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Optimal Contracts for Wind Power Producers in Electricity Markets

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Abstract— **This paper is focused on optimal contracts for an independent wind power producer in conventional electricity markets. Starting with a simple model of the uncertainty in the production of power from a wind turbine farm and a model for the electric energy market, we derive analytical expressions for optimal contract size and corresponding expected optimal profit. We also address problems involving overproduction penalties, cost of reserves, and utility of additional sensor information. We obtain analytical expressions for marginal profits from investing in local generation and energy storage.**

Index Terms— **Renewable Energy, Smart Grid, Energy Storage, Electricity Markets**

I. Introduction

Global warming, widely regarded as one of the most critical problems we face, has led to great emphasis on clean energy sources such as solar, wind, and geothermal. Many nations have set ambitious goals for the share of renewable energy in their overall energy portfolio. Wind energy is expected to be a major contributor to the realizing these goals. The significant uncertainty and inherent variability in wind power imposes major challenges in integrating this source into the electricity grid at deep penetration levels.

In this paper, we focus on the scenario in which wind power producers (WPP) must sell their energy using contract mechanisms in *conventional* electricity markets. Our goal is to formulate and solve problems of optimal contract sizing, value of sensor information, value of local auxiliary generation, value of storage, and cost of increased reserves needed to accommodate the uncertainty caused by wind integration. We start with a simple stochastic model for wind power production and a model for the conventional electricity market. With these models, we derive explicit formulae for optimal contract size and the optimal expected profit. Our results cleanly capture the trade-off between penalty for contract deviation and the need to *spill* some of the wind energy to increase the probability of meeting the contract. We analyze the case when the wind power plant cannot reduce its output by including a penalty for overproduction and derive a formula for the optimal contract size and the expected optimal profit. We show that extra information from meteorological models and data increases the expected optimal profit. We consider the scenario in which the ISO schedules a capacity reservation to "hedge" against potential shortfalls arising from contracts offered by the WPP. Again, we derive a formula for optimal contract size, assuming the cost of capacity reservation is transferred to the WPP. We formulate and solve problems to address the marginal value of investing in local generation and energy storage.

The remainder of this paper is organized as follows: In Section 2, we provide more detailed background on wind energy and electricity markets. Our problem formulation is described in Section 3 and our main results are contained in Sections 4 and 5. We conduct an empirical study of our strategies on wind power data obtained from Bonneville Power Authority in Section 6. Concluding comments and discussion of current and future research are contained in Section 7. Due to space constraints all proofs are omitted and may be found in [3].

II. Background

A. Wind Energy

Wind energy is a very rapidly growing source of renewable energy [11]. In 2009, global new installations of wind power exceeded 38 GW and it ranked first among all sources for new electricity production capacity (wind represented 25% of new capacity addition). While USA ranks first among all nations in installed wind power capacity (35 GW), it is increasing dramatically in Europe and Asia. In North America, there is a significant geographic mismatch between the areas of high onshore wind energy production potential and the major electric energy consumption centers. This mismatch becomes much less important for less mature offshore wind power technologies.

Inherent variability of power output is the most significant difference between wind power generators and most traditional power generators. This variability occurs at various time scales: hourly, daily, monthly, and annually. Since loads are also uncertain and variable, the issue of wind power variability does not cause major problems when the proportion of wind energy is small relative to the total generation. However, with plans to increase the share of clean energy to 20-30% and beyond in many parts of the world, integration of wind into the existing electric power system presents major engineering, economic and societal challenges. Recently, National Renewable Energy Laboratory, has released two major reports [8], [10] on integration of large amounts of wind power (20-30%) into the Eastern and Western electric

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grid interconnections in North America. These studies show that limitations on the transmission system, increased need for reserves, impact of unpredicted large ramps, uncertainty in unit commitment and load following, limited accuracy in wind forecasting, coordination among and conflicting objectives of independent power producers, system operators, and regulatory agencies, are some of the major issues in achieving increased penetration of wind and solar energy. Although large scale energy storage can potentially compensate for the variation in the wind energy, the current high capital cost of storage is a major barrier in this direction. However, there is significant interest in new storage technologies and it is likely that grid scale storage will become an important component of the overall system in years to follow.

Integration of wind power into the power system has been the subject of many academic and industry studies. Morales et al [18] formulate and solve a short term optimal trading strategy problem for a wind power producer. They show how their problem reduces to a linear programming problem. However, the issue of uncertainty is dealt with via creation of scenarios in a tree format. For some earlier work along this approach, see [1], [16], [19]. Cavallo [6], [7] has studied compressed air energy storage for utility scale wind farms, Greenblatt et al [12] compared gas turbines and compressed air energy storage (CAES) in the context of wind as part of baseload electricity generation. Economic viability of CAES in a wind energy systems in Denmark has recently been investigated in [15].

Prediction of wind power generation is vitally important in integrating wind power into the grid. Generally speaking, prediction errors decrease with shortening of the prediction horizon and expansion of the geographic area over which averaging is done. For recent papers on wind power prediction, see [13], [20]. Large ramps in power output (for example, going from full power to almost zero in an hour) do occur and cause major challenges in the operation of the power system. Prediction of these ramp events is both challenging and important and is subject of our current research.

B. Electricity Markets

We assume that the wind power producer is part of a power pool participating in electricity markets that are cleared by an external entity, such as an ISO or RTO. A common trading structure ([17], [18], [14]) consists of two successive *ex-ante* markets: a day-ahead (DA) forward market and a real-time (RT) spot market. The DA market permits participants to bid and schedule energy transactions for the following day. Depending on the region, the DA market closes for bids and schedules by 10 AM and clears by 1 PM on the day prior to the operating day. The schedules cleared in the DA market are financially binding and are subject to deviation penalties. As the schedules submitted to the DA market are cleared well in advance of the operating day, a RT spot market is employed to ensure the balance of supply and demand in real-time by allowing market participants to adjust their DA schedules based on more accurate wind and load forecasts. The RT market is cleared five to 15 minutes before the operating interval, which is on the order of five minutes.

For those market participants who deviate from their scheduled transactions agreed upon in the *ex-ante* markets, the ISO normally employs an *ex-post* deterministic settlement mechanism to compute asymmetric imbalance prices. This asymmetric pricing scheme for penalizing energy deviations reflects the energy imbalance of the control area as a whole and the *ex-ante* clearing prices. For example, if the overall system imbalance is negative, those power producers with a positive imbalance with respect to their particular schedules will receive a more favorable price than those producers who have negatively deviated from their schedules, and vice-versa.

For a more detailed analysis of electricity market systems in different regions, we refer the reader to [4], [5], [22].

III. Models: Markets and Wind Power

A. Wind Energy Model

Wind power $w(t)$ is modeled as a scalar-valued stochastic process. For a fixed $t \in \mathbb{R}$, $w(t)$ is random variable whose cumulative distribution function (CDF) is assumed known and defined as $F(w,t) = \mathbb{P}(w(t) \leq w)$. The distribution $F(w,t)$ has support $[0,1]$, because the wind power is assumed normalized by the wind power plant's nameplate capacity. The corresponding density is denoted by $f(w,t)$.

In this paper, we will work with marginal distributions defined on the time interval $[t_0, t_f]$ of width $T = t_f - t_0$. Of particular importance are the *time-averaged* density and distribution defined as

$$f(w) = \frac{1}{T}\int_{t_0}^{t_f} f(w,t)dt \qquad (1)$$

$$F(w) = \frac{1}{T}\int_{t_0}^{t_f} F(w,t)dt = \int_0^w f(x)dx \qquad (2)$$

Also, define $F^{-1} : [0,1] \to [0,1]$ as the *quantile function* corresponding to the CDF F. More precisely, for $\beta \in [0,1]$, the β^{th}- quantile of F is given by

$$F^{-1}(\beta) = \inf\{x \in [0,1] \ : \ \beta \leq F(x)\} \qquad (3)$$

The quantile function corresponding to the time-averaged CDF will play a central role in our results.

B. Market Model

The market model considered in this paper consists of a *single* ex-ante DA forward market with an ex-post imbalance penalty for scheduled contract deviations. Contracts offered in the DA market are structured as power levels that are piecewise constant over contract intervals [typically hour long]. In the absence of energy storage capabilities for possible price arbitrage, the decision of how much constant power to offer over any individual hour-long time interval is independent of the decision for every other time interval. Hence, the problems decouple with respect to contract intervals and our analysis focuses on the problem of optimizing a constant power contract C scheduled to be delivered continuously over a single time interval $[t_0, t_f]$.

We define p (\$/MW-hour) as the clearing price in the forward market and q (\$/MW-hour) as the imbalance penalty price for [uninstructed] contract shortfalls. The wind power producer (WPP) is assumed to be a price taker in the forward market, because the WPP is assumed to have a zero marginal cost of production.

Remark 3.1: In this formulation p and q are assumed to be fixed and known. However, this assumption can be relaxed to p and q random and time varying without affecting the tractability of the results as long as they are assumed to be independent of the wind process $w(t)$. Also, most of the results to follow can be generalized to the case in which p and q belong to a class of functions concave in C and convex in the deviation $C - w(t)$, respectively. □

The profit acquired, the energy shortfall, and the amount of energy spilled by the WPP over the time interval $[t_0, t_f]$ are defined respectively as

$$\Pi(C, w) = \int_{t_0}^{t_f} pC - q\left[C - w(t)\right]^+ dt \quad (4)$$

$$\Sigma_-(C, w) = \int_{t_0}^{t_f} \left[C - w(t)\right]^+ dt \quad (5)$$

$$\Sigma_+(C, w) = \int_{t_0}^{t_f} \left[w(t) - C\right]^+ dt \quad (6)$$

where $x^+ := \max\{x, 0\}$. As wind power $w(t)$ is modeled as a random process, we will be concerned with the *expected* profit $J(C)$, the *expected* energy shortfall $S_-(C)$ and the *expected* amount of spilled (curtailed) wind energy, $S_+(C)$:

$$J(C) = \mathbb{E}\ \Pi(C, w) \quad (7)$$

$$S_-(C) = \mathbb{E}\ \Sigma_-(C, w) \quad (8)$$

$$S_+(C) = \mathbb{E}\ \Sigma_+(C, w) \quad (9)$$

Here, the expectation is taken with respect to the random wind power process $w = \{w(t) \mid t_0 \le t \le t_f\}$.

IV. Main Results: Optimal Contract Sizing

A. Conventional Markets

We begin by defining a profit maximizing contract C^* as

$$C^* = \arg\max_{C \ge 0} J(C). \quad (10)$$

Theorem 4.1: Let $\gamma := p/q$. Define the time-averaged distribution $F(w)$ as in (2).

(a) An optimal contract C^* is given by the γ^{th}- quantile of the time-averaged distribution F, i.e.

$$C^* = F^{-1}(\gamma) := \inf\left\{x \in [0, 1]\ :\ \gamma \le F(x)\right\}. \quad (11)$$

(b) The expected profit, the shortfall, and the spillage corresponding to a contract C^* are given by

$$J(C^*) = J^* = qT \int_0^\gamma F^{-1}(w) dw \quad (12)$$

$$S_-(C^*) = S_-^* = T \int_0^\gamma \left[C^* - F^{-1}(w)\right] dw \quad (13)$$

$$S_+(C^*) = S_+^* = T \int_\gamma^1 \left[F^{-1}(w) - C^*\right] dw \quad (14)$$

■

Remark 4.2: (*Non-uniqueness of C^**) Clearly, any contract C that solves $\gamma = F(C)$ is profit maximizing with respect to problem (10). Because the CDF F is only guaranteed to be monotone non-decreasing on it's domain $[0, 1]$, it may have intervals in its domain on which it is constant, which allows for non-uniqueness of the optimizer C^*. Hence, it is straight forward to see that C^* is unique if and only if the set

$$\Gamma(F, \gamma) := \{x \in [0, 1]\ :\ \gamma = F(x)\}$$

is a singleton. As stated in theorem 4.1-(a), a particular choice for an optimal contract is $C^* = F^{-1}(\gamma)$ – the γ^{th}- quantile of F. Although the optimal expected profit J^* is independent of the choice of $C^* \in \Gamma(F, \gamma)$, it is straightforward to see that $C^* = F^{-1}(\gamma)$ is the minimizer of the expected optimal shortfall S_-^* among all contracts $C \in \Gamma(F, \gamma)$. The opposite is true for S_+^*. The effect of alternative choices of C^* from $\Gamma(F, \gamma)$ on S_+^* and S_-^* is quantified as follows.

$$S_-(C^*) = S_-\left(F^{-1}(\gamma)\right) + \gamma\left(C^* - F^{-1}(\gamma)\right) \quad (15)$$

$$S_+(C^*) = S_+\left(F^{-1}(\gamma)\right) - (1 - \gamma)\left(C^* - F^{-1}(\gamma)\right) \quad (16)$$

for $C^* \in \Gamma(F, \gamma)$. □

Remark 4.3: (*Graphical Interpretation*) Theorem 4.1-(b) provides explicit characterizations of the optimal expected profit J^*, energy shortfall S_-^*, and energy spilled S_+^*. These three quantities can be graphically represented as areas bounded by the mean CDF $F(w)$ as illustrated in Figure 1 for $\gamma = 0.5$.

$$A_1 = (1/qT)\ J^*$$

$$A_2 = (1/T)\ S_-^*$$

$$A_3 = (1/T)\ S_+^*$$

From Figure (1), it is apparent that a reduction of "statistical dispersion" in the time-averaged distribution $F(w)$ will result in an increase in optimal expected profit (A_1) and a decrease in the optimal expected energy shortfall and spillage ($A_2,\ A_3$) – all of which are favorable consequences. □

Remark 4.4: (*Price Elasticity of Supply*) Under certain assumptions, the quantile rule (11) in Theorem 4.1 can be interpreted as the *supply curve* for the WPP. Of primary importance is the assumption that the WPP is a price taker in the DA forward market, ensuring that it weilds no influence over the market price. This is reasonable given the relatively low penetration of wind energy in existing markets. For a fixed deviation penalty price q, one can interpret the optimal

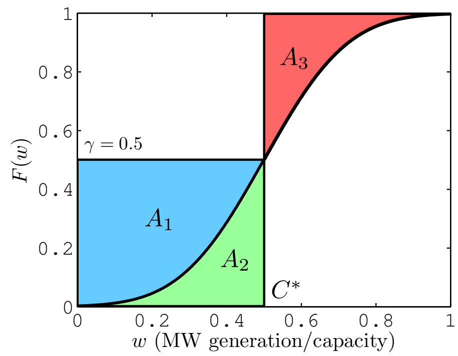

Fig. 1. Graphical interpretation of (A_1) optimal profit J^*, (A_2) deficit S_-^*, and (A_3) spillage S_+^* where $\gamma = 0.5$.

quantile rule (11) as indicating the amount of energy that the WPP is willing to supply at a price p. Specifically, the supply curve is given by

$$C(p) = F^{-1}\left(\frac{p}{q}\right)$$

The total energy that the WPP is willing to supply at price p over the time interval $[t_0, t_f]$ is then TC. With this explicit characterization of the WPP's supply curve, the price elasticity of supply, E_C, can be readily derived as

$$E_C := \frac{d \ln C(p)}{d \ln p} = \frac{\gamma}{F^{-1}(\gamma)} \frac{dF^{-1}(\gamma)}{d\gamma} = \frac{\gamma}{Cf(C)}.$$

□

Remark 4.5: (*Role of* γ) The price ratio $\gamma = p/q$ plays a critical role in implicitly controlling the probability of shortfall with respect to optimal offered contracts $C^* = F^{-1}(\gamma)$. Consider the scenario in which the ISO has direct control over the shortfall deviation penalty price q. As the penalty price q becomes more *harsh,* (i.e., larger), the price ratio γ decreases – resulting in smaller offered contracts C^*. This follows from the fact that the quantile function $F^{-1}(\gamma)$ is non-decreasing in γ (non-increasing in q). Consequently, the probability of shortfall $F(C^*, t)$ with respect to the optimal contract C^*, is non-increasing in q. □

Remark 4.6: (*Spillage*) The expected optimal shortfall S_-^* can be further interpreted as the expected amount of energy supplied by the ISO to balance the shortfalls in the WPP's contractual obligation. A straightforward corollary of Theorem 4.1 is that the expected optimal shortfall S_-^* and spillage S_+^* are monotonically nondecreasing and nonincreasing in γ, respectively. This makes explicit the claim that some wind energy must be spilled in order to reduce the amount of operational reserve capacity needed to hedge against uncertainty in the wind power. □

B. The Role of Information

It is of vital importance to understand the effect of information [such as available implicitly through forecasts] on expected optimal profit. Consider a random variable Y that is correlated to the wind process $w(t)$. The random variable Y can be interpreted as an observation of a meteorological variable relevant to the wind. Define

$$J^*(y) = qT \int_0^{\gamma} F^{-1}(w|y) dw.$$

where $F(w|y) := \frac{1}{T} \int_{t_0}^{t_f} F(w,t|y)dt$ and $F(w,t|y)$ is the CDF of $w(t)$ conditioned on the realization $Y = y$.

Theorem 4.7: $\mathbb{E}[\ J^*(Y)\ ] \geq J^*$ ■

Remark 4.8: Information helps in the metric of expected profit. Figure 1 offers some intuition as to how a reduction in "statistical dispersion" of the CDF F results in increased expected optimal profit. Future work is aimed at quantifying the marginal improvement of expected optimal profit with respect to information increase in various metrics of dispersion (e.g. entropy, interquartile range, variance). □

C. Optimal Contract with Penalty for Overproduction

Although wind power plant curtailment capabilities are continually improving, many existing wind power plants have insufficient curtailment capability. Consequently, these plants may be subject to imbalance penalties due to *overproduction* with respect to the offered contract C. To account for this added cost, consider an augmented profit function that explicitly accounts for the effect of contract sizing on wind energy spillage. Let λ (\$/MW-hours) be the asymmetrical ($\lambda \neq q$) penalty price for overproduction.

$$J_S(C) \quad = \quad \mathbb{E}\ [\ \Pi(C,w) - \lambda \Sigma_+(C,w)\ ] \qquad (17)$$

Define a profit maximizing contract C_S^* as

$$C_S^* = \arg\max_{C \geq 0} J_S(C) \qquad (18)$$

Theorem 4.9: Let $\gamma := (p+\lambda)/(q+\lambda)$. Define the time-averaged distribution $F(w)$ as in (2).

(a) An optimal contract C_S^* is given by the γ^{th}- quantile of the time-averaged distribution F, i.e.

$$C_S^* = F^{-1}(\gamma) := \inf\{x \in [0,1]\ :\ \gamma \leq F(x)\}. \quad (19)$$

(b) The expected profit, shortfall, and spillage corresponding to a contract C_S^* are given by

$$J_S(C_S^*) = qT \int_0^{\gamma} F^{-1}(w)dw - \lambda T \int_{\gamma}^{1} F^{-1}(w)dw \quad (20)$$

$$S_-(C_S^*) = T \int_0^{\gamma} \left[C_S^* - F^{-1}(w)\right] dw \qquad (21)$$

$$S_+(C_S^*) = T \int_{\gamma}^{1} \left[F^{-1}(w) - C_S^*\right] dw \qquad (22)$$

■

D. Optimal Contract with Cost of Reserves

In order to maintain reliable operation of the electric grid, the ISO must schedule operating reserve power to balance potential contract shortfalls arising from uncertainty in the wind power. We now consider the scenario in which the cost of the reserve power margin is explicitly transferred to the WPP. Although this is a departure from current deterministic approaches to scheduling reserve margins, we assume that the ISO schedules enough reserve power capacity to account for the WPP's worst-case shortfall with probability larger than $1-\epsilon$, where $\epsilon \in [0, 1)$. More precisely, given an offered contract C on interval $[t_0, t_f]$, the ISO schedules reserve capacity $R(C, \epsilon)$ satisfying

$$R(C,\epsilon) = \min R \quad \text{s.t.} \quad \mathbb{P}\left(R \leq [C - w(t)]^+\right) \leq \epsilon$$

for all $t \in [t_0, t_f]$. It is straightforward to show that the reserve capacity $R(C, \epsilon)$ is given by

$$R(C,\epsilon) = C - \min_t F^{-1}(\epsilon, t).$$

The cost of reserve capacity to the WPP can be explicitly accounted for in an augmented expected profit criterion $J_R(C)$:

$$J_R(C) \quad = \quad \mathbb{E}\ [\ \Pi(C,w) - q_R R(C,\epsilon)\ ] \tag{23}$$

In this context, q_R and q can be interpreted as *capacity* and *energy* prices respectively. Define a profit maximizing contract C_R^* as

$$C_R^* = \arg\max_{C \geq 0} J_R(C) \tag{24}$$

Theorem 4.10: Let $\gamma := (p - q_R)/(q)$. Define the time-averaged distribution $F(w)$ as in (2).

(a) An optimal contract C_R^* is given by the γ^{th}- quantile of the time-averaged distribution F, i.e.

$$C_R^* = F^{-1}(\gamma) := \inf\left\{x \in [0,1] \ : \ \gamma \leq F(x)\right\}. \tag{25}$$

(b) The expected profit corresponding to a contract C_R^* are given explicitly by

$$J_R\left(C_R^*\right) = q_R\left[\min_t F^{-1}(\epsilon,t)\right] + qT\int_0^\gamma F^{-1}(w)dw \tag{26}$$

■

Remark 4.11: It is interesting to note that the optimal contract size C_R^* offered by the WPP *does not depend on the risk level* ϵ. However, the expected profit certainly does depend on the risk level ϵ. □

V. Main Results: Marginal Profits

A. Local Generation

Now assume that the WPP has at its disposal a fast-acting *local* power plant with power capacity L and an operational cost q_L (\$/MW-hour). Hence, the local plant can be used to cover contract shortfalls up to a power limit of L. Fiscal benefit is derived from the assumption that the price of operating the local plant is less than the imbalance shortfall penalty price, $q_L < q$. These assumptions can be captured naturally through an augmented penalty price profile $\phi : \mathbb{R} \to \mathbb{R}_+$.

$$\phi(x) = \begin{cases} qx & x \in (L,\infty) \\ q_L x & x \in [0,L] \\ 0 & x \in (-\infty,0) \end{cases} \tag{27}$$

It is natural then to define expected profit under a contract C over a time interval $[t_0, t_f]$ as

$$J_L(C) = \mathbb{E}\int_{t_0}^{t_f} pC - \phi\left(C - w(t)\right)dt \tag{28}$$

Define the profit maximizing contract C_L^* as

$$C_L^* = \arg\max_{C \geq 0} J_L(C) \tag{29}$$

Theorem 5.1: Let $\gamma := p/q$. Define the time-averaged density $f(w)$ and distribution $F(w)$ as in (1) and (2).

(a) An optimal contract C_L^* is given by any solution C of

$$\gamma = F(C) + \frac{(q-q_L)}{q}L\left[f(C-L) - \frac{F(C)-F(C-L)}{L}\right].$$

(b) The marginal expected optimal profit with respect to L is given by

$$\frac{dJ_L(C_L^*)}{dL} = (q-q_L)Lf(C_L^* - L)$$

■

Remark 5.2: For L small, we have $C_L^* \approx F^{-1}(\gamma)$. □

B. Energy Storage

As wind energy penetration levels increase, energy storage will play a critical role in facilitating the firming of wind power contracts in conventional electricity markets. We now investigate the impact of energy storage capabilities on revenue for a WPP in our framework. The problem is formulated as a stochastic optimal control problem. For technical simplicity in exposition, we consider a discrete time formulation. The contract interval $[t_0, t_f]$ is divided into N intervals of size h indexed by k. Wind power is modeled as a discrete-time stochastic process $\{w_k \mid k \in \mathbb{N}\}$ with CDF $F(w,k) = \mathbb{P}(w_k \leq w)$. Define

$$F(w) = \frac{1}{N}\sum_{k=0}^{N-1} F(w,k)$$

Consider the following linear difference equation as a dynamic model for a generic energy storage system.

$$e_{k+1} = (1+\alpha h)e_k + h\left[\eta_{\text{in}}P_{k,\text{in}} - \frac{1}{\eta_{\text{ext}}}P_{k,\text{ext}}\right] \tag{30}$$

subject to the following constraints

$$0 \leq \quad e_k \quad \leq \overline{e} \tag{31}$$

$$0 \leq \quad P_{k,\text{in}} \quad \leq \overline{P}_{\text{in}} \tag{32}$$

$$0 \leq \quad P_{k,\text{ext}} \quad \leq \overline{P}_{\text{ext}} \tag{33}$$

The energy contained in the storage system at time k is denoted by e_k. The level of power extraction (injection) from (into) the storage system at time k is denoted by $P_{k,\text{ext}}$ ($P_{k,\text{inj}}$). The parameter $\alpha \leq 0$ is the dissipation coefficient on the stored energy, while $\eta_{\text{in}}, \eta_{\text{ext}} \in [0,1]$ model power injection and extraction efficiencies, respectively. Note that h is chosen such that $|1+\alpha h| < 1$ for stability considerations.

Define the storage decision vector $u_k = [P_{k,\text{ext}},\ P_{k,\text{inj}}]$. We assume that e_k and w_k are observed. For a particular time k, all of the information from the past relevant to the future is contained in the current storage state e_k and all past observed wind power realizations $w^k := \{w_i \mid i = 0, \cdots, k\}$. Hence, we consider storage operation policies of the form

$$u_k = g_k\left(e_k, w^k\right)$$

where g_k is constrained to belong the set of *feasible* feedback policies guaranteeing that constraints (31) - (33) are satisfied. Let $g := \{g_0, \cdots, g_{N-1}\}$ and let $\mathcal{G}$ denote the set of all feasible feedback policies g. Our objective is to find an optimal storage operation policy g^* and contract C^* that maximize the expected profit criterion:

$$J(g,C) = \mathbb{E}\sum_{k=0}^{N-1} phC - qh\left[C - w_k + P^g_{k,\text{in}} - P^g_{k,\text{ext}}\right]^+. \tag{34}$$

First note, that if $\overline{e} = 0$ we have no available storage and the optimal contract is simply

$$C^* = F^{-1}(\gamma)$$

analogous to Theorem 4.1.

The marginal optimal expected profit with respect to the storage capacity $\frac{dJ^*}{d\overline{e}}$ can be analytically computed for $\overline{e}$ *small*. First define the random variable $\xi(C,N)$ as the *number of times* that the random process w_k crosses the real value C from above for $k = 0, \cdots, N-1$. The random variable $\xi(C,N)$ is constructed as follows. Define the binary stochastic process $v_k(C)$

$$v_k(C) = \begin{cases} 1, & w_k \geq C \\ 0, & w_k < C \end{cases}$$

It follows that $\xi(C,N)$ is given by

$$\xi(C,N) = \sum_{k=0}^{N-2} \mathbf{1}\{v_k > v_{k+1}\}$$

Theorem 5.3: Let $\gamma := p/q$. Assume that (1) the storage system is perfectly efficient ($\alpha = 0, \eta_{\text{in}} = \eta_{\text{ext}} = 1$), (2) no constraints on power extraction or injection, and (3) $e(0) = 0$.

Then the marginal expected optimal profit with respect to $\overline{e}$ at the origin is given by

$$\left.\frac{dJ^*}{d\overline{e}}\right|_{\overline{e}=0} = q\,\mathbb{E}[\ \xi(C^*,N)\ ] \tag{35}$$

$$= q\sum_{k=0}^{N-2} \mathbb{P}\left(v_k > v_{k+1}\right) \tag{36}$$

■

Remark 5.4: The quantity $\mathbb{E}[\ \xi(C,N)\ ]$ is an important property of the stochastic process w and is intimately connected to spectral properties of w. □

VI. Empirical Studies

Using a wind power time series data set provided by the Bonneville Power Administration (BPA), we are in a position to illustrate the utility and impact of the theory developed in this paper.

A. Data Description

The data set consists of a time series of measured wind power aggregated over the 14 wind power generation sites in the BPA control area [2]. The wind power is sampled every 5 minutes and covers the 2008 and 2009 calendar years. Accompanying the measured wind power is a time series of *rolling* one hour-ahead forecasts sampled at the same frequency. To account for additional wind power capacity coming online over the 2-year horizon, all of the data are normalized by the aggregate nameplate power capacity of the wind farms.

B. Empirical Probability Model

As stated earlier, wind power is modeled as a continuous time stochastic process whose marginal cumulative distribution is denoted by $F(w,t)$. While the identification of stochastic models that accurately capture the statistical variability in wind power is of critical importance, this is not the focus of our paper. We will make some simplifying assumptions on the underlying physical wind and measurement process to facilitate our analysis.

A1: The wind process $w(t)$ is assumed to be first-order cyclostationary in the strict sense with period $T_0 = 24$ hours – i.e $F(w,t) = F(w,t+T_0)$ for all t [23], [9]. Thus, we are ignoring the effect of seasonal variability.

A2: For a fixed time τ, the discrete time stochastic process $\{w(\tau + nT_0) \mid n \in \mathbb{N}\}$ is independent in time (n).

Fix a time $\tau \in [0, T_0]$ and consider a finite length sample realization of the discrete time process $z_\tau(n) := w(\tau + nT_0)$ for $n = 1, \cdots, N$. Using this data set, we take the empirical

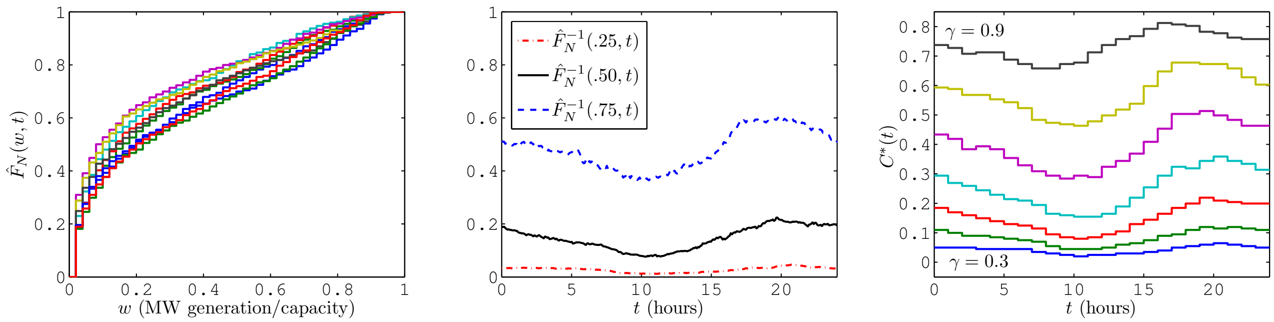

Fig. 2. (a) Empirical CDFs $\hat{F}_N(w,\tau)$ for nine equally spaced times throughout the day, (b) Trajectory of the empirical median $\hat{F}_N^{-1}(.5,t)$ and its corresponding interquartile range $[\hat{F}_N^{-1}(.25,t), \hat{F}_N^{-1}(.75,t)]$, (c) Optimal contracts offered in the DA market for various values of $\gamma = 0.3, 0.4, \cdots, 0.9$.

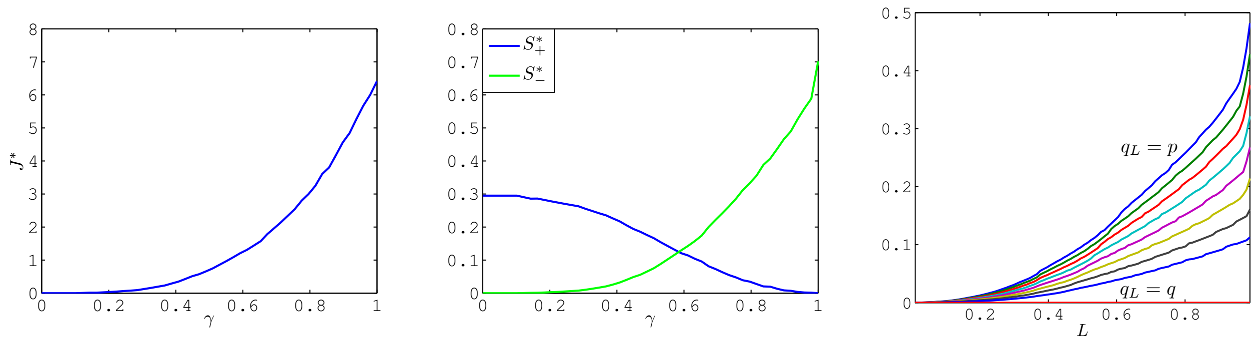

Fig. 3. (a) Optimal expected profit J^* as a function of γ, (b) Optimal expected energy shortfall and spillage for the 12^{th} hour interval, as a function of γ, (c) Marginal expected optimal profit with respect to power capacity L of a local generation plant for various operational costs $q_L \in [p, q]$.

distribution $\hat{F}_N(w,\tau)$ as an approximation of the underlying distribution $F(w,\tau)$:

$$\hat{F}_N(w,\tau) = \frac{1}{N}\sum_{i=n}^{N} \mathbf{1}\{z_\tau(n) \leq w\} \tag{37}$$

Invoking the strong law of large numbers under the working assumptions, it can be shown that the $\hat{F}_N(w,\tau)$ is consistent with respect to $F(w,\tau)$. Figure 2 (a) depicts nine representative marginal empirical distributions identified from the BPA data set described earlier. Note that the times corresponding to the nine distributions are equally spaced throughout the day to provide a representative sample. Figure 2 (b) depicts the trajectory of the empirical median $\hat{F}_N^{-1}(.5,t)$ and its corresponding interquartile range $[\hat{F}_N^{-1}(.25,t), \hat{F}_N^{-1}(.75,t)]$.

C. Optimal Contracts in Conventional Markets

Using empirical wind power distributions identified from the BPA wind power data set, we are now in a position to compute and appraise optimal day-ahead (DA) contracts offered by a representative Oregon wind power producer (WPP) participating in the idealized market system described in Section IV-A. We are also able to examine the effect of γ on J^*, S_-^*, and S_+^* using this particular characterization of wind uncertainty. The following empirical studies assume a contract structure $\{[t_{i-1}, t_i), C_i\}_{i=1}^{24}$, where $[t_{i-1}, t_i)$ is of length one hour for all i.

Remark 6.1: (*Optimal DA Contracts*) Figure 2 (c) depicts optimal contracts $(C_1^*, \cdots C_{24}^*)$ for various price ratios $\gamma = 0.3, 0.4, \cdots, 0.9$. As expected, as the price ratio $\gamma = p/q$ decreases, the optimal contract C^* decreases. From Figure 2 (c), it is evident that WPPs will tend to offer larger contracts during morning/night periods when wind speed is typically higher than during mid-day (as indicated by Figure 2 (b)). □

Remark 6.2: (*Profit, Shortfall, and Spillage*) Figures 3 (a) and (b) demonstrate the effect of the *risk-controlling* price ratio γ on the optimal expected profit, energy shortfall, and energy spillage. The units of S_-^* and S_+^* are (MW-hour)/(nameplate capacity), while the units of J^* are in \$/(q · nameplate capacity). When $\gamma = 1$, the WPP sells all of its energy production at price $p = q$. In this situation, the expected profit per hour (see Figure 3 at $\gamma = 1$) of approximately $\frac{6.4}{24}$ equals the ratio of average production to nameplate capacity. This number is consistent with typical values of the wind production *capacity factor*. The spillage S_+^* and shortfall S_-^* are relatively insensitive to variations in γ [for $\gamma \in [0, 0.1]$] because the marginal empirical distributions are steep here. □

D. Local Generation

We now consider the optimal contract sizing formulation in section V-A. Figure 3 (c) depicts the marginal expected optimal profit with respect to local generation power capacity

L. As $q \rightarrow q_L$, the marginal value of local generation diminishes.

VII. Conclusion

In this paper we have formulated and solved a variety of problems on optimal contract sizing for a wind power producer operating in conventional electricity markets. Our results have the merit of providing key insights into the trade-offs between a variety of factors such as prices for under and over production, cost of reserves, value of storage and local generation, etc. In our current and future work, we will investigate a number of intimately connected research directions: improved forecasting of wind power, optimization of reserve margins, making wind power dispatchable, network aspects of renewable energy grid integration, and new market structures for facilitating integration of renewable sources. We are also studying the important case of markets with recourse where the producer has opportunities to adjust bids in successive stages. We are also developing large scale computational simulations which can be used to test the behavior of of simplified analytically tractable models and suggest new avenues for research applicable to real-world grid-scale problems.

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