Data assimilation for the inner ring current using RAM-SCB

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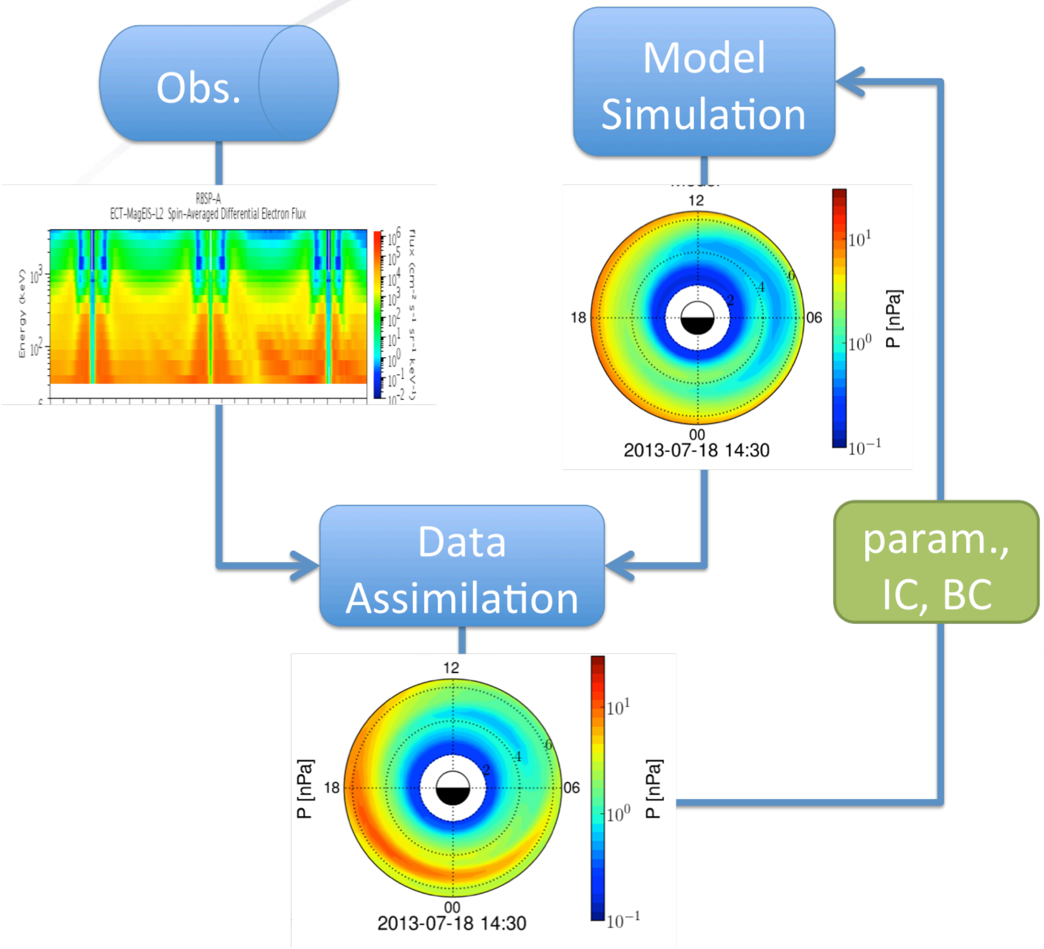
Data Assimilation



- Models have approximate physics.
- Data is typically sparse.
- **Data Assimilation combines the two to improve our estimate of the true state.**

We will combine fluxes from RAM-SCB with flux data from the Van Allen Probes.

Ensemble Kalman Filter

- EnKF is Monte Carlo approx. to Kalman filter: uses ensemble of simulation to estimate uncertainty about the state (fluxes)
- Data and model are combined using covariances (Model and data are approximated by Gaussian)
- Successfully used in weather forecasting, hurricane prediction, etc.
- EnKF is non-intrusive (doesn't require model modification or derivatives), easy to implement.

Ensemble Kalman Filter

x contains the state, fluxes (size ~ 10^7)

M is the forward model, RAM-SCB

y contains the observations, Van Allen probes (size ~ 100s)

H interpolates the model to the observation

K calibrates the adjustment based on the covariance of the ensembles (**P**) and the observation error (**R**).

P is the empirical covariance of the ensemble of states.

N is size of ensembles (~30)

$$\mathbf{x}^f(t_i) = \mathbf{M}\left[\mathbf{x}^f(t_{i-1})\right]$$

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left(\mathbf{y}^o - \mathbf{H}\mathbf{x}^f\right)$$

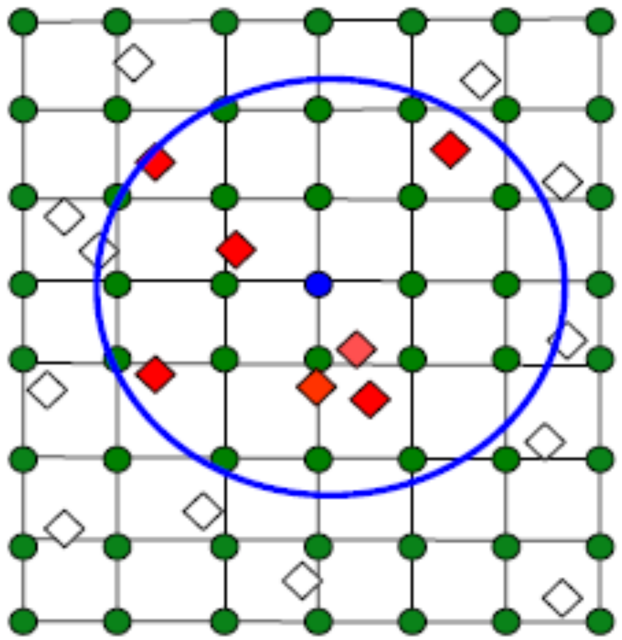
$$\mathbf{K} = \mathbf{P}\mathbf{H}^T\left(\mathbf{H}\mathbf{P}\mathbf{H}^T + \mathbf{R}\right)^{-1}$$

$$\mathbf{P} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\mathbf{x}_i^f - \overline{\mathbf{x}}\right)\left(\mathbf{x}_i^f - \overline{\mathbf{x}}\right)^T$$

The size of ensembles is much less than the size of the state, so the naïve estimate of P is not full rank. Modeling P is the challenge.

Local EnKF

- Update only parts of the state that are “close” to observed data.
- Localization operates an a small part of the state space for which a full rank covariance can be computed.
- Successfully developed and used for atmospheric models



Brian R. Hunt, Eric J. Kostelich, Istvan Szunyogh, *Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter*, Physica D, 230, 112-126, 2007

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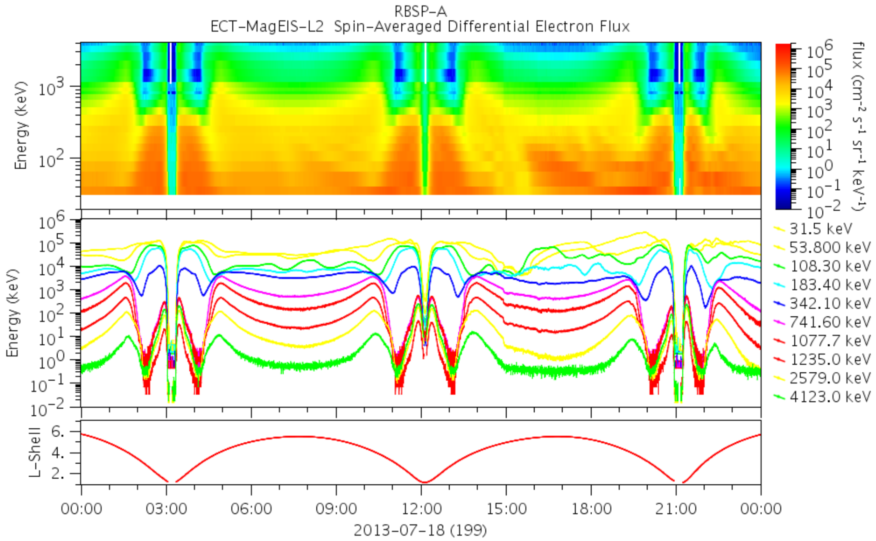
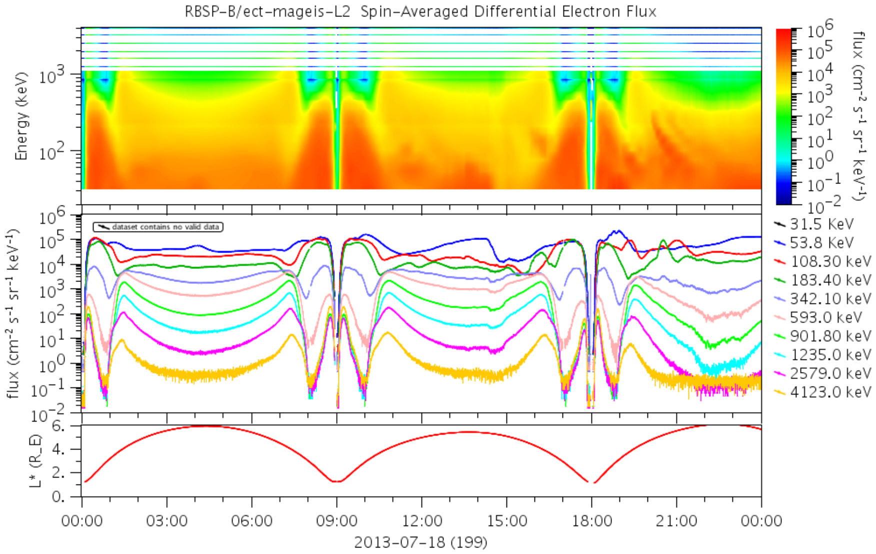
Example: 7/18/2013 Event





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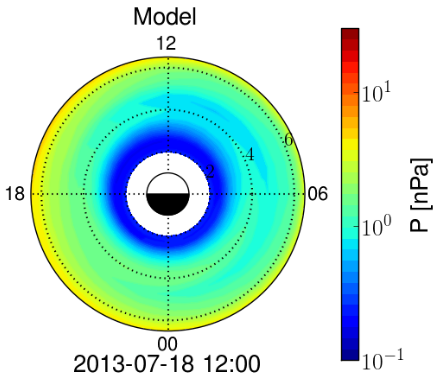
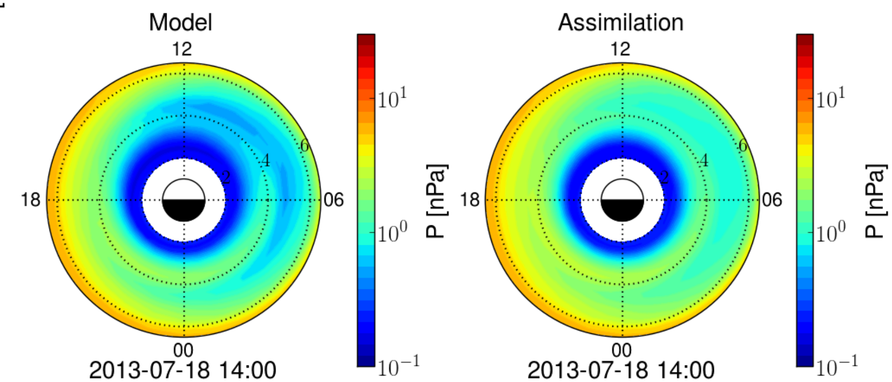
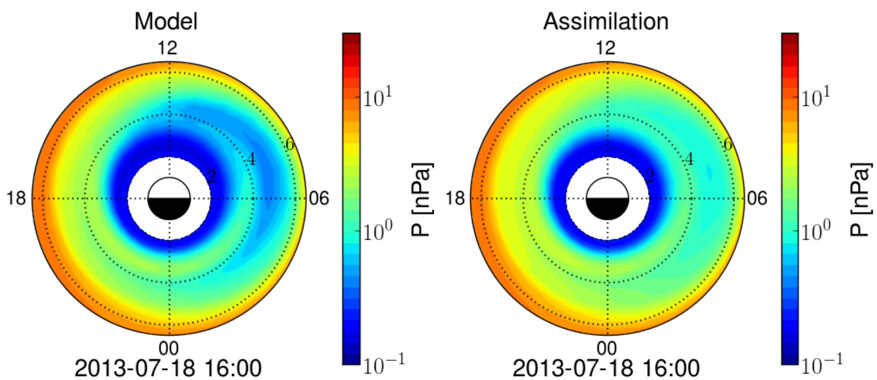
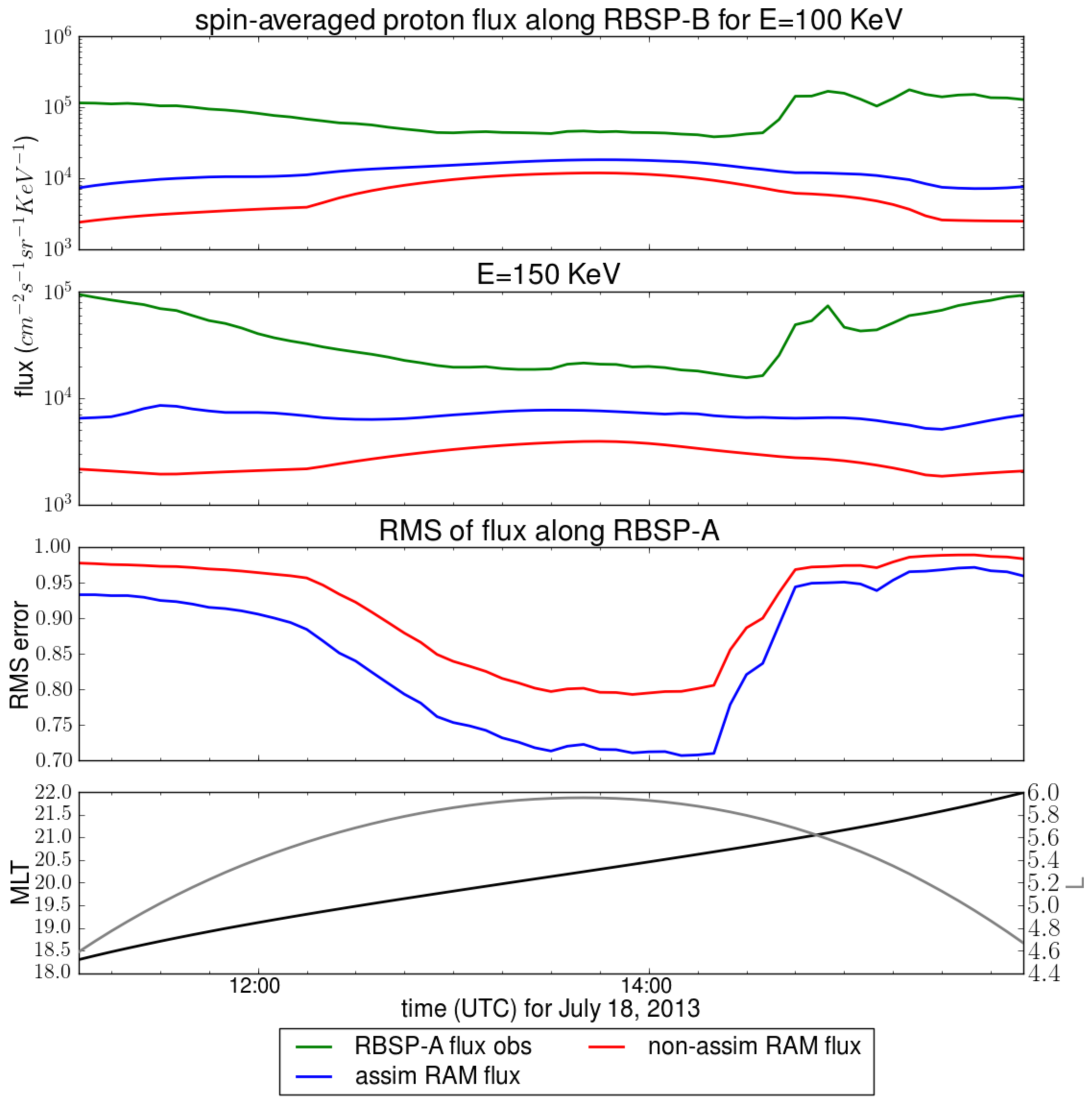
Local EnKF results (cont.)



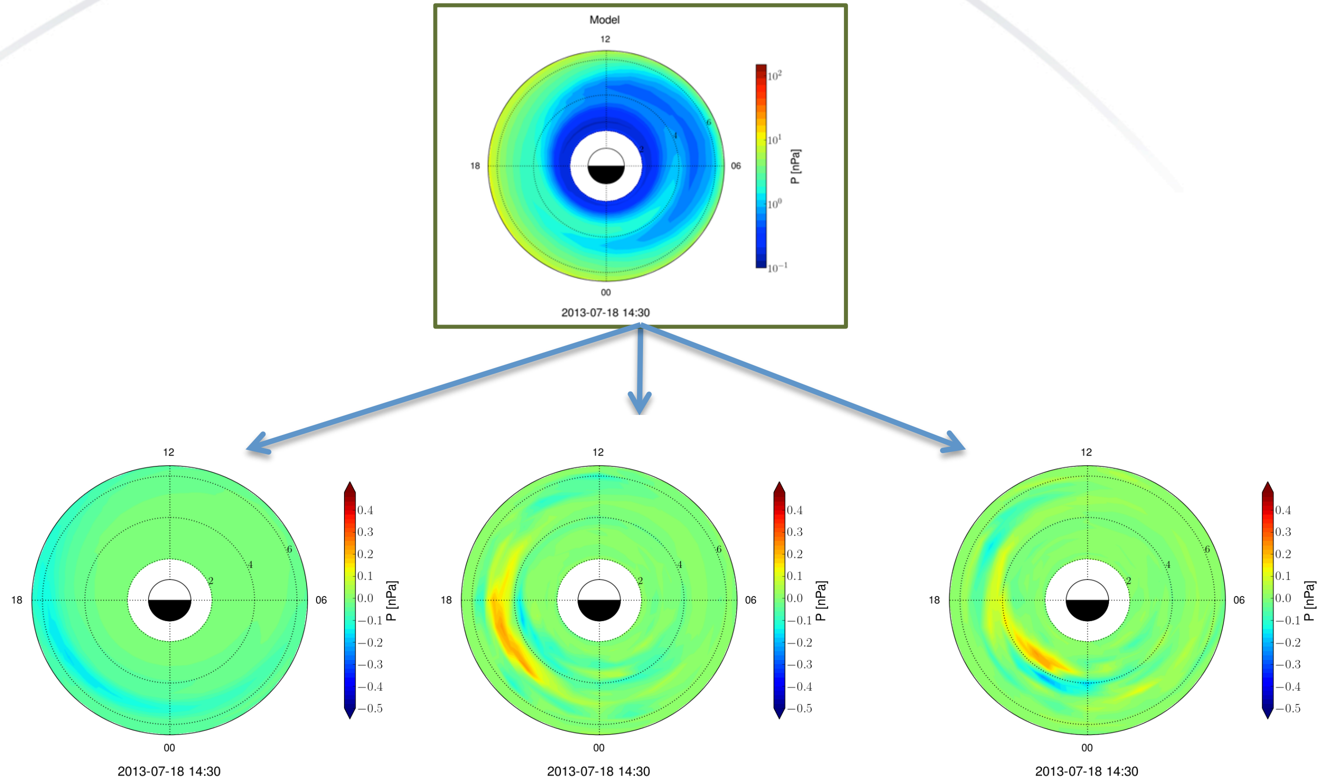




Local EnKF results (cont.)



New Developed Assimilation Method



Decompose the state into a combination of a small number of basis vectors. The state is replaced by the basis weights for which a full rank covariance can be computed. **Project onto basis and perform EnKF.**

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Reduced Basis EnKF Method

Define ensemble matrix $\mathbf{X}=[\mathbf{x}_1\ \mathbf{x}_2\ \ldots\ \mathbf{x}_N]$ and compute the singular value decomposition (PCA)

$$\mathbf{X} = \mathbf{U}\Sigma\mathbf{V}^T$$

define a basis

$$\mathbf{B} = \frac{1}{\sqrt{N-1}}\mathbf{U}_k\Sigma_k$$

project (weights)

$$\mathbf{w}_i = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{x}_i$$

Perform EnKF on the weights.

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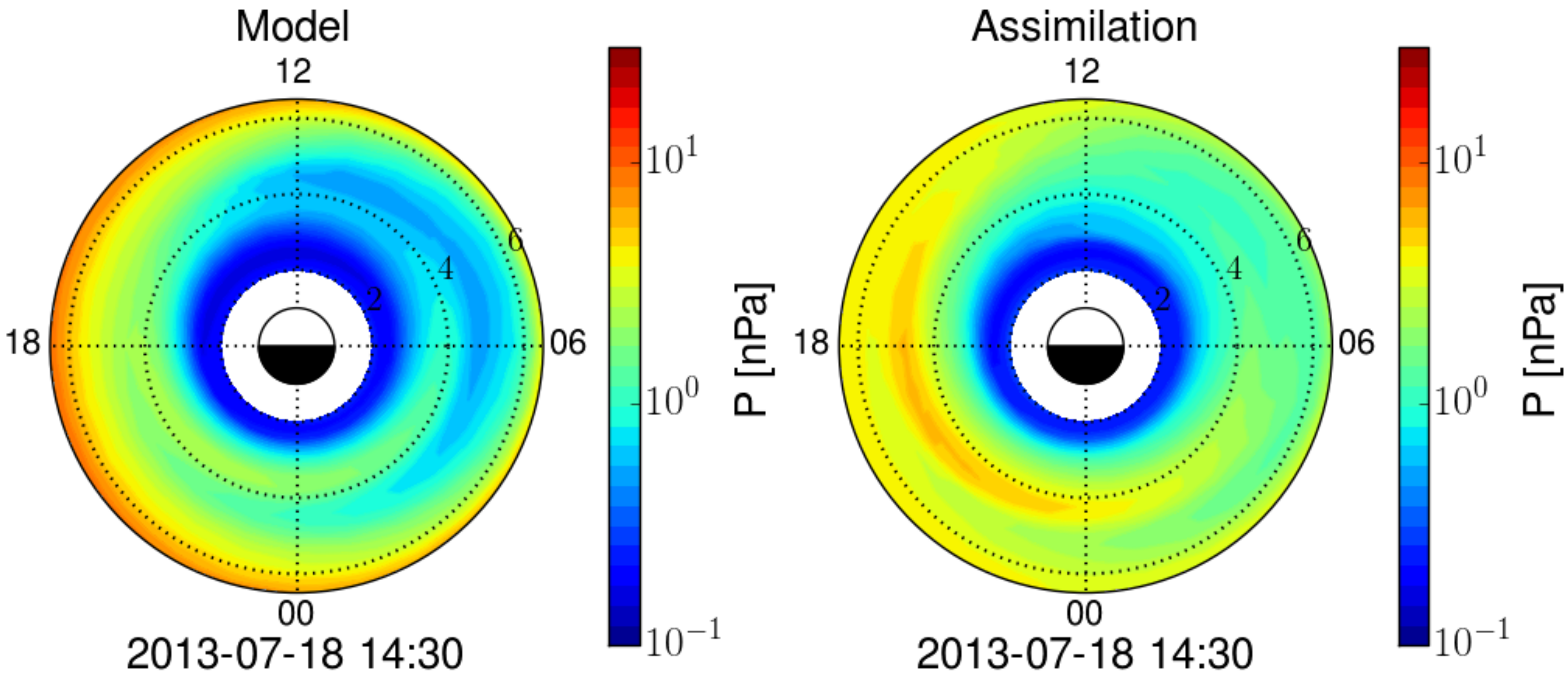
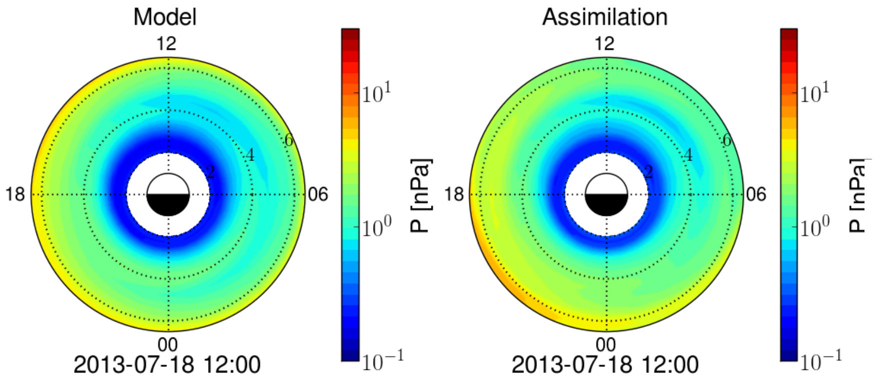
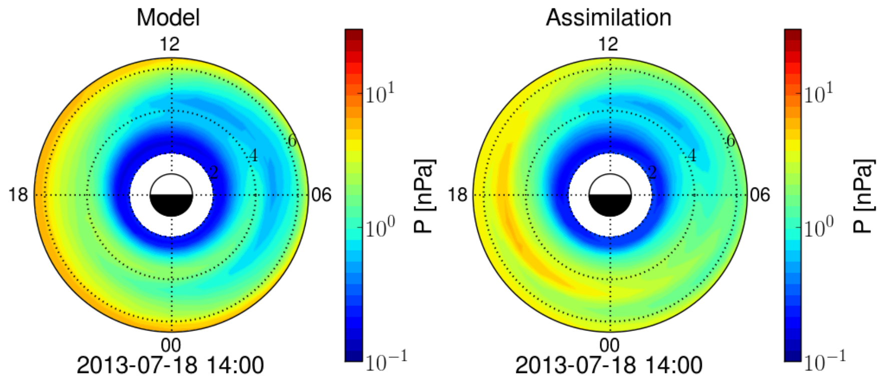
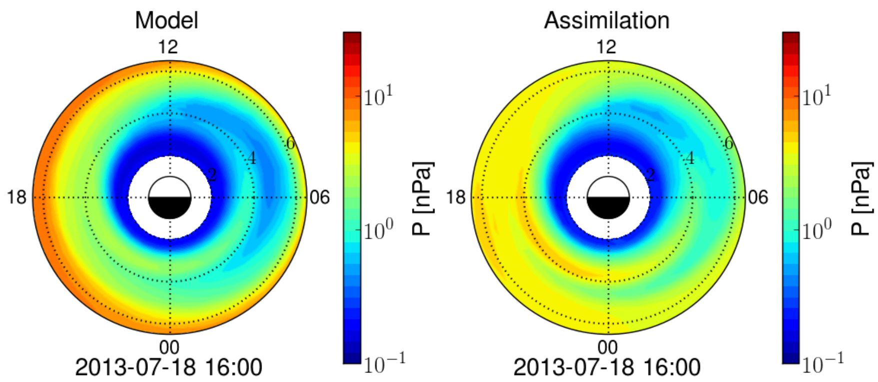
Reduced Basis EnKF Results



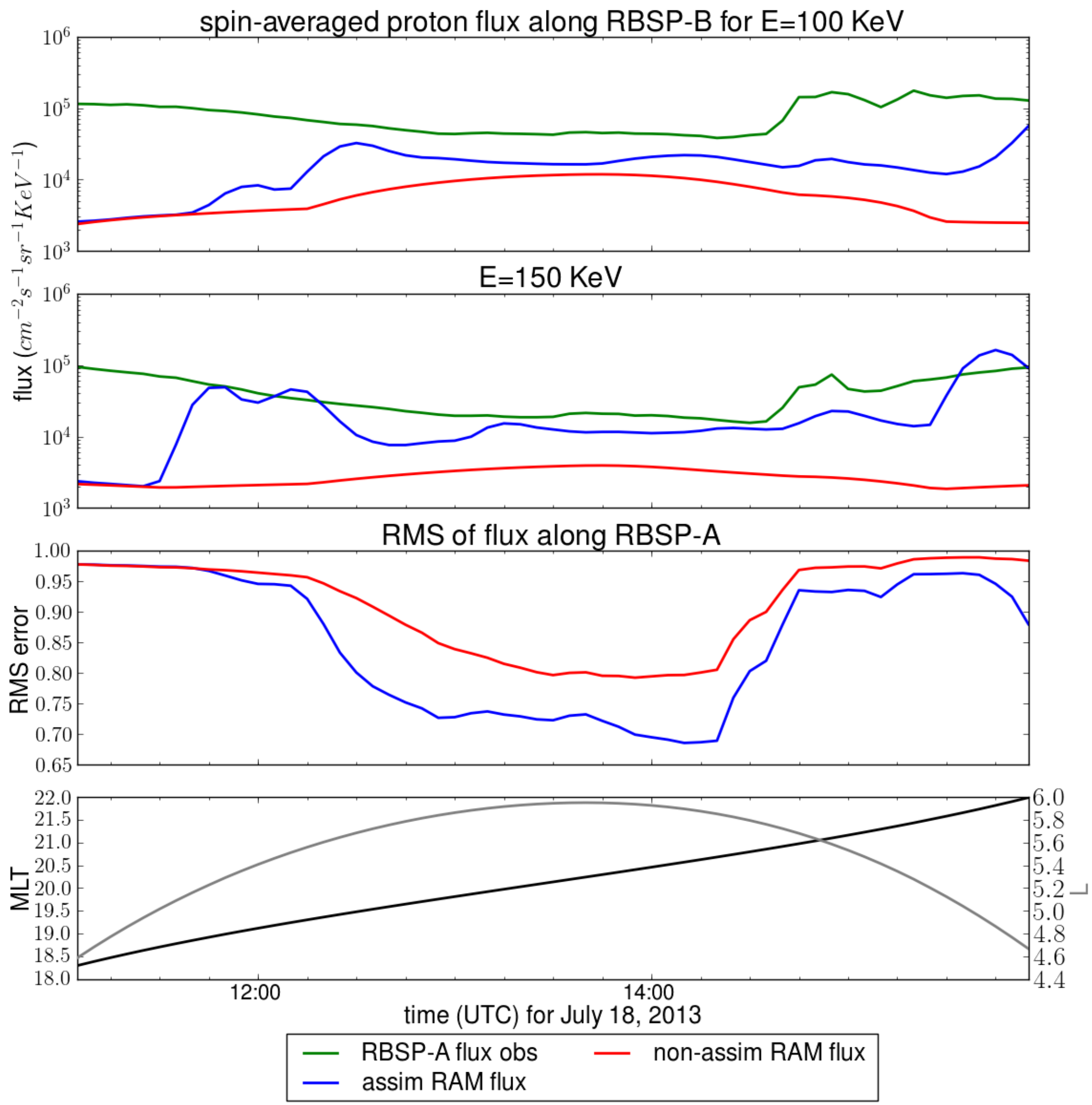
Reduced Basis EnKF Results







RMS error


spin-averaged proton flux along RBSP-B for E=100 KeV
flux $(cm^{-2}s^{-1}sr^{-1}KeV^{-1})$
E=150 KeV
RMS of flux along RBSP-A
RMS error
MLT
12:00
14:00
time (UTC) for July 18, 2013
RBSP-A flux obs
non-assim RAM flux
assim RAM flux

Summary

- Applied a Localized Ensemble Kalman Filter for RAM-SCB fluxes.
- LEnKF reduces error, but does not provide injection (not noticeable)
- Developed new assimilation algorithm using Singular Value Decomposition (SVD) to define new basis; captures main model signals
- Update weights of basis using EnKF
- Results look very promising, reduces error and provides an injection behaviour
- Outer boundary is important for injection, investigating empirical injection models on boundary

Future Work

- Use K_p and empirical model boundary model to build more physics intuition into the covariance modeling.
- Joint parameter (e.g. K_p) and state estimation.
- Couple assimilated RAM-SCB with BATS-R-US, provide simulations with more realistic outer boundary conditions

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