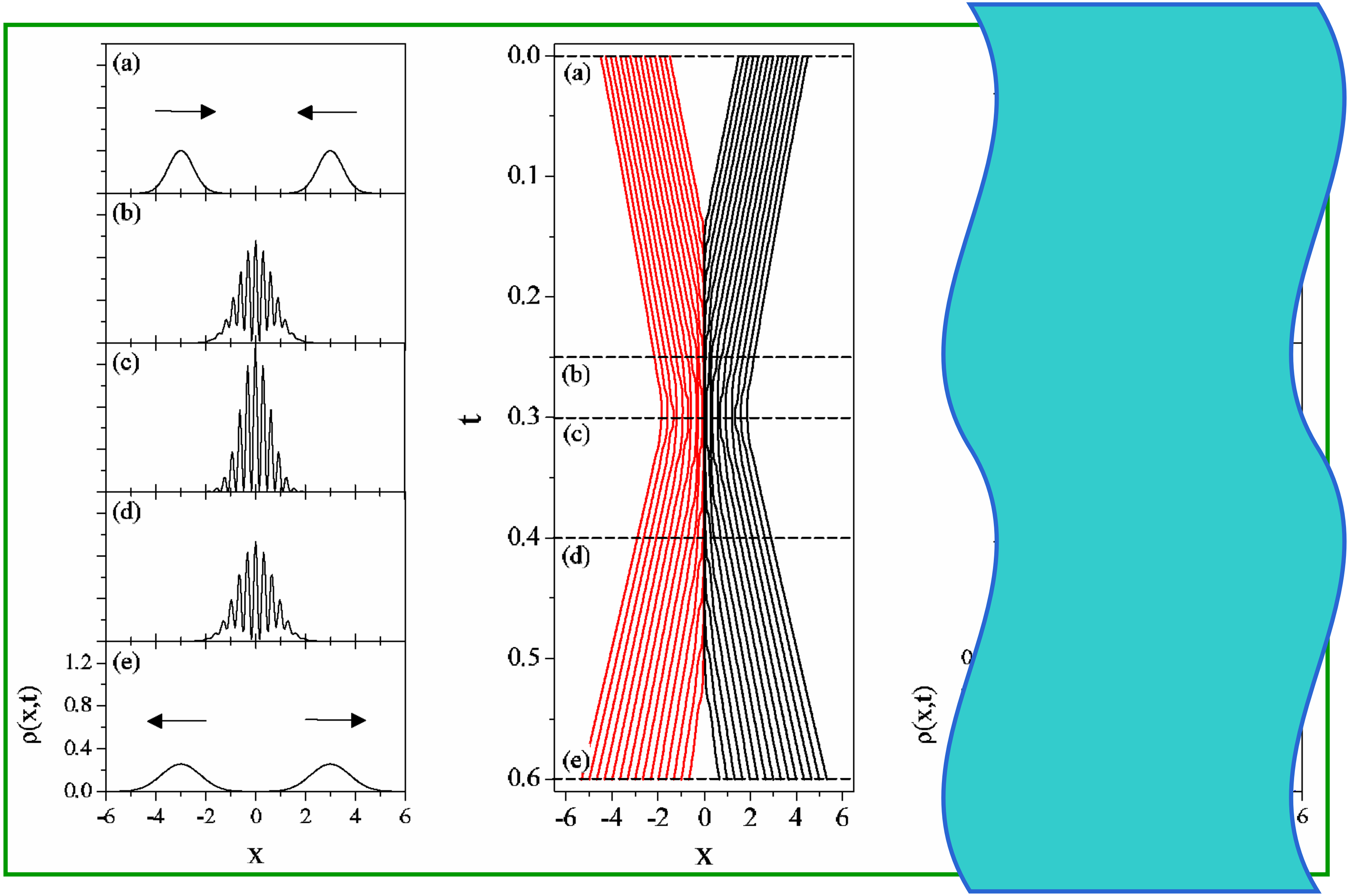
The superposition principle revisited



A trajectory based understanding of quantum interference

Sanz and Miret-Artés, *J. Phys. A* (submitted, 2008); arxiv:quant-ph/0806.2105

The superposition principle revisited



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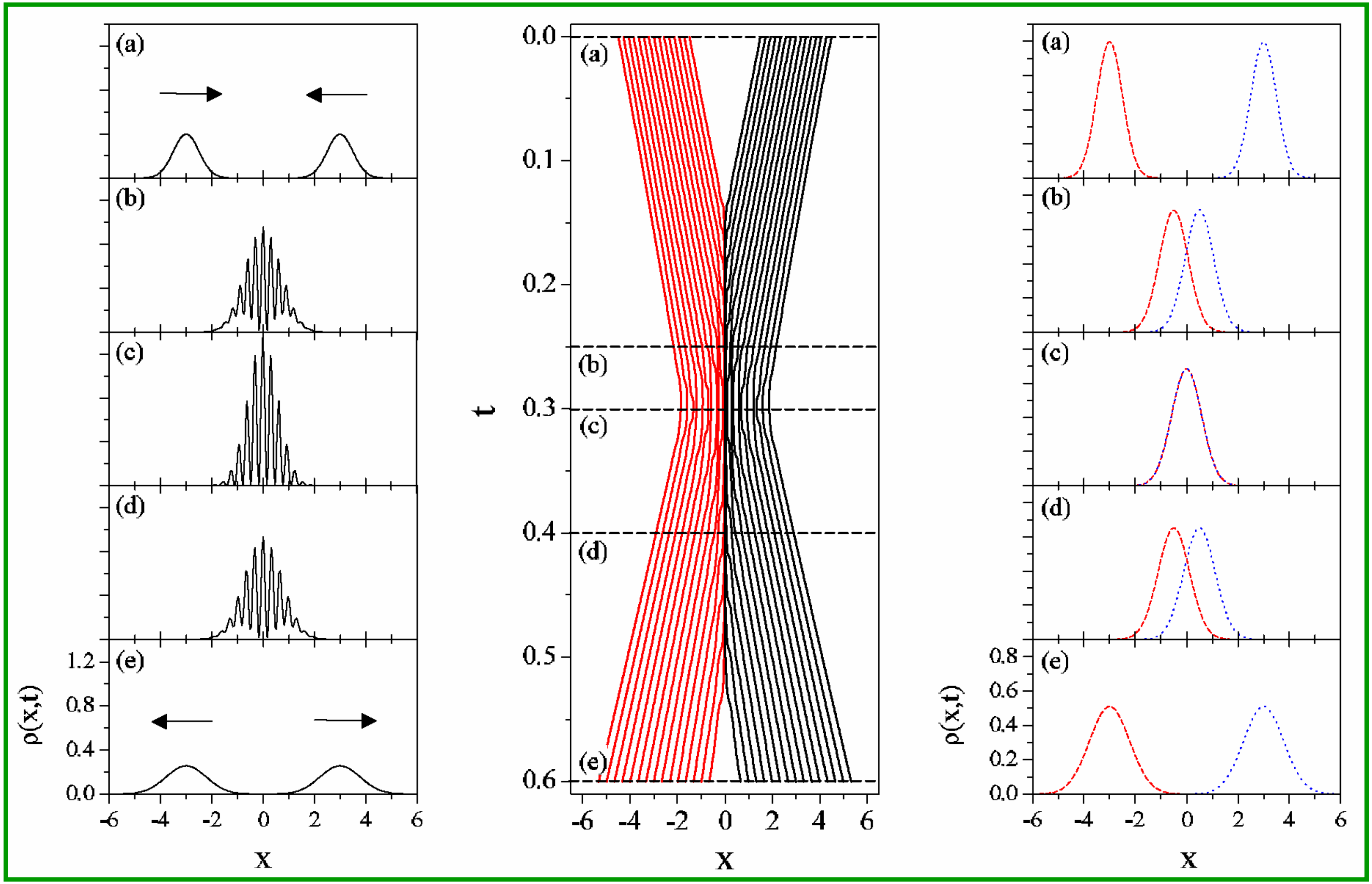

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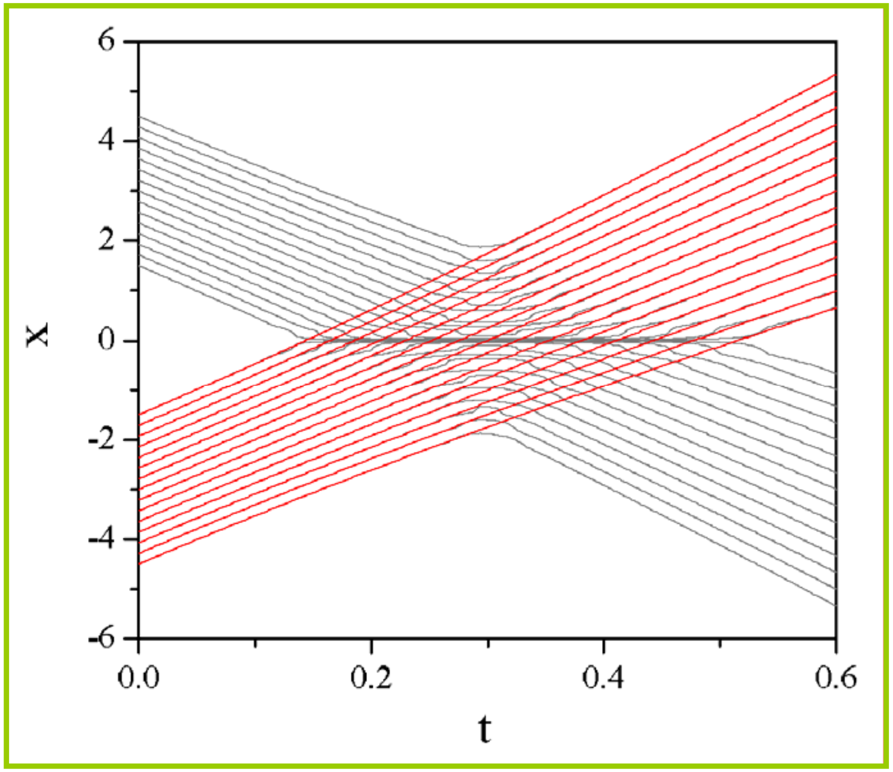

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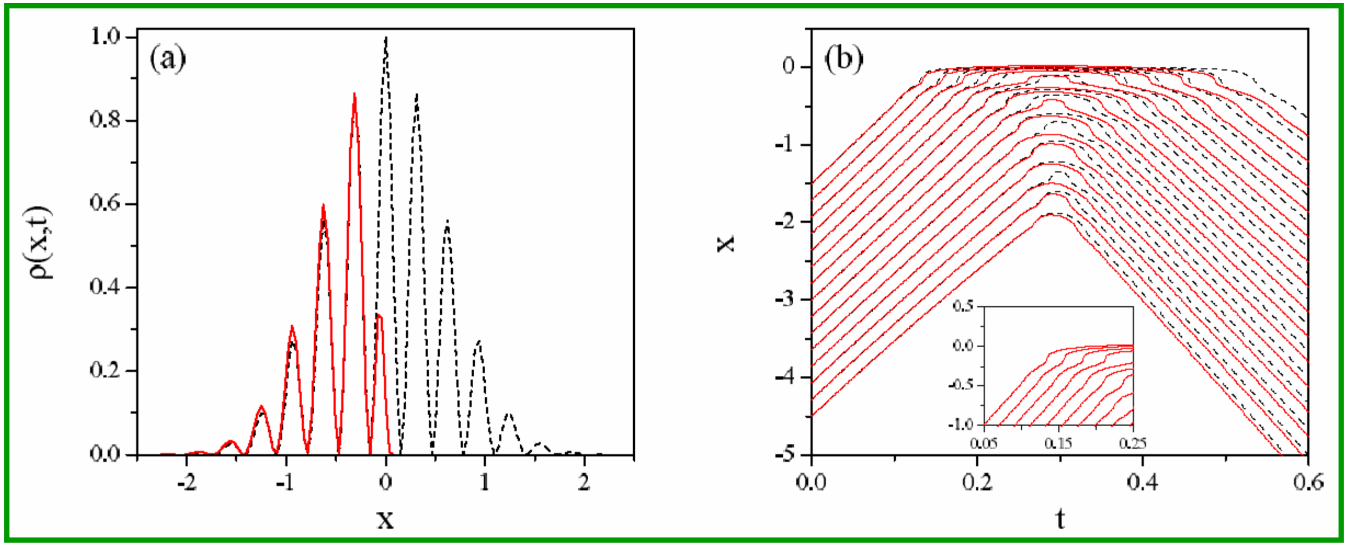

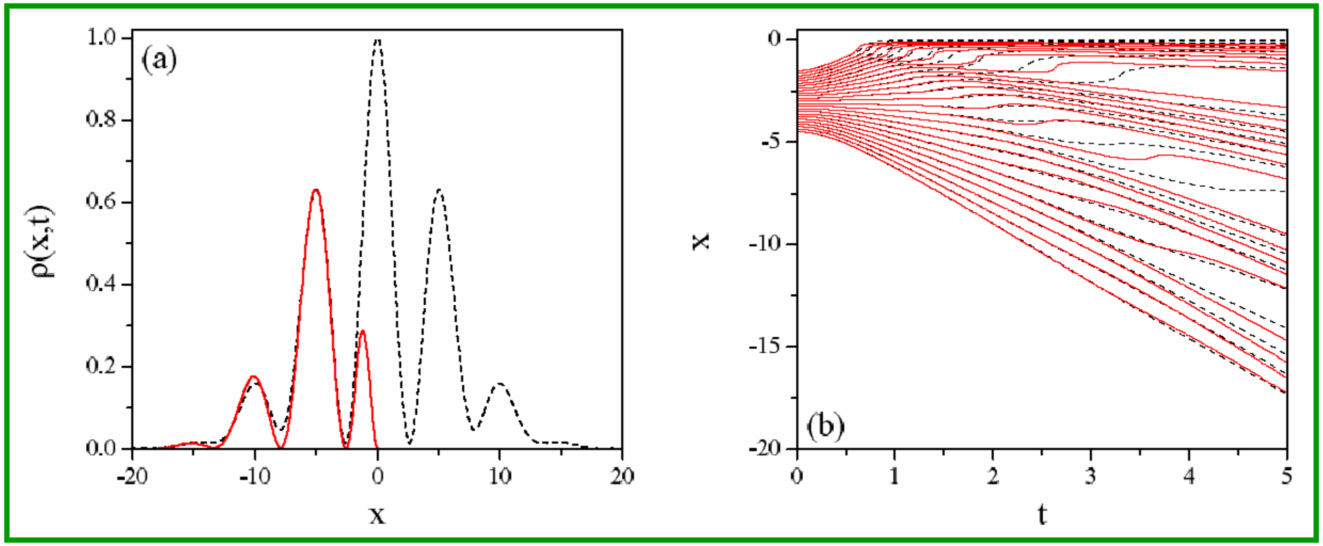

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1, 2, … N-slit diffraction. The Talbot effect

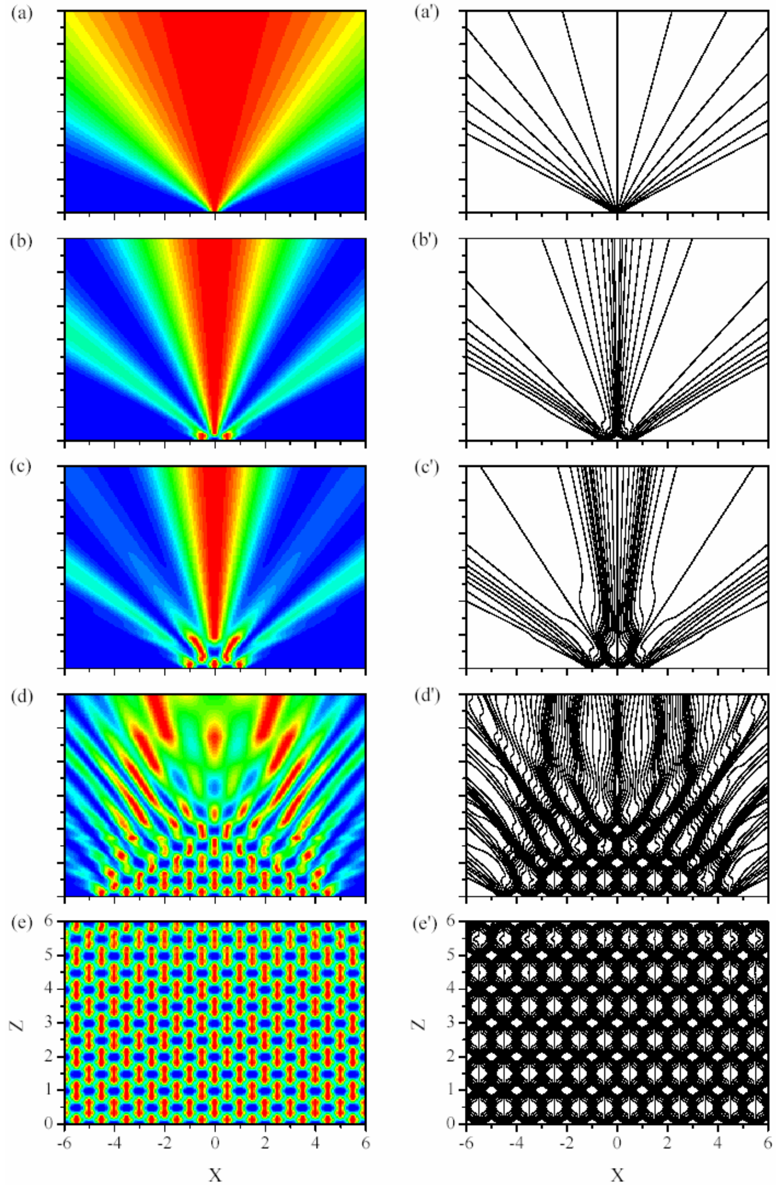
N = 1

N = 2

N = 3

N = 10

N = 50



A causal look into the quantum Talbot effect

Sanz and Miret-Artés, *J. Chem. Phys.* **126**, 234106 (2007)

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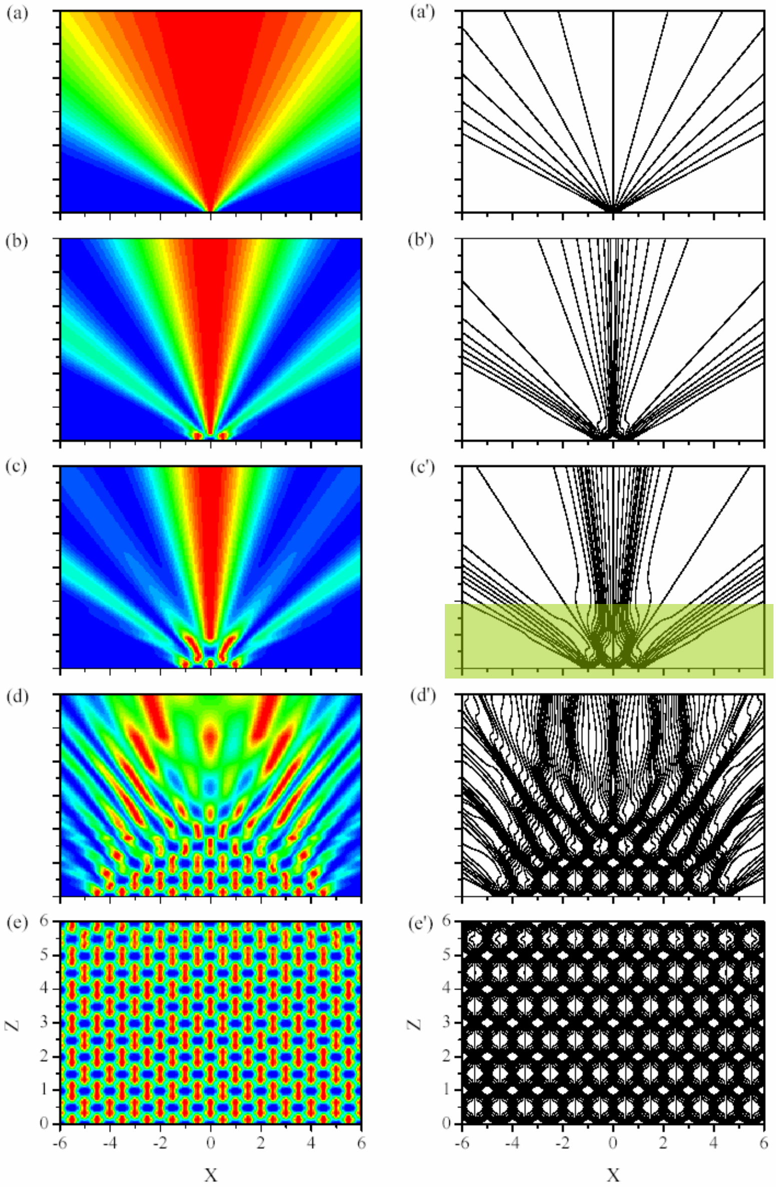

Fresnel

(convergence of wave-packet calculations)

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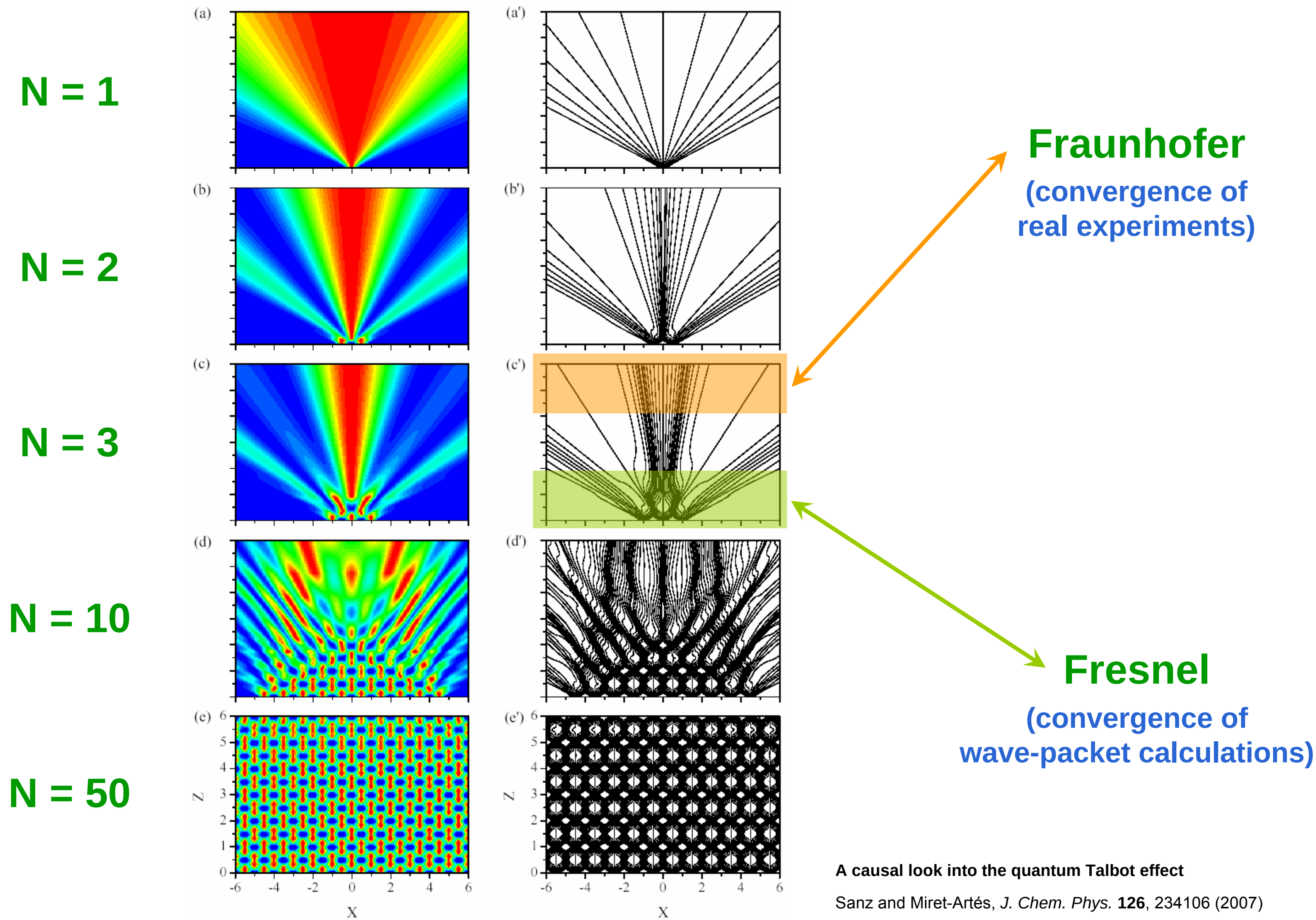

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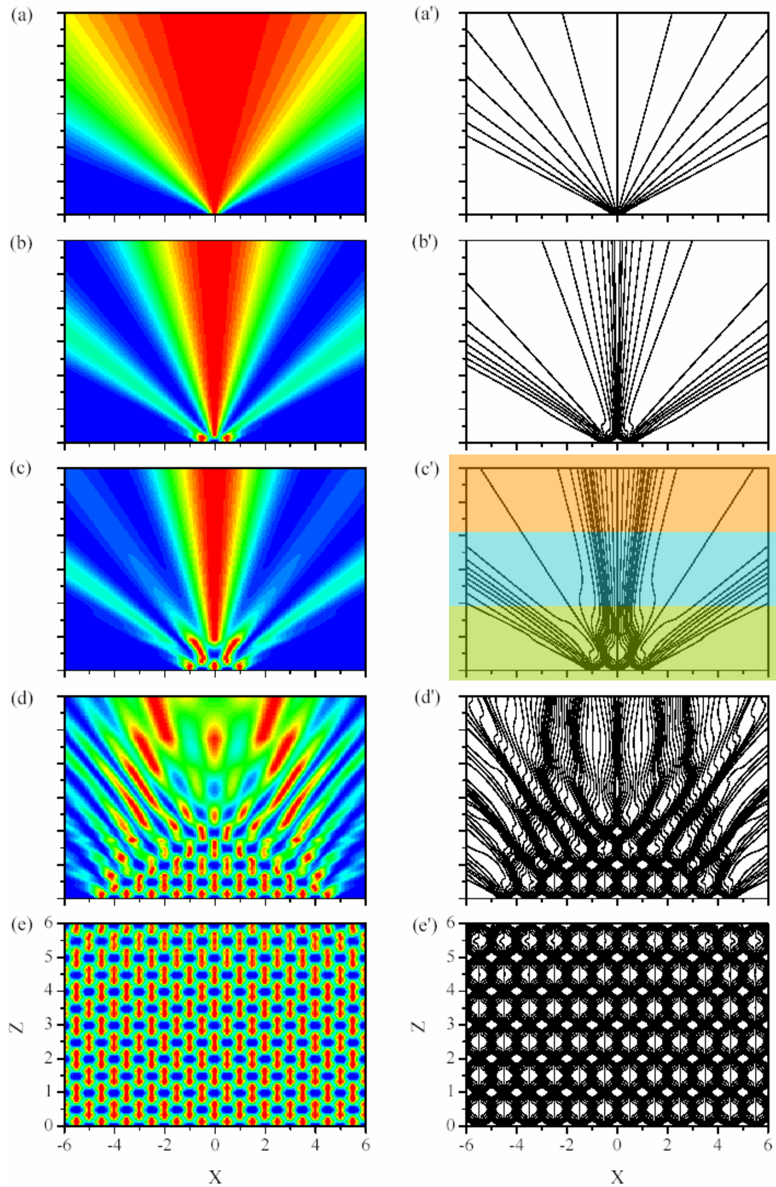


Fraunhofer

(convergence of real experiments)

Transition

Fresnel

(convergence of wave-packet calculations)

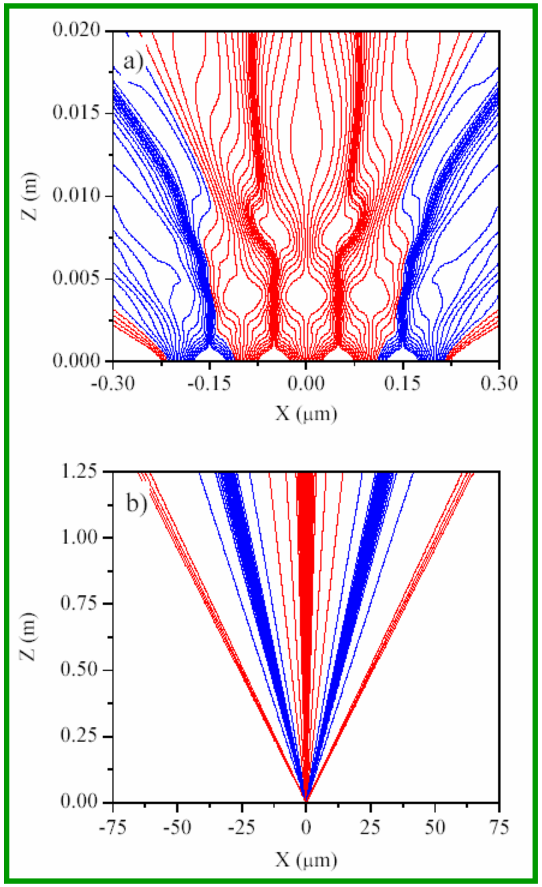
A causal look into the quantum Talbot effect

Sanz and Miret-Artés, *J. Chem. Phys.* **126**, 234106 (2007)

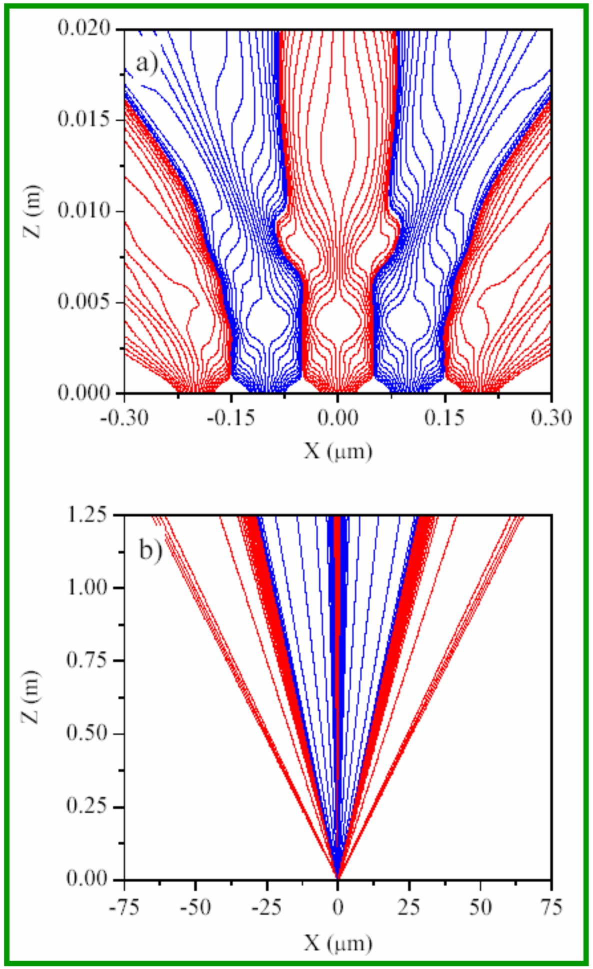
1, 2, … N-slit diffraction. The Talbot effect

The trajectories contributing to each diffraction peak can be associated with a specific *slit* (A) or, the other way around, one can determine the contribution of each *slit* to each final diffraction peak (B)

A



B



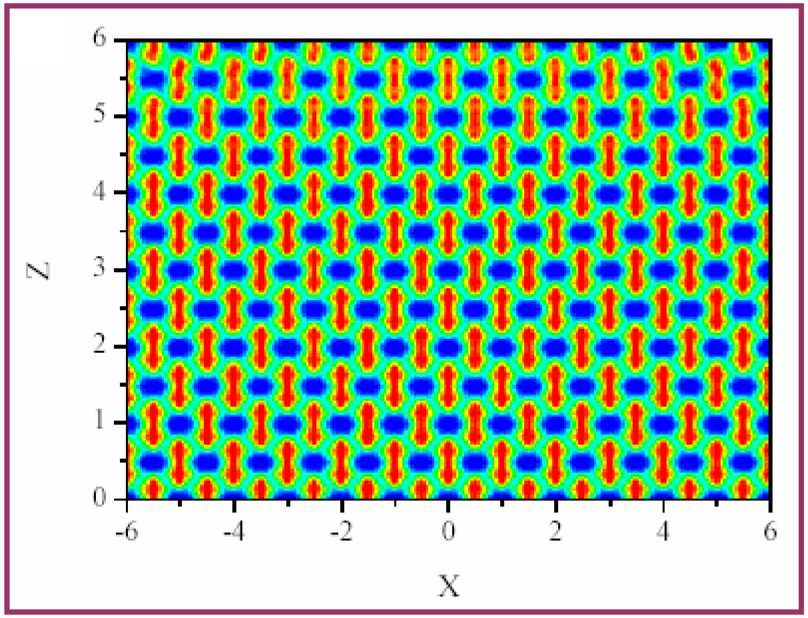
Particle diffraction studied using quantum trajectories

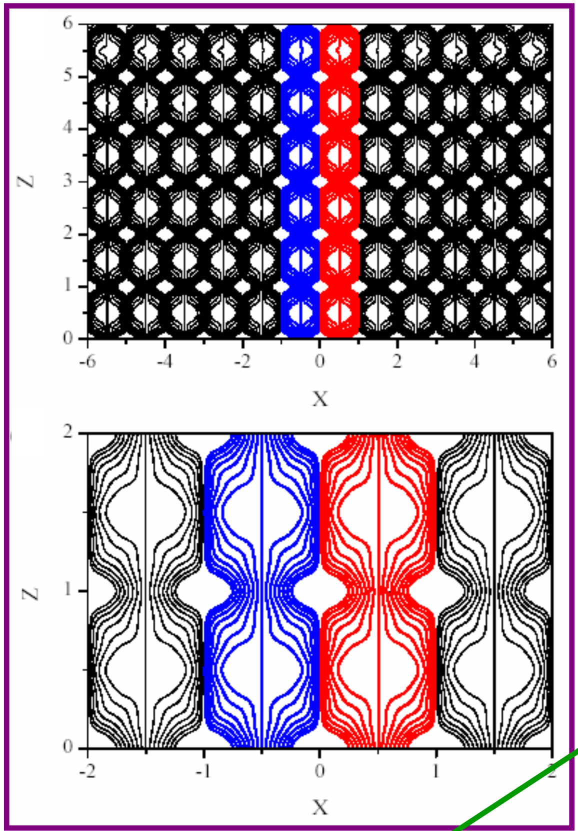
Sanz, Borondo and Miret-Artés, *J. Phys.: Condens. Matter* 14, 6109 (2002).

1, 2, … N-slit diffraction. The Talbot effect

When the number of *slits* becomes infinity, the Fresnel region also extends to infinity and we observe the Talbot effect (a near-field affect)

Talbot structure or quantum carpet





periodicity in x: d

periodicity in z: $2z_T = \dfrac{2d^2}{\lambda}$

Channel structure

multimode cavities

A causal look into the quantum Talbot effect

Sanz and Miret-Artés, *J. Chem. Phys.* **126**, 234106 (2007)

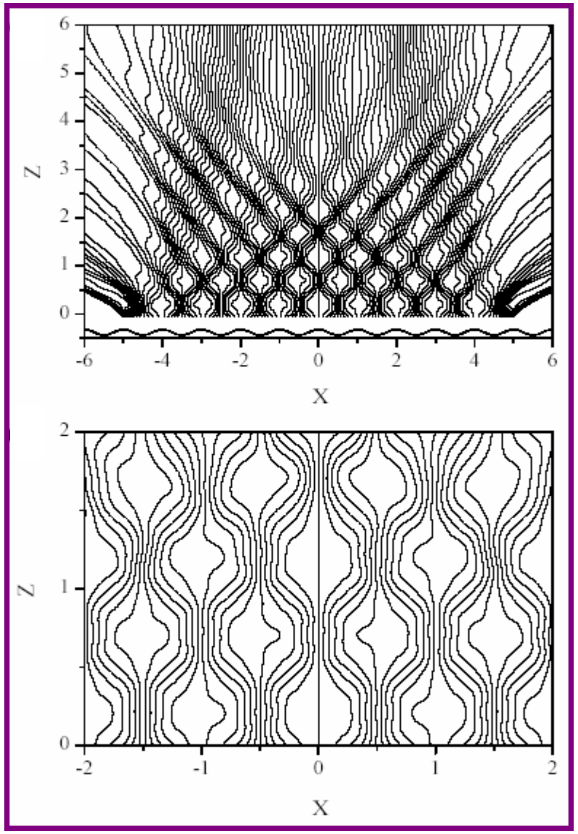
1, 2, … N-slit diffraction. The Talbot effect

Analogously, in elastic surface scattering problems, when the number of *unit cells (= slits)* becomes infinity, the Fresnel region also extends to infinity and we observe the Talbot-Beeby effect (an also near-field affect)

$$z_T = \frac{d^2}{\tilde{\lambda}}$$

$$\tilde{\lambda}(x,z) = \frac{2\pi\hbar}{\sqrt{2m[E_z - V(x,z)]}}$$

$$\lambda_{eff} = \frac{\lambda}{\sqrt{1 + D/E_z}}$$



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Causal trajectories description of atom diffraction by surfaces

Sanz, Borondo and Miret-Artés, *Phys. Rev. B* 61, 7743 (2000)

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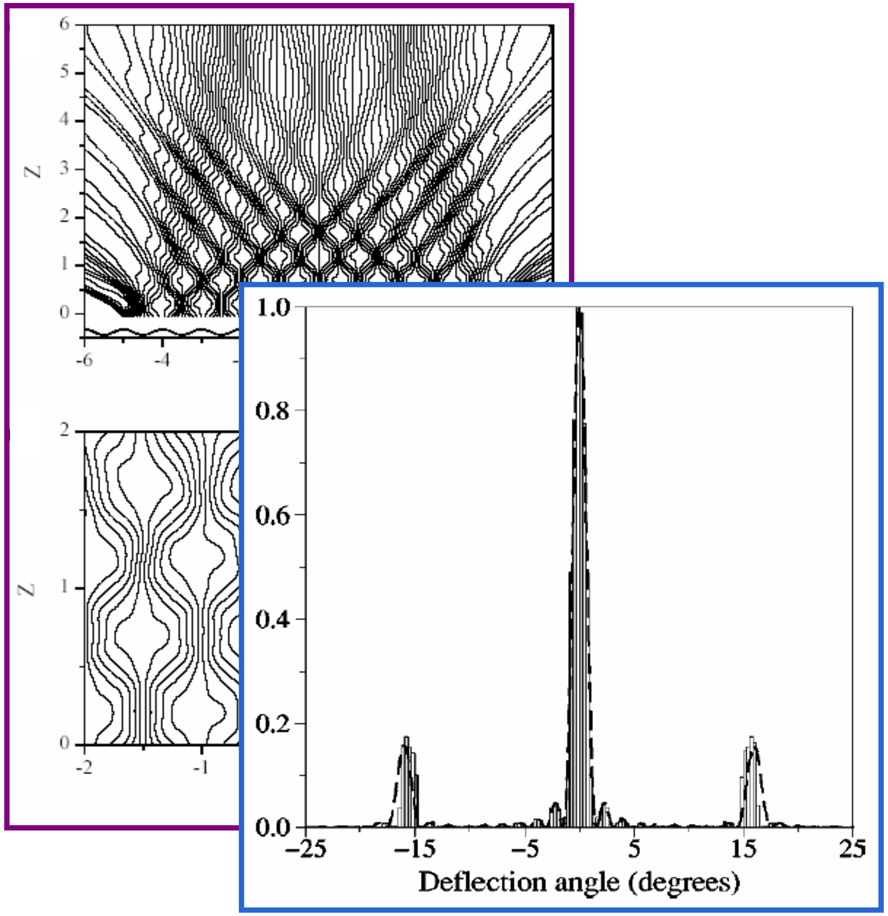

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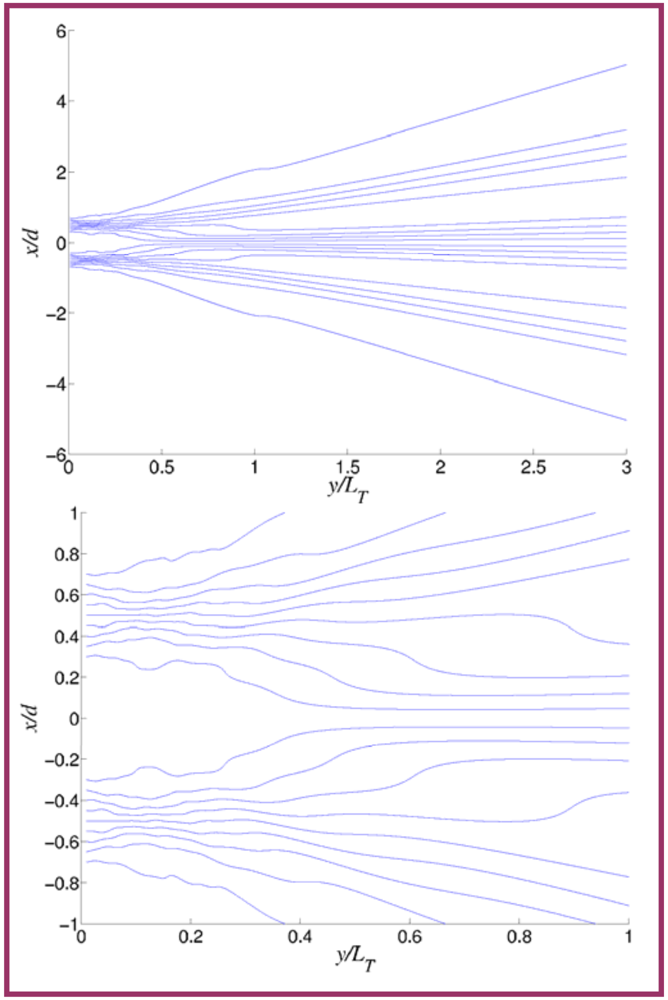
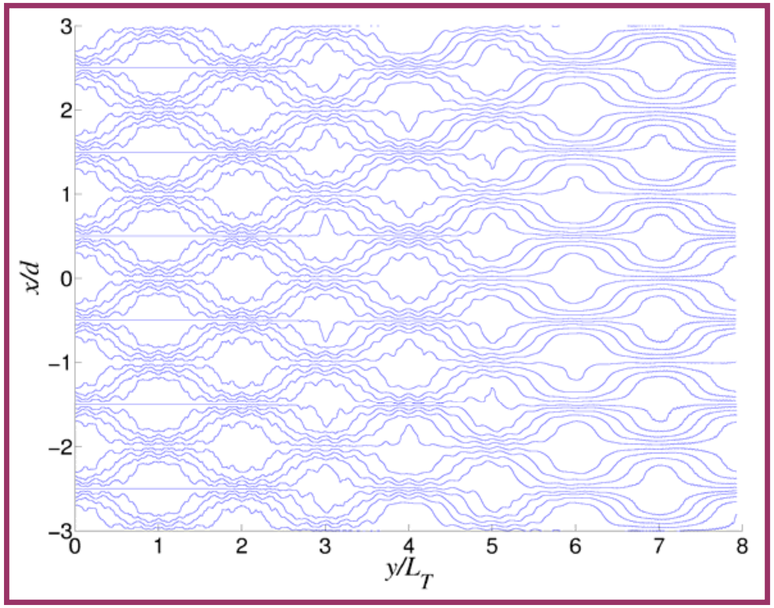
1, 2, … N-slit diffraction. The Talbot effect

The same can be found when working with photons (EM waves) instead of massive particles





Trajectory aspects of electromagnetic waves: A prescription to determine photon paths

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Fractal Bohmian mechanics

Quantum fractals in boxes

M V Berry

H H Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, UK

Received 22 April 1996

Abstract. A quantum wave with probability density $P(\boldsymbol{r}, t) = |\Psi(\boldsymbol{r}, t)|^2$, confined by Dirichlet boundary conditions in a D-dimensional box of arbitrary shape and finite surface area, evolves from the uniform state $\Psi(\boldsymbol{r}, 0) = 1$. For almost all positions $\boldsymbol{r} = x_1, x_2 \ldots x_D$, the graph of the evolution of P is a fractal curve with dimension $D_{\text{time}} = 7/4$. For almost all times t, the graph of the spatial probability density P is a fractal hypersurface with dimension $D_{\text{space}} = D + 1/2$. When $D = 1$, there are, in addition to these generic time and space fractals, infinitely many special 'quantum revival' times when P is piecewise constant, and infinitely many special spacetime slices for which the dimension of P is 5/4. If the surface of the box is a fractal with dimension $D - 1 + \gamma$ $(0 \leqslant \gamma < 1)$, simple arguments suggest that the dimension of the time fractal is $D_{\text{time}} = (7 + \gamma)/4$, and that of the space fractal is $D_{\text{space}} = D + 1/2 + \gamma/2$.

Berry, *J. Phys. A* **29**, 6617 (1996)

Fractal Bohmian mechanics

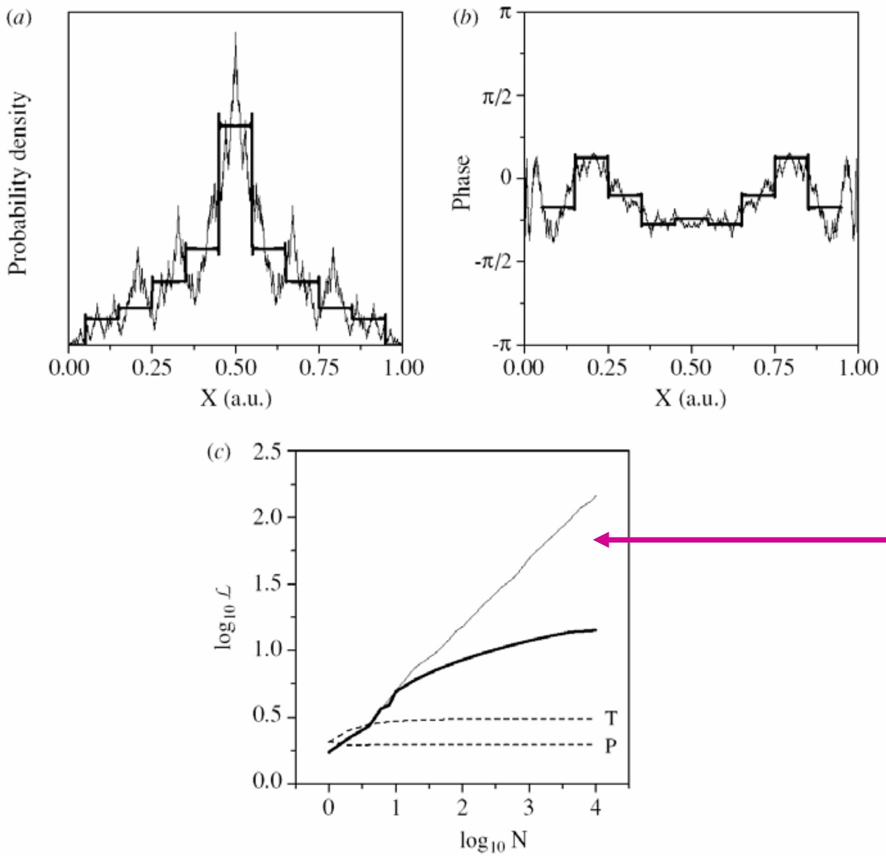

Figure 1. Probability density (*a*) and phase (*b*) associated with a highly delocalized particle in a box at $t = T/\sqrt{2}$ (thin solid line) and $t = 0.7T$ (thick solid line). (*c*) Measure of the fractal dimension of the probability densities displayed in part (*a*). To compare, measures of the fractal dimension of initial probability densities associated with triangular (T) and parabolic (P) wavefunctions are also shown.

fractal behavior

A Bohmian approach to quantum fractals

Sanz, *J. Phys. A* **38**, 6037 (2005)

Fractal Bohmian mechanics

Incompleteness of trajectory-based interpretations of quantum mechanics

Michael J W Hall

Theoretical Physics, IAS, Australian National University, Canberra ACT 0200, Australia

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Published 22 September 2004
Online at stacks.iop.org/JPhysA/37/9549
doi:10.1088/0305-4470/37/40/015

Abstract
Trajectory-based approaches to quantum mechanics include the de Broglie–Bohm interpretation and Nelson's stochastic interpretation. It is shown that the usual route to establishing the validity of such interpretations, via a decomposition of the Schrödinger equation into a continuity equation and a modified Hamilton–Jacobi equation, fails for some quantum states. A very simple example is provided by a quantum particle in a box, described by a wavefunction that is initially uniform over the interior of the box. For this example, there is no corresponding continuity or modified Hamilton–Jacobi equation, and the space-time dependence of the wavefunction has a known fractal structure. Examples with finite average energies are also constructed.

Hall, *J. Phys. A* **37**, 9549 (2004)

Fractal Bohmian mechanics

Bohmian mechanics can indeed be generalized to account for fractal quantum states, the corresponding trajectories being fractal curves

Fractal quantum dynamics:

$$\Psi_t(x;N) = \sum_{n=1}^{N} c_n \xi_n(x) e^{-iE_n t/\hbar}$$

$$\dot{x}_N(t) = \frac{\hbar}{m} \mathrm{Im}\left\{ \Psi_t^{-1}(x;N) \frac{\partial \Psi_t(x;N)}{\partial x} \right\}$$

$$\longrightarrow \quad \begin{cases} \Psi_t(x) \equiv \lim_{N\to\infty} \Psi_t(x;N) \\ x_t \equiv \lim_{N\to\infty} x_N(t) \end{cases}$$

A Bohmian approach to quantum fractals

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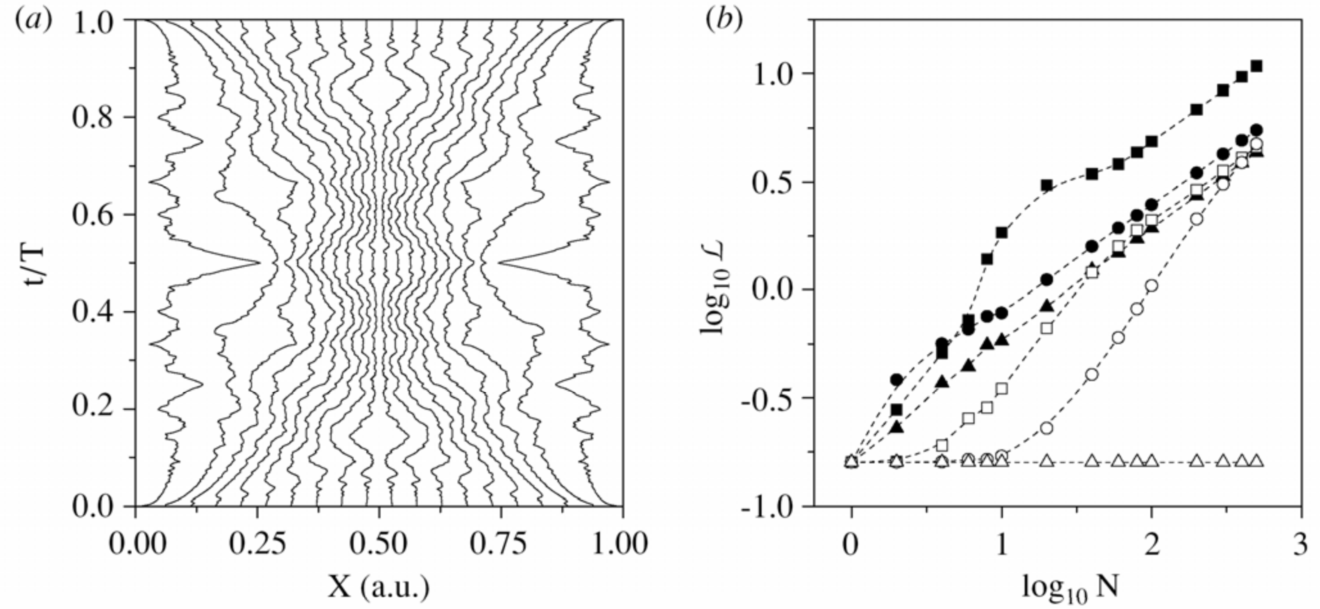

Figure 2. (*a*) QF-trajectories associated with a highly delocalized particle in a box. (*b*) Measure of the fractal dimension of a sample of QF-trajectories with initial positions: $x_0 = 0.01$ (■), $x_0 = 0.1$ (●), $x_0 = 0.4$ (▲), $x_0 = 0.49$ (□), $x_0 = 0.499$ (○), and $x_0 = 0.5$ (△).

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Many-body systems and reduced trajectories

wave function $\Psi(\vec{r})$

$$\vec{J}(\vec{r}) = \frac{\hbar}{m}\,\mathrm{Im}\left[\Psi^*(\vec{r})\nabla_{\vec{r}}\Psi(\vec{r})\right]$$

$$\rho(\vec{r}) = \Psi^*(\vec{r})\Psi(\vec{r})$$

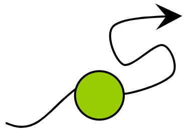
$$\dot{\vec{r}} = \frac{\vec{J}(\vec{r})}{\rho(\vec{r})}$$

density matrix $\rho(r,\vec{r}\,') = \langle r | \Psi \rangle\langle \Psi | \vec{r}\,' \rangle$

$$\vec{J}(\vec{r}) = \frac{\hbar}{m}\,\mathrm{Im}\left[\nabla_{\vec{r}}\,\rho(r,\vec{r}\,')\right]_{\vec{r}\,'=\vec{r}}$$

$$\rho(\vec{r}) = \mathrm{Re}\left[\rho(r,\vec{r}\,')\right]_{\vec{r}\,'=\vec{r}}$$

$$\dot{\vec{r}} = \frac{\vec{J}(\vec{r})}{\rho(\vec{r})}$$



A quantum trajectory description of decoherence

Sanz and Borondo, *Eur. Phys. J. D* 44, 319 (2007)

Many-body systems and reduced trajectories

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density matrix $\rho(r,\vec{r}\,') = \langle r|\Psi\rangle\langle\Psi|\vec{r}\,'\rangle$

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$$\rho(\vec{r}) = \mathrm{Re}\left[\rho(r,\vec{r}\,')\right]_{\vec{r}\,'=\vec{r}}$$

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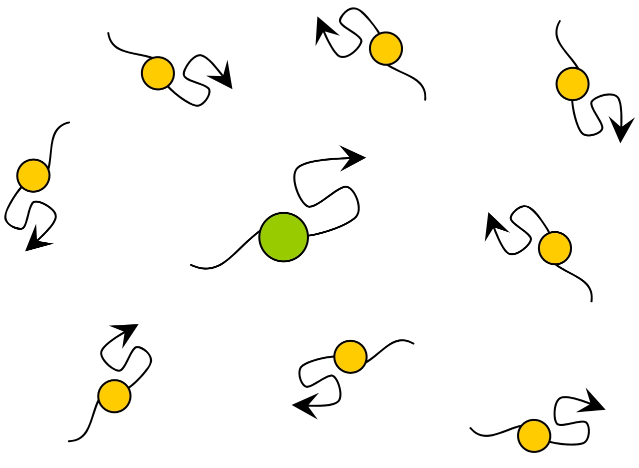


reduced density matrix

$$\tilde{\rho}(r,\vec{r}\,') = \int \langle r, r_1, r_2, \cdots, r_N|\Psi\rangle\langle\Psi|\vec{r}\,', r_1, r_2, \cdots, r_N\rangle dr_1 dr_2 \cdots dr_N$$

$$\left.\begin{aligned}\tilde{\vec{J}}(\vec{r}) &= \frac{\hbar}{m}\,\mathrm{Im}\left[\nabla_{\vec{r}}\tilde{\rho}(r,\vec{r}\,')\right]_{\vec{r}\,'=\vec{r}} \\ \tilde{\rho}(\vec{r}) &= \mathrm{Re}\left[\tilde{\rho}(r,\vec{r}\,')\right]_{\vec{r}\,'=\vec{r}}\end{aligned}\right\} \longleftrightarrow \left\{\begin{aligned}\dot{\tilde{\rho}} + \nabla\tilde{\vec{J}}(\vec{r}) &= 0 \\ \tilde{\vec{J}} &= \tilde{v}\tilde{\rho}\end{aligned}\right.$$

$$\tilde{v} = \dot{\vec{r}} = \frac{\tilde{\vec{J}}(\vec{r})}{\tilde{\rho}(\vec{r})}$$

A quantum trajectory description of decoherence

Sanz and Borondo, *Eur. Phys. J. D* 44, 319 (2007)

Many-body systems and reduced trajectories

A simple example:

$$\left|\Psi\right\rangle = \left|\Psi^{(0)}\right\rangle \otimes \left|E_0\right\rangle \longrightarrow \left|\Psi\right\rangle_t = c_1\left|\psi_1\right\rangle_t \otimes \left|E_1\right\rangle_t + c_2\left|\psi_2\right\rangle_t \otimes \left|E_2\right\rangle_t$$

$$\hat{\tilde{\rho}}_t = \sum_i {}_t\left\langle E_i\right|\hat{\rho}_t\left|E_i\right\rangle_t$$

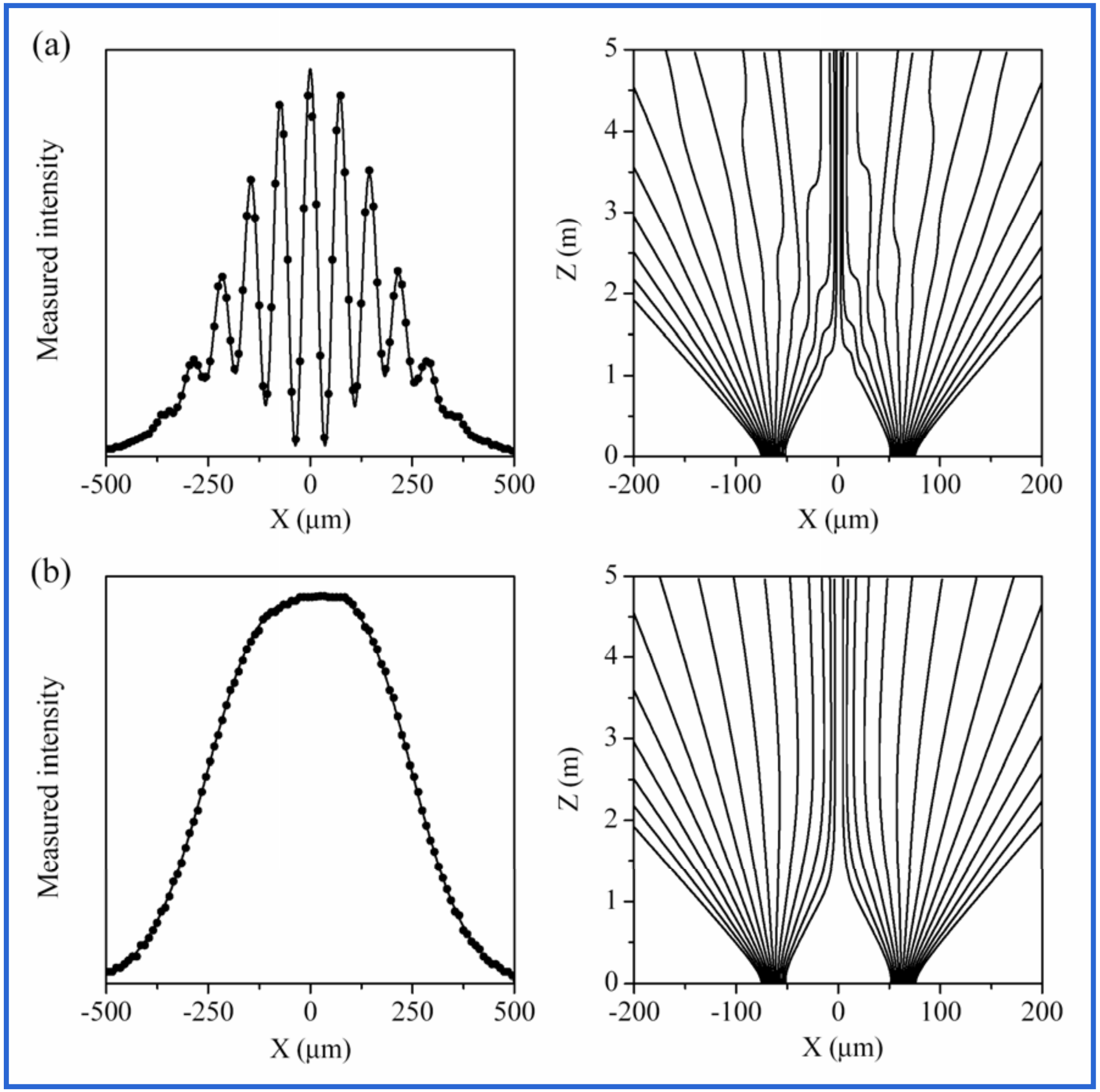
$$|\alpha_t| = \left|{}_t\left\langle E_2 | E_1\right\rangle_t\right| \approx e^{-t/\tau_c}$$

$$\boxed{\begin{aligned}
&\rho_t(\vec{r}) \sim |c_1|^2 \left|\Psi_1\right|_t^2 + |c_2|^2 \left|\Psi_2\right|_t^2 + 2\Lambda_t(\alpha_t)\,|c_1||c_2|\left|\Psi_1\right|\left|\Psi_2\right|\cos\delta_t \\
&\dot{\vec{r}} = \frac{(1+|\alpha_t|^2)\hbar}{2im\tilde{\rho}_t}\left\{|c_1|^2\left[\Psi_{1t}^*\nabla\Psi_{1t} - \Psi_{1t}\nabla\Psi_{1t}^*\right] + |c_2|^2\left[\Psi_{2t}^*\nabla\Psi_{2t} - \Psi_{2t}\nabla\Psi_{2t}^*\right]\right\} \\
&+ \frac{\hbar}{im\tilde{\rho}_t}\left\{\alpha_t c_1 c_2^*\left[\Psi_{2t}^*\nabla\Psi_{1t} - \Psi_{1t}\nabla\Psi_{2t}^*\right] + \alpha_t^* c_1^* c_2\left[\Psi_{1t}^*\nabla\Psi_{2t} - \Psi_{2t}\nabla\Psi_{1t}^*\right]\right\}
\end{aligned}}$$

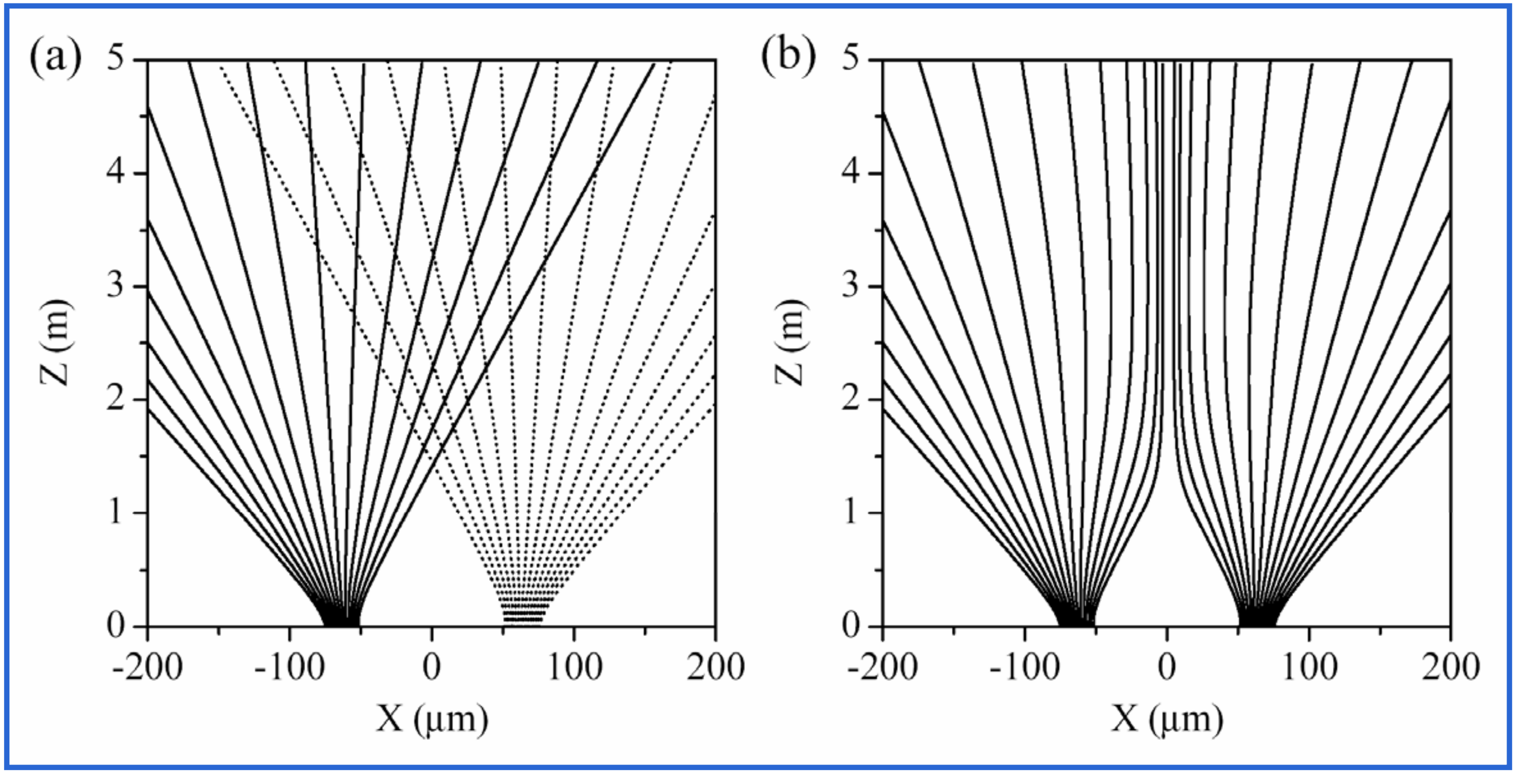
A quantum trajectory description of decoherence

Sanz and Borondo, *Eur. Phys. J. D* 44, 319 (2007)

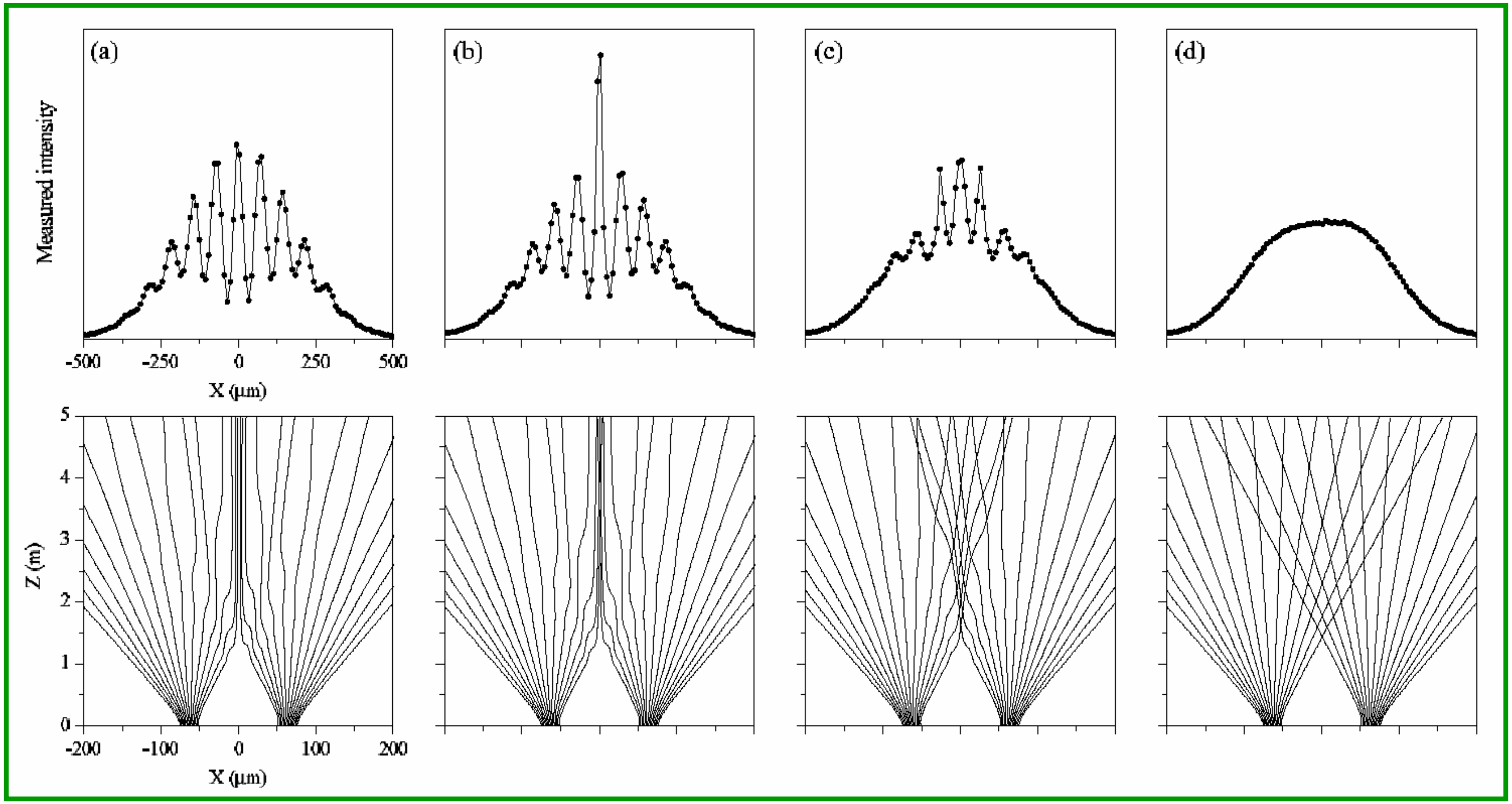
Many-body systems and reduced trajectories



Many-body systems and reduced trajectories



Many-body systems and reduced trajectories



Conclusions

Bohmian mechanics provides a robust and consistent framework to analyze and understand the dynamical behavior of quantum systems, which allows to treat particles as in classical mechanics (i.e., as individual entities) and, at the same time, to observe the well-known wave-like behaviors characteristic of the standard version of quantum mechanics.

Bohmian mechanics thus constitutes an important tool to create the quantum intuition necessary to think the quantum world, and particularly to better understand the physics underlying real experiments.

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