Drift kinetics within a global MHD model: a progress report

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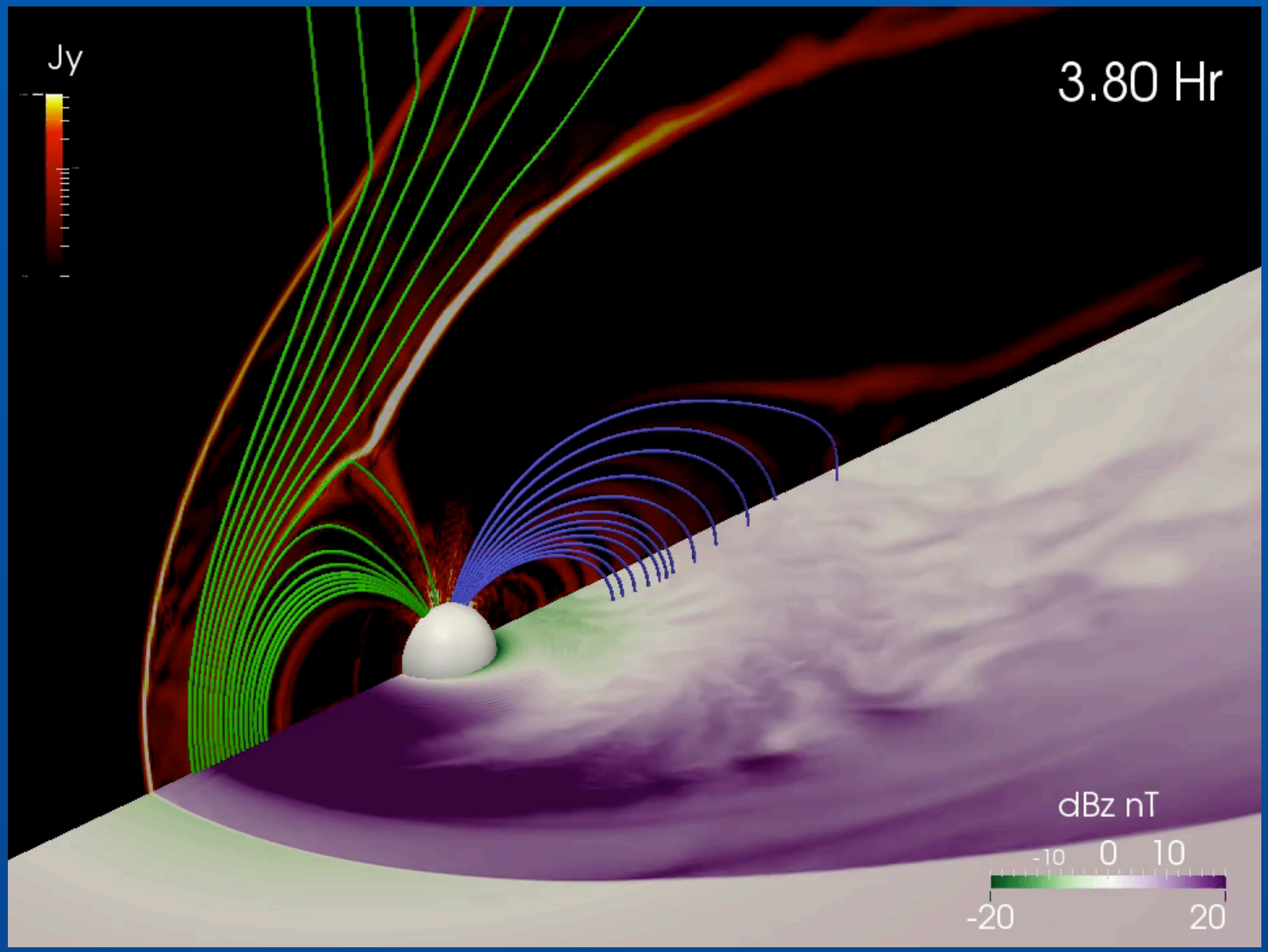
- What's being added - most FLR effects
  - Particle drifts, Hall electric field, gyroviscosity

- Approach is brute force using a multifluid MHD code
  - Similar in assumptions to Rice Convection Model
  - Multiple energy channels

- Outline
  - Motivation
  - Method
  - Initial Results

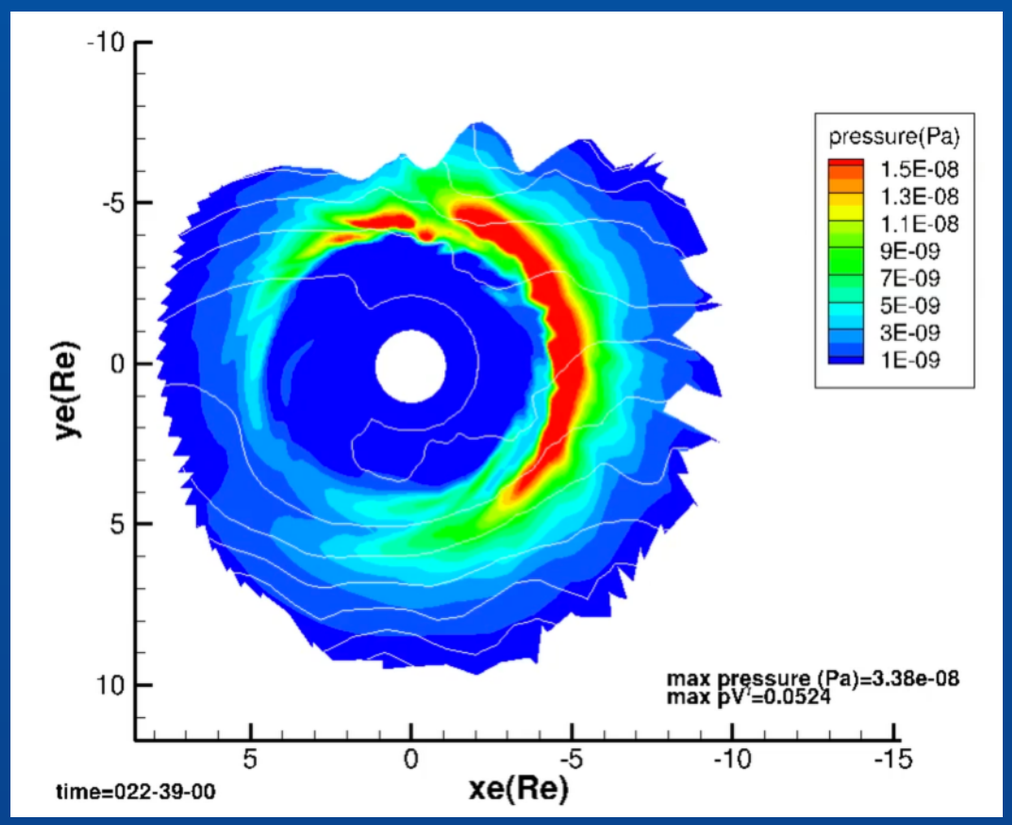
Motivation

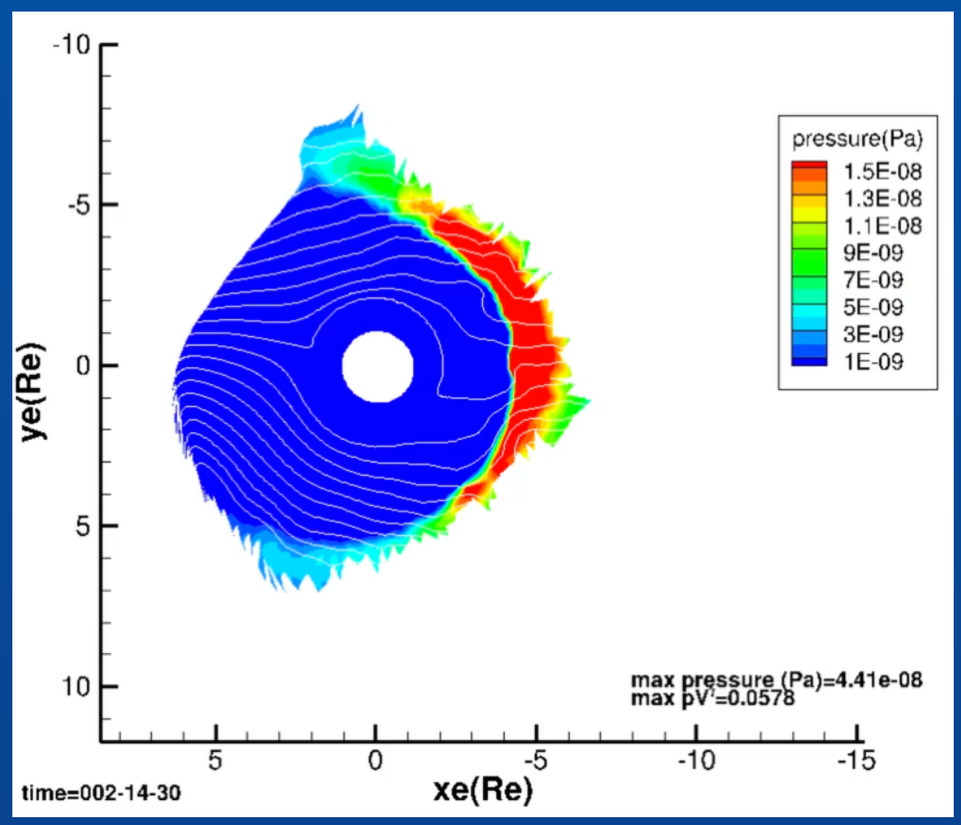
- Much of the structure is mesoscale or smaller
- What sets the scales:
  - ideal instabilities tend to have growth rates increasing with k
  - Increased resolution gives smaller scales without limit
  - Suspect ion FLR scales produce the scales of BBF's, for example



Motivation II

- Coupling to Rice Convection model is hindered by incompatible physics
  - Inertial effects limit the spatial range over which the RCM can accept MHD input
  - The current LFM to RCM input is a Maxwellian which doesn't reflect actual population in the tail





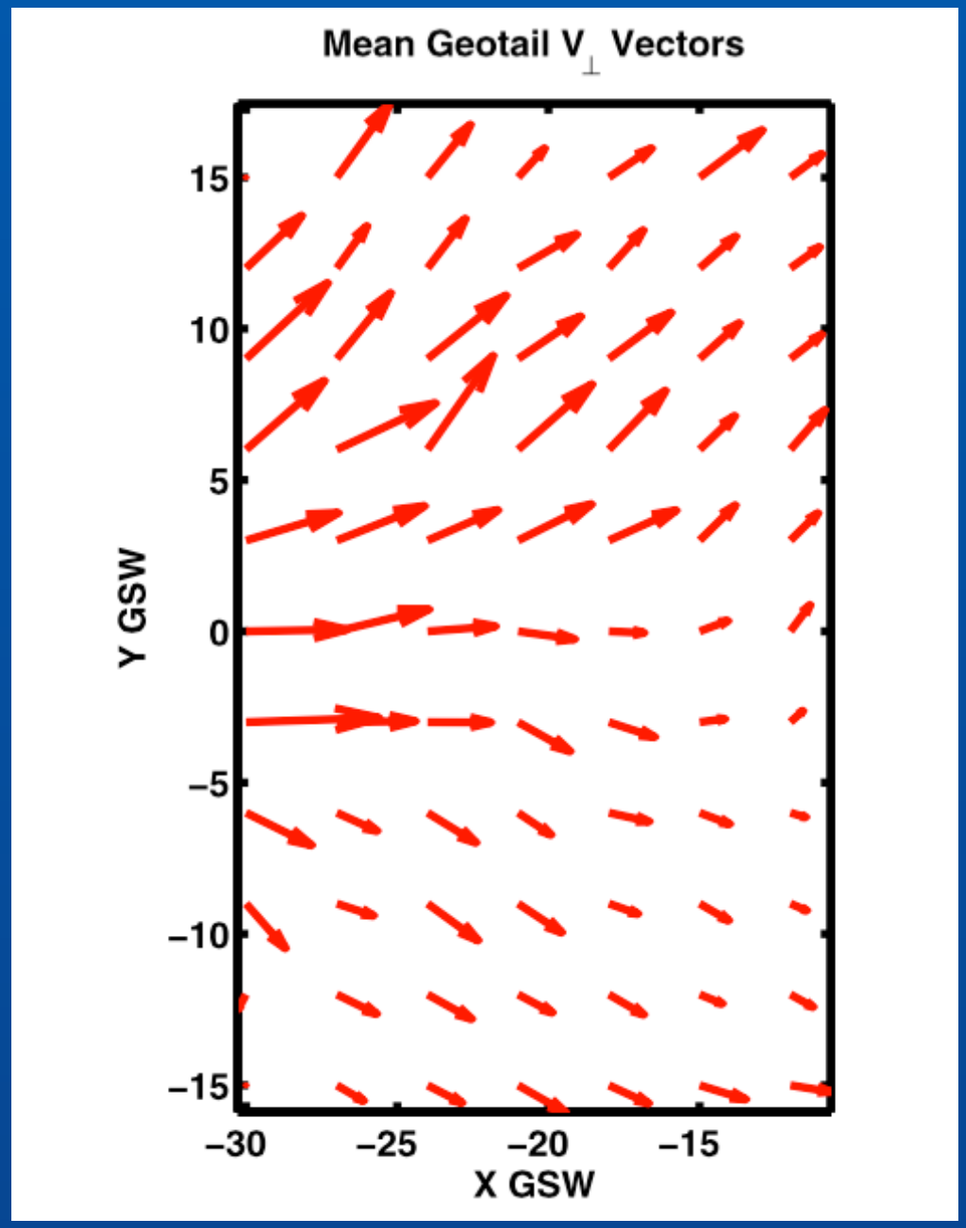
Motivation III

- Drifts are not dominant in the tail but they can be important

- Example: 10nT field, 2 keV proton.
  - Larmor radius 1/10 R_E
  - Drift speed = 30/L kms
  - V_E = E(mV/m) 100 km/s

- Geotail averages show asymmetry in direction of ion drift

- MHD calculations are approaching d_i
  - Does increasing resolution buy you better physics
  - Maybe put effort into something else



Guild *et al.*, JGR, **113,** 2008

Method

- Assumptions
  - At any given thermal energy the distribution function is isotropic
  - But, each energy can have a different velocity (drift) centroid
- These are essentially the same as the RCM makes
- Split distribution into discrete energy species
- No assumption is made about the parallel velocity at a given energy
- The model is essentially one in which the energy scattering rate is very slow, but pitch energy scattering is very fast
- Because the parallel velocity for each energy is unconstrained, distributions can look very different from a skewed Maxwellian

- Use the Vlasov equation and moments

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

- Note that the Lorentz force contains the gyro timescale
  - usually assume $\Omega_i >> 1/\tau_{MHD}$
  - suggests expansion in terms of

$$\epsilon \sim \Omega_i \tau_{MHD},\ r_L/L$$

  - but this isn't necessary
  - will rough out the perturbative approach

- Split the species velocity into

$$\mathbf{v} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \epsilon\mathbf{v}_d + \mathbf{v}_{||} = \mathbf{v}_E + \epsilon\mathbf{v}_d + \mathbf{v}_{||}$$

- Then the moment equations become

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) &= 0 \\
\frac{\partial \rho\mathbf{v}}{\partial t} + \nabla\cdot\rho\mathbf{v}\mathbf{v} + \nabla P + \epsilon\nabla\cdot\Pi_{gv} + \frac{nq}{\epsilon}(\mathbf{E}+\mathbf{v}\times\mathbf{B}) &= 0 \\
\frac{Ds}{Dt} &= 0
\end{aligned}$$

- We can write the Lorentz force as

$$\mathbf{F}_{Lorentz} = \frac{nq}{\epsilon}\mathbf{v}_d \times \mathbf{B}$$

- This is the drift force that in lowest order gives all species the same perpendicular velocity

- Momentum and Ohm's Law
  - Momentum contains both ExB drift and sum of the drifts
  - But, to lowest order the bulk speed is ExB/B^2
  - Thus, the drifts are specified by finding the bulk speed

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \rho \mathbf{v}\mathbf{v} + \nabla P - \mathbf{j} \times \mathbf{B} = 0$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \mathbf{j}/ne + \nabla P_e/ne$$

$$\mathbf{j} = \Sigma_\alpha n_\alpha q_\alpha \mathbf{v}_d^\alpha + \nabla P_e/ne$$

- Once we know the drifts to lowest order we can correct the individual velocities

$$\mathbf{v}_\perp^\alpha = \mathbf{v}_E + \mathbf{v}_d^\alpha$$

$$\mathbf{v}_\perp^\alpha = \mathbf{v}_{bulk} - \frac{1}{\rho}\Sigma_\beta n_\beta m_\beta \mathbf{v}_d^\beta + \mathbf{v}_d^\alpha$$

- The new velocities can be used to correct the convective terms in the equations

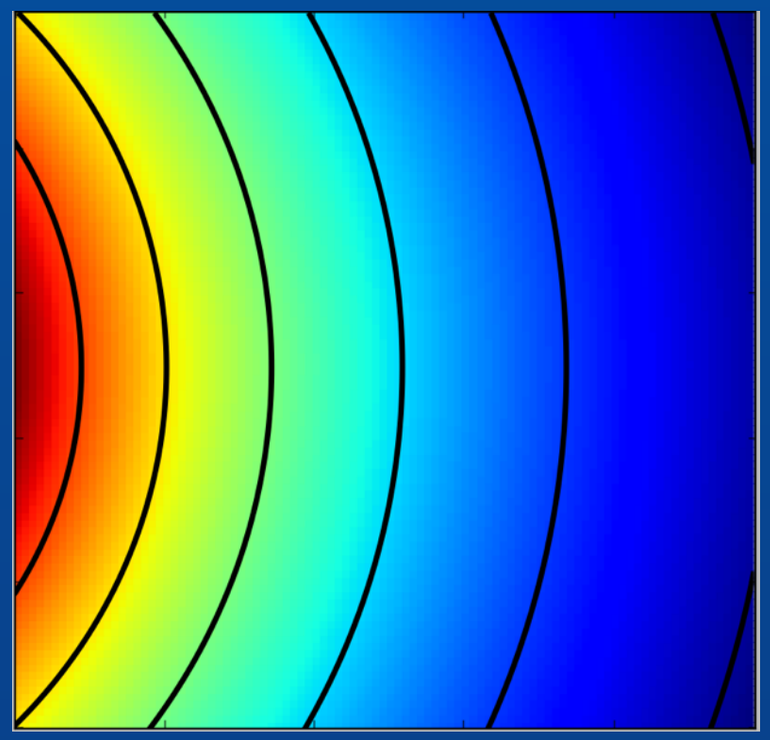
Hall term and the Biermann Battery

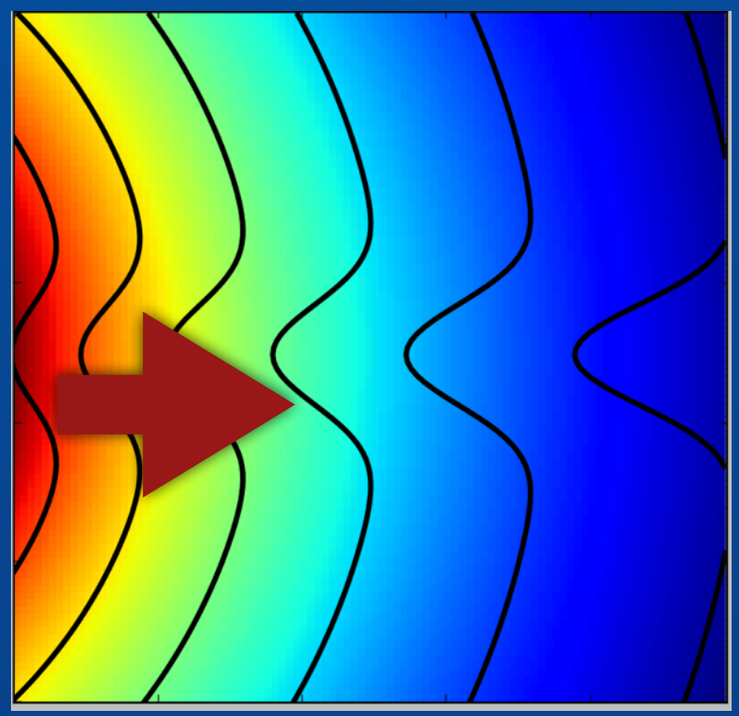
- Coupled ring current models by and large ignore the Hall effect - this is inconsistent
  - The velocity coming out of the momentum equation contains both the ExB and diamagnetic drifts, but Faraday's Law usually uses the total
  - The drifts can be much larger than ExB, so we need to include jxB/ne
  - To a good approximation in the inner magnetosphere $\nabla P = -\vec{j} \times \vec{B}$
  - substituting we get

$$\frac{\partial \vec{B}}{\partial t} = -\frac{1}{n^2 e} \nabla n \times \nabla P$$

  - If the flow is smooth probably no effect, but if, for example, we have fingers:

No effect



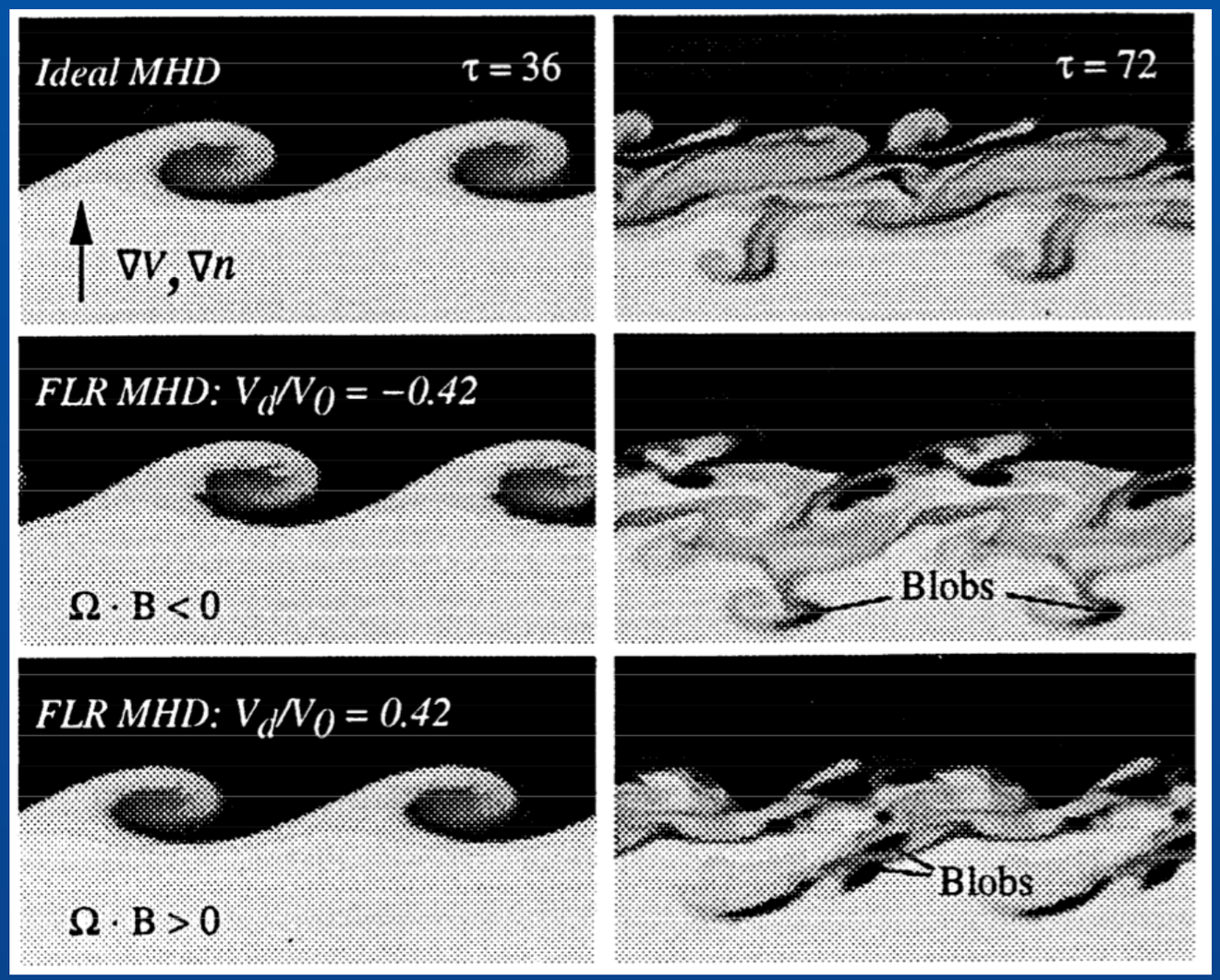


Some effect

Pros and Cons

- Advantages:
  - Handles inertial terms automatically
  - Keeps field-aligned behavior
  - Effectively provides a physically reasonable closure for the integrated equations
    - *e.g.,* sum over species implies a heat flux for the ensemble
- Disadvantages
  - Computationally intensive
  - As approach d_i scales it's, not clear that expansion works
    - Some hope for the plasma sheet (note Usadi *et al.,* JGR, **101**, 1996)
    - More related to the assumptions about the distribution function than the gyro scales
  - Electrons aren't handled well.

- Gyroviscosity is usually ignored in magnetospheric fluid models
  - finite Larmor radius effects other than the Hall term are usually dropped
  - Except for very simple cases the GV tensor is very messy, requires tensor velocity moments up to fourth order



Huba(GRL, **23**, 1996) did KH calculation with Hall term and gyro viscosity

Huba and Winske (Phys. Plasma, **5**, 1998) compared hybrid and FLR MHD with mixed results

- For a single energy, isotropic distribution the gyro viscous tensor reduces to something manageable

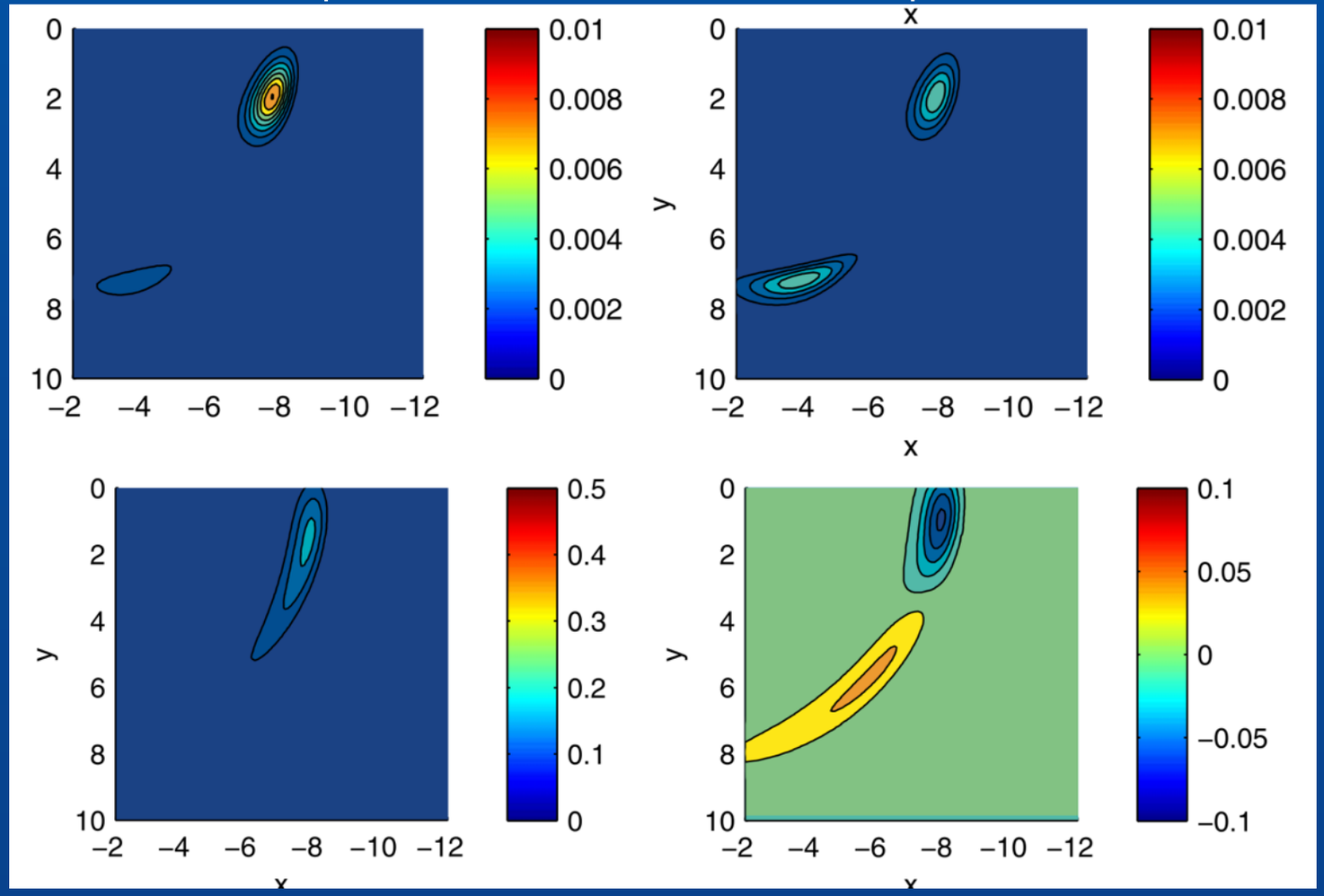
- When sum over energy channels the bulk properties recover higher order moment corrections

$$\Pi_{gv} = \frac{P}{4\Omega_i}[(\mathbf{b} \times \mathcal{W}) \cdot (\mathcal{I} + 3\mathbf{bb}) + (\mathcal{I} + 3\mathbf{bb}) \cdot (\mathcal{W} \times \mathbf{b})]$$

$$\mathcal{W} = (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$$

Drifts

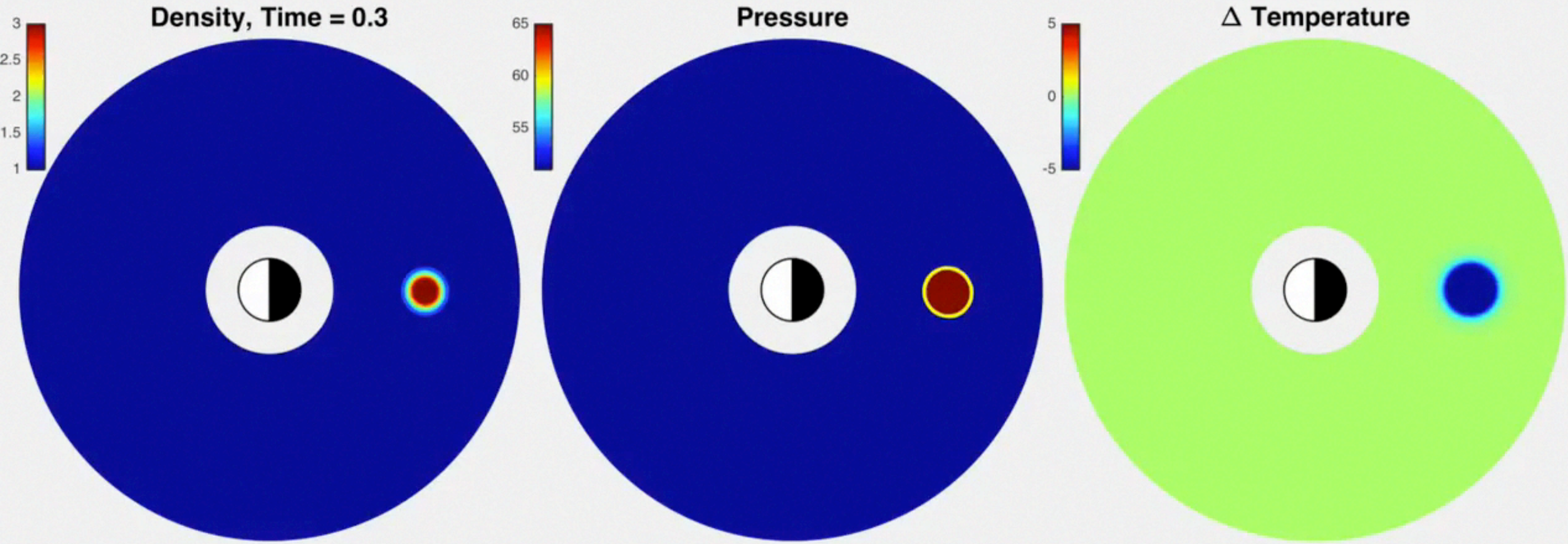
- Song *et al.* (JGR, **113**,2008), a cautionary tale
  - Compared the effort of Heinemann and Wolf (JGR, 106,2001) to provide a single fluid model for inner magnetosphere drifts to The RCM
  - Results were not encouraging
  - Test problem of drifts in a dipole field
    - drifting blob (Gaussian in space) - fluid model breaks into two separate blobs doesn't spread out like RCM



Fluid model
density, pressure

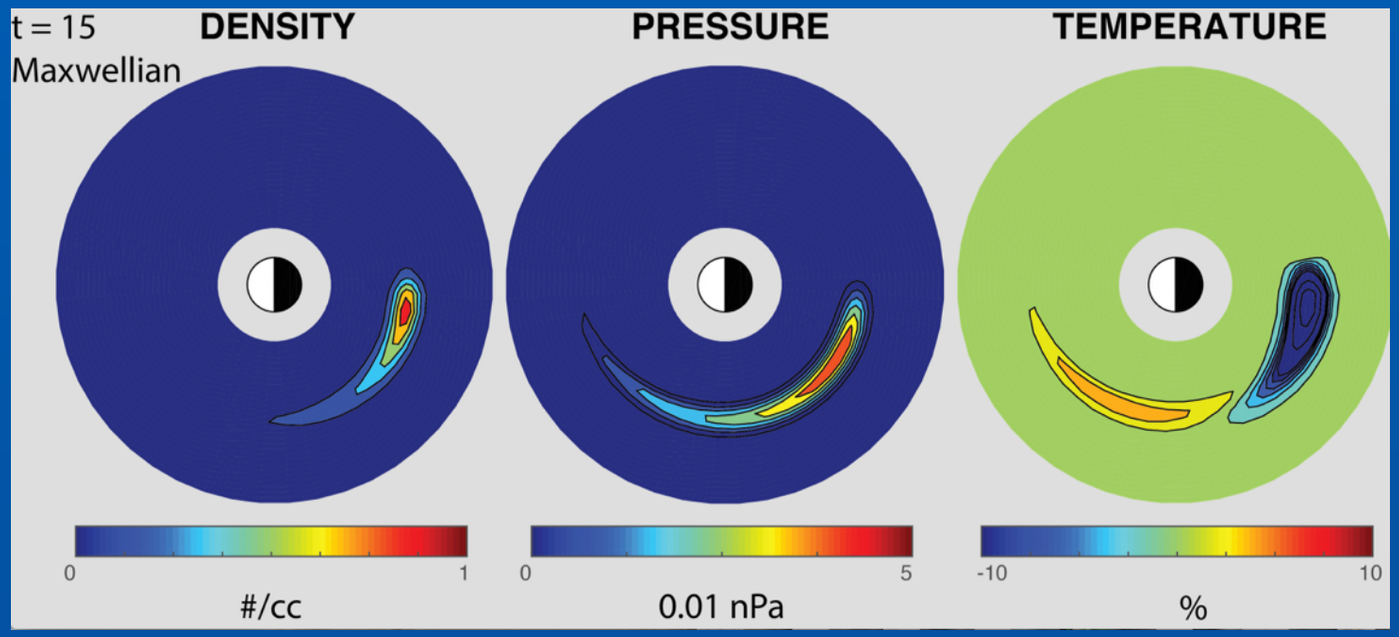
RCM
density,temperature

MHD drifts

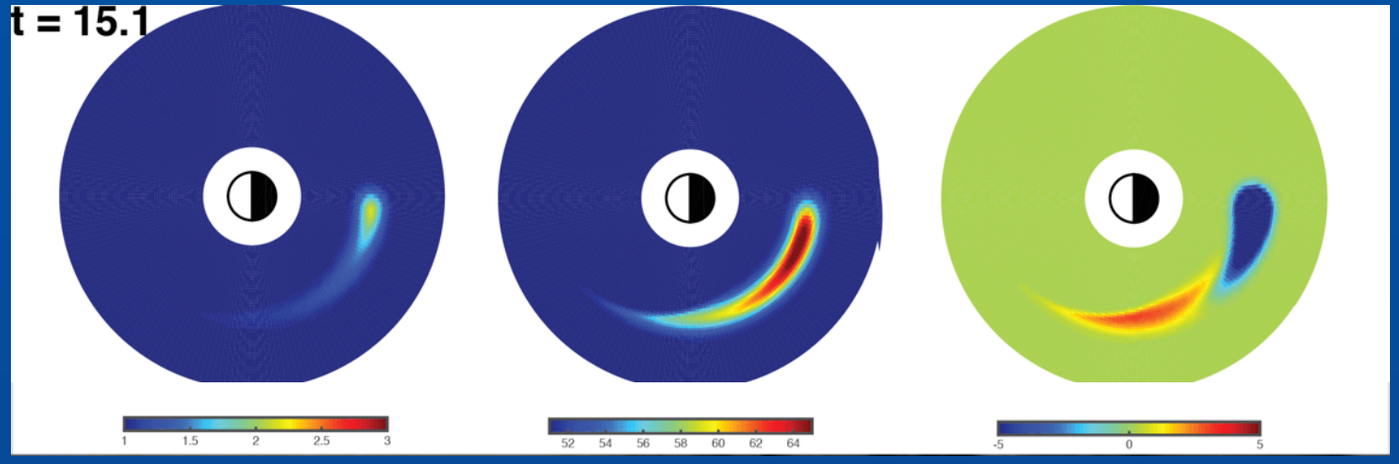


- Basically same as Song et al problem.
- Blob at 5.5 R_E , 64 species, 0.1 R_E resolution
- Higher energy particles drift almost entire way around Earth

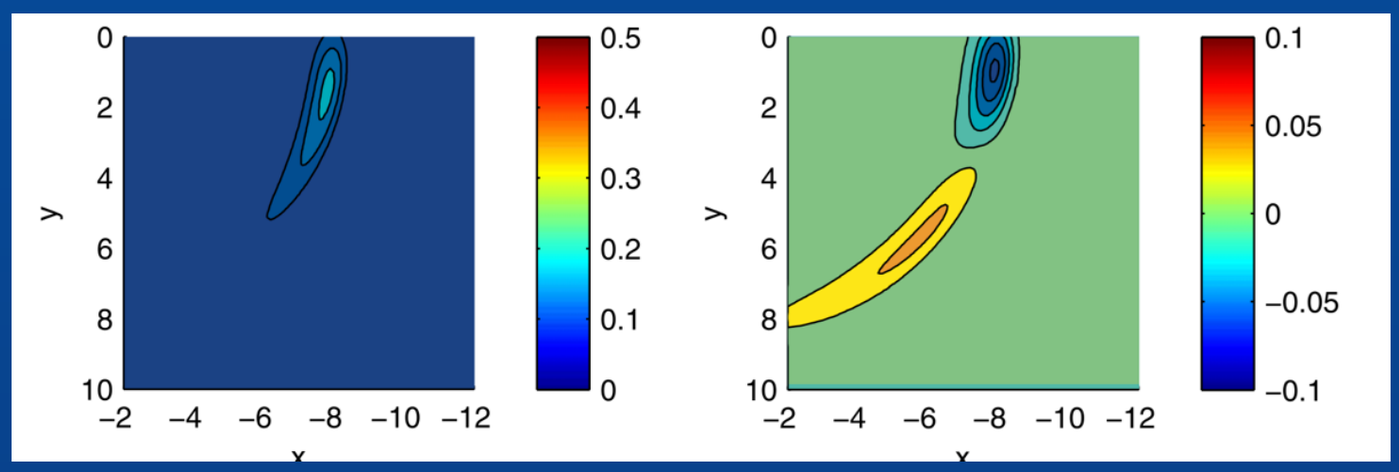
Comparisons



Low res MHD, 0.2 R_E, 15 species



Movie frame

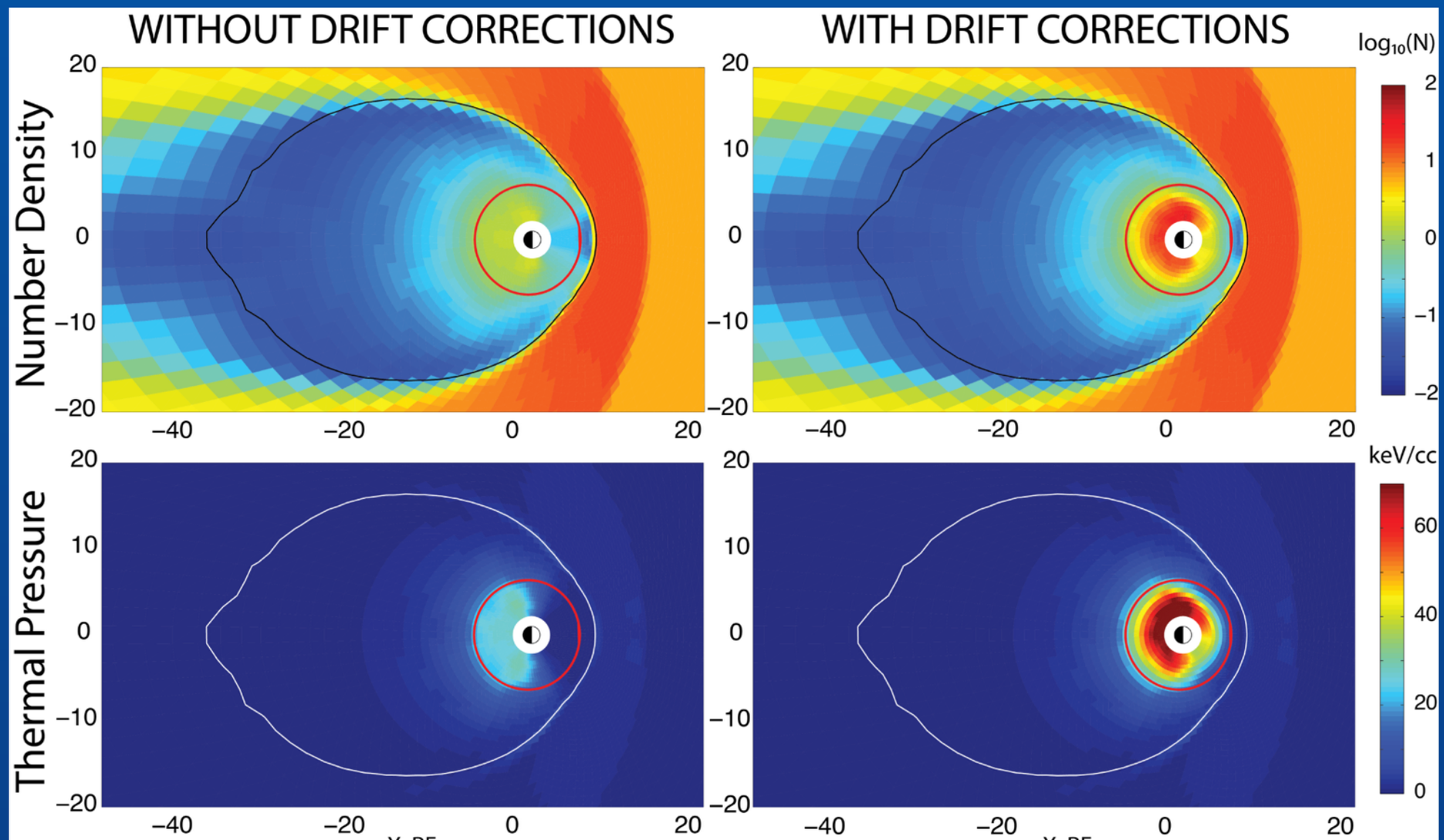


Song *et al.* 200 species left frame - density, right - temperature

Ring current simulations

- Preliminary - doing one-way coupling
  - use LFM fields to push drifting distribution, e.g. (∇P rather than ∇B
  - 16 energy channels
  - base run, V_{sw} = 400 km/s, n = 5 cm^{-3}, B_z = -15 nT

- Show development of ring current
  - Too close, too weak
  - Probably resolution related and lack of self-consistency

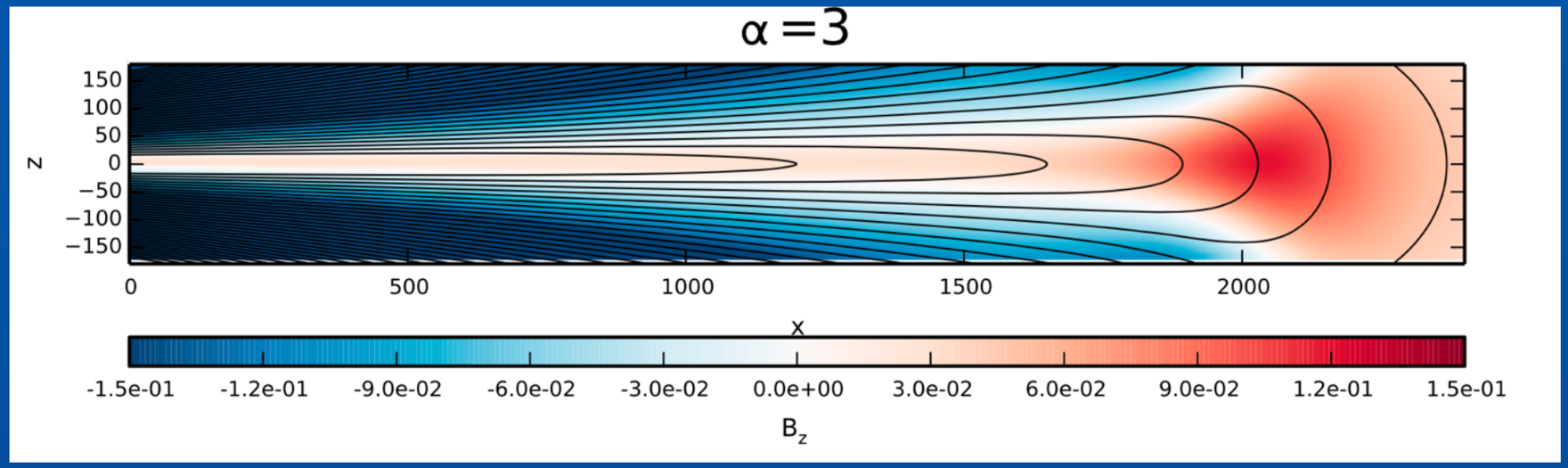


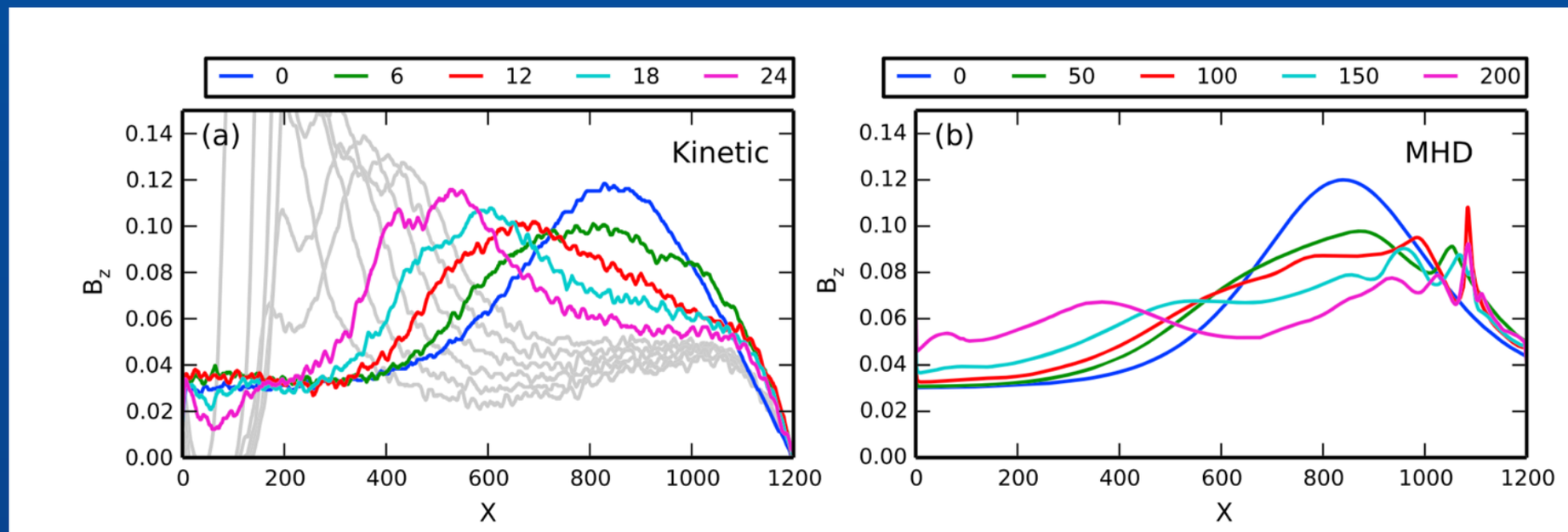
To Do List

- Full implementation in the Multi-Fluid LFM
- Anisotropic formalism
- Modify RCM coupling to take more energy info
  - Plan is to use both codes for what they do best
- 2 and 3D reconnection studies
  - Have been working on Hall only version
- Particle tracing in simulations
  - validation / improvements
  - energization

- Series of 2D equilibria that may allow collisionless tearing (Schindler and Sitnov)

- Some similarities between fully kinetic and ideal MHD
  - how do FLR effects change the MHD picture





Summary

- Developed a formalism for self-consistent application of drifts to MHD in a magnetospheric context
- Diamagnetic drifts in code are consistent with RCM results
- Initial application to the magnetosphere is promising