Can the Velocity in Bohmian Mechanics Be Measured?

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Bohmian Mechanics

$$\psi = \psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)$$

$$Q: \qquad \mathbf{Q}_1, \ldots, \mathbf{Q}_N$$

$Q \leftrightarrow$ Primitive Ontology (PO)

$\psi \leftrightarrow$ not PO

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi\,,$$

$$H = -\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla_k^2 + V,$$

$$\frac{d\mathbf{Q}_k}{dt} = \frac{\hbar}{m_k}\mathrm{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(\mathbf{Q}_1\ldots,\mathbf{Q}_N)$$

$$p = \hbar \boldsymbol{k}$$

$\nearrow$ time evolution for ψ

$\searrow$ time evolution for Q

$$dQ/dt = \nabla S/m$$

$$(\psi = Re^{iS/\hbar})$$

Implications of Bohmian mechanics:

- familiar (macroscopic) reality
- quantum randomness (Dürr, G, Zanghì)
- absolute uncertainty
- operators as observables
- the wave function of a (sub)system
- collapse of the wave packet
- formal scattering theory (T. Moser)
- identical particles: bosons and fermions

Equivariance

$$\left(\rho^{\psi}\right)_t = \rho^{\psi_t}$$

$$\rho^{\psi}(q) = |\psi(q)|^2$$

$$\rho_{t_0}(q) = |\psi_{t_0}(q)|^2 \textit{ at some time } t_0 \Longrightarrow$$
$$\rho_t(q) = |\psi_t(q)|^2 \textit{ for all } t$$

quantum equilibrium $\rho_{qe}(\mathbf{q}) = |\psi(\mathbf{q})|^2$

thermodynamic equilibrium $\rho_{eq}(\mathbf{v}) \propto e^{-\frac{1}{2}m\mathbf{v}^2/kT}$

$$\partial_t |\psi(x,t)|^2 = -\mathrm{div}\, j^{\psi}(x,t)$$

Deotto and Ghirardi, *Found. Phys.* **28**, 1–30 (1998)

The Uncertainty Principle

Weak Measurements

Yakir Aharonov, David Albert, and Lev Vaidman

Phys. Rev. Lett. **60**, 1351–1354 (1988)

Howard Wiseman

New Journal of Physics **9**, 165 (2007)

$$v(x) \equiv \lim_{\tau \to 0} \mathsf{E}[x_{\mathsf{strong}}(\tau) - x_{\mathsf{weak}} | x_{\mathsf{strong}}(\tau) = x]/\tau$$

$$ {}_{\langle\phi|}\left\langle \widehat{A}_{\mathrm{w}} \right\rangle_{|\psi\rangle} = \mathrm{Re}\frac{\langle\phi|\widehat{A}|\psi\rangle}{\langle\phi|\psi\rangle} $$

$$ {}_{\langle\phi|U(\tau)}\left\langle \widehat{A}_{\mathrm{w}} \right\rangle_{|\psi\rangle} = \mathrm{Re}\frac{\langle\phi|U(\tau)\widehat{A}|\psi\rangle}{\langle\phi|U(\tau)|\psi\rangle} $$

$$ v(x) = \lim_{\tau\to 0} \tau^{-1}\left[x - \mathrm{Re}\frac{\langle x|U(\tau)\widehat{X}|\psi\rangle}{\langle x|U(\tau)|\psi\rangle}\right] $$

$$ v(x) = v^{\psi}(x) \equiv \frac{j^{\psi}(x)}{|\psi(x)|^{2}} $$

$$ j^{\psi}(x) = (\hbar/m)\mathrm{Im}\,\psi(x)\nabla\psi(x) $$

"...a particular $\mathbf{j}$ is singled out if one requires that $\mathbf{j}$ be determined *experimentally* as a *weak value*, using a technique that would make sense to a physicist with no knowledge of quantum mechanics. This "naively observable" $\mathbf{j}$ seems the most natural way to define $\mathbf{j}$ operationally. Moreover, I show that this operationally defined $\mathbf{j}$ equals the standard $\mathbf{j}$, so, assuming $\dot{\mathbf{x}} = \mathbf{j}/P$ one obtains the dynamics of BM. It follows that the possible Bohmian paths are naively observable from a large enough ensemble."
(Howard Wiseman)

?

(1) A "weak measurement of velocity" in Bohmian mechanics is, in a reasonable sense, a *genuine* measurement of velocity.

(2) The same thing is true for the variants of Bohmian mechanics based on a velocity formula different from the Bohmian one.

(3) Bohmian mechanics and the variants referred to in (2) are empirically equivalent to each other—and to standard quantum mechanics. In particular, for all of them the result of a "weak measurement of velocity" is given by the Aharonov-Albert-Vaidman formula given above, and hence by the formula for velocity in Bohmian mechanics.

(4) It is impossible to measure the velocity in Bohmian mechanics.

?

Weak Measurement

$$\Phi(y) \sim e^{-\frac{y^2}{4\sigma^2}}$$

$$\psi(x)\Phi(y) \to \psi(x)\Phi(y-x)$$

$$\int dx\, \psi(x)|x\rangle|\Phi\rangle \to \int dx\, \psi(x)|x\rangle|\Phi\rangle_x$$

$$\psi_{0+}(x) = \psi_Y(x) \equiv \psi(x)\Phi(Y-x)$$

$$\rho^Y(y) = \int dx |\psi(x)|^2 |\Phi(y-x)|^2$$

$$\psi_{\mathrm{Y}}(x) \approx \Phi(Y)\psi(x)$$

$$\psi_{0+}(x) \approx \psi(x)$$

$$\mathbb{E}(Y) \equiv \int y\rho^{Y}(y)dy = \int x\rho^{X}(x)dx \equiv \mathbb{E}(X)$$

$$\rho^{Y}(y\,|X = x) = \frac{\rho^{X,Y}(x,y)}{\rho^{X}(x)} = \frac{|\psi(x)|^2|\Phi(y-x)|^2}{|\psi(x)|^2} = |\Phi(y-x)|^2$$

$$\mathbb{E}(Y|X = x) \equiv \int y\rho^{Y}(y\,|X = x) = x$$

$$v(x) \equiv \lim_{\tau \to 0} \mathsf{E}[x - x_{\mathsf{weak}} | x_{\mathsf{strong}}(\tau) = x]/\tau$$

$$v(x) = \lim_{\tau \to 0} \mathbb{E}\left(x - Y | X(\tau) = x\right)/\tau$$

$$v(x) = \lim_{\tau \to 0} \left[x - \mathbb{E}\left(Y | X(\tau) = x\right)\right]/\tau$$

$$X(\tau) \approx X + v^{\psi_{0+}}\tau$$

$$v^{\psi_{0+}} \approx v^{\psi}(x)$$

$$X(\tau) \approx X + v^{\psi}(X)\tau \approx X + v^{\psi}(X(\tau))\tau$$

$$\{X(\tau) = x\} \approx \left\{X = x - v^{\psi}(x)\tau\right\}$$

$$\begin{aligned}\mathbb{E}(Y|X(\tau)=x) \;&\approx\; \mathbb{E}(Y|X=x-v^{\psi}(x)\tau) \\ &=\; x-v^{\psi}(x)\tau\end{aligned}$$

$$v(x)=\lim_{\tau\to 0}\left[x-\mathbb{E}(Y|X(\tau)=x)\right]/\tau\approx v^{\psi}(x)$$

A More Careful Analysis

$$\psi_{0+}(x) = \psi(x)\Phi(Y - x)$$

$$v^{\psi_{0+}} = v(x, Y)$$

$$X(\tau) \approx X + v(X(\tau), Y)\tau$$

$$v(x) \approx \lim_{\tau \to 0} \left[x - \mathbb{E}(Y|X = x - v(x, Y)\tau)\right]/\tau$$

If

$$v^{\psi\Phi_y} = v^{\psi}$$

then

$$\begin{aligned} \rho^Y(y|X(\tau) = x) &= \rho^Y(y|X = x - v^{\psi}(x)\tau) \\ &= |\Phi|^2\Big(y - [x - v^{\psi}(x)\tau]\Big) \,. \end{aligned}$$

In general,

$$\begin{aligned} \rho^Y(y\,|X(\tau) = x) &\approx |\Phi|^2\Big(y - [x - v^{\psi}(x)\tau]\Big) \\ &+ (v^{\psi}(x) - v_B^{\psi}(x))\tau \cdot \nabla_x |\Phi|^2\,(y - x)\,. \end{aligned}$$

Suppose v^ψ defines a variant of Bohmian mechanics for which the condition

$$v^{\psi\phi} = v^\psi$$

holds for all (differentiable) real-valued functions ϕ, or at least for a collection of such functions that is "gradient-total," i.e., such that at every point $x \in \mathbb{R}^3$, the collection of vectors $\nabla\phi(x)$ spans $\mathbb{R}^3$. Then $v^\psi = v^\psi_B$.

The Impossibility of Measuring the Velocity in Bohmian Mechanics

$$\psi = \psi_{\mathsf{re}} + i\psi_{\mathsf{im}}$$

$$\psi_{\mathsf{re}} \otimes \Phi_0 \to \Psi_{v=0}$$

$$\psi_{\mathsf{im}} \otimes \Phi_0 \to \Psi'_{v=0}$$

$$\psi \otimes \Phi_0 \to \Psi_{v=0} + i\Psi'_{v=0}$$

Linear versus nonlinear measurements

Nonlinear: the initial state of the apparatus $\Psi_{\text{app}} = \Psi^{\psi}$ or the interaction $H_{\text{int}} = H^{\psi}$ depends upon the state ψ of the system

A final moral concerns terminology. Why did such serious people take so seriously axioms which now seem so arbitrary? I suspect that they were misled by the pernicious misuse of the word 'measurement' in contemporary theory. This word very strongly suggests the ascertaining of some preexisting property of some thing, any instrument involved playing a purely passive role. Quantum experiments are just not like that, as we learned especially from Bohr. The results have to be regarded as the joint product of 'system' and 'apparatus,' the complete experimental set-up. But the misuse of the word 'measurement' makes it easy to forget this and then to expect that the 'results of measurements' should obey some simple logic in which the apparatus is not mentioned. The resulting difficulties soon show that any such logic is not ordinary logic. It is my impression that the whole vast subject of 'Quantum Logic' has arisen in this way from the misuse of a word. I am convinced that the word 'measurement' has now been so abused that the field would be significantly advanced by banning its use altogether, in favour for example of the word 'experiment.'
(John Bell)

In the new, post-1925 quantum theory the ‘anarchist’ position became dominant and modern quantum physics, in its ‘Copenhagen interpretation’, became one of the main standard bearers of philosophical obscurantism. In the *new* theory Bohr’s notorious ‘complementarity principle’ enthroned [weak] inconsistency as a basic ultimate feature of nature, and merged subjectivist positivism and antilogical dialectic and even ordinary language philosophy into one unholy alliance. After 1925 Bohr and his associates introduced a new and unprecedented lowering of critical standards for scientific theories. This led to a defeat of reason within modern physics and to an anarchist cult of incomprehensible chaos.
(Lakatos, Criticism and the Growth of Knowledge, p. 145, 1965)

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Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. (John Stewart Bell, 1986)

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Some reactions to Bohmian mechanics

The fact . . . that Bohm's model was pushed aside while all sorts of weird ideas flourished is very interesting, and I hope that one fine day a historian or sociologist of science takes a close look at the matter. (Paul Feyerabend, 1993; letter to David Peat)

Thus, unless one allows the existence of contextual hidden variables with very strange mutual influences, one has to abandon them—and, by extension, ‘realism’ in quantum physics—altogether. (Gregor Weihs, The truth about reality, Nature, February 2007)

Over the years, a number of hidden variable theories have been proposed, to supplement q.m.; they tend to be cumbersome and implausible, but never mind—until 1964 the program seemed eminently worth pursuing. But in that year J.S. Bell proved that *any* local hidden variable is *incompatible* with quantum mechanics. (Griffiths)

Attempts have been made by Broglie, David Bohm, and others to construct theories based on hidden variables, but the theories are very complicated and contrived. For example, the electron would definitely have to go through only one slit in the two-slit experiment. To explain that interference occurs only when the other slit is open, it is necessary to postulate a special force on the electron which exists only when that slit is open. Such artificial additions make hidden variable theories unattractive, and there is little support for them among physicists. (Britannica)

In Putnam ([1965]), I rejected Bohm's interpretation for several reasons which no longer seem good to me. Even today, if you look at the Wikipedia encyclopaedia on the Web, you will find it said that Bohm's theory is mathematically inelegant. Happily, I did not give that reason in Putnam ([1965]), but in any case it is not true. The formula for the velocity field is extremely simple: you have the probability current in the theory anyway, and you take the velocity vector to be proportional to the current. There is nothing particularly inelegant about that; if anything, it is remarkably elegant!

. . . the de Broglie-Bohm 'pilot-wave' viewpoint (e) appears to have the clearest ontology among all those which do not actually alter the predictions of quantum theory. Yet, it does not, in my opinion, really address the measurement paradox in a clearly more satisfactory way than the others do. (R. Penrose)

As I see it, (e) [Bohmian mechanics] may indeed gain conceptual benefit from its two levels of reality—having a firmer ‘particle’ level of the reality of the configuration of the system, as well as a secondary ‘wave’ level of reality, defined by the wavefunction ψ, whose role is to guide the behaviour of the firmer level. But it is not clear to me how we can be sure, in any situation of actual experiment, which level we should be appealing to. My difficulty is that there is no parameter defining which systems are, in an appropriate sense, ‘big’, so that they accord with more classical ‘particle-like’ or ‘configuration-like’ pictures, and which systems are ‘small’, so that the ‘wavefunction-like’ behaviour becomes important . . . But . . . it seems to me that some measure of scale is indeed needed, for defining when classical-like behaviour begins to take over from small-scale quantum activity. In common with the other quantum ontologies in which no measurable deviations from standard quantum mechanics is expected, the point of view (e) does not possess such a scale measure, so I do not see that it can adequately address the paradox of Schrödinger’s cat. (R. Penrose)

No experimental consequences are drawn from [the Bohmian description] other than the standard predictions of the QM formalism, so whether one regards it as a substantive resolution of the apparent paradox or as little more than a reformulation of it is no doubt a matter of personal taste [the present author inclines towards the latter point of view]. (Anthony Leggett, 2003)

Objections to Bohmian mechanics

- Bohmian mechanics makes predictions about results of experiments different from those of orthodox quantum theory so it is wrong.

- Bohmian mechanics makes the same predictions about results of experiments as orthodox quantum theory so it is untestable and therefore meaningless.

- Bohmian mechanics is mathematically equivalent to orthodox quantum theory so it is not really an alternative at all.

- Bohmian mechanics is more complicated than orthodox quantum theory, since it involves an extra equation.

- Bohmian mechanics requires the postulation of a mysterious and undetectable quantum potential.

- Bohmian mechanics requires the addition to quantum theory of a mysterious pilot wave.

- Bohmian mechanics, as von Neumann has shown, can't possibly work.

- Bohmian mechanics, as Kochen and Specker have shown, can't possibly work.

- Bohmian mechanics, as Bell has shown, can't possibly work.

- Bohmian trajectories are crazy, since they may be curved even when no classical forces are present.

- Bohmian trajectories are crazy, since a Bohmian particle may be at rest in stationary quantum states.

- Bohmian trajectories are crazy, since a Bohmian particle may be at rest in stationary quantum states, even when these are large-energy eigenstates.

- Bohmian trajectories are surrealistic.

- Bohmian mechanics, since it is deterministic, is incompatible with quantum randomness.

- Bohmian mechanics, since it is deterministic, is incompatible with free will.

- Bohmian mechanics is nonlocal.

- Bohmian mechanics is unintuitive.

- Bohmian mechanics is the many-worlds interpretation in disguise.

- Bohmian mechanics is nonrelativistic.

- Bohmian mechanics is a childish regression to discredited classical modes of thought.

- Bohmian mechanics involves a classical reality (not a quantum one).

- Bohmian mechanics involves a classical ontology.